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BALLET BALANCE STRATEGIES

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Abstract

Animating physically realistic human characters is challenging, since human observers are highly tuned to recognize human cues such as emotion and gender from motion patterns. The main contribution of this paper is a new model firmly based on biomechanics. The model has been developed to animate balance and basic movements of a ballet dancers. It is supported by computer simulated experiments and is showing good agreement with biomechanical measurements of real-life dancers.

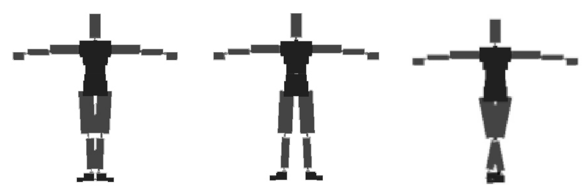
KE: Pertective, what impact does the work have?

Keywords: Biomechanics, Balance Strategy, Weight shift strategy, Control Mechanism.

Nomenclature

The following symbols are used in this paper:

τ	Joint torque
$\theta, \Delta\theta$	Joint angle and angular update
k_s, k_d	Spring constants
$\vec{r}_i$	Position of body part i
$\vec{r}_{cm}, \vec{r}_{cp}$	Position of center of mass and pressure
m_i, M	Mass of body part i and total mass
p_j	Contact point
n_j	Normal force at p_j
$\vec{f}_j$	Contact force at p_j



(a) (b) (c)

Figure 1: **KE: I think it is good to be explicit about telling where these images came from** Three of the basis ballet positions shown in our simulator: (a) First, (b) Second, and (c) Fifth. The feet are positioned in the frontal plane in all poses.

Introduction

A long term goal of computer graphics is to increase realism and believability into computer generated animations and pictures [1, 31, 18, 5, 37, 4]. With improved rendering techniques, the lack of physical realism and believability is becoming increasingly obvious and annoying to the common observer, and one accompanying long term goal in animation is to increase physical realism by using physics to model plausible behavior and movement of computer models. This known as physics-based animation.

This paper studies biomechanical and ballet inspired balance and weight shifting strategies and is based on original work in [35]. Ballet is a balance art and is a prime focus for learning about human balance and weight shifting strategies. We will describe how to make a dynamic animation from the first of the basic positions in Figure 1 to obtain a quiet standing on one toe. To achieve the goal the articulated figure moves through the four sub-goals shown in Figure 2: Balancing on two legs, weight shift to the supporting leg, balancing on one leg, and balancing on the toe of one leg. The new model is firmly based on biomechanics and is supported by computer simulated experiments showing good agreement with biomechanical measurements of real-life dancers.

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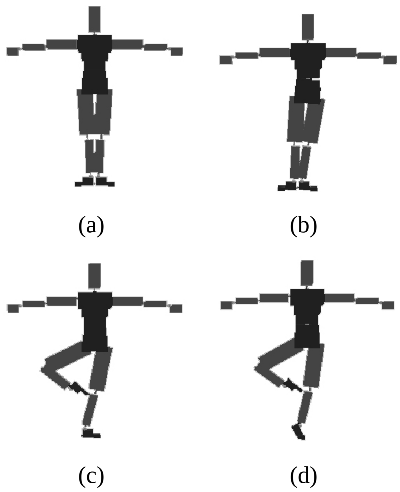


Figure 2: **KE: I think it is good to be explicit about telling where these images came from** Images from our simulator illustrating shifting the weight from both legs in first pose (a) to the left leg (b), to a one-legged stand (c), and finally to a one-toed (d) quiet standing.

Survey of past work

The study of balance has been performed in at least three separate areas of research: Biomechanics, Robotics, and Animation. We will in the following highlight some research from these areas.

Biomechanics and the study of ballet: Ballet is an art-form, where balance plays a central role. Classical ballet techniques are throughly described in the literature, see e.g. [17, 43]. Biomechanical studies of ballet has mainly been studied through injury cases, e.g. [13]. In The studying of human balance there is **KE: are?** two theories: Either we balance by controlling the center of mass directly or indirectly by controlling the center of pressure [28]. Empirical investigations have shown that the velocity of center of mass plays a role in balancing [44]. Empirical studies on real humans have been performed on balancing of humans versus the position of the center of mass [34] **KE: Balancing humans versus center-of-pressure strategy, don't you mean center-of-mass versus center-of-pressure?** . Weight shifts have been measured using a force-platform [32, 33]. Finally, a thorough measurement of the properties of mass and inertia of human body parts may be found in [10].

Robotics: Early work on balancing robots may be found in [41]. Weight shift strategies for walking are often performed through dynamic walking machines, where there is more or less degree of control involved in the walking cycle [29, 24, 40]. Recently, a number of spectacular and stable humanoid robots have been produced starting with the Honda robot [22]. Especially in relation to our work, the center of pressure has recently been introduced as a stable control mechanism for balancing robots [21, 20].

Animation: A very early mentioning of simulated human motion is given in [2], and early implementations may be found in [16, 30]. The implementation of dancing models is only scarcely discussed in the literature, however one exception is [8]. A major inspiration for our work has been [26, 46], where center of mass is used to control the balance and motion of a humanoid model of varying complexity. Alternative to center of mass strategies are inverse kinematics e.g. [27], energy models e.g. [7], and learning approaches e.g. [11].

The Biomechanics of Quiet Standing

A human in quiet standing may be modeled by an articulated figure [26, 46] consisting of a set of joints and a set of links representing body parts. The set of possible joints consist of revolute (1 Degree of Freedom (DOF)), universal (2 DOF), and ball-and-socket (3 DOF) joints. In this work, we have used Wooten's model [46], which contains 28 DOFs, and uses real measurements of the mass, m_i, center of mass, $\vec{r}_i$, and moments of inertia of all the body parts [10]. The ankle and hip joints is of particular importance for this paper, and they are modeled by an universal joint and a ball-and-socket joint respectively.

Traditionally in biomechanics and anatomy motion orientation is described in three planes: The Sagittal (x-axis), the Transverse (y-axis) and the Frontal (z-axis) plane. These planes are illustrated in Figure 3. Measurements of angles and positions are traditionally also performed in these planes [32] by projection onto the respective planes and axes. Typical projections are: The position of the center of mass, the position of the center of pressure, angle of joints, and the direction of gravity. To work with the projec-

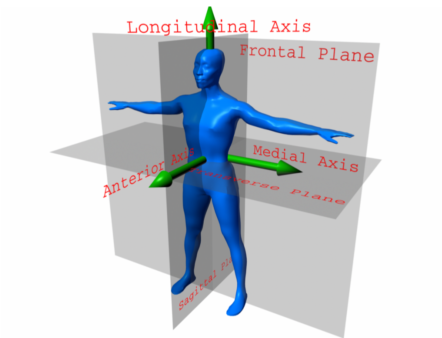


Figure 3: Sagittal, Transverse, and Frontal planes together with Medial, Longitudinal and Anterior axis. **KE: Do something about text**

tions rather than the underlying 3D geometry, allows for comparison with the substantial biomechanical literature.

Muscles are used to move and sustain posture of the human skeleton, and our articulated figure is supplemented by an actuator system, which applies joint torques according to a simple damped angular spring model,

$$\tau = k_s^{\text{muscle}} \left(\theta^{\text{target}} - \theta^{\text{current}} \right) - k_d^{\text{muscle}} \dot{\theta}^{\text{current}}. \quad (1)$$

In the equation, τ is the length of the torque vector, θ^{target} and θ^{current} are target and current angles, $\dot{\theta}^{\text{current}}$ is the current velocity of the angle, and k_s^{muscle} and k_d^{muscle} are spring and damping constants.

A balance control strategy is a function that determine updates, $\Delta\theta$'s, based on the current state of the articulated figure, θ^{current}'s, such that the model will converge towards a desired state of quiet balance. I.e. the strategy iteratively determines new parameter values, θ^{new}'s as,

$$\theta^{\text{new}} = \theta^{\text{current}} + \Delta\theta. \quad (2)$$

In the rest of this article, it will be assumed that the model is placed on a planar floor, and the contact between the floor and the feet is represented by a set of coplanar contact points, $\vec{p}_j$. The support polygon is defined as the 2D convex hull of all the contact points. The center of mass is of the human model is defined as

$$\vec{r}_{\text{cm}} = \frac{1}{M} \sum_i^N m_i \vec{r}_i, \quad (3)$$

where $M = \sum_i^N m_i$ is the total mass, N is the number of body parts in the model, m_i is the individual weights of the body parts, and r_i are their locations. Note that center of mass is not fixed w.r.t. any location of the body during motion of the individual body parts. The center of pressure is defined as

$$\vec{r}_{\text{cp}} = \frac{1}{||\vec{n}||} \sum_j^K ||\vec{n}_j|| \vec{p}_j, \quad (4)$$

where $\vec{n} = \sum_j^K \vec{n}_j$ is the total normal force acting on the human model, $\vec{n}_j$ is the normal force applied to the human model at the j'th contact point $\vec{p}_j$, and K is the number of contact points.

For simplicity, a spring model for the floor contact forces is used, where the contact force of the j'th contact is

$$\vec{f}_j = k_s^{\text{contact}} \left(\vec{p}_j^{\text{ initial}} - \vec{p}_j \right) - k_d^{\text{contact}} \dot{\vec{p}}_j. \quad (5)$$

In the equation, $\vec{p}_j^{\text{ initial}}$ is the initial point of contact, $\vec{p}_j$ is the current contact point, $\dot{\vec{p}}_j$ is the velocity of the current contact point, and k_s^{contact} and k_d^{contact} are spring and damping constants. The vector, $\vec{n}_j$, is calculated as the projection of $\vec{f}_j$ onto the contact normal of the floor, and the tangential part is a simple model of frictional force.

Contact forces can only be repulsive, and attractive contact forces, f_j, are therefore set to zero. Slipping is obtained by setting $\vec{p}_j^{\text{ initial}}$ equal to $\vec{p}_j$, when the magnitude of the tangential force component exceeds a multiple of the magnitude of the normal force component, $||\vec{f}_{\text{friction}}|| > -\mu ||\vec{n}||$.

The line of gravity is defined as the line going from the center of mass to the ground in the direction of the gravitational field. The point of intersection between the line of gravity and the floor is referred to as the projection of the center of mass. These concepts are illustrated in Figure 4.

Balance is defined as an objects ability to maintain quiet standing, where quiet standing is the state, where the projection of the center of mass is kept within the support polygon [23, 26, 46]. The implication is that the greater support polygon, the lower center of mass, the more stable the balance and vice versa. The human body has a highly placed center of mass over a rather small support polygon, and as such the human body behaves as an inverted pendulum. This will be further developed later.

In the remainder of this paper, control strategies for maintaining quiet standing and moving an articulated figure from one pose of quiet standing to another will be discussed.

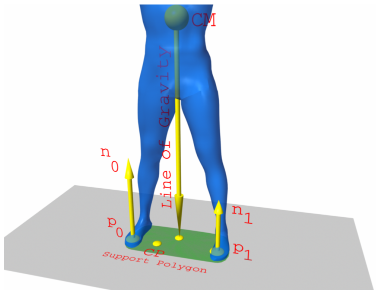


Figure 4: Illustration of biomechanical definitions: Center of mass, center of pressure, support polygon and line of gravity. **KE: Bigger, difficult to read.**

Balance Strategies: Mass Center versus Pressure Center

In the following, two strategies for balance inspired by Biomechanics will be compared: The center of mass strategy and the center of pressure strategy. The analysis are done on an articulated figure in 3D based on [46] standing with parallel feet. The balance is controlled by the ankle [44].

The center of mass strategy is the traditional balance strategy for dynamic animation, where the angular change is controlled as a function of the projection of the center of mass onto the support plane [26, 46]:

$$\Delta\theta = k_s^{\mathrm{cm}} \left(r_{\mathrm{cm}}^{\mathrm{current}} - r_{\mathrm{cm}}^{\mathrm{target}} \right) - k_d^{\mathrm{cm}} v_{\mathrm{cm}}, \quad (6)$$

where $\Delta\theta$ is the angular change of the ankle in the Sagittal plane, $r_{\mathrm{cm}}^{\mathrm{current}}$ and $r_{\mathrm{cm}}^{\mathrm{target}}$ **KE: target position is living in 2D plane, not in 3D space, right?** are the projections of the current and target positions of the center of mass onto the anterior axis, v_{cm}, is the velocity vector of the center of mass projected onto the anterior axis, and k_s^{cm} and k_d^{cm} are control parameters.

In the center of pressure strategy, the model uses the center of pressure to control the center of mass. The goal of this strategy is to calculate a desired position of the center of pressure, and use this for controlling the muscles.

$$\begin{aligned} \Delta r_{\mathrm{cp}} &= k_s^{\mathrm{cm}} \left(r_{\mathrm{cm}}^{\mathrm{current}} - r_{\mathrm{cm}}^{\mathrm{target}} \right) - k_d^{\mathrm{cm}} v_{\mathrm{cm}}, && (7) \\ r_{\mathrm{cp}}^{\mathrm{target}} &= \Delta r_{\mathrm{cp}} + r_{\mathrm{cm}}, && (8) \\ \Delta\theta &= k_s^{\mathrm{cp}} \left(r_{\mathrm{cp}}^{\mathrm{current}} - r_{\mathrm{cp}}^{\mathrm{target}} \right) - k_d^{\mathrm{cp}} v_{\mathrm{cp}}, && (9) \end{aligned}$$

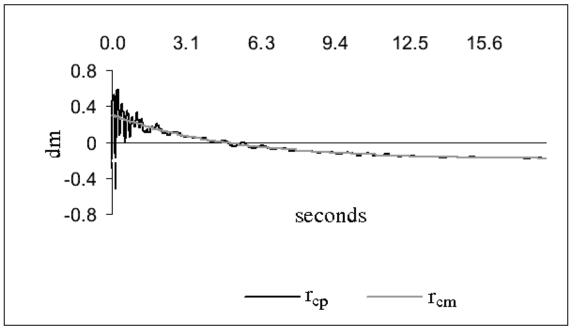
where Δr_{cp} is the positional change of the projection of the center of pressure onto the anterior axis, $r_{\mathrm{cp}}^{\mathrm{current}}$ and $r_{\mathrm{cp}}^{\mathrm{target}}$ are the projections of the current and target positions of the center of projection onto the anterior axis, and k_s^{cp} and k_d^{cp} are control parameters.

The center of pressure theory can be understood by looking at the human balance like the balance of an inverted pendulum. The center of mass is the top and on the floor is the center of pressure. When the positions are right above each other, it's a perfect balance **KE: Unproven statement, cite?** . In order to change the balance to a new desired position. The center of mass can start an acceleration towards the new point by moving the center of pressure in the opposite direction. The same happens if you balance a match on your finger. **KE: There might also be some analogy to center of boyance, i.e. why ships are stable.**

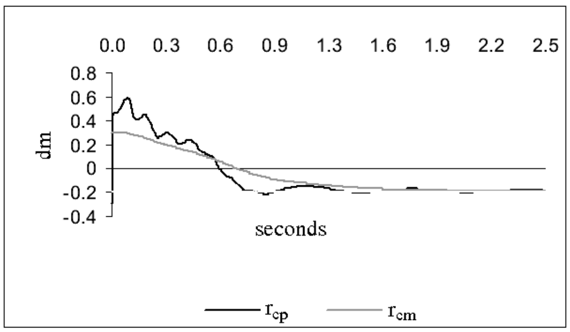
Figure 5 shows the resulting dynamics of the projected center of mass and the center of pressure on the anterior axis. The movement of the projection of the center of mass on the articulated figure start in a position on the heels to the desired position on the center of the feet (the center of the support area **KE: polygon?**). The articulated figure manages to balance using both strategies, but the center of pressure strategy requires only approximately 1.5 sec. while the center of mass strategy requires almost 10 sec. to get a balance in the desired position. In addition, the center of pressure strategy also has the smallest amplitude of the oscillations of the center of pressure, which means that it has the best control **KE: by what definition?** over the contact with the ground. The articulated figure is also balancing in frontal plane, but in this plane the projection of the center of mass starts in the desired position in the center. We obtained similar results for the same tests on another model [35] than Wooten's [46].

Dynamics of a Ballet Dancer

Quiet standing on the toe of one leg is central in all ballet training. This is demanding, since the dancer has to balance on a very small support polygon while at the same time looking at ease. We have developed strategies for obtaining a balance on the left leg with the right foot by the left knee as illustrated in Figure 2. The pose is used, when dancers turn in a pirouette, and it is perhaps the most basic of all balances in ballet training. Both in quiet balance and in



(a)



(b)

Figure 5: Comparison of the dynamics of the ankle for the two strategies: (a) Center of mass strategy and (b) Center of pressure strategy. r_{cp} is the center of pressure and r_{cm} is the center of mass. Motion is restricted to the Sagittal plane.

most of the basic ballet exercises, the legs are strictly separated into the working leg (doing the exercise) and the supporting leg. Shifting the weight between the legs is basic, and it should preferably be done without drawing the attention of the audience.

Weight Shifting Strategies

The weight shifting strategy described in the following was inspired by the analysis of real dancers presented in the literature [32]. They showed that angle changes in the hips and ankles are nearly identical during a weight shift, and that the center of pressure starts moving towards the working leg and end up being on the supporting leg. The last result is only explainable from the center of pressure balance strategy.

To shift the weight, we designated the left ankle to be the controlling joint using (9). **KE: The remaining part of this pharagraph is a bit heavy to read, perhaps some picture help?** The desired position of the projection of the center of mass where on the center of the supporting legs foot. The angular changes in the hip joints where calculated from the angular changes in the left ankle. The right ankle controlled the position of the right foots center of pressure.

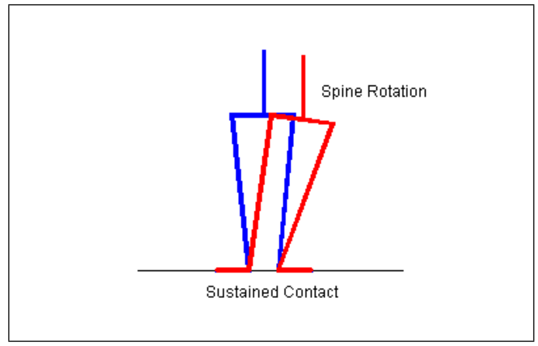


Figure 6: A rotation in the ankles with straight legs implies a rotation in the hip, and a rotation in the spine is required in order to keep the upper body vertical.

Ballet aesthetics requires that the upper body is kept parallel to the line of gravity during a weight shift in the frontal plane with both feet fixed on the floor. It has been claimed [32] that dancers keep their upper body vertical by a counter rotation in the hip joints, however this is only physically possible when the legs are parallel as illustrated in Figure 6 **KE: This figure do not show anything about parallel legs, it shows spine rotation, do we need another figure?** . Ballet dancers compensate for the hip rotation by a counter rotation in the lumbar region of the lower back [17], and therefore a control function in the spine is required. Dancers control the body center by the stomach muscles. Experience has shown that these stomach muscles are extremely important for aesthetic motion of the articulated figure [35], thus a control function in the pelvis is used to inhibit rotation in the Sagittal plane. Both control functions are modeled using a spring law similar to (1).

The final weight shift of the articulated figure is shown in Figure 2. The resulting articulated figure agrees with measurements performed on real dancers [32] as follows: The measured angle changes are nearly identical during the weight shift, and the center of pressure starts to move towards the right foot and ends in a position on the left foot. The last result are showed in Figure 5 **KE: I hope I got**

the right reference here? . On the dm-axis zero is between the center of the legs **KE: I rewritten this sentence, did I get it right?** , the negative numbers are the foot of the working leg and the positive are foot of the supporting leg.

Movement to Quiet Standing on One Toe

The final movements to obtain a one-toed quiet standing is achieved by lifting the non-supporting leg, and shifting from a foot stand to a toe stand, see Figure 2. It is not difficult to raise the leg, however the major challenge is to keep balance on a very small support polygon.

Two strategies have been developed: A strategy for raising the right leg, and a strategy for making a weight shift to the toe. Similarly to the weight shifting balance strategy, both are based on the center of pressure strategy.

KE: Maybe to pictures illustration content of this pharagraph? To raise the leg, the left ankle is kept as the controlling joint, and the center of pressure strategy is used to keep the balance on the center of the left foot. To shift the weight to one toe, the center of pressure strategy is used in two steps: Firstly, for the controlling ankle joint, and secondly to control the toe joint, when the ankle has been straightened.

When the supporting area **KE: polygon** is getting smaller the human bodys small deviations from a stiff pendulum makes it necessary to have a control of the center of mass at the left hip joint to maintain the balance.

Lifting the right leg to an aesthetically pleasing pose is performed using a spring law. The spine and pelvis are controlled as explained in the previous section.

The analysis of the results shows a motion of the center of pressure, when the right foot let go of the floor contact. This is not entirely in agreement with measurements on real dances. Otherwise the result shows a stable line.

Discussion

This paper has shown how you can use results from biomechanics for developing strategies for dynamic animations of an articulated figure of a human. The focus has been on balancing strategies. The comparison of the two balance strategies: Center of mass strategy, and the center of pressure strategy showed far the best results for the last one which we have not seen it used for dynamic animation before. We showed that using the center of pressure strategy makes it possible to develop strategies for a articulated figure of a human to balance in a very complicated position. The center of pressure strategy is useful in all dynamic animations of a balancing human. But if the projection of the center of mass is outside the area of support **KE: support polygon?** , it will not be possible to use this strategy alone to get the articulated figure back in balance.

We found out that when the human body is not simply a stiff inverted pendulum it is necessary to use another joint than just the ankle to control balance. This happens when one leg is lifted from the ground because there is movement in the upper body.

From a ballet point of view, controlling the center of pressure is also a way of controlling the contact of the feet with the ground. Ballet-dancers are very much aware of the relation between their feet and the ground, since it strongly influences their balance, their stance, and the audience's impression of the dancer's body.

Balance and weight shifting are the most basic techniques in ballet. Future steps in our research will be to develop strategies for exercises on one leg.

KE: Implications/perspective of this work?

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