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Poincaré and Cosmic Space: Curved or not?

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Résumé: En 1870, la géométrie non euclidienne avait été établie comme un domaine de recherche mathématique, mais pas encore comme un domaine pertinent pour l’espace réel habité par les étoiles et les nébuleuses. Elle intéressait beaucoup moins les physiciens et les astronomes que les mathématiciens et les philosophes. Bien que la plupart des astronomes aient pris la géométrie euclidienne séculaire pour acquise, au cours des décennies suivantes, quelques-uns d’entre eux, dont K.F. Zöllner, S. Newcomb et K. Schwarzschild, ont suivi les traces du pionnier N.I. Lobachevsky en envisageant sérieusement la possibilité que l’espace cosmique puisse être courbé. Du point de vue conventionnaliste de H. Poincaré, la discussion n’avait aucun sens puisqu’il n’y avait aucun moyen par lequel les observations ou autres données empiriques pouvaient déterminer la géométrie de l’espace. Le point de vue de Poincaré était partagé par quelques astronomes, mais son impact sur l’astronomie était limité. L’article examine la question telle qu’elle a été discutée avant l’avènement de la théorie de la relativité générale.

Abstract: By 1870, non-Euclidean geometry had been established as a mathematical research field but was yet to be considered relevant to the real space inhabited by stars and nebulae. It was of much less interest to physicists and astronomers than to mathematicians and philosophers. Although most astronomers took the age-old Euclidean geometry for granted, during the following decades a few of them such as K.F. Zöllner, S. Newcomb and K. Schwarzschild followed in the footsteps of the pioneer N.I. Lobachevsky by seriously considering the possibility that cosmic space might be curved. From the H. Poincaré’s conventionalist perspective the discussion was meaningless since there was no way in which observations or other empirical data could determine the geometry of space. Poincaré’s view was shared by a few astronomers, but its impact on astronomy was limited. The paper examines the question as it was discussed prior to the advent of the general theory of relativity.
1 Introduction

In a famous address on “Geometry and Experience” delivered to the Prussian Academy of Science in 1921, Einstein [1982, 233] reflected on the relationship between the mathematical and physical sciences. “As far as the propositions of mathematics refer to reality, they are not certain”, he stated, “and as far as they are certain, they do not refer to reality”. He found it useful to distinguish between two versions of geometry, what he called “purely axiomatic geometry” and “practical geometry”. Whereas the former did not refer to the experienced world, the latter was “evidently a natural science” of great interest to physicists and astronomers. In Einstein’s cosmological model of 1917, based on his new general theory of relativity, cosmic space was assumed to be spherical, and this was what he had in mind:

According to the view advocated here, the question whether the continuum has a Euclidean, Riemannian, or any other structure is a proper question of physics which must be answered by experience, and not a question of a convention to be chosen on grounds of mere expediency. [...] Therefore the question whether the universe is spatially finite or not seems to me an entirely meaningful question in the sense of practical geometry. I do not even consider it impossible that this question will be answered before long by astronomy. [Einstein 1982, 238–239]

Although Einstein did not mention Poincaré by name, he did it indirectly when rejecting the view that the geometry of space was merely a convention. Einstein elaborated on his view concerning physics and geometry in an article four years later, this time explicitly referring to Poincaré as an advocate of the view that “geometry alone contains no statements about objects of reality, but only geometry together with physics” [Einstein 1925].

The philosophical and historical dimensions of the concept of space have been scrutinized in a number of scholarly works, e.g., [Bonola 1955], [Torretti 1978], [Gray 1989], [Jammer 1993]. However, few of them pay much attention to how the possibility of curved space was received by astronomers and physicists in the pre-Einstein era, which is the subject of the present paper. More specifically, apart from an introductory section on early non-Euclidean geometry, I review the responses of the few astronomers, physicists and mathematicians who between 1872 and 1908 expressed an interest in the question [see also Walter 1997].

Although by the end of the nineteenth century most astronomers were conversant with non-Euclidean geometry, the large majority of them considered it irrelevant with regard to what interested them, namely the space encompassing the celestial bodies [Kragh 2012a]. This kind of space described by “practical geometry” clearly belonged to the domain of astronomy, whereas the abstract and empty space of the mathematicians was generally thought to be something
the astronomers could safely ignore. The prominent American astronomer Simon Newcomb spoke for the majority of his colleagues when he warned against “the tendency among both geometers and psychologists to talk about space as an entity in itself” [Newcomb 1898, 5]. At the time non-Euclidean geometry had almost no explanatory force and could be tested observationally only in principle, but not in practice. No wonder, then, that astronomers saw no reason to abandon the traditional flat space which so admirably had served astronomy since Antiquity. This was also the opinion of Poincaré, although his arguments were very different from those of the astronomers.

2 The slow rise of non-Euclidean geometry

Disregarding the early anticipations of Karl Friedrich Gauss, the field of non-Euclidean geometry was effectively founded by works published independently in 1829-1831 by János Bolyai in Hungary and Nikolai Ivanovich Lobachevsky in Russia [Bonola 1955]. Of the two pioneers, Lobachevsky is the more important from an astronomical point of view, since he alone took seriously the possibility that the supposed Euclidean structure of real space could be tested by means of astronomical observations. Contrary to Bolyai, he had a solid training in astronomical theory and practice and he served for a period as director of the Kasan University Observatory [Vucinich 1962].

In Lobachevsky’s hyperbolic or “imaginary” geometry, space was curved with a negative curvature, $K < 0$. As he pointed out in his Russian memoir of 1829 entitled “On the Principles of Geometry” [Lobachevsky 1899], this implies that the angle sum of a celestial triangle is less than 180°, a prediction which in principle can be tested by precise measurements of stellar parallaxes. Considering the triangle spanned by the Sun, the Earth and the star Sirius, and using available (but unreliable) data, Lobachevsky was forced to admit that the deviation from the flat Euclidean case was much smaller than the estimated error of observation [Brylevskaya 2008]. Although Euclidean geometry thus seemed to be a perfect approximation for all practical purposes, Lobachevsky did not conclude that cosmic space was in fact Euclidean. In a later work called Pangeometry published in Russian in 1855 and translated into French the following year, he derived another interesting consequence of hyperbolic space [Lobachevsky 2010]: If space is Euclidean ($K = 0$) the parallax angle tends toward zero as the distance to the star increases, whereas in the hyperbolic case there is a minimum parallax for all stars irrespective of their distances from the Earth.

Neither Gauss, Bolyai nor Lobachevsky recognized the possibility of positively curved space with a radius of curvature given by $R^2 = \frac{1}{K}$ or, alternatively, a curvature constant $k = +1$ instead of $k = -1$ as in hyperbolic geometry. This class of non-Euclidean geometry was introduced by Bernhard Riemann in a famous but cryptic Habilitation lecture given in Göttingen on
June 10, 1854, in which he pointed out that, in this case, space is finite and yet it has no boundaries:

In the extension of space-construction to the infinitely great, we must distinguish between unboundedness and infinite extent. [...] If we assume independence of bodies from position, and therefore ascribe to space constant curvature, it must necessarily be finite provided this curvature has ever so small a positive value. [Riemann 1873, 36]

However, whereas Riemann was interested in what he called “the metric relations of space in the infinitesimal small”, he only referred in passing—and without mentioning Lobachevsky—to the space of the astronomers. He thought that questions about the global properties of the space of the stellar universe were metaphysical and could be left to the philosophers. Moreover, his address was only published post mortem in 1867 and became generally known only after William Clifford translated the lecture into English in 1873. It eventually came to be seen as a visionary address on a possible geometrization of physics [Gray 2007, 187–201]; on Clifford’s role, see [Farwell & Knee 1990].

Non-Euclidean geometry circulated at first slowly in the mathematical community, but since the mid-1870s the subject began to attract much interest, not only among mathematicians but also among philosophers and the general public. One example is provided by the uncompromising atheist and materialist philosopher Eugen Karl Dühring, who was aware of the contemporary discussion of non-Euclidean geometries. However, he found it all wrong to ascribe physical reality to space itself, and in a book published in 1875 he scornfully dismissed the new ideas about curved space as “mathematical mysticism”, even “religious stupidity” [Dühring 1875, 67]. To mention but one more example, the new geometry appeared prominently in a passage in Fyodor Dostoevsky’s classic novel The Brothers Karamazov published 1879–1880. In one of the passages, Ivan Karamazov confides to his brother Alyosha that he does not understand the nature of God any better than he understands those mathematicians who “dare to dream that two parallel lines, which according to Euclid can never meet on Earth, may meet somewhere in infinity” [Dostoevsky 2003, 274].

The increasing popularity of the subject is documented in a careful bibliography by Duncan Sommerville [1911], according to which more than 4000 titles were published on non-Euclidean geometry and related subjects during the period from 1870 to 1910. It should be noted, though, that a

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1. All translations are mine, unless otherwise indicated.

2. Sommerville’s bibliography may be compared with the more than thirty-year older bibliography by George Bruce Halsted, an American mathematician who was a central figure in the dissemination of non-Euclidean geometry and the translator of several of the classic works in this area. Halsted [1878] was more mathematically restricted than the one of Sommerville and it comprised only 174 titles.
substantial part of the many titles was on other aspects of “hypergeometry”, an epithe
which also included speculations about a fourth spatial dimension or even higher-dimentional spaces. Also worth of mention is the national distribution of the publications listed in Sommerville’s bibliography: German (28.6%), French (22.0%), Italian (21.1%), English (18.0%), other languages (10.3%). Although non-Euclidean geometry was popular indeed in the fin de siècle era, the scholarly interest was largely limited to mathematicians and philosophers. Only a dozen physicists and astronomers felt tempted to investigate the remote possibility that real space might deviate from Euclid’s geometry.

3 Zöllner’s closed universe

With one noteworthy exception, Riemann’s suggestion of a spherically closed universe failed to attract the attention of contemporary astronomers. As noted by a German philosopher, the hypothesis “has usually been met with either an incredulous shake of the head or a contemptuous laughter” [Vaihinger 1875, 216]. The exception was the German astrophysicist Karl Friedrich Zöllner, a professor in Leipzig in “physical astronomy” known in particular for his pioneering works on astrophotometry, solar physics and stellar evolution [Kragh 2012b].

In a book from 1872 with the title Über die Natur der Cometen, Zöllner not only presented an electrical theory of comets but also, in a separate chapter, an essay “On the finitude of matter in infinite space”. In this cosmological essay, he argued that cosmic space was closed in Riemann’s sense and that the hypothesis offered a new and convincing explanation of Heinrich Olbers’ so-called paradox of the dark night sky. He was the first scientist to apply non-Euclidean geometry to a cosmological and not only to an astronomical context, and also the first to demonstrate that the new geometry, in this case Riemann’s, actually possessed explanatory power. Paraphrasing Riemann, Zöllner wrote:

It seems to me that any contradictions will disappear if we ascribe to the constant curvature of space but a positive value, however small. [...] The assumption of a positive value of the spatial curvature measure involves us in no way in contradictions with.

3. The name is a misnomer for two reasons. First, the problem with an infinite number of stars had been noted by Kepler as early as 1610, and in 1744 the Swiss astronomer Jean-Philippe Loys de Chésaux analyzed it in detail, for which reason it is sometimes referred to as the Chésaux-Olbers paradox. Second, during the nineteenth century it was not considered paradoxical, but as an argument for an absorbing medium filling interstellar space. The eponymous label “Olbers’ paradox” may first have been used in [Bondi 1952, 21]. For full accounts, see [Jaki 1969] and [Harrison 1987].
the phenomena of the experienced world if only this value is taken to be sufficiently small. [Zöllner 1872, 308]

The traditional answer to the old problem of the dark night sky, including Olbers’ of 1826, was to turn it into a non-problem by assuming interstellar space to be filled with a rarefied light-absorbing medium, but Zöllner considered this to be an \textit{ad hoc} explanation. His alternative was to adopt Riemann’s geometry, which implied but a finite number of stars in the entire universe. But wasn’t this just another \textit{ad hoc} hypothesis? Not according to Zöllner, for he was convinced that the idea of a closed space also explained many other phenomena, some of them on an astronomical scale and others related to microphysics.

Zöllner’s argument in favour of a Riemannian universe made almost no impact at all on contemporary astronomers and physicists. The cosmological hypothesis was effectively forgotten until it was reinvented some three decades later. Among the reasons for the fate of Zöllner’s essay was that he got a very poor reputation after he, a few years later, entered what he called “transcendental physics” based on the hypothesis of a fourth space dimension of a spiritual nature [Zöllner 1878], [Staubermann 2001], [Kragh 2012b]. His extensive work in this area and insistence of a hidden dimension of space had no connection to his interest in non-Euclidean geometry.

While astronomers chose to ignore Zöllner’s excursion into cosmology (or were unaware of it), it attracted some attention among German philosophers, although none of them supported the hypothesis of a cosmic Riemannian space. Wilhelm Wundt, the pioneer of experimental psychology and a former assistant to Helmholtz, argued against the model of a finite-space and finite-matter universe which he criticized for treating space and time asymmetrically. As another argument he pointed out that the curved-spaced universe would be subject to the heat death and therefore imply an uncaused creation of the world, something he found unscientific and totally unacceptable. While Zöllner assumed the universe to be spatially limited, Wundt took it to be temporally unlimited, with no beginning and no end. His favoured world model contained a finite amount of matter located in an infinite Euclidean space. Despite his objections, Wundt valued the daring hypothesis of positively curved space as a provocative challenge to current philosophical thinking about the universe:

\begin{quote}
Zöllner’s attempt to introduce the concept of transcendental space into the field of cosmology may possibly belong to the most fruitful ideas that had been for a long time proposed in cosmology. [Wundt 1877, 119]
\end{quote}

Unaware of Zöllner’s cosmological essay, Newcomb was keenly interested in non-Euclidean astronomy from both a mathematical and astronomical point of view. In his widely read \textit{Popular Astronomy} first published in 1878, he discussed the possibility of curved physical space, which he considered a fascinating speculation but not more than that. As he pointed out, if space
were closed, the enormous output of solar heat would eventually return to the Sun after a long cosmic journey and thus provide a mechanism that secured the Sun—and with it the Earth—an infinite lifetime. However, fascinating as the scenario was, he did not believe in it. The hypothesis, he said, was “too purely speculative to admit of discussion” [Newcomb 1878, 504]. When Newcomb reviewed the subject twenty years later, he concluded—much as Lobachevsky had done a generation ago—that although the hypothesis of curved space was testable and therefore scientific, this was more in principle than in practice. The complete absence of observational support gave good reason to believe that cosmic space was in fact Euclidean:

Unfortunately, we cannot triangulate from star to star; our limits are the two extremes of the Earth’s orbit. All we can say is that, within those narrow limits, the measures of stellar parallax give no indication that the sum of the angles of a triangle in stellar space differs from two right angles. [Newcomb 1898, 7]

Almost all scientists in the pre-Einsteinian era shared Newcomb’s reservations. The consensus at the turn of the century is further exemplified by Ludwig Boltzmann, who in a lecture course given in Vienna discussed the possibility of a positively curved stellar universe. He found it fascinating that space as a whole might return into itself but characteristically refrained from committing himself to a cosmology of this kind. Although he found it “possible that measurements of the stars will prove space to be non-Euclidean”, he cautiously added that it was “not likely” [Fasol-Boltzmann 1990, 255]. Boltzmann’s voice was just one of many.

4 Poincaré’s conventionalist space

Although curved space excludes space from being flat, the question of whether physical space belongs to one or the other of the categories was not necessarily an either-or question. It could be side-stepped, as Poincaré famously did, by declaring it meaningless. Poincaré stated his conventionalist view of geometry and space in an article of 1891 in Revue générale des sciences pures et appliquées which the following year appeared in Nature in an English translation. His unorthodox view on non-Euclidean geometries received further

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4. The only scientist I know of who seriously believed that the universe is non-Euclidean was the American polymath Charles Sanders Peirce, who is today better known for his works in logic and philosophy. Peirce gave a number of arguments, some of them philosophical but others based on astronomical observations, why space is curved, and he corresponded with Newcomb on the matter. However, most of his arguments remained for a long time unpublished or were made public in obscure journals not read by astronomers and mathematicians. See [Kragh 2012a] and [Dipert 1977].
attention when the essay was reprinted in 1901 in the widely read and translated *La Science et l’Hypothèse*. “If geometry were an experimental science, it would not be an exact science”, he wrote, referring to what Einstein would later call “purely axiomatic geometry” (Section 1). After having stated his belief that geometrical axioms were conventions or “definitions in disguise”, Poincaré [2015, 65] addressed the supposed dichotomy between flat and curved space:

> Then what are we to think of that question: Is the Euclidean geometry true? It has no meaning. As well ask whether the metric system is true and the old measures false. [...] One geometry can not be more true than another; it can only be more *convenient*. Now, Euclidean geometry is, and will remain, the most convenient.5

One might imagine that astronomical measurements proved the sum of angles in a celestial triangle to be, for example, $176^\circ \pm 1^\circ$, but according to Poincaré [2015, 81] this would not amount to a definite disproof of Euclidean space since one could “modify the laws of physics and suppose that light does not travel rigorously in a straight line. [...] The Euclidean geometry has, therefore, nothing to fear from fresh experiments”.

So, Poincaré asserted that we can always maintain Euclidean geometry because the geometrical postulates are only testable in conjunction with associated hypotheses of a physical nature; if observations disagree with Euclid’s flat geometry, the blame may be put on the latter and these may be modified. Although Poincaré’s conventionalism differed from that of his contemporary Pierre Duhem, his argument can reasonably be considered an instance of the Duhem-Quine thesis [Ivanova 2015], [Bland 2011]. However, Poincaré’s reasoning not only protected Euclid’s space from empirical falsification but likewise Lobachevsky’s hyperbolic space (and also, although Poincaré did not mention it, Riemann’s spherical space). The problem of how to understand the claim that Euclidean geometry in particular is invulnerable to “fresh experiments” has been critically examined by Zahar [2001, 71–72], among others.

In an essay on the relativity of space, Poincaré invited his readers to imagine that “in the night all the dimensions of the universe become a thousand times greater”. How would one experience such an astonishing transformation the next morning? Poincaré [2015, 414] answered: “Well, I shall perceive nothing at all”. In his *La Valeur de la science*, Poincaré [2015, 237–238] elaborated. Aware that this kind of thought experiment had been popularized by the Belgian experimental psychologist Joseph Delboeuf [Torretti 1978, 298–301], he did not claim originality to it. In fact, Laplace had entertained the same

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speculation in his *Exposition du système du monde* from 1808 [Jammer 1993, 168–169], and so had Roger Boscovich [1966, 203], as early as 1758 in his main work *Theoria philosophiae naturalis*. They all argued that an expansion or contraction of all magnitudes in the universe would be unobservable. Of course, the thought experiment does not qualify as an anticipation of the expansion of the universe in the modern sense of the term. Note that Poincaré did not speak of the expansion of space itself, a notion which he would deem to be meaningless since his space had no physical properties and also no intrinsic metric.

As argued convincingly by Scott Walter [1997], Poincaré’s extreme conventionalism with regard to space was generally rejected or just ignored. Nonetheless, elements of his philosophy of space can be found in the writings of some prominent astronomers, sometimes with explicit reference to the French mathematician. One supporter was the distinguished Munich astronomer Hugo von Seeliger, a former pupil of Zöllner who from 1896 served as president for the German Astronomical Society. Seeliger denied that space exists physically in the same sense that the objects of the universe exist. He held space to be nothing but a concept introduced to order and coordinate our sensory experiences. In an article from 1913, he warned “in agreement with Poincaré” against the serious misapprehension that measurements can provide knowledge of which kind of space we live in. This he considered to be plainly impossible, for “Space as such has no properties whatsoever” [Seeliger 1913].

It is more surprising to find a prominent astrophysicist and cosmologist advocating Poincaré’s concept of space many years after general relativity had changed the scene of cosmic space or space-time. Edward Arthur Milne constructed in the 1930s an ambitious and controversial non-Einsteinian system of the world which for more than a decade was influential and much discussed. Considering space to be nothing but an abstract system of reference, he declared the question of space curvature to be irrelevant to scientific cosmology. Physical space could have no structure, curved or not, and nor could space itself expand or contract. In a monograph of 1935, he summarized his position as follows:

The space used for the description of phenomena is essentially arbitrary and at the disposal of the observer. The phrase “physical space” has no meaning. [...] Laws of nature and geometry are complementary; a modification of the one implies a modification of the other. [Milne 1935, 10]

He described this as “an old view [...] most clearly expressed, perhaps, by H. Poincaré”, after which he quoted a long passage in French from *La Science et l’Hypothèse*.

Poincaré’s “last astronomical gift”, as Schwarzschild [1913] called it, was the monograph *Hypothèses cosmogoniques* published in [1911]. In this influential

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6. For a careful examination of Poincaré’s work, see [Rhee 2018]. To Poincaré and contemporary scientists the word “cosmogony” did not refer to its literal meaning,
work the author only once referred to non-Euclidean space, namely when commenting on the consequences of the second law of thermodynamics for a closed and therefore finite universe:

If, for example, our world is not Euclidean but Riemannian, it would be finite and yet without limits; we would then have a finite, non-degrading system, which negates the principle of Carnot usually presumed. [Poincaré 1911, 254]

Without offering any further comments on the possibility of closed cosmic space, Poincaré expressed strong doubt that the universe was infinite in its temporal dimension, such as argued by the Swedish chemist and Nobel laureate Svante Arrhenius in his hugely popular Worlds in the Making. This book was published in 1908 and two years later it appeared in a French edition as L’Évolution des mondes [Arrhenius 1908].

While many astronomers believed the material universe to be roughly identical to the Milky Way, Arrhenius based his cosmology on what would later be called the perfect cosmological principle, the postulate that the large-scale appearance of the universe is the same at any location and at any time. Arrhenius, who took Euclidean space to be self-evident, concluded that the law of entropy growth did not apply on a cosmic scale. By invoking radiation pressure and various energy transfer processes between stars and nebulae he suggested that the universe would never run down in the thermodynamic sense [Kragh 2013]. However, according to Poincaré’s detailed analysis in Hypothèses cosmogoniques [Poincaré 1911, 239–256], [Rhee 2018, 391–405], what he called “Arrhenius’ demon” might only retard the universal heat death, perhaps for an exceedingly long time, but it could never escape it. Schwarzschild [1913] agreed with Poincaré that Arrhenius’ demon would not work as promised and after the criticism of the two distinguished scientists, Arrhenius’ cosmological hypothesis was essentially abandoned by astronomers and physicists.

namely the origin of the cosmos or universe in its entirety. Cosmogony as understood at the time was largely restricted to the formation and evolution of celestial objects ranging from the Moon and the solar system to stars and nebulae. This version of cosmogony is reflected in the titles of numerous works in the pre-1930 period, such as Charles Wolf’s Les Hypothèses cosmogoniques (1886) and James Jeans’ Astronomy and Cosmogony (1928). After about 1950, when scientific cosmology came to include the big-bang origin of the universe, the term cosmogony was rarely used any longer.

7. Poincaré [1911, lxviii] introduced Arrhenius’ demon in analogy with Maxwell’s famous thought experiment in which a so-called demon transfers molecules from a colder to a warmer body. The name “demon” was coined by William Thomson in a letter to Nature of 1874, whereas Maxwell originally, in a letter to Peter G. Tait of 1867, referred to it as “a very observant and neat-fingered being”.

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Contemporaries of Poincaré

Despite the great authority Poincaré held in matters of science, and in France in particular, some of his compatriots disagreed with his view that Euclidean geometry was indisputably to be preferred because of its simplicity and greater convenience. One of the critics was the mathematician Paul Barbarin, a secondary school (lycée) teacher in Bordeaux and a prolific writer on non-Euclidean geometry. Contrary to Poincaré, he was an empiricist in the sense that he believed that the geometry of space could be determined by means of astronomical observations. In 1902 he published a small monograph, La Géométrie non euclidienne, which came out in a second edition in 1908 and was reprinted as late as 1928.

Barbarin [1902, 81–86] included a chapter on what he called geometrical physics in which he suggested that our part of the universe might possibly be curved with a radius of curvature $R > 400,000$ AU (astronomical units; $1$ AU = 150 million km). In a paper two years earlier, he noted that the physicist or astronomer did not have the same kind of luxurious freedom as the mathematician when it came to the space of the actual universe. Barbarin [1900] approached the question of the structure of physical space by deriving formulae for celestial triangles that could in principle distinguish between the three rival geometries. However, he had to admit that his formulae were of no practical value as they relied on angle measurements much more accurate than $0'01$. As illustrated by its almost complete lack of astronomical data, Barbarin’s paper was primarily a geometrical exercise rather than a contribution to astronomy.

A contemporary of Poincaré and Barbarin, the mathematician and philosopher Auguste Calinon shared the latter’s more empiricist attitude according to which the different geometrical systems are not physically equivalent. As he pointed out in a paper of 1889, “In order to know which of these spaces contains the bodies we see around us, we must necessarily look to experience” [Calinon 1889, 594], English translation in [Čapek 1976, 297–303]. Calinon distinguished between three hypotheses concerning the geometry of the universe: (1) It is strictly Euclidean. (2) It is strictly non-Euclidean with constant curvature. (3) The curvature of space varies in time, implying that $R = R(t)$. The latter hypothesis he spelled out as follows:

Our space realizes different geometric spaces successively in time, either by deviating more or less from the Euclidean parameter, or by oscillating around a given parameter near the Euclidean parameter. In this last hypothesis, which is the most general hypothesis, the shapes of the bodies surrounding us are slowly being modified before our eyes at the same time as our space, since different spaces cannot contain the same shapes. [Calinon 1889, 595]
In another paper, Calinon [1893] came closer to Poincaré’s conventionalism by arguing that astronomical and other empirical problems might be approached by whatever geometry best suited to produce a simple solution. The choice of geometry might vary from one problem to another and from one area of the universe to another. Any of the three geometries might be applied to the actual world by a suitable hypothesis regarding the course of rays of light. “By itself, space is neither finite nor infinite”, he concluded in partial agreement with Poincaré.

Calinon’s writings were of a philosophical nature that failed to appeal to most physicists and astronomers. They did however attract the critical attention of Bertrand Russell [1897], who dismissed the idea of space curvature changing in time. Interestingly, the American mathematical physicist Howard P. Robertson, a specialist in general relativity and a pioneer of the expanding universe, knew about and apparently appreciated Calinon’s old paper. In an essay on the occasion of Einstein’s 70-year birthday, he quoted from the French paper published 33 years before Alexander Friedmann introduced the hypothesis that the space curvature of the universe varies in time.

Barbarin and Calinon shared with most of their contemporaries interested in the geometry of space that their writings were of a mathematical and philosophical orientation. In this respect they stand in stark contrast to the more elaborate and astronomically oriented works by two German astronomers in the same period, namely Karl Schwarzschild and Paul Harzer. The important contribution of 26-year-old Schwarzschild [1900] is today well known. Although Schwarzschild obtained his doctoral degree in 1896 under Seeliger, he disagreed fundamentally with his Doktorvater’s view concerning the nature of geometry and space. It is sufficient to say that despite Schwarzschild’s systematic analysis based on modern data and advanced computational methods his systematic investigation proved unable to decide whether cosmic space really possesses a positive or negative curvature. All Schwarzschild could do was to establish bounds for the curvature radius, which he estimated to be $R > 4 \times 10^6$ AU for the hyperbolic case and $R > 1.6 \times 10^8$ AU for the closed spherical case which he preferred from a philosophical and emotional point of view.

In a little-known memoir of 1908, Paul Harzer, professor of astronomy at the Christian-Albrecht University in Kiel, extended Schwarzschild’s investigation into a more detailed model of the closed stellar universe, which essentially meant the Milky Way System [Harzer 1908], [Walter 1997]. Like Schwarzschild, he emphasized that the question of the geometry of space could only be

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9. Schwarzschild’s paper based on a lecture given on 9 August 1900 to the Astronomical Society in Heidelberg, has been translated as “On the permissible curvature of space” in Classical and Quantum Gravity, 15 (1998), 2539–2544. For the historical context and connection to relativistic cosmology, see [Schemmel 2005].
answered by means of observations. Harzer’s material universe was enclosed in a finite cosmic space about 17 times that of the stellar system. This system contained the same number of stars but was compressed to a size approximately one half of what it had in models based on flat space, such as Seeliger’s. The size of the entire universe was given by the time it took for a ray of light to circumnavigate it, which Harzer estimated to be 8,700 years. Harzer took the model of a closed stellar universe no less seriously than Schwarzschild, but of course he realized that it was hypothetical and lacked the support of solid observational evidence. Consequently, his conclusion was cautious:

This picture includes no features that can be characterized as improbable. [...] But the picture speaks of the possibility of the finite space only, not of its reality, and as yet we have no evidence for this reality. [Harzer 1908, 266]

Things had not really changed since the days of Zöllner—or, for that matter, since the days of Lobachevsky.

### 6 Towards Einstein and beyond

By 1911, the year of publication of Poincaré’s work on cosmogony, astronomers and many other scientists were well aware that the geometry of cosmic space might possibly be non-Euclidean. One example from that year is a massive textbook on “cosmical physics” written by the Austrian meteorologist and geophysicist Wilhelm Trabert, a book with substantial chapters on astronomy and cosmology. It even mentioned the possibility of positively curved space. It is conceivable, Trabert [1911, 258], wrote, that “light, gravitation and electricity do not propagate in straight lines, but in circles”. As another example, consider the 1911 edition of the widely acclaimed *Newcomb-Engelmann Populäre Astronomie*, a much enlarged translation of Newcomb’s book edited and updated by leading German astronomers. This book too referred to Riemann’s closed space:

Although this interpretation of the finite space transcends our conceptions, still it does not contradict them. However, what experience can tell us about it is limited by the fact that the entire observable universe may be but a small fraction of the entire finite space. [Kempf 1911, 664]

In spite of their awareness of curved space, astronomers agreed that so far it was merely a remote possibility.

The situation only changed with Einstein’s general theory of relativity and especially with his cosmological field equations of 1917. In this context it is worth noting that Schwarzschild’s old paper of 1900 played a role in the origin of relativistic cosmology. As early as February 1916, before Einstein had begun
contemplating the cosmological consequences of his new theory of gravitation, Schwarzschild wrote him:

As concerns very large spaces, your theory has quite a similar position as Riemann’s geometry, and you are certainly not unaware that one obtains an elliptic geometry from your theory if one puts the entire universe under uniform pressure.\(^\text{10}\)

Einstein presented his closed cosmological model governed by general relativity theory to the Prussian Academy a year later.

While Poincaré, Newcomb and Seeliger maintained that empty space was an abstraction devoid of physical properties, most physicists in the period considered space without ether to be almost oxymoronic. The ubiquitous electromagnetic ether permeated everything, matter as well as space. H. A. Lorentz and other physicists in the early twentieth century spoke of the ether as were it equivalent to vacuum, if a vacuum far from nothingness. On the other hand, the popular belief in a dynamically active ether was rarely considered in astronomical or cosmological contexts and never, to my knowledge, associated with the new geometries of space. Poincaré [2015, 145–147] wrote about the ether in *La Science et l’Hypothèse* and elsewhere, but without clearly endorsing the belief in an ethereal medium permeating cosmic space. Like Lorentz but unlike Einstein, he believed that the ether was indispensable as a privileged frame of reference and yet he expressed some skepticism with regard to its reality: “It matters little whether the ether really exists; that is the affair of metaphysicians. The essential thing for us is that everything happens as if it existed, and that this hypothesis is convenient for the explanation of phenomena” [Poincaré 2015, 174], [Darrigol 2004].

To make a long story short, in 1916 Walther Nernst argued from quantum theory that empty space—which he identified with the ether—was filled with blackbody radiation of an extremely high energy density. Following a completely different route, in an address of 1933, Georges Lemaître derived from the equations of general relativity that the vacuum energy density was given by Einstein’s cosmological constant \(\Lambda\), namely by \(\rho = \frac{\Lambda c^2}{8\pi G}\) where \(G\) is the gravitational constant and \(c\) the speed of light [Kragh & Overduin 2014]. These two independent strands were eventually, many years later, unified in the modern concept of “dark energy” demonstrated observationally in 1998. Irrespective of whether cosmic space is Euclidean or not, contrary to what Poincaré believed it is known today to be physically real and the seat of most of the energy in the universe.

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\(^{10}\) Schwarzschild to Einstein, 6 February 1916, in [Einstein 1998, document 188]. See also [Schemmel 2005]. The elliptic space was first described by Felix Klein in a memoir of 1871. This kind of space, which was advocated by Schwarzschild in his paper of 1900 and later by Willem de Sitter, is not quite the same as spherical or Riemannian space. Both spaces are finite and with constant curvature, but for the same radius of curvature \(R\) the volumes differ. For spherical space the volume is \(2\pi^2 R^3\), and for elliptic space it is \(\pi^2 R^3\).
7 Conclusion

Although non-Euclidean geometry as a branch of mathematics dates from about 1830, it only entered astronomy in the last decades of the century and then slowly and hesitantly. The main reason why mainstream astronomers failed to take the new theories of curved space seriously was their lack of explanatory power. The traditional Euclidean space seemed entirely satisfactory, so why consider the exotic alternatives due to Lobachevsky and Riemann? Nonetheless, a handful of astronomers did contribute to what may be called pre-relativistic non-Euclidean astronomy, although none of them considered a curved-space universe more than an exciting hypothesis. They realized that while it could never be proved that space is strictly Euclidean, it could conceivably be proved that it is not.

An original but also, as it turned out, sterile answer to the question of the geometry of space was proposed by Poincaré, who thought of it as a pseudo-question. From his conventionalist perspective it simply made no sense to speak of space as were it describable by a particular geometry or endowed with physical qualities. However, Poincaré’s point of view was not generally accepted, neither in France nor abroad, and a minority of fin-de-siècle astronomers and mathematicians did investigate the possibility of curved cosmic space. Their works led to nothing, though. It was only with Einstein’s closed cosmological model of 1917 that non-Euclidean geometry came to play an indispensable role in the astronomical sciences. Two years later observations from the British solar eclipse expedition demonstrated for the first time that local space is curved around a gravitating body.

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