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Freeman J. Dyson (15 December 1923–28 February 2020) was one of the greatest and most influential scientists of our time. Born in England in 1923, he was appointed professor at the Institute for Advanced Study in Princeton (USA) in 1952, where he remained until the end of his career.

Dyson was an exceptionally visionary and original mathematician and theoretical physicist. Many of his important contributions to mathematical physics were, as we shall highlight below, published in the *Journal of Mathematical Physics*. It is therefore highly appropriate to honor his legacy with this special collection.

A vast range of topics in the sciences as well as many other subjects of relevance to society and humanity have been impacted by Dyson’s contributions. It will not be possible to highlight all of these in this short preface. We will focus on those areas close to mathematical physics that have inspired the selection of articles in this special issue.

While Dyson started his career with a few publications on number theory, he soon turned to theoretical and mathematical physics. His earliest contribution to theoretical physics was the groundbreaking work on quantum electrodynamics. Dyson unified the works of Feynman, Tomonaga, and Schwinger for which they received the Nobel Prize in 1965. He understood the issue of renormalization while at the same time realizing that the renormalized perturbation series could not be convergent. These early works laid a foundation for much of the vast body of later research on quantum field theory.

In the mid-1950s, Dyson made influential contributions to the statistical mechanical study of magnetism, viz., with his general theory of spin waves and his analysis of the phase transition and symmetry breaking in the quantum Heisenberg spin model. He later returned to these problems from a more rigorous point of view, e.g., in Ref. 7, where he studied a phase transition in a one-dimensional long range Ising model, and in the collaboration with Lieb and Simon that rigorously established the phase transition in the three-dimensional quantum anti-ferromagnetic Heisenberg model.

In 1957, inspired by work of Lee and Yang, Dyson published a paper on the ground state energy of a hard sphere gas of bosons in which he studied the asymptotics of the ground state energy in the dilute limit. This paper pioneered the rigorous mathematical work on many-body quantum mechanics which is today a highly active research field.

The work on the hard sphere Bose gas may also be seen as a precursor for the later celebrated work with Lenard on stability of matter. Stability of matter is one of the most fundamental consequences of quantum mechanics and the Pauli principle. It refers to the property that the ground state energy of normal charged matter, consisting of electrons and nuclei, is bounded below with a bound linear in particle number. As already mentioned, the Pauli exclusion principle, i.e., the fermionic character of electrons is important for stability of matter. If both the negatively and positively charged particles were bosons matter would be unstable as shown by Dyson in Ref. 13. It is remarkable that...
there had not been a heuristic derivation of stability of matter prior to the fully rigorous analysis by Dyson and Lenard. It is truly one of the cornerstones in mathematical physics. The proof of stability of matter was later simplified by Lieb and Thirring in Ref. 14.

Dyson indirectly played an important role in the study of quantum entropy and trace inequalities. In the paper5 by Wigner and Yanase, they study an expression for information contents of quantum states. Dyson is quoted in the paper for having suggested a generalization of Wigner–Yanase–Dyson skew information. Lieb10 later proved an important convexity property of the Wigner–Yanase–Dyson skew information and this was a key ingredient in the fundamental result of Lieb and Ruskai on strong subadditivity of quantum entropy.17

The last topic we want to address is the ground-breaking contribution of Dyson to random matrix theory.18–22 The study of random matrices may be seen as going back to Wishart23 and had been pioneered by Wigner in his study of energy spectra of nuclei.24 Dyson focused, in particular, on correlations of eigenvalues and the corresponding universality of energy-local correlation functions, which successfully described in terms of his famous sine kernel. In that context, a very influential tool introduced by Dyson is the Brownian-motion model for the eigenvalues of a random matrix. Moreover, Dyson understood how different classes of random matrix ensembles were naturally characterized by their symmetry properties and the relation this had to group representations. In particular, Dyson coined the celebrated three-fold way classification.25 While this work was published in the early 1960s, Dyson’s interest in spectra of random operators can actually be traced back to a paper of 1952,26 which even predates Wigner’s foundational paper.

It is a famous conjecture by Montgomery27 that the correlation functions of the high-lying zeros of Riemann’s zeta function agree with those of the eigenvalues of random complex Hermitian matrices, i.e., the Gaussian Unitary Ensemble. Montgomery credits Dyson for having drawn his attention to the random matrix ensembles. It is worth adding that in the more recent body of work by Keating and others the statistical properties of the Riemann zeros are actually modeled by the random eigenvalues of the Circular Unitary Ensemble, another paradigm introduced by Dyson.

Freeman Dyson received numerous prestigious prizes and awards. Among these are the Heineman prize, the Wolf Prize in Physics, the Templeton Prize, and the Henri Poincaré Prize.

The papers in this special collection relate to many of the contributions by Freeman Dyson. Some are written by collaborators or former colleagues of his.

The following papers relate to random matrix theory and eigenvalue statistics.27–41

The papers of Refs. 42–44 deal with Bose gases and stability of matter.

The subject of quantum entropy and trace inequalities is the subject of the papers of Refs. 45 and 46.

The papers of Refs. 47–52 are in the area of symmetry breaking and statistical mechanics.

Quantum field theory and perturbation theory is in Refs. 53 and 54.

Finally, Ref. 55, written by a former colleague of Dyson from the Institute for Advanced Study, is an example of mathematics applied in neuroscience and we believe Dyson, with his broad interest in the sciences, would have appreciated this contribution as well as all the other excellent papers in this collection.

REFERENCES

23. F. J. Dyson, “The threefold way classification.”