Numerical Python for Scalable Architectures
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Numerical Python for Scalable Architectures

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Abstract
In this paper, we introduce DistNumPy, a library for doing numerical computation in Python that targets scalable distributed memory architectures. DistNumPy extends the NumPy module[15], which is popular for scientific programming. Replacing NumPy with Dist-NumPy enables the user to write sequential Python programs that seamlessly utilize distributed memory architectures. This feature is obtained by introducing a new backend for NumPy arrays, which distribute data amongst the nodes in a distributed memory multiprocessor. All operations on this new array will seek to utilize all available processors. The array itself is distributed between multiple processors in order to support larger arrays than a single node can hold in memory.

We perform three experiments of sequential Python programs running on an Ethernet based cluster of SMP-nodes with a total of 64 CPU-cores. The results show an 88% CPU utilization when running a Monte Carlo simulation, 63% CPU utilization on an N-body simulation and a more modest 50% on a Jacobi solver. The primary limitation in CPU utilization is identified as SMP limitations and not the distribution aspect. Based on the experiments we find that it is possible to obtain significant speedup from using our new array-backend without changing the original Python code.

Keywords  NumPy, Productivity, Parallel language

1. Introduction
In many scientific and engineering areas, there is a need to solve numerical problems. Researchers and engineers behind these applications often prefer a high level programming language to implement new algorithms. Of particular interest are languages that support a broad range of high-level operations directly on vectors and matrices. Also of interest is the possibility to get immediate feedback when experimenting with an application. The programming language Python combined with the numerical library NumPy[15] supports all these features and has become a popular numerical framework amongst researchers.

The idea in NumPy is to provide a numerical extension to the Python language. NumPy provides not only an API to standardized numerical solvers, but a possibility to develop new numerical solvers that are both implemented and efficiently executed in Python, much like the idea behind the MATLAB[8] framework. NumPy is mostly implemented in C and introduces a flexible N-dimensional array object that supports a broad range of numerical operations. The performance of NumPy is significantly increased when using array-operations instead of scalar-operations on this new array.

Parallel execution is supported by a limited set of NumPy functions, but only in a shared memory environment. However, many scientific computations are executed on large distributed memory machines because of the computation and memory requirements of the applications. In such cases, the communication between processors has to be implemented by the programmer explicitly. The result is a significant difference between the sequential program and the parallelized program. DistNumPy eliminates this difference by introducing a distributed version of the N-dimensional array object. All operations on such distributed arrays will utilize all available processors and the array itself is distributed between multiple processors, which makes it possible to expand the size of the array to the aggregated available memory.

1.1 Motivation
Solutions to numerical problems often consist of two implementations: a prototype and a final version. The algorithm is developed and implemented in a prototype by which the correctness of the algorithm can be verified. Typical many iterations of development
are required to obtain a correct prototype, thus for this purpose a
high productivity language is used, most often MATLAB. How-
ever, when the correct algorithm is finished the performance of the
implementation becomes essential for doing research with the al-
gorithm. This performance requirement presents a problem for the
researcher since highly optimized code requires a fairly low-level
programming language such as C/C++ or Fortran. The final ver-
sion will therefore typically be a reimplementaiton of the prototype,
which involves both changing the programming language and par-
allellizing the implementation (Fig. 1a).

The overall target of DistNumPy is to provide a high productiv-
ity tool that meets both the need for a high productivity tool that
allows researcher to move from idea to prototype in a short time,
and the need for a high performance solution that will eliminate the
need for a costly and risky reimplementaiton (Fig. 1b). It should be
desirable to develop and implement an algorithm using a sim-
ple notebook and then effortlessly execute the implementation on a
cluster of computers while utilizing all available CPUs.

1.2 Target architectures

NumPy supports a long range of architectures from the widespread
x86 to the specialized Blue Gene architecture. However, NumPy is
incapable of utilizing distributed memory architectures like Blue
Gene supercomputers or clusters of x86 machines. The target of
DistNumPy is to close this gap and fully support and utilize dis-
tributed memory architectures.

1.3 Related work

Libraries and programming languages that support parallelization
on distributed memory architectures is a well known concept. The
existing tools either seek to provide optimal performance in parallel
applications or, like DistNumPy, seek to ease the task of writing
parallel applications.

The library ScalAPACK[2] is a parallel version of the linear
algebra library LAPACK[1]. It introduces efficient parallel opera-
tions on distributed matrices and vectors. To use ScalAPACK, an
application must be programmed using MPI[7] and it is the respon-
sibility of the programmer to ensure that the allocation of matrices
and vectors comply with the distribution layout ScalAPACK spec-
ifies.

Another library, Global Arrays[13], introduces a distributed
data object (global array), which makes the data distribution trans-
parent to the user. It also supports efficient parallel operations and
provides a higher level of abstraction than ScalAPACK. However,
the programmer must still explicitly coordinate the multiple
processes that are involved in the computation. The programmer
must specify which region of a global array is relevant for a given
process.

Both ScalAPACK and Global Arrays may be used from within
Python and can even be used in combination with NumPy, but it is
only possible to use NumPy locally and not with distributed
operations. A more closely integrated Python project IPython[16]
supports parallelized NumPy operations. IPython introduces a dis-
tributed NumPy array much like the distributed array that is intro-
duced in this paper. Still, the user-application must use the MPI
framework and the user has to differentiate between the running
MPI-processes.

Co-Array Fortran[14] is a small language extension of Fortran-95
for parallel processing on Distributed Memory Machines. It in-
troduces a Partitioned Global Address Space (PGAS) by extending
Fortran arrays with a co-array dimension. Each process can ac-
cept remote instances of an array by indexing into the co-array
dimensions. A similar PGAS extension called Unified Parallel C
(UPC)[3] extent the C language with a distributed array declara-
tion. Both languages provide a high abstraction level, but users still
program with the SPMD model in mind, writing code with the un-
derstanding that multiple instances of it will be executing cooper-
atively.

A higher level of abstraction is found in projects where the ex-
ecution, seen from the perspective of the user, is represented as a
sequential algorithm. The High Performance Fortran (HPF)[12]
programming languages provide such an abstraction level. How-
ever, HPF requires the user to specify parallelizable regions in the
code and which data distribution scheme the runtime should use.

The Simple Parallel R INTerface (SPRINT)[9] is a parallel
framework for the programming language R. The abstraction level
in SPRINT is similar to DistNumPy in the sense that the distribu-
tion and parallelization is completely transparent to the user.

2. NumPy

Python has become a popular language for high performance com-
puting even though the performance of Python programs is much
lower than that of compiled languages. The growing popularity is
because Python is used as the coordinating language while the com-
pute intensive tasks are implemented in a high performance lan-
guage.

NumPy[15] is a library for numerical operations in Python
which is implemented in the C programming language. NumPy
provides the programmer with an N-dimensional array object and
a whole range of supported array operations. By using the array
operations, NumPy takes advantage of the performance of C while
retaining the high abstraction level of Python. However, this also
means that no performance improvement is obtained otherwise e.g.
using a Python loop to traverse a NumPy array does not result in
any performance gain.

2.1 Interfaces

The primary interface in NumPy is a Python interface and it is pos-
sible to use NumPy exclusively from Python. NumPy also provides
a C interface in which it is possible to access the same function-
ality as in the Python interface. Additionally, the C interface also
allows programmers to access low level data structures like point-
ers to array data and thereby provides the possibility to implement
arbitrary array operations efficiently in C. The two interfaces may
be used interchangeably through the Python program.

2.2 Universal functions

An important mechanism in NumPy is a concept called Universal
function. A universal function (ufunc) is a function that operates
on all elements in an array independently. That is, a ufunc is a
vectorized wrapper for a function that takes a fixed number of scalar
inputs and produces a fixed number of scalar outputs. Using ufunc
can result in a significant performance boost compared to native
Python because the computation-loop is implemented in C.

2.2.1 Function broadcasting

To make ufunc more flexible it is possible to use arrays with
different number of dimensions. To utilize this feature the size of
the dimensions must either be identical or have the length one.
When the ufunc is applied, all dimensions with a size of one will
be broadcasted in the NumPy terminology. That is, the array will
be duplicated along the broadcasted dimension (Fig. 2).

It is possible to implement many array operations efficiently
in Python by combining NumPy’s ufunc with more traditional
numerical functions like matrix multiplication, factorization etc.

2.3 Basic Linear Algebra Subprograms

NumPy makes use of the numerical library Basic Linear Algebra
Subprograms (BLAS) [11]. A highly optimized BLAS implemen-
tation exists for almost all HPC platforms and NumPy exploits
this when possible. Operations on vector-vector, matrix-vector and matrix-matrix in NumPy.

3. DistNumPy

DistNumPy is a new version of NumPy that parallelizes array operations in a manner completely transparent to the user – from the perspective of the user, the difference between NumPy and DistNumPy is minimal. DistNumPy can use multiple processors through the communication library Message Passing Interface (MPI)[7]. However, we have chosen not to follow the standard MPI approach in which the same user-program is executed on all MPI-processes. This is because the standard MPI approach requires the user to differentiate between the MPI-processes, e.g. sequential areas in the user-program must be guarded with a branch based on the MPI-rank of the process. In DistNumPy MPI communication must be fully transparent and the user needs no knowledge of MPI or any parallel programming model. However, the user is required to use the array operations in DistNumPy to obtain any kind of speedup. We think this is a reasonable requirement since it is also required by NumPy.

The only difference in the API of NumPy and DistNumPy is the array creation routines. DistNumPy allow both distributed and non-distributed arrays to co-exist thus the user must specify, as an optional parameter, if the array should be distributed. The following describes the only difference between the creation of a standard array and a distributed array:

```python
# Non-Distributed
A = numpy.array([1,2,3])

# Distributed
B = numpy.array([1,2,3], dist=True)
```

3.1 Interfaces

There are two programming interfaces in NumPy – one in Python and one in C. We aim to support the complete MPI interface and a great subset of the C interface. However, the part of the C interface that involves direct access to low level data structures will not be supported. It is not feasible to return a C-pointer that represents the elements in a distributed array.

3.2 Data layout

Two-Dimensional Block Cyclic Distribution is a very popular distribution scheme and it is used in numerical libraries like ScalAPACK[2] and LINPACK[5]. It supports matrices and vectors and has a good load balance in numerical problems that have a diagonal computation workflow e.g. Gaussian elimination. The distribution scheme works by arranging all MPI-processes in a two dimensional grid and then distributing data-blocks in a round-robin fashion either along one or both grid dimensions (Fig. 3); the result is a well-balanced distribution.

Figure 2. Universal function broadcasting. The ufunc addition is applied on a 3x2 array and a 1x2 array. The first dimension of the 1x2 array is broadcasted to the size of the first dimension of the 3x2 array. The result is a 3x2 array in which the two arrays are added together in an element-by-element fashion.

Figure 3. The Two-Dimensional Block Cyclic Distribution of a matrix on a 2 x 3 grid of processors.

NumPy is not limited to matrices and vectors as it supports arrays with an arbitrary number of dimensions. DistNumPy therefore use a more generalized N-Dimensional Block Cyclic Distribution inspired by High Performance Fortran[12], which supports an arbitrary number of dimensions. Instead of using a fixed process grid, we have a process grid for every number of dimensions. This works well when operating on arrays with the same number of dimensions but causes problems otherwise. For instance in a matrix-vector multiplication the two arrays are distributed on different process grids and may therefore require more communication. ScalAPACK solves the problem by distributing vectors on two-dimensional process grids instead of one-dimensional process grids, but this will result in vector operations that cannot utilize all available processors. An alternative solution is to redistribute the data between a series of identical leveled BLAS operations using a fast runtime redistribution algorithm like [18] demonstrates.

3.3 Operation dispatching

The MPI-process hierarchy in DistNumPy has one MPI-process (master) placed above the others (slaves). All MPI-processes run the Python interpreter but only the master executes the user-program, the slaves will block at the import numpy statement.

The following describes the flow of the dispatching:

1. The master is the dispatcher and will, when the user applies a python command on a distributed array, compose a message with meta-data describing the command.
2. The message is then broadcasted from the master to the slaves with a blocking MPI-broadcast. It is important to note that the message only contains meta-data and not any actual array data.
3. After the broadcast, all MPI-processes will apply the command on the sub-array they own and exchange array elements as required (Point-to-Point communication).
4. When the command is completed, the slaves will wait for the next command from the master and the master will return to the user’s python program. The master will return even though some slaves may still be working on the command, synchronization is therefore required before the next command broadcast.

3.4 Views

In NumPy an array does not necessarily represent a complete contiguous block of memory. An array is allowed to represent a subpart of another array i.e. it is possible to have a hierarchy of arrays where only one array represent a complete contiguous block of memory and the other arrays represent a subpart of that memory.
Inspired by NumPy, DistNumPy implements an array hierarchy where distributed arrays are represented by the following two data structures.

- **Array-base** is the base of an array and has direct access to the content of the array in main memory. An array-base is created with all related meta-data when the user allocates a new distributed array, but the user will never access the array directly through the array-base. The array-base always describes the whole array and its meta-data such as array size and data type are constant.

- **Array-view** is a view of an array-base. The view can represent the whole array-base or only a sub-part of the array-base. An array-view can even represent a non-contiguous sub-part of the array-base. An array-view contains its own meta-data that describe which part of the array-base is visible and it can add non-existing 1-length dimensions to the array-base. The array-view is manipulated directly by the user and from the users perspective the array-view is the array.

Array-views are not allowed to refer to each other, which means that the hierarchy is flat with only two levels: array-base below array-view. However, multiple array-views are allowed to refer to the same array-base. This hierarchy is illustrated in Figure 4.

### 3.5 Optimization hierarchy

It is a significant performance challenge to support array-views that are not aligned with the distribution block size, i.e. an array view that has a starting offset that a not aligned with the distribution block size or represents a non-contiguous sub-part of the array-base. The difficulty lies in how to handle data blocks that are located on multiple MPI-processes and are not aligned to each other. Such problems can be handled by partitioning data blocks into sub-blocks that both are aligned and located on a single MPI-process. However, in this work we will not focus on problems that involve non-aligned array-views, but instead simply handle them by communicating and computing each array element individually.

In general we introduce a hierarchy of implementations where each implementation is optimized for specific operation scenarios. When an operation is applied a lookup in the hierarchy determines the best suited implementation for that particular operation. All operations have their own hierarchy some with more levels than others, but at the bottom of the hierarchy all operations have an implementation that can handle any scenario simply by handling each array element individually.

### 3.6 Parallel BLAS

As previously mentioned NumPy supports BLAS operations on vectors and matrices. DistNumPy therefore implements a parallel version of BLAS inspired by PBLAS from the ScaLAPACK library. Since DistNumPy uses the same data-layout as ScaLAPACK, it would be straightforward to use PBLAS for all parallel BLAS operations. However, to simplify the installation and maintenance of DistNumPy we have chosen to implement our own parallel version of BLAS. We use SUMMA[6] for matrix multiplication, which enable us to use the already available BLAS library locally on the MPI-processes. SUMMA is only applicable on complete array-views and we therefore use a straightforward implementation that computes one element at a time if partial array-views are involved in the computation.

### 3.7 Universal function

In DistNumPy, the implementation of ufunc uses three different scenarios.

1. In the simplest scenario we have a perfect match between all elements in the array-views and applying an ufunc does not require any communication between MPI-processes. The scenario is applicable when the ufunc is applied on complete array-views with identical shapes.

2. In the second scenario the array-views must represent a continuous part of the underlying array-base. The computation is parallelized by the data distribution of the output array and data blocks from the input arrays are fetched when needed. We use non-blocking one-side communication (MPI_Get) when fetching data blocks, which makes it possible to compute one block while fetching the next block (double buffering).

3. The final scenario does not use any simplifications and works with any kind of array-view. It also uses non-blocking one-side communication but only one element at a time.

### 4. Examples

To evaluate DistNumPy we have implemented three Python programs that all make use of NumPy’s vector-operations (ufunc). They are all optimized for a sequential execution on a single CPU and the only program change we make, when going from the original NumPy to our DistNumPy, is the array creation argument dist.

A walkthrough of a Monte Carlo simulation is presented as an example of how DistNumPy handles Python executions.

#### 4.1 Monte Carlo simulation

We have implemented an efficient Monte Carlo Pi simulation using NumPy’s ufunc. The implementation is a translation of the Monte Carlo simulation included in the benchmark suite SciMark 2.0[17],

```python
from numpy import *
(x, y) = (empty([S], dist=True),)
empty([S], dist=True)
(x, y) = (random(x), random(y))
(x, y) = (square(x), square(y))
z = (x + y) < 1
print add.reduce(z) * 4.0 / S #The result
```

Figure 5. Computing Pi using Monte Carlo simulation. S is the number of samples used. We have defined a new ufunc (ufunc_rand) to make sure that we use an identical random number generator in all benchmarks. The ufunc uses `rand()/(double)RAND_MAX` from the ANSI C standard library (stdlib.h) to generate numbers.
which is written in Java. It is very simple and uses two vectors with length equal the number of samples used in the calculation. Because of the memory requirements, this drastically reduces the maximum number of samples. Combining multiple simulations will allow more samples but we will only use one simulation. The implementation is included in its full length (Fig. 5) and the following is a walkthrough of a simulation (the bullet-numbers represents line numbers):

1: All MPI-processes interpret the import statement and initiate DistNumPy. Besides calling MPI_Init() the initialization is identical to the original NumPy but instead of returning from the import statement, the slaves, MPI-processes with rank greater than zero, listen for a command message from the master, the MPI-process with rank zero.

2-3: The master sends two CREATE ARRAY messages to all slaves. The two messages contain an array shape and unique identifier (UID), which in this case identifies \( x \) and \( y \), respectively. All MPI-processes allocate memory for the arrays and stores the array information.

4: The master sends two UFUNC messages to all slaves. Each message contains a UID and a function name ufunk_random. All MPI-processes apply the function on the array with the specified UID. A pointer to the function is found by calling PyObject_GetAttrString with the function name. It is thereby possible to support all ufuns from NumPy.

5: Again the master sends two UFUNC messages to all slaves but this time with function name square.

6: The master sends a UFUNC messages with function name add followed by a UFUNC messages with function name less than. The scalar 1 is also in the message.

7: The master sends a UFUNC, REDUCE messages with function name add. The result is a scalar, which is not distributed, and the master therefore solely computes the remainder of the computation and print the result. When the master is done a SHUTDOWN message is sent to the slaves and the slaves call exit(0).

4.2 Jacobi method

The Jacobi method is an algorithm for determining the solutions of a system of linear equations. It is an iterative method that uses a spitting scheme to approximate the result. Our implementation uses ufunk operations in a while-loop until it converges. Most of the implementation is included here (Fig. 6).

Figure 6. Iteratively Jacobi solver for matrix \( A \) with solution vector \( B \) both are distributed arrays. The import statement and the creation of \( A \) and \( B \) is not included here. tol is the maximum tolerated value of the diagonal-element with the highest value (dmax).
using more than 16 CPU-cores is very poor. The problem is memory bandwidth – since we use multiple CPU-cores per node when using more than 8 CPU-cores, the aggregated memory bandwidth of the Core 2 cluster does only increase up to 8 CPU-cores. The Nehalem cluster is a bit better because it has two memory buses per node, but using more than 16 CPU-cores will not increase the aggregated memory bandwidth.

5.2.1 Profiling of the Jacobi implementation
To investigate the memory bandwidth limitation observed in the Jacobi execution we have profiled the execution by measuring the time spend on computation and communication (Fig. 9). As expected the result shows that the percentages used with communication increases when the number of CPU-cores increases. Furthermore, a noteworthy observation is the almost identical communication overhead at eight CPU-cores and sixteen CPU-cores. This is because half of the communication is performed through the use of shared memory at sixteen CPU-cores, which also means that the communication, just like the computation, is bound by the limited memory bandwidth.

5.3 Newtonian N-body simulation
The result of the speedup benchmark is illustrated in Figure 10. Compared to the Jacobi method we see a similar speedup and CPU utilization. This is expected because the dominating operations are also simple ufuncs. Even though there are some matrix multiplications, which have a great scalability, it is not enough to significantly boost the overall scalability.

5.4 Alternative programming language
DistNumPy introduces a performance overhead compared to a lower-level programming language such as C/C++ or Fortran. To investigate this overhead we have implemented the Jacobi benchmark in C. The implementation uses the same sequential algorithm as the NumPy and DistNumPy implementations.

Executions on both architectures show that DistNumPy and NumPy is roughly 50% slower than the C implementation when executing the Jacobi method on one CPU-core. This is in rough runtime numbers: 21 seconds for C, 31 seconds for NumPy and 32 seconds for DistNumPy.

Obviously highly hand-optimized implementations have a clear performance advantages over DistNumPy. For instance by the use of a highly optimized implementation in C [10] demonstrates extreme scalability of a similar Jacobi computation – an execution by 16384 CPU-cores achieves a CPU utilization of 70% on a Blue Gene/P architecture.

5.5 Summary
The benchmarks clearly show that DistNumPy has both good performance and scalability when execution is not bound by the memory bandwidth, which is evident from looking at the CPU utilization when only one CPU-core per node is used. As expected the scalability of the Monte Carlo simulation is better than the Jacobi and the N-body computation because of the reduced communication requirements and more CPU-intensive ufunc operation.

The scalability of the Jacobi and the N-body computation is drastically reduced when using multiple CPU-cores per node. The problem is the complexity of the ufunc operations. As opposed to the Monte Carlo simulation, which makes use of a complex ufunc, the Jacobi and the N-body computation only use simple ufuncs e.g. add and multiplication.

As expected the performance of the C implementation is better than the DistNumPy implementation. However, by utilizing two CPU-cores it is possible to outperform the C implementation in the case of the Jacobi method. This is not a possibility in the case of the Monte Carlo simulation where the algorithm does not favor vectorization.

6. Future work
In its current state DistNumPy does not implement the NumPy interface completely. Many specialized operations like Fast Fourier transform or LU factorization is not implemented, but it is our intention to implement the complete Python interface and most of the C interface.
Figure 8. Speedup of the Jacobi solver. In graph (a) the two architectures uses a minimum number of CPU-cores per node. Added in graph (b) is the result of using multiple CPU-cores on a single node (SMP).

Figure 9. Profiling of the Jacobi experiment. The two figures illustrate the relationship between communication and computation when running on the Core 2 Quad architecture (a) and the Nehalem architecture (b). The area with the check pattern represent MPI communication and the clean area represent computation. Note that these figures relates directly to the Jacobi speedup graph (Fig 8a).
The performance of NumPy programs that make use of array-views that are not aligned with the distribution block size is very poor because each array element is handled individually. This is not a problem for a whole range of NumPy programs, including the experiments presented in this paper, since they do not use non-aligned array-views. However some operations, such as stencil operations, require non-aligned array-views and an important future work is therefore to support all array views with similar efficiency.

Other important future work includes performance and scalability improvement. As showed by the benchmarks, applications that are dominated by non-complex ufuncs easily become memory bounded. One solutions is to merge calls to ufuncs, that operate on common arrays, together in one joint operation and thereby make the joint operation more CPU-intensive. If it is possible to merge enough ufuncs together the application may become CPU bound rather than memory bound.

7. Conclusions

In this work we have successfully shown that it is possible to implement a parallelized version of NumPy[15] that seamlessly utilize distributed memory architectures. The only API difference between NumPy and our parallelized version, DistNumPy, is an extra optional parameter in the array creation routines.

Performance measurements of three Python program, which make use of DistNumPy, show very good performance and scalability. A CPU utilization of 88% is achieved on a 64 CPU-core Nehalem cluster running a CPU-intensive Monte Carlo simulation. A more memory-intensive N-body simulation achieves a CPU utilization of 91% on 16 CPU-cores but only 63% on 64 CPU-cores. Similar a Jacobi solver achieves a CPU utilization of 85% on 16 CPU-cores and 50% on 64 CPU-cores.

To obtain good performance with NumPy the user is required to make use of array operations rather than using Python loops. DistNumPy take advantage of this fact and parallelizes array operations. Thus most efficient NumPy applications should be able to benefit from DistNumPy with the distribution parameter as the only change.

We conclude that it is possible to obtain significant speedup with DistNumPy. However, further work is needed if shared memory machines are to be fully utilized as nodes in a scalable architecture.

References


