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A Resolution of the Purchasing Power Parity Puzzle: Imperfect Knowledge and Long Swings

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Abstract

Asset prices undergo long swings that revolve around benchmark levels. In currency markets, fluctuations involve real exchange rates that are highly persistent and that move in near-parallel fashion with nominal rates. The inability to explain these two regularities with one model has been called the "purchasing power parity puzzle." In this paper, we trace the puzzle to exchange rate modelers' use of the "Rational Expectations Hypothesis." We show that once imperfect knowledge is recognized, a monetary model is able to account for the puzzle, as well as other salient features of the data, including the long-swings behavior of exchange rates.

Keywords: PPP puzzle, Long Swings, Imperfect Knowledge, Rational Expectations Hypothesis

JEL Classification: F31, F41, G15, G17

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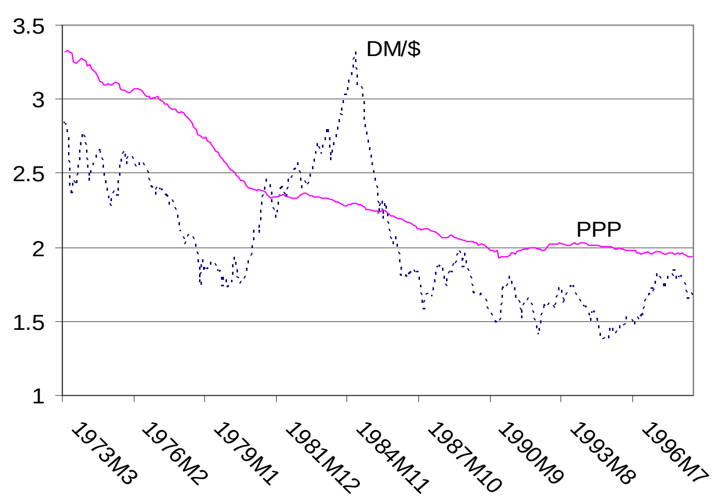
1 Introduction

Like all assets that trade freely in markets, floating currencies tend to undergo long swings that revolve around benchmark levels. This pattern is clearly evident in figure 1, which shows that the German mark-US dollar exchange rate moves away from purchasing power parity (PPP) for extended periods but eventually, at unpredictable moments of time, undergoes sustained movements back toward parity.[1] Most estimates of the half-life of PPP deviations—the number of years that a PPP deviation is expected to decay by 50 percent—are in the range of 3-5 years. Thus, while PPP deviations are ultimately bounded, they are highly persistent. Researchers have also found that real and nominal exchange rates move nearly one-for one over monthly or quarterly time horizons, which implies that the short-term volatility of real exchange rates (and thus of PPP deviations) is as high as the volatility of nominal exchange rates. Rogoff (1996, 2007) and others have pointed out that no single extant model of the open economy provides a satisfactory account of *both* the high persistence of real exchange rates and their near-parallel movement with nominal rates. The inability to explain these two empirical regularities in the context of one model is referred to as the "PPP puzzle."

Figure 1
DM/$ Exchange Rate: 1973-1998



[1] The PPP exchange rate in figure 1 is based on the *Big Mac* PPP exchange rate reported in the April 1990 issue of *The Economist* magazine (which was 1.96) and CPI-inflation-rate differentials from the IMF's *International Financial Statistics*.

In this paper, we trace the PPP puzzle to exchange rate modelers' use of the "Rational Expectations Hypothesis" (REH) to represent forecasting behavior. Market participants, policy makers, and economists themselves have imperfect knowledge about the causal mechanism driving macroeconomic outcomes. We show that once this fact is recognized, a traditional monetary model is able to account for the high persistence of real exchange rates *and* their near-parallel movement with nominal rates. The model also accounts for other salient features of the time series data, including the tendency of exchange rates to undergo long swings away from and toward PPP. Indeed, we resolve the PPP puzzle by modeling the long-swings behavior of currencies on the basis of imperfect knowledge.

International macroeconomists have advanced two broad classes of REH monetary models to account for exchange rate movements, those that assume all prices are fully flexible and those that rely on some type of nominal rigidity. Flexible-price monetary models emphasize shocks to taste and technology.[2] Because these shocks are thought to be highly persistent, flexible-price models are able to rationalize the slow adjustment of real exchange rates. However, they are unable to account for the near-parallel movement and high short-term volatility of real and nominal exchange rates.[3] Moreover, the tendency of real and nominal exchange rates to undergo long swings away from and toward PPP is puzzling in the context of flexible-price models. In discussing the currency swings of the 1980s, Rudiger Dornbusch argued that "[t]he events were too large and the reversal too sharp and complete to allude to mystical shifts in tastes and technology" (Dornbusch, 1989, p. 415).

To explain the near-parallel movement of real and nominal exchange rates over the short-term, most macroeconomists invoke the sticky-price model of Dornbusch (1976) or one of its New Open Economy Macroeconomics (NOEM) formulations.[4] With goods prices that are largely rigid at a point in time, monetary shocks, which cause jumps in the nominal exchange rate, imply comparable jumps in the real exchange rate. However, in these models, PPP deviations tend to dampen at a rate that is rigidly tied to the degree of price stickiness. Consequently, the large half-life estimates found in the literature are puzzling in the context of REH sticky-price models; no one believes that the adjustment in goods markets is so slow as to take 3-5 years to get only halfway

[2] See Stockman (1980, 1987), Lucas (1982), Helpman (1981), and Svensson (1985).

[3] For quantitative studies, see Cardia (1991) and Miller and Todd (1995).

[4] The seminal work in the NOEM literature is Obstfeld and Rogoff (1995, 1996, 1998, 2000). See also Svensson and Wijnbergen (1989), Betts and Devereux (1996, 2000).

back to equilibrium.[5]

Indeed, empirical evidence shows that goods prices adjust much more quickly. Engel and Morley (2001) report half-life estimates for price adjustment as small as one quarter, whereas for the real exchange rate they find the usual half-life estimates of 3-5 years. Frydman, *et al.* (2008) use the $I(2)$ framework of Johansen (1997, 2006) to estimate a VECM for the German mark-US dollar market. They find that PPP deviations have a half-life of five years, while goods-market adjustment has a half-life of two months.[6]

This evidence reveals that a resolution of the PPP puzzle requires a de-linking of real exchange rate movements from the adjustment of goods prices. However, the reliance of extant models on REH severely limits their ability to accomplish this objective. By design, the causal variables and the parameters of an REH representation of an individual's forecasting strategy are derivative of a model's specifications of preferences, constraints, and the processes governing the causal variables.[7] Thus, reliance on REH forces economists to seek explanations of the PPP puzzle by altering specifications of the non-expectational components of their models.[8]

In a recent attempt to resolve the PPP puzzle in a sticky-price REH model, Benigno (2004) shows that the adjustments of the real exchange rate and goods prices can be de-linked by endogenizing monetary policy.[9] In his calibration exercises, Benigno assumes a reasonable degree of price stickiness, but he also makes several special assumptions.[10]

[5] Quatitative studies of sticky-price monetary models show that the degree of price rigidity needed to generate enough persistence is much too high to be plausible. See Kollman (2001), Bergin and Feenstra (1999), and Chari, Kehoe, and McGratten (2000).

[6] Cheung, Lai, and Bergman (2004) also estimate a VECM and find faster speeds of adjustment in goods markets.

[7] The causal variables that enter an REH representation of forecasting behavior are limited to those that the economist uses in representing the non-expectational components of his model. Moreover, the parameters of this representation are restricted to be particular functions of the parameters of the model's specifications of preferences and constraints and the way that the policy and other causal variables unfold through time.

[8] The severe difficulties in searching for alternative explanations of time-series data on the basis of REH models are not limited to the PPP puzzle. For example, Mehra and Prescott (1985) pointed out that REH models are unable to explain the large magnitude of the "equity premium"—the historical-average return on stocks over risk-free bonds. In their quest to resolve this equity premiuim "puzzle," REH theorists have searched for increasingly more special specifications of preferences. See Epstein and Zin (1989, 1991), Constantinides (1990), Campbell and Cochrane (1999), and Barberis, Huang, and Santos (2001).

[9] See also Gali and Monacelli (2005), Engel and West (2006), and Engel, Mark, and West (2007).

[10] In Benigno's model, the price-setting parameters are independent of the Taylor rule specification. Rogoff (2007) points out that it may be implausible to assume that these parameters remain unchanged across the different settings of the policy reaction function used in the calibration exercise. Indeed,

He shows that the model implies a highly persistent stationary process for the real exchange rate that can replicate the large half-life found in the data. However, despite its special assumptions, the model does not quite match the observed volatility of the real exchange rate, which is either too high or too low depending on the particular parameter values used in the calibration.

Moreover, Benigno's results reveal how calibration exercises obscure the failure of the model to provide an adequate account of the time series data on exchange rates.[11] Johansen et al. (2008) shows that the near-I(1)—stationary, though persistent—processes generated by extant REH models are insufficiently persistent to account for the long-lasting currency swings and other empirical regularities during floating-rate periods.

Johansen et al. (2008 develops new test procedures to allow for shifts in the deterministic component and estimates a cointegrated I(2) model for goods prices, exchange rates, and interest rates for Germany and the US.[12] The paper shows that the I(2) model characterizes the different levels of persistence in the data significantly better than its I(1) counterpart: the null hypothesis of I(1) is rejected in favor of I(2) for relative goods prices, the nominal exchange rate, and short-term interest rates.[13] They also reject the hypothesis that the real exchange rate and the long-term interest rate differential are stationary; instead they find that they are near-I(2)—highly persistent I(1)—processes.[14]

The models of Begnino and others generate insufficient persistence of the real exchange rate and other macroeconomic variables for reasons directly connected to their reliance on REH. This representation rigidly ties individuals' exchange rate forecasts

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the references cited by Benigno himself (see page 496) report a wide range of estimates for the interest rate smoothing parameter and are strongly suggestive of its instability. In order to generate higher persistence in the real exchange rate, Beningo also makes use of other special assumptions. For example, he must assume that the degree of price adjusment differs across domestic and foreign firms within the same country.

[11] For forceful arguments against the use of calibration methods as a substitute for standard statistical inference in testing economic models, see Sims (1996).

[12] For an extensive discussion and analysis of cointegrated VAR models, see Juselius (2006).

[13] The new test procedures build on previous work in Johansen (1992, 1995, 1997, 2006), Rahbek et al. (1999), Paruolo (2000, 2002), Nielsen and Rahbek (2000).

[14] We also show that imposing the I(1) structure on the data significantly distorts information. These conclusions are consistent with several other studies that find I(2) trends in time-series data on, for example, exchange rates, goods prices, and money supplies. See, Johansen (1992), Juselius (1994), Kongsted (2003, 2005), Kongsted and Nielsen (2004), and Bacchiocchi and Fanelli (2005). Using the alternative methodology of spectral analysis, Jung (2007, p.383) finds a "dramatic failure of the [NOEM] model [to explain] business cycle frequency fluctuation in exchange rates."

to PPP; individuals invariably predict a tendency of the exchange rate to revert back to this benchmark. Consequently, the exchange rate tends to revert back to PPP following a jump, in say, the money supply. Exogenous shocks can push the exchange rate away from PPP over successive time periods. Under REH, these shocks are necessarily uncorrelated over time. Thus, individuals' forecasts and, therefore, the exchange rate can move away from parity over successive time periods if realizations of these shocks just happen to be of the same sign and of sufficiently large magnitude. However, the long lasting runs of random shocks that would be needed to explain the duration and magnitude of the long swings in exchange rates are too improbable. This is precisely why the statistical analysis of Johansen et al. (2008) rejects the stationary near-I(1) hypothesis in favor of a highly persistent near-I(2) process for the real exchange rate.[15] As Rudiger Dornbusch and Jeffrey Frankel put it, "the chief problem with the overshooting theory, indeed, with the more general rational expectations approach, is that it does not explain well the [long-swings] dynamics in exchange rates" (Dornbusch and Frankel, 1995, p. 16).[16]

This leads us to replace REH with an imperfect knowledge economics (IKE) representation of individuals' forecasting behavior in a monetary model. In this model, which is developed in Frydman and Goldberg (2007) (the FG model), individuals' exchange rate forecasts are no longer rigidly tied to the other components—preferences, constraints, and policy variables—of the model. This independent role for expectations delivers the extra persistence that is missing in REH models. However, it does so without presuming that individuals behave irrationally, as is the case in non-REH behavioral models. The FG and other IKE models avoid this presumption by recognizing that rational individuals in a world of imperfect knowledge revise their forecasting strategies, at least intermittently, over time and by modeling this change with mathematical conditions that are qualitative. As we show in this paper, although the FG model is based on qualitative conditions that do not prespecify exactly how an individual may change her forecast from one point in time to another, it nevertheless generates testable implications.

[15]See also Evans (1986) and Engel and Hamilton (1990), which show that the persistence stemming solely from the cumulation of random shocks buffeting the model cannot account for the curency swings we observe.

[16]The inability of standard monetary models to explain long swings in exchange rates has led international macroeconomists to rely on the REH bubble paths of sticky-price models. However, Frydman and Goldberg (2007, chapter 7) point out that the bubble paths of REH monetary models, as with their fundamental solutions, are inconsistent with key features of the long swings we actually observe in currency markets.

In the FG model, persistent swings in the exchange rate away from PPP occur because market participants have only imperfect knowledge about the mechanism that relates the exchange rate to a set of causal variables. Moreover, these swings are ultimately self-limiting; eventually, if the divergence from PPP were to become large enough, market participants would revise their forecasting strategies or policy makers would alter policy in ways that would trigger a sustained movement back toward parity.

The ability of the FG model to generate protracted currency swings, and thus highly persistent PPP deviations, does not depend on how quickly goods prices adjust to equilibrium levels. In the model, currency swings away from PPP arise not because of sticky goods prices, but because market participants' exchange rate forecasts, in the aggregate, tend to move persistently away from parity over some time periods. We show that such movements in individuals' forecasts de-links real exchange rate persistence in the model from the speed of nominal price adjustment. In fact, even if we assume that goods prices are fully flexible, the model continues to imply long-swings behavior and a large half-life of PPP deviations. The model is thus able to generate a highly persistent real exchange rate without the odd conclusion that goods prices also adjust very slowly. We show that even when goods prices are flexible, the model is also consistent with the near-parallel movement of the nominal and real exchange rate over the short-term.

Although the FG model of currency markets differs sharply from its extant counterparts in stopping short of fully prespecifying market participants' forecasting strategies, it nevertheless generates testable empirical implications. The model implies that the real exchange rate follows a random walk with a temporally unstable drift. The qualitative conditions used in the model to represent revisions of forecasting strategies restrict the way the unstable drifts unfold over time. Frydman et al. (2008) show that these conditions are sufficient to characterize the real exchange rate as a highly persistent near-I(2). In the present paper, we show that this IKE-based real exchange rate process implies a large half-life as typically defined in the literature. Our analysis derives an estimate of the half-life based on the usual AR(1) specification. Thus, FG's IKE model resolves the PPP puzzle: it accounts for *both* the high persistence of real exchange rates and their near-parallel movement with nominal rates.

The remainder of the paper is structured as follows. In section 2, we sketch a sticky-price monetary model due to Dornbusch (1976) and Frankel (1979) and show that it generates stationary but persistent processes for real and nominal exchange rates and goods prices. Section 3 shows how endogenizing monetary policy de-links real exchange rate movements from goods prices under REH. Section 4 introduces the IKE approach to modeling revisions of forecasting strategies and shows how the FG model implies long swings in the nominal and real exchange rate even with fully flexible goods prices. In section 5, we show that these currency swings are characterized by PPP deviations with a large half-life. Section 6 concludes with remarks concerning the empirical implications of our IKE resolution of the puzzle.

2 Puzzling Behavior in REH Sticky-Price Models

We begin by sketching the well-known difficulties of the traditional overshooting model of Dornbusch (1976) in resolving the PPP puzzle. In response to these problems, international macroeconomists have altered the non-expectational features of traditional sticky-price models, including NOEM's introduction of explicit intertemporal micro-foundations or endogenizing monetary policy. The literature has largely sidestepped what Dornbusch and Frankel (1995) conjectured may be the key problem with exchange rate models: it may not stem from particular assumptions concerning price adjustment, policy reaction, or other non-expectational components, but with the reliance on REH to represent forecasting behavior.

In this section, we trace the inability of the overshooting model and its modifications to provide a plausible resolution of the PPP puzzle to REH.[17] The Benigno (2004) model de-links real exchange rate movements from the adjustment of goods prices and, under some conditions, produces near-unit root representations for the real exchange rate. But, REH representations do not generate enough persistence to adequately account for the frequency with which currencies tend to undergo large and long lasting swings away from parity.

2.1 The Overshooting Model

Consider the following stochastic version of the two-country, sticky-price monetary model due to Dornbusch (1976) and Frankel (1979):

$$m_t = p_t + \phi y_t - \lambda i_t \tag{1}$$

[17] More broadly, Frydman and Goldberg (2007) argue that the empirical difficulties of REH models in explaining prices and risk in asset markets can be traced ultimately to the fundamental epistemological flaws of REH as a way to model forecasting behavior.

$$\widehat{s}_{t|t+1} - s_t = i_t \tag{2}$$

$$\Delta p_{t+1} = \delta \left[\alpha \left(s_t - p_t - q^{\text{PPP}} \right) - \eta \left(i_t - \hat{\pi}_t \right) \right] + E_t \Delta \bar{p}_{t+1} \tag{3}$$

$$m_t = \mu^m + m_{t-1} + v_t^m \quad \text{and} \quad y_t = \mu^y + y_{t-1} + v_t^y \tag{4}$$

where s_t is the domestic currency price of foreign exchange, m_t, p_t, y_t, and i_t denote the relative (domestic minus foreign) level of money supplies, good prices, income levels, and nominal interest rates, respectively, $\widehat{s}_{t|t+1}$ denotes an aggregate of market participants' point forecasts of the future exchange rate conditional on individuals' information sets and forecasting strategies, q^{PPP} denotes the relative PPP level of the real exchange rate, which we assume to be constant,[18] $\hat{\pi}_t$ is the market's assessment concerning the steady-state relative rate of inflation, μ^m and μ^y are drifts, which are typically assumed to be constant, v_t^m and v_t^y are mutually uncorrelated, mean-zero, i.i.d. errors, Δ is the first-difference operator, and an overbar denotes a steady-state value. Variables except for i_t and $\hat{\pi}_t$ are expressed in log-levels.

Equations (1)-(4) are well known and we offer little discussion of them. We note that the specification of price adjustment in equation (3), which depends on excess demand (the term in square brackets) and the expectation of the secular trend, assumes imperfect substitutability between domestic and foreign goods.[19] As such, equilibrium in the domestic and foreign goods markets is given by

$$s_t - p_t - q^{\text{PPP}} = \frac{\eta}{\alpha} \left(i_t - \hat{\pi}_t \right) \tag{5}$$

This specification shows that goods market clearing implies a relationship between the real exchange rate and real interest rate. Traditional sticky price models represent $\widehat{s}_{t|t+1}$ and $\hat{\pi}_t$ with REH. This representation leads to the well known result that the model's steady state is characterized by international Fisher parity (that is, $i_t - \hat{\pi}_t = 0$), and thus PPP. The short-run dynamics of the model is then anchored to this steady state.

By design, REH specifies individuals' forecasts of the aggregate variables and the predictions of the model on the aggregate level to be one and the same. Because goods prices are sticky, an unanticipated monetary shock, v_t^m, causes a jump in both the

[18] Dornbusch (1976), Frankel (1979), and many others assume absolute PPP, thereby setting $q^{PPP} = 0$. However, price levels across countries are based on different baskets of goods and services. In general, then, $q^{PPP} \neq 0$.

[19] Mussa (1982) and Obstfeld and Rogoff (1984) have pointed out that the specification in equation (3) ignores anticipated disturances. Because this issue plays no role in our analysis, we follow Dornbusch (1976) and others and ignore it.

nominal and real exchange rate away from PPP. The resulting excess demand and excess supply in the domestic and foreign goods markets, respectively, cause p_t to move back toward PPP, thereby reducing the disequilibria.

Under REH, the model ties its prediction of the change in the nominal exchange rate to the adjustment of goods prices. It predicts, therefore, that the nominal exchange rate tends to move back to PPP at every point in time:

$$\Delta s_{t+1} = \theta_1 \left(\bar{s}_t^{\text{RE}} - s_t \right) + \hat{\pi}^{\text{RE}} + v_{t+1}^s \tag{6}$$

where the steady-state values $\bar{s}_t^{\text{RE}} = \overline{p}_t^{\text{RE}} + q^{\text{PPP}} = \bar{s}_t^{\text{PPP}}$, $\hat{\pi}^{\text{RE}} = E\Delta\overline{p}^{\text{RE}} = E\Delta\bar{s}^{\text{RE}} = \mu^m - \phi\mu^y$, $\overline{p}_t^{\text{RE}} = m_t - \phi y_t + \lambda\hat{\pi}^{\text{RE}}$ imply PPP and international Fisher parity, $v_{t+1}^s = \frac{1+\theta_1\lambda}{\theta_1\lambda}\left(v_{t+1}^m - \phi v_{t+1}^y \right)$ is a mean zero, i.i.d. error, $0 < \theta_1 < 1$ is one minus the stable root of the system,[20] and the superscript "RE" denotes an REH representation. Consequently, REH constrains individuals' point forecasts of the exchange rate to imply a movement back toward PPP at every point in time according to the following fixed rule:

$$\widehat{s}_{t|t+1}^{\text{RE}} - s_t = \theta_1 \left(\bar{s}_t^{\text{RE}} - s_t \right) + \hat{\pi}^{\text{RE}} \tag{7}$$

A similar equation holds for relative goods prices:

$$\Delta p_{t+1} = \theta_1 \left(\bar{p}_t - p_t \right) + \hat{\pi}^{\text{RE}} \tag{8}$$

Since $q_t = s_t - p_t$, equations (7) and (8) imply the following REH representation for the real exchange rate:

$$\Delta q_{t+1} = \theta_1 \left(q^{\text{PPP}} - q_t \right) + v_{t+1}^s \tag{9}$$

For countries in which secular trends in goods prices are small relative to money and income shocks, the representations in (6) and (9) imply near-parallel movements of the nominal and real exchange rates. Thus, the model is consistent with the finding that the short-term conditional volatilities of real and nominal exchange rates are of comparable magnitudes. However, the very features of the model that deliver this implication make it difficult to rationalize a large half-life of real exchange rate movements.

[20] One minus the stable root $\theta_1 = \frac{\delta(\alpha\lambda+\eta)}{2\lambda} + \frac{\left[\left(\frac{\delta(\alpha\lambda+\eta)}{\lambda} + \frac{4\delta\alpha}{\lambda}\right)\right]}{2} > 0$. It is usual to assume that goods prices do not adjust fast enough to imply oscillatory behavior, that is, $\theta_1 > 1$. The required condition is $\delta < \frac{\lambda}{\alpha(1+\lambda)+\eta}$.

To see this, we take the time-t expectation of equations (8) and (9):

$$\frac{E_t\Delta\left(p_{t+1}-\bar{p}_{t+1}\right)}{p_t-\bar{p}_t}=\frac{E_t\Delta\left(q_{t+1}-q^{\text{PPP}}\right)}{q_t-q^{\text{PPP}}}=-\theta_1 \tag{10}$$

These expressions show that the trend rates of adjustment of goods prices to equilibrium levels and the real exchange rate to PPP are one and the same. The vast majority of empirical estimates of θ_1 imply a half-life of PPP deviations in the range of 3-5 years. Thus, for the traditional sticky-price model to be consistent with the time series evidence, goods prices would have to adjust unreasonably sluggishly to their equilibrium levels following a monetary disturbance.

2.2 De-linking Goods Prices From the Real Exchange Rate Under REH

The failure of the traditional sticky price model to explain both the high persistence and high volatility of the real exchange rate makes clear that a model capable of accounting for both of these findings must de-link real exchange rate movements from the adjustment of goods prices. Because REH specifies $\widehat{s}^{\text{RE}}_{t|t+1}$ as an output of rather than an input to an economist's model, de-linking requires a modification of the non-expectational components of the model.

Benigno (2004) shows that such a de-linking can be accomplished if the overshooting model is reformulated as a NOEM model with Calvo (1983) price adjustment and endogenous monetary policy. The model's use of a Taylor-type reaction function allows for interest rate smoothing. Engel, Mark and West (2007) (EMW) construct a simplified version of this model and show that the real exchange rate is proportional to the policy errors of the domestic monetary authority relative to its foreign counterpart:[21]

$$q_t-q^{\text{PPP}}=cu_t \tag{11}$$

where u_t denotes the relative policy error and c depends inversely on the degree of price sluggishness; if prices are assumed to be perfectly flexible, $c=0$.

Equation (11) shows that real exchange rate persistence can occur if goods prices are to some extent sluggish and the relative policy error is itself persistent. For example, EMW suppose that u_t evolves according to:

[21] EMW build on Galí and Monacelli (2005), and Engel and West (2006). They show that their simplified model captures the key features of the Beningo model.

$$u_t = \phi u_{t-1} + \epsilon_t \quad (12)$$

where $0 < \phi < 1$ and ϵ_t is mean zero, i.i.d.. This, in turn, implies the following real exchange rate equation:

$$\Delta q_{t+1} = (\phi - 1)\left(q_t - q^{\text{PPP}}\right) + c\epsilon_{t+1} \quad (13)$$

Thus, if policy errors are persistent enough, that is, ϕ is close to unity, the real exchange rate will be characterized by a large half-life. Moreover, because the parameter that represents the degree of price sluggishness enters only through c in this model, the real exchange rate is completely de-linked from the speed of price adjustment. Thus, the model can account for a persistent real exchange rate without implying that goods prices adjust unreasonably sluggishly.

However, the EMW and Benigno models sketched in this section suffer from the same basic flaw as the original overshooting model: they do not produce sufficient persistence to explain the currency swings we actually observe in markets. Equation (9) shows that, even if goods prices were assumed to be very sluggish, the original overshooting model would produce at best a near-unit root (near-I(1)) process. Equation (13) shows that this is also the case for NOEM models with some, not necessarily large, price sluggishness and endogenous monetary policy. A near-I(1) variable trends stochastically, that is, undergoes successive movements in the same direction, because realizations of the exogenous errors (v^s_{t+1} or ϵ_{t+1}) just happen to be sufficiently large and of the same sign for several time periods.

The insufficient persistence generated by near-I(1) models is apparent when one considers fluctuations in the U.S. dollar markets for the euro, British pound, and Japanese yen. These markets, which are the largest, are each characterized by at least one swing in q_t away from PPP that lasts 3 years or more and that involves departures from PPP of more than 40 percent in every decade of floating (the 1970s, 1980s, 1990s, 2000s).[22] It is clear that the frequency and magnitude of these swings are too great to be explained by a stationary process, even if near-I(1). Not surprisingly, Johansen et al. (2008) rejects the stationary near-I(1) hypothesis in favor of a highly persistent near-I(2) process for the real exchange rate. As we mentioned in the introduction, other studies, which include Evans (1986), Engel and Hamilton (1990), and Jung (2007), also

[22] See Frydman and Goldberg (2007), which use the German mark prior to the intoduction of the euro in January 1999.

find that the I(1) class of models provides an inadequate account of the currency swings we actually observe.

3 Recognizing Imperfect Knowledge

In the remainder of this paper, we build on Frydman and Goldberg (2007, 2008) and pursue an alternative explanation of fluctuations that accounts for long swings and that resolves the PPP puzzle. To this end, we drop REH and the constraint that individuals' exchange rate forecasts are rigidly anchored to the model's other components. We show how forecast revisions provide an additional source of persistence in the real exchange rate that by design is missing from REH models.

In real-world markets, individuals have imperfect knowledge of the causal mechanism driving outcomes. Economists themselves have constructed many different models. Thus, the aggregate of market participants' forecasting strategies differs from the strategy that is implied by any one REH model. We show that departures of $\widehat{s}_{t|t+1}$ from $\widehat{s}^{\mathrm{RE}}_{t|t+1}$ lead to a de-linking of trend changes in the real exchange rate from the speed of adjustment in goods markets. Moreover, unlike in the overshooting and NOEM models, movements of $\widehat{s}_{t|t+1}$ can involve extended swings away from PPP that are followed by persistent movements back toward parity. This feature of the IKE model enables us to account for the long-lasting currency swings that are characteristic of floating-rate regimes. We show in section 5 that the added persistence that comes from such swings leads to a real exchange rate process that exhibits a large half-life without the odd conclusion that goods prices adjust too slowly.

3.1 An Individual's Forecasting Strategy

In order to formalize what we mean by imperfect knowledge, we begin with the following general representation of an individual's point forecast of the exchange rate:

$$\widehat{s}^{i}_{t|t+1} = \hat{\beta}^{i}_{t} x^{i}_{t} + \hat{\rho}^{i} s_t \tag{14}$$

where the vector x^i_t and s_t represent the variables that individual i uses in forming her forecasts and $\hat{\beta}^i_t$ and $\hat{\rho}^i$ are the parameters that she attaches to these variables.[23]

We refer to the parameters $\hat{\beta}^i_t$ and $\hat{\rho}^i$, the composition of the causal variables, x^i_t, and the probability distribution of these variables as the structure of an economist's

[23] Relaxing the assumption of a constant $\hat{\rho}^i$ does not alter the main conclusions of the analysis. See chapter 14 in Frydman and Goldberg (2007).

representation of forecasting behavior. An economist formalizes his assumptions on how an individual forms and revises her forecasting strategies with restrictions that constrain the structure of (14) and its change. That much is common to all extant approaches to modeling forecasting behavior, including REH, behavioral, and IKE models.

In this paper, we assume that the causal variables in (14), like m_t and y_t, follow random walks with drift. This enables us to focus on the role of revisions of forecasting strategies in driving fluctuations in the model.[24]

As for changes in the structure of the representation in (14), the vast majority of economists construct models that disregard the fact that individuals in real-world financial markets revise their forecasting strategies, at least intermittently; these models impute to market participants exactly the same forecasting strategy at every point in time. The overshooting and NOEM models of the preceding section impose this invariance restriction and thus constrain the structure of (14) to be the same for each t.[25] Moreover, because these models represent forecasting strategies with REH, they select the causal variables and parameters to ensure consistency between their predictions on the aggregate and individual levels: x_t^i includes only those variables given by the economist's own semi-reduced-form model and $\hat{\beta}_t^i$ and $\hat{\rho}^i$ are particular functions of the parameters of this model. For example, in the overshooting model sketched above, $x_t^i = [m_t \;\; y_t \;\; \hat{\pi}^{\text{RE}} \;\; q^{\text{PPP}}]$, $\hat{\beta}^i = \left[\theta_1 \, \theta_1\phi \, 1 + \theta_1\lambda \, 1\right]$, and $\hat{\rho}^i = 1 - \theta_1$ for all i.

However, in a world of imperfect knowledge, market participants make use of diverse strategies to forecast future market outcomes. At each point in time, for example, some individuals may well base their exchange rate forecasts solely on the PPP exchange rate. But, this variable is merely one of many fundamental factors that a market participant might reasonably rely on in forming her forecast. Thus, the structure that adequately represents an individual's forecasting strategy differs from the one implied by an REH model, that is, $\widehat{s}_{t|t+1}^i \neq \widehat{s}_{t|t+1}^{\text{RE}}$.

[24] A more complete analysis of the monetary model under imperfect knowledge would allow for changes in the m_t, y_t, and x_t processes.

[25] Such invariance characterizes not only conventional REH representations, but also most of the extant behavioral representations of forecasting behavior. For references and a formal analysis of this striking similarity between REH and behavioral models, see chapters 4 and 6 in Frydman and Goldberg (2007).

3.1.1 Trend Changes in an Individual's Point Forecast

Profit-seeking motivates *rational* participants in financial markets to find new ways to forecast future outcomes; how and when they revise their forecasting strategies is to some extent non-routine. Indeed, the decision to revise one's strategy depends on many factors, including prior forecasting success, economic and political developments, emotions, or, as we will suggest shortly, the size of the departure of the exchange rate from PPP.[26]

The representation in (14) represents revisions of forecasting strategies through their impact on the semi-reduced-form component of an individual's forecast, which we denote by $\widehat{s}^{a^i}_{t|t+1} = \hat{\beta}^i_t x^i_t$.[27] The total change in this forecast between consecutive points of time can be written as:

$$\widehat{s}^{a,i}_{t|t+1} - \widehat{s}^{a,i}_{t-1|t} = \Delta\hat{\beta}^i_t x^i_t + \hat{\beta}^i_{t-1}\Delta x^i_t \tag{15}$$

where revisions of $\hat{\beta}^i_t$ are assumed to involve jumps at a point in time. Thus, $\Delta\hat{\beta}^i_t = \hat{\beta}^i_t - \hat{\beta}^i_{t-1} \neq 0$ represents a change at time t. Because the PPP puzzle is cast in terms of the rates at which goods prices and the real exchange rate tend toward particular values, we need to decompose the total change in (15) into a trend change, which we denote by $\mathcal{T}\widehat{s}^{a^i}_{t|t+1}$, and random deviations from this trend change. If we were to impose the invariance restriction on the model, so that $\Delta\hat{\beta}^i_t = 0$ for all t, the total change in $\widehat{s}^{a^i}_{t|t+1}$ could be written as:

$$\widehat{s}^{a,i}_{t|t+1} - \widehat{s}^{a,i}_{t-1|t} = \mathcal{T}\widehat{s}^{a^i}_{t|t+1} + \hat{\beta}^i \nu^{x^i}_t \tag{16}$$

where $\mathcal{T}\widehat{s}^{a^i}_{t|t+1} = \hat{\beta}^i \mu^{x^i}$, which is equal to the time $t-1$ expectation of $\Delta\widehat{s}^{a,i}_{t|t+1}$ conditional on no change in $\hat{\beta}^i_t$ at time t, that is, $E_{t-1}\left[\Delta\widehat{s}^{a,i}_{t|t+1}|\Delta\hat{\beta}^i_t = 0\right]$, and μ^{x^i} and $\nu^{x^i}_t$ are the drift and errors of the x^i_t process, respectively.[28] In this case, the trend change in $\widehat{s}^{a^i}_{t|t+1}$ would be constant and only stem from the drifts in the causal variables.

[26] Nonetheless, IKE holds out the possibility that individual decision making does display some regularity. But, that this regularity can at best be captured with qualitative conditions. See below for a formal representation.

[27] In general, IKE allows for the set of variables that are used for forcasting to change over time. The specification in (15) allows for such behavior defining x^i_t to include all of the variables used at evry point in time and setting some of the parameters in $\hat{\beta}^i_t$ to zero.

[28] Consequently, in the case of an invariant structure, $\mathcal{T}\widehat{s}^{a^i}_{t|t+1} = E_{t-1}\Delta\widehat{s}^{a,i}_{t|t+1}$, where the expectation is conditional on a fixed $\hat{\beta}^i$ and time $t-1$ information.

Once we recognize that market participants' and economists' knowledge is imperfect, the trend change in (16) varies over time; it depends on how the structure of the representation changes, that is, on $\Delta\hat{\beta}_t^i$. This trend change is given by

$$\mathcal{T}\widehat{s}_{t|t+1}^{a^i} = \Delta\hat{\beta}_t^i x_t^i + E_{t-1}\left[\Delta\widehat{s}_{t|t+1}^{a,i}|\Delta\hat{\beta}_t^i = 0\right] \quad (17)$$

where we note that both $\Delta\hat{\beta}_t^i x_t^i$ and $E_{t-1}\left[\Delta\widehat{s}_{t|t+1}^{a,i}|\Delta\hat{\beta}_t = 0\right] = \hat{\beta}_{t-1}^i \mu^{x^i}$ vary over time.[29]

To derive any implications for time-series data, an economist must impose restrictions on the way an individual's forecasting strategy unfolds over time, that is, on the two components of its trend change, $\mathcal{T}\widehat{s}_{t|t+1}^{a^i}$. IKE does so by imposing only qualitative constraints on $\Delta\hat{\beta}_t^i$. But, what we show next is that even if we fully prespecify the imperfection of knowledge, the model implies a de-linking of Δq_t and Δp_t.

3.2 A De-linking of the Real Exchange Rate From Goods Prices

We now show that jettisoning REH opens new channels for de-linking trend changes in q_t from the adjustment of p_t to equilibrium. To simplify our analysis, we assume that individuals' x_t^i's include only exogenous variables.[30] This assumption, together with equations (1)-(4) and (14), imply the following equations of motion:[31]

$$\Delta z_{t+1} = \vartheta\left(\bar{z}_t - z_t\right) + E_t\left[\Delta\bar{z}_{t+1}|\Delta\hat{\beta}_{t+1} = 0\right] + J_{t+1} \quad (18)$$

[29] It may seem natural to define $\mathcal{T}\widehat{s}_{t|t+1}^{a^i}$ not as in (17), but as $\Delta\hat{\beta}_t^i\left(x_{t-1}^i + \mu^{x^i}\right) + \hat{\beta}_{t-1}^i \mu^{x^i}$. This alternative formulation, however, is not a trend change because $\Delta\hat{\beta}_t^i$ and $\nu_t^{x^i}$ are, in general, correlated. As such, deviations from this alternative formulation are not random and do not have mean zero.

[30] Relaxing this assumption, as with a constant $\hat{\rho}^i$, does not alter the main conclusions of the analysis. See Frydman and Goldberg (2007, chapter 14).

[31] Because goods prices are assumed to be rigid at a point in time, we replace $E_t\Delta\bar{p}_{t+1}$ with $E_t\left[\Delta\bar{p}_{t+1}|\Delta\hat{\beta}_{t+1} = 0\right]$ in equation (3).

where $z_t = \left[s_t\, p_t\, i_t\, q_t\right]$, $0 < \vartheta = \frac{\delta[\alpha+(1-\hat{\rho})(\alpha\lambda+\eta)]}{\lambda(1-\hat{\rho})} < 1$ is one minus the root of the system,[32] J_t is a matrix of jump terms.

$$J_t = \begin{bmatrix} \frac{1}{(1-\hat{\rho})\lambda}\left[v^m_{t+1} - \phi v^y_{t+1} + \lambda\left(\hat{\beta}_t \nu^x_{t+1} + \Delta\hat{\beta}_{t+1} x_{t+1}\right)\right] \\ 0 \\ \frac{1}{\lambda}\left(v^m_{t+1} - \phi v^y_{t+1}\right) \\ \frac{1}{(1-\hat{\rho})\lambda}\left[v^m_{t+1} - \phi v^y_{t+1} + \lambda\left(\hat{\beta}_t \nu^x_{t+1} + \Delta\hat{\beta}_{t+1} x_{t+1}\right)\right] \end{bmatrix} \tag{19}$$

which are analogous to v^s_{t+1} in equation (6), but now recognize that $\hat{\beta}_t$ can change at a point in time, and the steady-state (goods-market-clearing) values are

$$\bar{s}_t = \bar{s}^{\mathrm{RE}}_t + \frac{\eta + \alpha\lambda}{G}\left(\widehat{s}^{a}_{t|t+1} - \widehat{s}^{\mathrm{RE}a}_{t|t+1}\right) \tag{20}$$

$$\bar{p}_t = \bar{p}^{\mathrm{RE}}_t + \frac{\alpha\lambda}{G}\left(\widehat{s}^{a}_{t|t+1} - \widehat{s}^{\mathrm{RE}a}_{t|t+1}\right) \tag{21}$$

$$\bar{q}_t = q^{\mathrm{PPP}} + \frac{\eta}{G}\left(\widehat{s}^{a}_{t|t+1} - \widehat{s}^{\mathrm{RE}a}_{t|t+1}\right) \tag{22}$$

$$\bar{i}_t = \hat{\pi} + \frac{\alpha}{G}\left(\widehat{s}^{a}_{t|t+1} - \widehat{s}^{\mathrm{RE}a}_{t|t+1}\right) \tag{23}$$

such that $G = \alpha + (1-\hat{\rho})(\eta + \alpha\lambda) > 0$, $\widehat{s}^{a}_{t|t+1} = \hat{\beta}_t x_t$ and $\widehat{s}^{\mathrm{RE}a}_{t|t+1}$ are the semi-reduced-form parts of $\widehat{s}_{t|t+1}$ and $\widehat{s}^{\mathrm{RE}}_{t|t+1}$, respectively, and $\hat{\pi}$ is assumed to be constant[33]. The vector x_t and s_t represent the union of variables that individuals use in forming their forecasts and $\hat{\beta}_t$ and $\hat{\rho}_t$ are weighted averages of the parameters that they attach to these variables.[34]

Like in the overshooting model, the time paths in (18)-(23) depend on a short-run adjustment term, a steady-state (goods-market clearing) level, and, except for the p_t equation, jump terms. Moreover, this dynamic system is also one dimensional, implying that in the absence of further exogenous shocks and revisions of forecasting strategies (that is, $J_{t+1} = 0$), the real exchange rate and goods prices, as well as all of the other

[32] Like with REH, we assume that if goods prices are sluggish, their speed of adjustment is not too fast to imply oscillatory behavior, that is, $\vartheta < 1$. See footnote 20.

[33] From equation (7), $\widehat{s}^{\mathrm{RE}a}_{t|t+1} = (1-\hat{\rho})\,\bar{s}^{\mathrm{RE}}_t + \hat{\pi}^{\mathrm{RE}}$, where we replace θ with $1-\hat{\rho}$. Equations (20)-(23) each omit a stationary term involving $\hat{\pi}$ and $\hat{\pi}^{\mathrm{RE}}$ that equals zero under REH. Also, equation (23) omits a second constant term that also equals zero under REH. See Frydman and Goldberg (2007). The assumption of a constant $\hat{\pi}$ enables us to highlight the role of exchange rate expectations in driving currency swings. A more complete analysis would consider alternative specifications for $\hat{\pi}$.

[34] Frydman and Goldberg (2007) use wealth shares as aggregation weights.

endogenous variables of the system, revert back to their steady-state levels at the same rate, ϑ:

$$\frac{E_t\left[\Delta\left(p_{t+1}-\bar{p}_{t+1}\right)|\Delta\hat{\beta}_{t+1}=0\right]}{p_t-\bar{p}_t}=\frac{E_t\left[\Delta\left(q_{t+1}-\bar{q}_{t+1}\right)|\Delta\hat{\beta}_{t+1}=0\right]}{q_t-\bar{q}_t}=-\vartheta \tag{24}$$

Thus, jettisoning REH does not lead to differing speeds at which p_t and q_t respond to departures from steady-state values. Although the EMW and Benigno modifications of the overshooting model imply that these speeds differ, they share a key feature with their overshooting predecessor: steady-state values imply PPP.

In sharp contrast, equations (20)-(23) show that this is not the case once REH is abandoned. In a world of imperfect knowledge, $\widehat{s}^a_{t|t+1}$ influences $\bar{s}_t$ and $\bar{p}_t$ differently.[35] Consequently, $\bar{q}_t=\bar{s}_t-\bar{p}_t$ also depends on individuals' exchange rate forecasts; movements of $\widehat{s}^a_{t|t+1}$ relative to $\widehat{s}^{\mathrm{RE}_a}_{t|t+1}$ cause $\bar{q}_t$ to move either away from or toward PPP levels.

It is not difficult to understand the intuition behind departures of the model's steady-state values from PPP. Once one recognizes imperfect knowledge, goods market equilibrium in (5) no longer implies PPP. An increase in $\widehat{s}^a_{t|t+1}$, for example, leads market participants to bid up $\bar{s}_t$. This domestic currency depreciation does create excess demand for domestic goods and a rise in relative goods prices. But, $\bar{p}_t$ increases less than one-for-one with $\bar{s}_t$ because money market equilibrium requires a rise in $\bar{i}_t$; with imperfect substitutability between domestic and foreign goods ($\eta\neq 0$), the rise in $\bar{i}_t$ helps to restore goods-market equilibrium. Under imperfect knowledge, then, the changes in the steady-state values of the model are inconsistent with PPP. Instead, they involve increases in both the real exchange rate, $\bar{s}_t-\bar{p}_t$, and the real interest rate, $\bar{i}_t-\hat{\pi}_t$.[36]

With steady-state values no longer rigidly tied to PPP, the link found in the original overshooting model between the rate at which the real exchange rate tends toward PPP and the speed of adjustment in goods markets is broken.

[35] Equations (20)-(23) show that if all individuals somehow adhered to the REH forecasting strategy endlessly (so that $\widehat{s}^a_{t|t+1}=\widehat{s}^{\mathrm{RE}_a}_{t|t+1}$ for all t), then their exchange rate forecasts would influence $\bar{s}_t$ and $\bar{p}_t$ in a parallel way.

[36] In Frydman et al. (2008), we indeed find evidence of an equilibrium relationship beyween the real exchange rate and the real interest rate.

3.2.1 Trend Changes in Goods Prices and the Real Exchange Rate

Under imperfect knowledge, the trend rate of change of the real exchange rate *toward* q^{PPP} is not given by $-\vartheta$, but by

$$\frac{\mathcal{T}\left(q_{t+1}-q^{\text{PPP}}\right)}{q_t-q^{\text{PPP}}}=\frac{-\vartheta\left(q_t-\bar{q}_t\right)}{q_t-q^{\text{PPP}}}+\frac{H\left(\mathcal{T}\bar{q}_{t+1}\right)}{q_t-q^{\text{PPP}}}=\psi_{t+1} \tag{25}$$

where

$$H\left(\mathcal{T}\bar{q}_{t+1}\right)=E_t\left[\Delta q_{t+1}|\Delta\hat{\beta}_{t+1}=0\right]+\frac{G}{\left(1-\hat{\rho}\right)\lambda}\left(\mathcal{T}\bar{q}_{t+1}-E_t\left[\Delta q_{t+1}|\Delta\hat{\beta}_{t+1}=0\right]\right)$$

This expression shows that the trend rate of change of q_t toward PPP not only differs from $-\vartheta$, but this rate varies over time.[37] Equation (25) also shows that departures of $\bar{q}_t$ from PPP break the link between ψ_{t+1} and ϑ through two channels: the short-run adjustment of the system and the trend change in the steady-state real exchange rate, the first and second terms in the expression, respectively.

By sharp contrast to its REH counterpart, both of these channels could imply a tendency for the real exchange rate to move away from PPP at any point in time. It is clear from equation (22) that depending on individuals' exchange rate forecasts, although q_t may lie above q^{PPP} at a point in time, it may lie below its steady-state value, $\bar{q}_t$. In this case, short-run adjustment would place upward pressure on q_t to move away from PPP in the ensuing period, that is, $\frac{-\vartheta(q_t-\bar{q}_t)}{q_t-q^{\text{PPP}}}>0$. It is also clear that the trend change in individuals' exchange rate forecasts could also imply upward pressure on q_t. Thus, once one recognizes imperfect knowledge in a traditional sticky-price monetary model, the trend rate of change of the real exchange rate is not only de-linked from the speed of adjustment of goods prices, but this trend rate could imply further departures from PPP.

The fact that the de-linking of ψ_{t+1} and ϑ occurs through both the short-run and steady-state components of the model implies that, unlike in the EMW and Benigno models, the assumption of sticky goods prices is not needed for the result. In the case of fully flexible goods prices, $\psi_{t+1}=\frac{H(\mathcal{T}\bar{q}_{t+1})}{q_t-q^{\text{PPP}}}\neq-\vartheta$. Thus, even without nominal rigidities of any kind in the model, ψ_{t+1} could be positive and the real exchange rate could tend to move away from PPP in any period.

[37] As in the overshooting model, if $\bar{q}_t$ were to equal q^{PPP} at all t, $\mathcal{T}\bar{q}_{t+1}$ would equal zero and ψ_t would be a constant and equal to $-\vartheta$.

3.3 Parallel Movements of Nominal and Real Exchange Rates Even With Flexible Goods Prices

The monetary model with imperfect knowledge is also consistent with near-parallel movements of nominal and real exchange rates over the short term. International macroeconomists often view such behavior as compelling evidence in favor of open-economy models that assume nominal rigidities of some kind.[38] However, once one recognizes the imperfection of knowledge, the monetary model is compatible with near-parallel movements of s_t and q_t regardless of whether goods prices are assumed to be sticky or flexible.

Consider first the sticky-price case. The equations for Δs_t and Δq_t follow from the system in (18) and are

$$\Delta s_{t+1} = \vartheta\left(\bar{s}_t - s_t\right) + E_t\left[\Delta \bar{s}_{t+1} | \Delta \hat{\beta}_{t+1} = 0\right] + J_{1_{t+1}} \tag{26}$$

$$\Delta q_{t+1} = \vartheta\left(\bar{q}_t - q_t\right) + E_t\left[\Delta \bar{q}_{t+1} | \Delta \hat{\beta}_{t+1} = 0\right] + J_{1_{t+1}} \tag{27}$$

where from equations (20) and (22) and the specification of $\widehat{s}^a_{t|t+1}$,

$$E_t\left[\Delta \bar{s}_{t+1} | \Delta \hat{\beta}_{t+1} = 0\right] = \hat{\pi}^{\mathrm{RE}} + \frac{\eta + \alpha\lambda}{G}\left(\hat{\beta}_t \mu^x - (1-\hat{\rho})\,\hat{\pi}^{\mathrm{RE}}\right) \tag{28}$$

$$E_t\left[\Delta \bar{q}_{t+1} | \Delta \hat{\beta}_{t+1} = 0\right] = \frac{\eta}{G}\left(\hat{\beta}_t \mu^x - (1-\hat{\rho})\,\hat{\pi}^{\mathrm{RE}}\right) \tag{29}$$

and $J_{1_{t+1}}$ denotes the first and fourth cells of the J_{t+1} matrix in (19).[39]

Like in the original overshooting model, shocks to the causal variables, which enter the model through J_{1_t}, lead to one-for-one movements in s_t and q_t. Equations (26) and (27) show that this is also the case with revisions of forecasting strategies, which also enter through $J_{1_{t+1}}$.

As before, the trend changes in s_t and q_t conditional on no change in structure also differ. With imperfect knowledge, this difference depends on the size of α relative to η, that is, on the relative impacts of changes in the real exchange rate and real interest rates on excess demand in the goods markets, respectively. If α is small relative to η, which the literature on the J-curve suggests is true, then $\hat{\pi}^{\mathrm{RE}}$ will account for much of

[38] For example, see Obstfeld and Rogoff (1996), chapter 9.

[39] We have used the fact that $\Delta \widehat{s}^{\mathrm{RE}a}_{t|t+1} = (1-\hat{\rho})\,\hat{\pi}^{\mathrm{RE}}$. See footnote 33.

the difference between the trend changes in equations (28) and (29).[40] As before, then, if the secular trend in goods prices is modest, the sticky price monetary model under imperfect knowledge implies near-parallel movements s_t and q_t.

Such behavior also arises in the model under flexible goods prices. In this case, the time paths for the endogenous variables of the model are given by equations (20)-(23). The equations for $\Delta\bar{s}_t$ and $\Delta\bar{q}_t$ are

$$\Delta\bar{s}_{t+1} = \hat{\pi}^{\text{RE}} + \frac{\eta + \alpha\lambda}{G}\left(\Delta\widehat{s}^{a}_{t+1|t+2} - (1-\hat{\rho})\left(\hat{\pi}^{\text{RE}} + v^{m}_{t+1} - \phi v^{y}_{t+1}\right)\right) \tag{30}$$

$$\Delta\bar{q}_{t+1} = \frac{\eta}{G}\left(\Delta\widehat{s}^{a}_{t+1|t+2} - (1-\hat{\rho})\left(\hat{\pi}^{\text{RE}} + v^{m}_{t+1} - \phi v^{y}_{t+1}\right)\right) \tag{31}$$

These equations show that movements of $\Delta\bar{s}_{t+1}$ and $\Delta\bar{q}_{t+1}$ depend on shocks to money and income, $v^{m}_{t+1} - \phi v^{y}_{t+1}$, as well as on movements of $\widehat{s}^{a}_{t|t+1}$, regardless of whether they arise from revisions of forecasting strategies or from shocks to the causal variables, v^{x}_{t+1}. As with sticky prices, the equations show that parallel movements of s_t and q_t require a modest secular trend in goods prices and that the relative magnitude of α/η matters. Again, if α is small relative to η, then a movement in $\widehat{s}^{a}_{t|t+1}$ or a shock to money or income will lead to a movement in $\bar{s}_t$ that is large relative to the movement in $\bar{p}_t$ (see equation 21) and thus, is associated with a near-parallel movement in $\bar{q}_t$. In the limit, as $\alpha \to 0$, the impact of changes in $\widehat{s}^{a}_{t|t+1}$ on $\bar{p}_t$ approaches zero. Thus, if excess demand in the goods markets is not sensitive to the real exchange rate, which is what the evidence indicates, then the monetary model with fully flexible goods prices generates near-parallel movements in nominal and real exchange rates.

3.4 Currency Swings and Flexible Goods Prices

As with de-linking and near-parallel movements, the ability of the monetary model with imperfect knowledge to generate currency swings away from PPP does not require the assumption of sticky goods prices. Again, this is because currency swings arise from the impact of forecasting behavior on the steady-state component of the model.

To highlight this result, we examine the implication of currency swings under the assumption of fully flexible goods prices.[41] Equations (20) and (22) show that the ability

[40] Empirical evidence shows that over the short-term, real exchange rate movements are associated with small substitution effects. See Meade (1981), Moffet (1989), Marquez (1991), and Hooper and Marquez (1995). The VAR estimates in Johansen *et al.* (2008) and Frydman *et al.* (2008) indicate that $\frac{\alpha}{\eta}$ is roughly .01.

[41] For the case of sticky goods prices, see chapter 14 of Frydman and Goldberg (2007).

of the model to generate currency swings depends on the behavior of $\left(\widehat{s}^{a}_{t|t+1} - \widehat{s}^{\mathrm{RE}a}_{t|t+1}\right)$:

$$\Delta \bar{q}_t = \frac{\eta}{G} \left(\Delta \widehat{s}^{a}_{t|t+1} - \Delta \widehat{s}^{\mathrm{RE}a}_{t|t+1} \right) \tag{32}$$

If, for example, revisions of forecasting strategies and movements in the causal variables led to a tendency for $\widehat{s}^{a}_{t|t+1}$ to rise relative to $\widehat{s}^{\mathrm{RE}a}_{t|t+1}$ over some extended period of time, that is, $\left(\Delta \widehat{s}^{a}_{t|t+1} - \Delta \widehat{s}^{\mathrm{RE}a}_{t|t+1} \right) > 0$, then s_t and q_t would also tend to rise over that period. Moreover, such a swing in s_t and q_t would end once the swing in individuals' forecasts ended.

Thus, to model currency swings with the flexible-price monetary model in equations (1)-(4) and (14), we need to model movements in the aggregate of individuals' point forecasts of the exchange rate. We continue to assume that x_t follows a random walk with constant drift. As for modeling $\Delta \hat{\beta}_t$, it is useful to consider first the implications of imposing the invariance restriction, that is, setting $\Delta \hat{\beta}_t = 0$ for all t.

3.4.1 An Unbounded Swing Away from PPP

With no revisions of forecasting strategies, the one-period change in individuals' point forecasts is given by $\Delta \widehat{s}^{a}_{t|t+1} = \hat{\beta} \mu^x + \hat{\beta} \nu^x_t$. The trend change in this aggregate forecast, $\hat{\beta} \mu^x$, is thus constant and, in general, differs from the trend change that would be obtained under REH, $(1 - \hat{\rho})\, \hat{\pi}^{\mathrm{RE}}$. Consequently, the invariance restriction and a fixed money-growth rule, together with the assumption of imperfect knowledge imply that individuals' point forecasts will tend to move in one direction or the other, relative to $\widehat{s}^{\mathrm{RE}a}_{t|t+1}$, endlessly

To see the implications of such behavior, suppose that this trend change in forecasts is positive, so that $E \left(\Delta \widehat{s}^{a}_{t|t+1} - \Delta \widehat{s}^{\mathrm{RE}a}_{t|t+1} \right) > 0$ at every point in time. Equation (32) shows that the trend change in the real exchange rate will also be positive and constant, that is, q_t will also tend to move up every period by the same magnitude without bound. It is easy to see from equation (20) that s_t will also undergo an unbounded upswing. Moreover, if the swings in q_t and s_t were initially toward PPP, then these prices would eventually shoot through this benchmark and begin trending away from parity from the other side. Equation (23) shows that these swings in q_t and s_t are associated with corresponding swings in nominal and real interest rates and thus a breakdown of international Fisher parity.

The analysis makes clear that such unbounded swings would arise in the model even if we were to assume that market participants's forecasting strategies were based solely on macroeconomic fundamentals. Indeed, an unbounded swing in s_t away from parity would arise even if x_t included only those fundamentals that drive the REH forecast, that is, m_t and y_t. Thus, a currency swing away from PPP occurs in the monetary model not because goods prices are sticky or market participants ignore macroeconomic fundamentals in forming their forecasts, but because knowledge about the causal mechanism driving exchange rates is imperfect.

3.4.2 Bounded Instability of Asset markets

Imperfect knowledge leads to de-linking, parallel movements, and an unbounded currency swing away from PPP. However, modeling imperfect knowledge with invariant representations suffers from the same difficulties as REH models: both are unable to explain the currency swings we actually observe and both presume gross irrationality. Like in other asset markets, the exchange rate experience of the last three decades is characterized by swings that, although protracted, are bounded and eventually reverse themselves over subsequent time periods.

Equation (32) shows that to generate bounded swings in the monetary model, the trend change in individuals' forecasts must eventually switch direction. But, for this to occur, the invariance restriction must be dropped: either individuals revise their forecasting strategies or one or more of the drifts underpinning the causal variables shift so as to change the sign of $E_t\left(\Delta\widehat{s}^{a}_{t|t+1} - \Delta\widehat{s}^{\mathrm{RE}_a}_{t|t+1}\right)$. Moreover, to escape the presumption that market participants are grossly irrational, we must stop short of fully prespecifying revisions of forecasting strategies.[42]

In the next section, we continue to assume a fixed policy environment, that is, the drifts behind the causal variables are constants. This allows us to focus on modeling revisions of forecasting strategies.

4 An IKE Representation of Forecasting Behavior

To model individuals' forecasting behavior, Frydman and Goldberg (2007) explore the implications of a well-documented phenomenon that psychologists call "conservatism:" individuals tend to revise the ways that they form their beliefs about uncertain outcomes gradually, relative to some baseline.[43]

While market participants may tend to behave conservatively, conservatism is a

[42] For a rogorous demonstration that jettisoning fully predetermined representations is required to model currency fluctuations without the presumption of irrationality see Frydmand and Goldberg (2007, 2008).

[43] See Edwards (1968) and Shleifer (2000).

regularity that is at best uneven and qualitative.[44,45] We would not expect an individual to behave conservatively forever: eventually, the unfolding historical record on market outcomes, changes in the social context, including policy, or the sheer creativity in thinking about the future, may lead a market participant to revise her forecasting strategy in a more substantial, non-conservative way. Moreover, in those periods when an individual does behave conservatively, we would not expect her to do so in exactly the same way across those periods.

The FG model represents these features of conservatism by placing qualitative conditions on the representation in (14). In doing so, the model is able to account for the swings we observe in currency markets, which are not only bounded, but, as with conservatism, are also uneven in magnitude and duration. By imposing qualitative conditions on forecasting behavior, the FG model is also able to represent the imperfection of knowledge without the presumption that individuals are irrational.[46]

4.0.3 Conservatism as a Qualitative Regularity

How one would formalize conservatism depends on the context. In general, it requires a specification of both the formation of beliefs and the baseline against which revisions of those beliefs are judged.[47]

In the context of the FG model, the way in which an individual forms her beliefs—her forecast—is represented by (14). Expression (16) specifies the change in this forecast, which we rewrite as follows

$$\widehat{s}^{a,i}_{t|t+1} - \widehat{s}^{a,i}_{t-1|t} = \Delta\hat{\beta}^i_t x^i_t + E_{t-1}\left[\Delta\widehat{s}^{a,i}_{t|t+1}|\Delta\hat{\beta}^i_t = 0\right] + \hat{\beta}^i_t \nu^{x^i}_t \tag{33}$$

[44] This seems to be the case with other empirical findings uncovered by behavioral economists. For example, they report much evidence that participants in financial markets often rely on technical trading rules in deciding when to take open poisitons. However, there is also much evidence that the importance individuals place on such strategies varies over time. See Schulmeister (2006, 2008) and references therein. By contrast, we would expect that the regularities that characterize individuals' preferences—for example, the importance of fairness or loss aversion in individual decision making—may be more enduring.

[45] Barberis, Shleifer, and Vishny (1998) also appeal to conservatism in modeling an individual's forecasting behavior. But, to generate sharp predictions, they formulate conservative behavior with a fixed rule that presumes that individuals under-react to earnings announcements in exactly the same way at every point in time. Consequently, they presume that market participants forego obvious profit opportunities endlessly.

[46] For a rigorous demonstration of how IKE models are able to avoid the presumption of irrationality, see Frydman and Goldberg (2007, 2008).

[47] In Edwards's experiments, conservatism was defined as updating that is too slow relative to a baseline defined by the updating associated with Bayes' rule.

We define the baseline change as the updating that would occur if an individual decided to leave her forecasting strategy unchanged. Such "status quo" change in the forecast is given by $E_{t-1}\left[\Delta\widehat{s}^{a,i}_{t|t+1}|\Delta\hat{\beta}^i_t = 0\right] + \hat{\beta}^i\nu^{x^i}_t$. Conservatism is then defined in terms of the change of $\widehat{s}^i_{t|t+1}$ that arises from revisions of an individual's forecasting strategy, relative to this status-quo change. Because the PPP puzzle is cast in terms of the trends in goods prices and the real exchange rate, Frydman and Goldberg formalize conservatism in terms of what they call the baseline drift, that is, the trend in the status quo change, $\hat{\beta}^i_{t-1}\mu^{x^i}$.

The FG model makes use of two conservative conditions to represent how an individual may revise her forecasting strategy. One of these conditions restricts revisions of $\hat{\beta}^i_t$ so that their impact on the level of $\widehat{s}^{a,i}_{t|t+1}$ is smaller in size than the baseline drift, that is,

$$\left|\Delta\hat{\beta}^i_t x^i_t\right| < \delta_t \tag{34}$$

where $|\cdot|$ denotes an absolute value and $\delta_t = \left|\hat{\beta}^i_{t-1}\mu^{x^i}\right|$ is the magnitude of the baseline drift.

A revision of $\hat{\beta}^i_t$ at a point in time, say $t = \tau$, impacts not only the change in an individual's forecast between $\tau - 1$ and τ, but also the baseline change between τ and $\tau + 1$, that is, $\hat{\beta}^i_\tau\mu^{x^i}$. The second conservative condition restricts revisions of $\hat{\beta}^i_t$ so that the baseline drifts in two consecutive periods have the same sign. Given that $\hat{\beta}^i_t\mu^{x^i} = \hat{\beta}^i_{t-1}\mu^{x^i} + \Delta\hat{\beta}^i_t\mu^{x^i}$, the following condition ensures that if $\hat{\beta}^i_{t-1}\mu^{x^i}$ implies an upward or downward movement in $\widehat{s}^{a,i}_{t|t+1}$, so will $\hat{\beta}^i_t\mu^{x^i}$:

$$\left|\Delta\hat{\beta}^i_t\mu^{x^i}\right| < \delta_t \tag{35}$$

The constraints (34) and (35) embody the idea that when an individual decides to revise her strategy, she is reluctant to do so in ways that would alter the level or baseline drift of her forecast too much from what would be associated with the status quo change.

The baseline drift in FG's formulation of conservatism is the change that is produced in standard models, which arises from the mere updating due to movements of causal variables. Economists sometimes recognize that changes in causal variables may lead to revisions of individuals' forecasting strategies. But, when they do, they rely on preexisting rules, like Bayes' formula. By contrast, the present IKE formulation recognizes that individuals do not endlessly obey pre-existing rules in deciding on when and how to alter their forecasting strategies. Indeed, the decision to revise one's forecasting strategy depends on many factors, including prior forecasting success, economic and political developments, emotions, or, as we will suggest shortly, the size of the departure of the exchange rate from PPP.

The conservative restrictions in (34) and (35) leave room for non-rule-based revisions by restricting neither the causal variables that may enter the representation in (14), nor how exactly these variables might matter. Moreover, they do not constrain the trend change in an individual's forecast to be small in magnitude, only that this change is conservative relative to the status quo, in which the causal variables drive change. Consequently, if changes in the causal variables between two points in time were large, the change in $\widehat{s}^{a,i}_{t|t+1}$ could also be large.

Nonetheless, the conservative restrictions in (34) and (35) place sufficient structure on the analysis. Frydman and Goldberg (2007, 2008) show that any time period in which the revisions of an individual's forecasting strategy are conservative and trends in the causal variables remain unchanged, will be characterized by movements of $\widehat{s}^{a,i}_{t|t+1}$ that are, on average, in the same direction.

To see this, suppose that the baseline drift behind an individual's forecast is initially positive between $t = \tau - 1$ and τ, that is, $\hat{\beta}^{i}_{\tau-1}\mu^{x^i} > 0$. This positive drift implies that if the individual decides to leave her forecasting strategy unchanged at τ, so that $\Delta\hat{\beta}^{i}_{\tau} = 0$, her forecast will tend to rise between $\tau - 1$ and τ. With imperfect knowledge, however, an individual revises her strategy at least intermittently. Revisions can either reinforce or impede the positive change in $\widehat{s}^{a,i}_{t|t+1}$ that is due to the movements of the causal variables. But, if revisions satisfy the constraint in (34), their impact on $\widehat{s}^{a,i}_{t|t+1}$ will be smaller than that of the underlying drift and $\widehat{s}^{a,i}_{t|t+1}$ will tend to move up between $\tau - 1$ and τ.

The tendency of $\widehat{s}^{a,i}_{t|t+1}$ to rise may persist between τ and $\tau + 1$. The matter depends partly on whether the baseline drift behind $\widehat{s}^{a,i}_{t|t+1}$ remains positive, that is, whether $\hat{\beta}^{i}_{\tau}\mu^{x^i} > 0$. But, if the revision of $\hat{\beta}^{i}_{\tau}$ satisfies the constraint in (35), this will be the case. Thus, if an individual revises her forecasting strategy at time $\tau + 1$ and this revision satisfies the constraint in (34), the tendency of $\widehat{s}^{a,i}_{t|t+1}$ to rise will endure between τ and $\tau + 1$. Moreover, it is clear that this tendency for $\widehat{s}^{a,i}_{t|t+1}$ to rise will persist as long as the revisions of individual i's forecasting strategy are conservative, as defined by the conditions in (34) and (35).

4.1 Uneven and Bounded Long Swings

Findings in psychology suggest that individuals are often reluctant to revise their decision-making strategies in dramatic ways. And macroeconomic fundamentals often trend in particular directions for years at a time. Consequently, the FG model implies not only that the exchange rate will tend to undergo swings in one direction, but that these swings may be quite protracted.

The key to the FG model's ability to account for the type of long swings we observe in currency markets is that it recognizes that conservatism is a qualitative regularity whose duration is uneven; the model does not fully prespecify when and how long market participants might behave conservatively. Allowing for the possibility that individuals abandon conservative revisions of their forecasts from time to time is important for explaining the partial, but significant countermovements that characterize currency swings away from PPP: such movements begin and end because individuals revise their forecasting strategies in non-conservative and non-reinforcing ways. More broadly, the uneven nature of conservatism is crucial in accounting for the currency swings in figure 1, whose duration and magnitude are also uneven.

Moreover, because an IKE representation does not determine exactly when individuals' forecasting behavior may be adequately characterized by conservatism, it does not determine exactly when an exchange rate swing might begin or end. This feature enables the FG model to allow for both bulls and bears in the market at every point in time and yet avoid the inconsistency problem that was identified by Robert Lucas and that gave rise to the REH revolution.[48]

Although the FG model leaves open the exact timing of when an exchange rate swing might begin or end, it does imply that swings away from PPP are self limiting: eventually such periods are followed by sustained movements back in the direction of parity. This implication follows partly from the model's use of a new specification of the premium on foreign exchange, which relates the risk of potential losses from speculation not to the variance of foreign exchange returns, as is usually the case, but to the gap between an individual's forecast of the exchange rate and its historical benchmark level.[49] With this specification of risk, the model continues to imply currency swings of uneven duration and magnitude. However, while revisions of forecasting strategies and

[48] For a discussion of this issue, see Frydman and Goldberg (2007, 2008).

[49] This formulation of risk replaces the usual assumptions of risk aversion and expected utility theory with endogenous prospect theory. Endogenous prospect theory provides a way to represent the experimental findings of Kahneman and Tversky (1979) and others in a world of imperfect knowledge.

trends in the causal variables may, for example, lead foreign exchange bulls to bid the exchange rate further above PPP, they simultaneously become more concerned about the capital losses that they would incur if a sustained counter-movement were to arise. Moreover, if the swing away from PPP were to continue, bulls would eventually become so concerned about a reversal that they would no longer revise their forecasting strategies in conservative ways. At that point, they would either reduce their long positions or abandon them altogether, which would precipitate a reversal in the exchange rate.[50]

5 A Large Half-Life of PPP Deviations

By recognizing imperfect knowledge, the FG model is able to account for near-parallel movements of nominal and real exchange rates without requiring that goods prices adjust unreasonably sluggishly to their market clearing levels. Indeed, this is the case even if goods prices are assumed to be perfectly flexible.

By sharp contrast to extant REH models, which imply near-I(1) behavior, the FG model is also able to account for the long swings that we actually observe in currency markets. Frydman, et al. (2008) show that the FG model implies a near-I(2) process for the real exchange rate. This, in turn, implies that PPP deviations have a large half life.

However, because REH models imply at most I(1) trends, extant empirical studies of the half-life of PPP deviations are based mostly on estimating an AR(1) model:[51]

$$(q_t - q^{\mathrm{PPP}}) = \rho(q_{t-1} - q^{\mathrm{PPP}}) + \eta_t \tag{36}$$

where η_t white noise, ρ is estimated by OLS, and the half-life is given by $\frac{\log(0.5)}{\log(\rho)}$. The properties of OLS estimates of ρ depend on the process generating $q_t - q^{\mathrm{PPP}}$. In the context of the FG model with flexible goods prices, this process is given by (22), which we rewrite as follows:

$$(q_t - q^{\mathrm{PPP}}) = (q_{t-1} - q^{\mathrm{PPP}}) + \xi_t + \varepsilon_t \tag{37}$$

[50] The implication of self-limiting swings also follows form the behavior of policy makers, who tend to alter trends of policy variables when the exchange rate moves too far away from parity. See Frydman and Goldberg (2007, chapter 14).

[51] Murray and Papell (2003) and Rossi (2003) estimate the half-life of the real exchange rate on the basis of an autoregressive specification of order $p > 1$ and continue to find a large half-life. Allowing for higher-order AR processes would not change the conclusions in this section.

where the drift is given by

$$\xi_t = \frac{\eta}{G}\left(\Delta\hat{\beta}_t x_t + \hat{\beta}_{t-1}\mu^x - \mu^{\text{RE}}\right) \tag{38}$$

$\mu^{\text{RE}} = \mu^m - \phi\mu^y$, and ε_t depends on the error terms of the x_t, m_t, and y_t processes.

We now show that if the real exchange rate process were adequately characterized by our IKE-based monetary model, but an AR(1) model were used for estimation nonetheless, one would indeed obtain an estimate of ρ close to unity, thereby implying a large half life.

To introduce some notation, assume that we have a number, n, of long swings, and we let $0 = T_0 < T_1^* < T_2^* < \cdots < T_n^* = T$ denote the points of time at which non-conservative revisions of forecasting strategies occur, leading to a change in the direction of the long swing. Then $T_i = T_i^* - T_{i-1}^*$ denotes the length of the ith period, I_i, of conservative behavior or the length of the ith long swing.

With imperfect knowledge, $\Delta\hat{\beta}_t$ is, in general, not zero and, thus, the drift ξ_t is time varying. A swing away or toward PPP occurs during periods in which forecast revisions satisfy the conservative conditions in (34) and (35). For example, we consider the first such period spanning $t = 1, ..., T_1$ and rewrite the drift for any t during this subperiod as

$$\Delta q_t = \xi_t + \varepsilon_t = \alpha_1 + \zeta_t, \quad t = 1, \ldots, T_1 \tag{39}$$

where $\alpha_1 = T_1^{-1}\sum_{t=1}^{T_1}\xi_t$ is the average of the drifts during the first period and $\zeta_t = \xi_t - \alpha_1$ denotes deviations from this average. More generally, we define the average in period i as

$$T_i^{-1}\sum_{t\in I_i}(\xi_t + \varepsilon_t) = \bar{\xi}_i + \bar{\varepsilon}_i = \alpha_i$$

and the deviations from the average,

$$\zeta_t = \xi_t + \varepsilon_t - \bar{\xi}_i + \bar{\varepsilon}_i, t \in \mathcal{I}_i$$

The following assumptions are implied by conservative restrictions on revisions of forecasting strategies and the fact that the observed swings in the exchange rate are long:

Assumption 1: We assume that the deviations ζ_t are small in the sense that $\alpha_i^{-2}T_i^{-1}\sum_{t\in\mathcal{I}_i}\zeta_t^2$ is small.

Assumption 2: We assume that the swings are long in the sense that T_i are large.

Assumption 1 follows from the conservative conditions in (34) and (35), which imply that inside a period of conservatism, there is no change in the sign of the trend in the real exchange rate. Consequently, during those periods, the variation of ξ_t is small compared to the average trend. Assumption 2 merely says that the currency swings are long-lasting, for example, the 2-4 years that is actually observed would be sufficient for our result.

We show in the appendix that under assumptions 1 and 2, the OLS estimate of the root in equation (36) $\hat{\rho}_{OLS} \to 1$, thereby implying that estimates of the half-life will tend to be large.

6 The IKE Resolution of the Puzzle and Empirical Implications

The IKE monetary model of currency swings resolves the PPP puzzle by according market participants' forecasts an autonomous role in driving outcomes. In doing so, REH's rigid connection between individuals' exchange rate forecasts and PPP is severed. This enables us to de-link real exchange rate movements from the adjustment of goods prices and generate enough persistence to account for the long-lasting swings and large half lives that characterize floating currencies.

Despite the qualitative nature of the constraints that it uses to represent forecasting behavior, the FG model not only resolves the PPP puzzle, but it generates other testable implications. Frydman et al. (2008) provide empirical support for our IKE resolution of the PPP puzzle by testing key implications of FG model against those of its REH counterpart. We have already discussed how, in the context of the traditional monetary model, IKE and REH lead to differing implications for time series data: under IKE, q_t is near-I(2) and the half-life of q_t is not tied to the speed of adjustment in goods markets, whereas under REH, q_t is at most near-I(1) and p_t and q_t tend to converge to PPP at the same rate. Frydman et al.'s (2008) VAR analysis rejects the implications of the model under REH in favor of those under IKE at very high significance levels.

Frydman et al. (2008) also show that whereas the real exchange rate and real interest rate differential are separately near-I(2) under IKE, goods market equilibrium implies that these variables co-move over time; as forecasting behavior leads to long-lasting swings in the nominal exchange rate, goods prices adjust so as to imply a cointegrating relationship between q_t and $i_t - \hat{\pi}_t$. By contrast, the monetary model under REH implies that PPP and international Fisher parity are individually cointegrating relationships. Our VAR analysis rejects that q_t and $i_t - \hat{\pi}_t$ are separately I(0), in favor of the alternative hypothesis that they are near-I(2) and cointegrated.[52]

[52] The VAR studies of Juselius (1995) and Juselius and MacDonald (2004) also find that the real exchange rate and the real interest rate differential are individually non-stationary and cointegrated.

Appendix

A regression of q_t on q_{t-1} and a constant gives the regression coefficient

$$\begin{aligned}\hat{\rho}_{OLS} &= \frac{\sum_{t=1}^{T} q_t(q_{t-1} - \bar{q}_T)}{\sum_{t=1}^{T}(q_{t-1} - \bar{q}_T)^2} = 1 + \frac{\sum_{t=1}^{T} \Delta q_t(q_{t-1} - \bar{q}_T)}{\sum_{t=1}^{T}(q_{t-1} - \bar{q}_T)^2} \\ &= 1 + \frac{\sum_{t=1}^{T} \Delta q_t q_{t-1} - (q_T - q_0)\bar{q}_T}{\sum_{t=1}^{T} q_{t-1}^2 - T\bar{q}_T^2},\end{aligned}$$

where $\bar{q}_T = T^{-1}\sum_{t=1}^{T} q_{t-1}$. We note from the identity

$$\sum_{t=1}^{T}(\sum_{i=1}^{t-1}\phi_i)\phi_t = \frac{1}{2}[(\sum_{t=1}^{T}\phi_t)^2 - \sum_{t=1}^{T}\phi_t^2]$$

that

$$\sum_{t=1}^{T} q_{t-1}\Delta q_t = \frac{1}{2}[(\sum_{t=1}^{T}\Delta q_t)^2 - \sum_{t=1}^{T}(\Delta q_t)^2] = \frac{1}{2}[(q_T - q_0)^2 - \sum_{t=1}^{T}(\Delta q_t)^2].$$

We then find because $\sum_{t\in\mathcal{I}_i}\zeta_t = 0$, that

$$q_t = \sum_{j=1}^{i-1} T_i a_i + (t - T_{i-1}^*)a_i + \sum_{s\in\mathcal{I}_i, s\leq t}\zeta_s$$

which can be well approximated by

$$q_t^0 = \sum_{j=1}^{i-1} T_i a_i + (t - T_{i-1}^*)a_i$$

since $\sigma_{\zeta i}$ is small compared to a_i, according to Assumption 1. We next find after some algebra

$$\begin{aligned}\sum_{t=1}^{T} q_t^0 &= \sum_{i=1}^{n} T_i[\sum_{j=1}^{i-1}\alpha_j T_j + \frac{1}{2}(T_i + 1)a_i], \\ \sum_{t=1}^{T}(q_t^0)^2 &= \sum_{i=1}^{n} a_i^2\frac{T_i(T_i+1)(2T_i+1)}{6} + \sum_{1=1}^{n}(\sum_{j=1}^{i-1}\alpha_j T_j)(\sum_{j=1}^{i-1}\alpha_j T_j + a_i(T_i+1))T_i.\end{aligned}$$

It is seen that q_T^0 is linear in T_i, $\sum_{t=1}^{T} q_t^0$ is quadratic, and $\sum_{t=1}^{T}(q_t^0)^2$ is cubic in T_i.

Therefore, the denominator of $\hat{\rho}_{OLS} - 1$ is of the order of T_i^3 and the numerator is of the order of T_i^2, so the ratio will be small if the lengths of the swing periods T_i are large.

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