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B-type anomaly coefficients for the D3-D5 domain wall

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A B S T R A C T
We compute type-B Weyl anomaly coefficients for the domain wall version of $\mathcal{N} = 4$ SYM that is holographically dual to the D3-D5 probe-brane system with flux. Our starting point is the explicit expression for the improved energy momentum tensor of $\mathcal{N} = 4$ SYM. We determine the two-point function of this operator in the presence of the domain wall and extract the anomaly coefficients from the result. In the same process we determine the two-point function of the displacement operator.

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1. Introduction

Tractable interacting boundary conformal field theories in four dimensions have been advertised for in the program which aims to study conformal anomalies in the presence of boundaries [1,2]. A class of such theories is constituted by the domain wall versions of $\mathcal{N} = 4$ SYM that are dual to the D3-D5 or the D3-D7 probe brane models with flux [3–7]. In these theories a subset of the scalar fields of $\mathcal{N} = 4$ SYM gets a non-zero vacuum expectation value (vev) in the form of a Nahm pole [8] on one side of a co-dimension one defect. In the D3-D5 case the classical solution conserves half of the supersymmetries of $\mathcal{N} = 4$ SYM [9] whereas in the D3-D7 case supersymmetry is lost [6]. Both types of solutions preserve conformal symmetry on the defect itself, thus providing us with a defect conformal field theory (dCFT), and the presence of the non-vanishing vevs implies that numerous correlation functions are non-trivial already at tree level or at the level of a few contractions.

These domain wall versions of $\mathcal{N} = 4$ SYM have been studied mainly for their integrability properties. The domain wall can be represented as a spin chain boundary state in the form of a matrix product state or a valence bond state and the computation of correlation functions (mainly one-point functions) can be formulated as the computation of the overlap between the boundary state and a Bethe eigenstate [10–12]. The D3-D5 domain wall has proven to be integrable at all loop orders [13–16]. Conversely, one of the two D3-D7 domain wall setups seems to be integrable only at leading order [17], while the other is not integrable at all [18]. Integrability has also been studied from the string theory perspective in [19,20]. For a review of these developments we refer to [21–23].

In the present letter, we advocate the D3-D5 domain wall model as a tractable interacting four dimensional boundary CFT where anomaly coefficients of the stress tensor trace anomaly can be explicitly calculated and conjectured relations between them tested.

An example of a relation between anomaly coefficients is the equality of the two type B anomaly coefficients for a supersymmetric co-dimension two defect CFT in four dimensions [24]. One of the coefficients appears in the one-point function of the stress tensor and the other in the two-point function of the displacement operator which hence are related. A similar relation between these two correlation functions is conjectured to hold in higher dimensions for defects of co-dimension larger than or equal to two [24].

For a defect of co-dimension one the one-point function of the stress tensor vanishes as only spin-less operators can have non-vanishing one-point functions in this case [25,26]. On the other hand, the two-point function of the displacement operator will in general be non-vanishing. For a flat co-dimension one defect the two-point function of the displacement operator can be extracted in a simple manner from the stress tensor two-point function. The

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stress tensor two-point function itself is very constrained by symmetries. More precisely, it is completely fixed up to three unknown functions of a certain conformal ratio [27]. These functions encode information about the two-point function of the displacement operator as well as certain anomaly coefficients of the theory. One of the unknown functions in a certain limit becomes equal to the B-type anomaly coefficient of the KVV term, denoted as b_2,t and in another limit to the anomaly coefficient denoted as c [1]. It was noticed that for free theories b = 8c [28]. In our domain wall model this relation is not fulfilled.

We determine explicitly the three unknown functions entering the two-point function of the stress tensor for the D3-D5 defect CFT at the leading order in the 't Hooft coupling. This allows us to determine the above mentioned anomaly coefficient and the two-point function of the displacement operator.

Our letter is organized as follows. We start by introducing in more precise terms the domain wall version of N = 4 dual to the D3-D5 probe brane system with flux in section 2. In section 3 we express the improved stress tensor of N = 4 SYM explicitly in terms of the fundamental fields of the theory. Subsequently, in sections 4 and 5 we derive respectively the two-point function of the stress tensor in the presence of the domain wall and the two-point function of the associated displacement operator. Finally, section 6 contains our conclusions. Certain definitions as well as various details of our computations can be found in Appendix A–Appendix C.

2. The D3-D5 domain wall

We start from the Lagrangian density of \( \mathcal{N} = 4 \) SYM theory which reads:

\[
\mathcal{L}_{\mathcal{N}=4} = \frac{2}{g_{YM}^2} \text{tr} \left\{ -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} (D_{\mu} \varphi_i)^2 + i \bar{\psi}_\alpha \gamma^\mu \psi_\alpha \right. \\
\left. + \frac{1}{4} [\varphi_i, \varphi_j]^2 + \sum_{i=1}^{6} G_{\alpha\beta}^{i} \bar{\psi}_\alpha [\varphi_i, \psi_\beta] + \sum_{i=4}^{6} G_{\alpha\beta}^{i} \bar{\psi}_\alpha \gamma^5 [\varphi_i, \psi_\beta] \right\},
\]

(1)

where \( \bar{\psi}_\alpha \equiv \gamma^\nu D_\nu \psi_\alpha \), \( \psi_\alpha \equiv \gamma^\nu D_\nu \varphi_i \) and

\[ F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu], \quad D_\mu f \equiv \partial_\mu f - i [A_\mu, f]. \]

(2)

and the definitions of the matrices \( \gamma^\mu \) and \( G^i \) can be found in [29]. All the fields (gluons, scalars, fermions) of the Lagrangian (1) carry adjoint u(N) color indices as follows:

\[ A_\mu = A_\mu^a T^a, \quad \varphi_i = \varphi_i^a T^a, \quad \psi_\alpha = \psi_\alpha^a T^a, \]

(3)

where \( a = 1, \ldots, N^2 - 1, \mu = 0, \ldots, 3, i = 1, \ldots, 6 \) and \( \alpha = 1, \ldots, 4 \). The basic properties of the \( N \times N \) generators \( T^a \) of \( u(N) \) have been collected in Appendix A. The details of the convention for the spinors, which are of Majorana-Weyl type, can be found in [29], appendix C. The equations of motion that follow from the action (1) are:

\[ D^\mu F_{\mu\nu} = i [D_\nu \varphi_i, \varphi_i], \quad D^\mu D_{\nu} \varphi_i = [\varphi_j, [\varphi_j, \varphi_i]]. \]

(4)

These equations have a number of “fuzzy funnel” solutions which break the PSU(2,2|4) global symmetry of \( \mathcal{N} = 4 \) SYM keeping either 1/2 supersymmetry (D3-D5 case) or none (D3-D7 case). The simplest and most studied of these constitute the dual of the D3-D5 probe brane system with flux and reads with \( i = 1, 2, 3 \)

\[ A_\mu = 0, \quad \psi_\alpha = 0, \]

(5)

\[ \varphi_i = \varphi_i^x (x_3) = \frac{1}{x_3^4} \left( t_{i,k}(x) Q_{0,N-k}(x) \right) \]

(6)

\[ \varphi_{i+3} = 0, \]

(7)

where the matrices \( t_i \) furnish a k-dimensional irreducible representation of \( su(2) \):

\[ [t_i, t_j] = i e_{ij} t_i, \quad i, j = 1, 2, 3, \]

(8)

and where we will assume that \( x_3 > 0 \). Furthermore, we will assume that \( k \geq 2 \). We shall briefly comment on the special cases \( k = 1 \) and \( k = 0 \) later. In the string theory dual k is a flux parameter. Quantizing around this classical solution one finds that only the field components in the \( (N-k) \times (N-k) \) block of the fields that are present in the \( x_3 > 0 \) region can propagate to the \( x_3 < 0 \) region. This way, we effectively get a domain wall set-up where the gauge group of the theory is \( U(N-k) \) for \( x_3 < 0 \) and (broken) \( U(N) \) for \( x_3 > 0 \), with the full \( U(N) \) gauge symmetry only being recovered as \( x_3 \to \infty \). The set-up obviously breaks translational invariance in the \( x_3 \) direction but conformal symmetry is conserved on the \( x_3 = 0 \) domain wall leaving us with a \( O(4/4) \) symmetric dCFT.

3. The improved stress tensor of \( \mathcal{N} = 4 \) SYM

The stress tensor of \( \mathcal{N} = 4 \) SYM can be computed from the action (1) by the canonical prescription:

\[
T_{\mu\nu} = \frac{\partial \mathcal{L}}{\partial \partial_{\mu} A_{\rho}} \partial_\nu A_{\rho} + \frac{\partial \mathcal{L}}{\partial \partial_{\mu} \varphi_i} \partial_\nu \varphi_i + \frac{\partial \mathcal{L}}{\partial \partial_{\mu} \psi_\alpha} \partial_\nu \psi_\alpha + \\
+ \frac{\partial \mathcal{L}}{\partial \partial_{\mu} \psi_\alpha} \partial_\nu \gamma_5 \psi_\alpha - g_{\mu\nu} \mathcal{L},
\]

(9)

but it is neither symmetric, nor traceless, nor conserved. Following Callan, Coleman and Jackiw [31], an improved version of the stress tensor can be constructed by applying a series of transformations to the canonical formula (11).

\[
\Theta_{\mu\nu} = \frac{2}{g_{YM}^2} \text{tr} \left\{ -F_{\mu}^\epsilon F_{\nu}\epsilon - \frac{2}{3} (D_{\mu} \varphi_i) (D_\nu \varphi_i) + \\
\left[ \frac{1}{4} \psi_i D_{(\mu} D_{\nu)} \right] + \\
\frac{i}{2} \bar{\psi}_\alpha \gamma_{(\mu} D^\nu \psi_\alpha \right\} - g_{\mu\nu} \mathcal{L},
\]

(10)

where \( a_{(\mu\nu)} \equiv (a_{\mu
u} + a_{\nu\mu})/2 \) and \( f_\mu^\epsilon g \equiv f (\partial_\mu g) - (\partial_\mu f) g \). Furthermore,

\[ \mathcal{L} = \frac{1}{g_{YM}^2} \text{tr} \{ F_{\mu\nu} F^{\mu\nu} \}, \quad S = \int dx^4 \sqrt{-g} \mathcal{L}. \]

(11)

which leads to a manifestly symmetric stress tensor. Note however that the fermionic terms of the action must be varied with respect to the vierbein field \( e_\mu \) instead of the graviton \( g^{\mu\nu} = e_\mu^\alpha e_\nu^\beta \).
\[ \Lambda = 2 \frac{g_{\text{YM}}}{g_{\text{YM}}} \cdot \text{tr} \left\{ -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{6} (D \mu \phi \psi)^2 \right\}. \tag{13} \]

Details are provided in Appendix B. Apart from being manifestly symmetric, the stress tensor (12) is also on-shell traceless and conserved:
\[ \Theta_{\mu \nu} = \Theta_{\nu \mu}, \quad g^{\mu \nu} \Theta_{\mu \nu} = 0, \quad D \mu \Theta_{\mu \nu} = 0. \tag{14} \]

The (unregularized) 2-point function of the improved stress tensor can be written in terms of the inversion tensors \( I_{\mu \nu}, I_{\mu \nu \rho \sigma} \) \[ \Theta_{\mu \nu} (x) \Theta_{\rho \sigma} (y) \] \[ = \frac{C_T}{4 \pi^4 s^3} \cdot \text{tr} \left\{ I_{\mu \nu \rho \sigma} \right\}. \tag{15} \]

The 16 contributions of the stress tensor are protected within \( \mathcal{N} = 4 \) SYM (see e.g., [35] and references therein).

4. Stress tensor two-point function of the defect CFT

The stress tensor one-point function of the defect CFT can be directly computed by plugging the fuzzy-funnel solution (8) into the formula (12). As only scalars have non-zero vevs it is enough to consider the contribution to the stress tensor coming from scalars which reads
\[ \Theta_{\mu \nu \text{ (scalars)}} = \frac{2}{4} \frac{g_{\text{YM}}}{g_{\text{YM}}} \cdot \text{tr} \left\{ -\frac{2}{3} (\partial \mu \phi \psi) (\partial \nu \phi \psi) + \frac{1}{3} \phi \psi (\partial \mu \partial \nu \phi \psi) + \frac{1}{6} \delta_{\mu \nu} \left[ (\partial \phi \psi)^2 + \frac{1}{2} (\phi \psi)^2 \right] \right\}. \tag{19} \]

and which is also obviously symmetric, on-shell traceless and conserved. We find that the classical value of the stress tensor vanishes
\[ \Theta_{1 \mu \nu}^{\text{cl}} = 0, \tag{20} \]

as it has to for a co-dimension one defect [27,25]. The leading contribution to the connected part of the stress tensor two-point function consists of a single Wick contraction and is of order \( g_{\text{YM}}^{-2} \). More specifically
\[ \left\langle \Theta_{\mu \nu} (x) \Theta_{\rho \sigma} (y) \right\rangle = N \left( \lambda^{-1} + \lambda^{0} + \ldots \right), \tag{21} \]

where the \( \lambda \) Hooft coupling is defined as \( \lambda = g_{\text{YM}}^2 N \). Expanding the fields around the classical solution (8),

\[ A_\mu = \tilde{A}_\mu, \quad \psi_\alpha = \tilde{\psi}_\alpha, \quad \phi_i (x) = \phi_i^{(1)} (x_3) + \tilde{\phi}_i (x), \tag{22} \]

for \( i = 1, \ldots, 6 \), it follows that the leading correction to the stress tensor (12) takes the form:
\[ \Theta_{\mu \nu}^{(1)} (x) = \frac{1}{g_{\text{YM}}^2} \frac{4}{3} \text{tr} \left\{ \left( \frac{1}{x_3} \left( n_\mu n_\nu - \delta_{\mu \nu} \right) \right) \tilde{\phi}_i + n_\mu \partial_\mu \tilde{\phi}_i + n_\mu \partial_\mu \tilde{\phi}_i + \frac{n_\mu \partial_\mu \tilde{\phi}_i}{2} \partial_\nu \tilde{\phi}_i + \frac{x_3}{2} \partial_\nu \partial_\mu \tilde{\phi}_i \right\}. \tag{23} \]

where \( n_\mu \equiv \delta_{\mu 3} \) denotes the normal vector to the defect. The expression (23) can be inserted into (21). The two-point function only obtains contributions from the \( k \times k \) block of the quantum fields. This block is conveniently expanded in terms of the fuzzy \( SU(2) \) spherical harmonics as explained in [36,29] and summarized in Appendix C. The expression for the bulk two-point function of the stress tensor (21) contains the Wick-contracted quantity
\[ \text{tr} [t \tilde{\phi}_i] \cdot \text{tr} [t \tilde{\phi}_j] = c_k \cdot K^{5/2} (x, y) = \frac{c_k}{4 \pi^2 y_3} \left[ \frac{2 F_1 (2, 3, 6; -1)}{\xi^3 (1 + \xi)} \right] \]

and its derivatives. The conformal ratios \( \xi, \nu \) and the constants \( c_k \) are defined as:
\[ \xi \equiv \frac{s^2}{4 x_3 y_3}, \quad \nu \equiv \frac{s}{1 + \xi}, \quad c_k \equiv \frac{k (k^2 - 1)}{4}. \tag{25} \]

The expression (24) is obtained by using the scalar propagator of the references [29,37] (see (C.2) in Appendix C) and the colour factors (C.9)–(C.10). Performing all contractions we arrive at the following result
\[ \left\langle \Theta_{\mu \nu}^{(1)} (x) \Theta_{\rho \sigma}^{(1)} (y) \right\rangle = \frac{1}{s^3} \left\{ \left( X_\mu X_\nu - \frac{g_{\mu \nu}}{4} \right) (Y_\rho Y_\sigma - \frac{g_{\rho \sigma}}{4}) A (v) + \left( X_\mu Y_\rho I_{\nu \sigma} + X_\nu Y_\sigma I_{\mu \rho} \right) + X_\mu Y_\rho I_{\nu \sigma} - \frac{g_{\mu \nu} Y_\rho Y_\sigma - g_{\rho \sigma} X_\nu X_\mu + \frac{1}{4} g_{\mu \nu} g_{\rho \sigma}}{4} B (v) + \right\} \]

which has the generic form specified in [27,25] where
\[ X_\mu \equiv \nu \left( \frac{2 x_3}{s^2} s_\mu - n_\mu \right), \quad Y_\rho \equiv - \nu \left( \frac{2 x_3}{s^2} s_\rho + n_\rho \right). \tag{28} \]

Symmetry arguments fix the two-point function up to the three functions \( A (v), B (v) \) and \( C (v) \). The functions are not independent but fulfil the relations [25]
\[ \left( \frac{d}{d v} - 4 \right) (C + 2 B) = - \frac{1}{2} (A + 4 B) - 4 C, \tag{29} \]
\[ \left( \frac{d}{d v} - 4 \right) (3 A + 4 B) = 2 A - 24 B. \tag{30} \]

For our domain wall set-up these functions take the form
\[ A (v) = 4 \nu \left( 6 v^6 + 3 v^4 + v^2 \right), \tag{31} \]
\[ B (v) = - \nu \left( 3 v^6 - v^4 - 2 v^2 \right), \tag{32} \]
\[ C (v) = \nu v^2 \left( v^2 - 1 \right)^2, \tag{33} \]
where
\[ \gamma \equiv \frac{32c_k}{9\pi^2g_{_{YM}}} \] (34)

Far away from the boundary we recover the \( \mathcal{N} = 4 \) two-point function which vanishes at the linearized level and is only nonzero at quadratic order [16]. Indeed, the functions \( A, B, C \) and the stress tensor two-point function (27) all vanish for \( \xi, \nu \rightarrow 0 \).

Our derivation of the two-point function of the stress tensor is strictly speaking only valid for \( k \geq 2 \) but the result indicates that the correlation function should vanish for the special cases \( k = 0, 1 \). This can indeed be seen to be the case. For \( k = 1 \) there are no vevs, and instead one has to hand impose boundary conditions on the \( 1 \times (N - 1) \) and \( (N - 1) \times 1 \) and \( 1 \times 1 \) blocks of fields, Dirichlet or Neumann, following a certain recipe that ensures supersymmetry [37,12]. In this case the leading contribution to the two-point function involves two contractions and is of higher order in \( \lambda \) than the one for \( k \geq 2 \). The case \( k = 0 \) is a completely different set-up there are no particular boundary conditions on the bulk fields but additional fundamental fields living on the defect and interacting among themselves and with the bulk fields [38]. Also in this case the two-point function of the stress tensor involves two contractions and is of higher order in \( \lambda \).

In [1] the anomaly coefficient of the \( \mathcal{K}^{YM} \) term, denoted as \( b_2 \), was argued to be given by the expression
\[ b_2 = \frac{2\pi^4}{15} \cdot \alpha(1), \] (35)
with
\[ \alpha(\nu) = d - 1 \cdot \left[ (d - 1)(A(\nu) + 4B(\nu)) + dC(\nu) \right], \] (36)
and with \( d \) being the dimension, i.e. \( d = 4 \) in our case. Making use of the relations (31)–(33) we find that for the \( D3-D5 \) domain wall
\[ \alpha(1) = \frac{9}{16} \cdot A(1) = \frac{20k(k^2 - 1)}{\pi^2g_{_{YM}}}, \quad b_2 = \frac{8\pi^2}{3g_{_{YM}}} \cdot k(k^2 - 1). \] (37)

It has been observed that for free theories \( \alpha(1) = 2\alpha(0) \) or equivalently \( b_2 = 8c \) [28,1]. This relation is clearly not fulfilled in our interacting case for \( k \geq 2 \) where (at leading order) \( \alpha(0) = 0 \) and \( \alpha(1) \neq 0 \).

5. Displacement operator

The displacement operator \( \mathbb{D} \) can be defined from the divergence of the improved (scalar) stress tensor (19) as follows:
\[ \delta \mathcal{L} = g_{\mu \nu} \delta \bar{\theta}_{\mu \nu} \delta \mathbb{D}, \] (38)
where \( g_{\mu \nu} \equiv (0, 0, 0, 1) \) is the unit normal to the \( x_3 = 0 \) plane boundary. Integrating over \( x_3 \) from \( 0^- \) to \( 0^+ \) and bearing in mind the conformal invariance of the defect we find
\[ \mathbb{D}(x) = \lim_{x_3 \rightarrow 0^+} \Theta_{33}(x, x_3) - \lim_{x_3 \rightarrow 0^-} \Theta_{33}(x, x_3), \] (39)
with \( x \equiv (x_0, x_1, x_2) \). The leading order contribution to the two-point function of the displacement operator can thus be directly read off from (27):
\[ \left( \mathbb{D}^{(1)}(x) \mathbb{D}^{(1)}(y) \right) = \lim_{x_1, y_1 \rightarrow 0^+} \lim_{x_3, y_3 \rightarrow 0^+} \Theta_{33}(x, x_3) \Theta_{33}(y, y_3) = \frac{\alpha(1)}{8}, \] (40)
where \( \alpha(1) \) was given in (37). The terms involving \( \lim x_3 \rightarrow 0^- \) are of higher order in \( \lambda \).

6. Conclusion

We have illustrated that the \( D3-D5 \) domain wall version of \( \mathcal{N} = 4 \) SYM is a tractable, interacting four dimensional co-dimension one defect CFT where correlation functions involving the stress tensor and hence certain anomaly coefficients can be explicitly calculated. So far we evaluated the two-point function of the stress tensor as well as its associated displacement operator and extracted the coefficient \( b_2 \). In particular, we found that \( b_2 \neq 8c \) for this theory. It might be interesting to compute the three-point function of the stress tensor and extract the other B-type anomaly coefficient for this case, \( b_1 \).

The present defect CFT should also be amenable to the boundary conformal bootstrap program. Thus one could imagine that the explicit perturbative calculation of the leading order contribution to the two-point functions as pursued in this paper in combination with the all loop results for the one-point functions [14,15] could be used as input to start a bootstrap procedure to determine higher loop correlation functions. First steps in the direction of developing such a program were taken in [37].

Correlation functions of the \( D3-D5 \) and \( D3-D7 \) domain wall versions of \( \mathcal{N} = 4 \) SYM can be studied from the strong coupling string perspective as well. One-point functions of chiral primaries can be explicitly matched between gauge theory and string theory using supersymmetric localization [16] and show, for an operator of length \( 2l \), a scaling \( k^{2l+1}/\lambda^l \) for weak coupling and large flux [39]. It would be interesting if one could reproduce from holography the weak coupling, large flux scaling of the two-point function observed here and in [37]. The holographic computation of two-point functions for the specific \( D3-D5 \) probe brane set-up considered here was addressed in [40]. A computation of holographic two-point functions for generic Karch-Randall set-ups, not specifying the brane or supergravity action, has been pursued in e.g. [41–47].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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Appendix A. Color decompositions

All the fields of \( \mathcal{N} = 4 \) SYM carry adjoint \( u(N) \) colour indices (3), where \( u(N) = su(N) \times u(1) \). The fundamental representation of \( su(N) \) is spanned by of \( N^2 - 1 \) traceless hermitian \( N \times N \) generators. These are normalized as
\[ tr[T^a T^b] = \delta^{ab}, \] (A.1)
while they also satisfy the $su(N)$ Fierz identity:
\[
(T^a)_{mn} (T^a)_{\tau \sigma} = \delta_{mn} \delta_{\tau \sigma} - \frac{1}{N} \delta_{m\tau} \delta_{n\sigma}. \tag{A.2}
\]

The extra $u(1)$ generator that is needed to obtain the fundamental representation of $u(N)$ is proportional to the identity matrix:
\[
(T^0)_{mn} = \frac{1}{\sqrt{N}} \delta_{mn}. \tag{A.3}
\]

The $N^2$ hermitian generators of $u(N)$ still satisfy (A.1), whereas the $u(N)$ Fierz identity becomes:
\[
(T^0)_{mn} (T^a)_{\tau \sigma} = \delta_{mn} \delta_{\tau \sigma}. \tag{A.4}
\]

Appendix B. Improved stress tensor

The recipe for computing the improved stress tensor can be found in e.g. [33]. It can be adapted to a covariant context as follows:
\[
\Theta_{\mu \nu} = T_{\mu \nu} + D^\rho (X_{\rho \mu \nu} - X_{\rho \nu \mu} + Y_{\rho \mu \nu} - Y_{\rho \nu \mu}) - Y_{\nu \mu \rho} + D_{\mu \nu \sigma} Z^\sigma, \tag{B.1}
\]
where $T_{\mu \nu}$ is the stress tensor (10) or (11), and $X$ and $Z$ must be determined so that
\[
T_{[\mu \nu]} = -D^\rho X_{\rho \mu \nu}, \quad X_{\rho \mu \nu} = -X_{\rho \nu \mu}, \quad D^\mu T_{\mu \nu} = D^\rho D^\sigma Y_{\rho \mu \nu} - Y_{\rho \mu \nu} - D^\rho Y_{\mu \nu \rho} - D^\rho Y_{\nu \mu \rho} + D^\rho D^\sigma Z_{\rho \mu \nu}, \tag{B.2}
\]
\[
D^\mu T_{\mu \nu} = 2D^\mu X_{\rho \mu \nu} + 2D^\rho Y_{\rho \mu \nu} + 2D^\mu Y_{\rho \nu \mu} - D^\mu Y_{\mu \nu \rho} + D^\rho D^\sigma Z_{\rho \mu \nu}, \tag{B.3}
\]
\[
Z_{\mu \nu} = Z_{\nu \mu}, \tag{B.4}
\]
and $D_{\mu \nu \sigma}$ is defined in $d$-dimensional spacetime as
\[
D_{\mu \nu \sigma} = \frac{1}{d-2} \left( g_{\mu \rho} (D_\rho D_\sigma) D_\nu + g_{\nu \rho} (D_\rho D_\sigma) D_\mu - g_{\mu \rho} (D_\rho D_\sigma) D_\nu - g_{\nu \rho} (D_\rho D_\sigma) D_\mu \right) - \frac{1}{(d-1)(d-2)} \left( D_{\mu \nu} D_\sigma - g_{\mu \nu} D_\sigma \right). \tag{B.5}
\]
We have also used the definitions
\[
a_{\mu \nu} = \frac{a_{\mu \nu} + a_{\nu \mu}}{2}, \quad a_{[\mu \nu]} = \frac{a_{\mu \nu} - a_{\nu \mu}}{2}. \tag{B.7}
\]
By construction, the improved stress tensor (B.1) is on-shell conserved, symmetric and traceless:
\[
D^\mu \Theta_{\mu \nu} = 0, \quad \Theta_{\mu \nu} = 0, \quad g^\mu \nu \Theta_{\mu \nu} = 0. \tag{B.8}
\]

Appendix C. Scalar propagators in the D3-D5 dCFT

The fluctuations of the upper $k \times k$ blocks of the scalar fields are expanded in fuzzy $su(2)$ spherical harmonics as follows:
\[
[\tilde{\phi}]_{l_{1} l_{2} \cdots} = \sum_{l=1}^{k} \sum_{m=-l}^{l} \left[ \tilde{\phi}^{m}_{l} \right]_{l_{1} l_{2} \cdots} (\tilde{\phi})_{l, m}. \tag{C.1}
\]

The corresponding propagators are given by [29]:
\[
\langle (\tilde{\phi}_{l} \tilde{\phi}_{l}^{m} \tilde{\phi}_{l}^{m'} \rangle = (\tilde{\phi}_{l} \tilde{\phi}_{l}^{m} \tilde{\phi}_{l}^{m'} \rangle = (\tilde{\phi}_{l} \tilde{\phi}_{l}^{m} \tilde{\phi}_{l}^{m'} \rangle = \sum_{l=1}^{k} \sum_{m=-l}^{l} \left[ \tilde{\phi}^{m}_{l} \right]_{l_{1} l_{2} \cdots} (\tilde{\phi})_{l, m}. \tag{C.1}
\]
\[
\langle (\tilde{\phi}_{l} \tilde{\phi}_{l}^{m} \tilde{\phi}_{l}^{m'} \rangle = \sum_{l=1}^{k} \sum_{m=-l}^{l} \left[ \tilde{\phi}^{m}_{l} \right]_{l_{1} l_{2} \cdots} (\tilde{\phi})_{l, m}. \tag{C.1}
\]

\[
\langle \tilde{\phi}_{l} \tilde{\phi}_{l}^{m} \tilde{\phi}_{l}^{m'} \rangle = \sum_{l=1}^{k} \sum_{m=-l}^{l} \left[ \tilde{\phi}^{m}_{l} \right]_{l_{1} l_{2} \cdots} (\tilde{\phi})_{l, m}. \tag{C.1}
\]

\[
\langle \tilde{\phi}_{l} \tilde{\phi}_{l}^{m} \tilde{\phi}_{l}^{m'} \rangle = \sum_{l=1}^{k} \sum_{m=-l}^{l} \left[ \tilde{\phi}^{m}_{l} \right]_{l_{1} l_{2} \cdots} (\tilde{\phi})_{l, m}. \tag{C.1}
\]

References
