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Jespersen, Thomas Sand; Grove-Rasmussen, Kasper; Paaske, Jens; Muraki, K.; Fujisawa, T.; Nygård, Jesper; Flensberg, Karsten

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Gate-Dependent Orbital Magnetic Moments in Carbon Nanotubes

T. S. Jespersen,1,* K. Grove-Rasmussen,1,2,* K. Flensberg,1 J. Paaske,1 K. Muraki,2 T. Fujisawa,3 and J. Nygård1

1Niels Bohr Institute & Nano-Science Center, University of Copenhagen, Universitetsparken 5, DK-2100 Copenhagen, Denmark
2NTT Basic Research Laboratories, NTT Corporation, 3-1 Morinosato-Wakamiya, Atsugi 243-0198, Japan
3Research Center for Low Temperature Physics, Tokyo Institute of Technology, Ookayama, Meguro, Tokyo 152-8551, Japan

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We investigate how the orbital magnetic moments of electron and hole states in a carbon nanotube quantum dot depend on the number of carriers on the dot. Low temperature transport measurements are carried out in a setup where the device can be rotated in an applied magnetic field, thus enabling accurate alignment with the nanotube axis. The field dependence of the level structure is measured by excited state spectroscopy and excellent correspondence with a single-particle calculation is found. In agreement with band structure calculations we find a decrease of the orbital magnetic moment with increasing electron or hole occupation of the dot, with a scale given by the band gap of the nanotube.

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The response of an electron in a quantum dot to an applied magnetic field is determined by the coupling to the electron spin—the Zeeman effect—and the coupling to the orbital magnetic moment of the electron. In carbon nanotubes the electrons encircle the circumference of the tube with a resulting magnetic moment pointing along the nanotube axis which thus couples to the parallel component of an applied magnetic field. This was first studied by Minot et al. [1], for the first few carriers in small band-gap nanotubes where the resulting orbital g factor, \( g_{\text{orb}} \), was shown to reflect the nanotube diameter. Recently, the surprising discovery of a strong spin-orbit interaction in nanotubes [2–10] has spurred renewed interest in the use of carbon nanotubes as templates for spin qubits addressable by electric fields [11,12]. One new possibility is to control the spin-orbit magnetic field and the parallel component of an applied magnetic field by moving the quantum dot along a curved nanotube segment [13].

This calls for a better understanding of the relation between the nanotube band structure and the inherited properties of nanotube quantum dots. Here we extend the work of Ref. [1] by investigating the dependence of \( g_{\text{orb}} \) on the electron or hole occupation of a nanotube quantum dot tuned by a potential \( V_g \) on a nearby electrostatic gate. We find \( g_{\text{orb}} \) decreasing with electron or hole filling as the circumferential velocity of the electrons decrease and we show that while the nanotube diameter determines the value of \( g_{\text{orb}} \) for the first carrier, the nanotube band gap \( \Delta_g \) sets the scale of its gate dependence.

It is well established that the low energy electron dispersion for nanotubes can be obtained from the graphene dispersion by imposing periodic boundary conditions in the circumferential direction of the nanotube [14]. Ignoring spin-orbit effects this leads to \( E(k) = \pm \hbar v_F \sqrt{k^2 + k_{||}^2} \), where \( v_F \approx 10^6 \text{ m/s} \) is the Fermi velocity of graphene, \( k_{||} \) is the component of the wave vector along the nanotube axis and the offset \( k_g \) between the quantization lines and the Dirac points of the graphene dispersion, results in a band gap \( \Delta_g = \hbar v_F k_g \) for the nanotube [15]. A magnetic field \( B_\parallel \) applied parallel to the nanotube axis adds an Aharonov-Bohm phase to the electron wave function and shifts the circumferential quantization lines by \( k_{\phi} = eB_\parallel D/4\hbar \) where \( D \) is the nanotube diameter. Letting \( s = \pm 1 \) denote the spin \( \uparrow \), \( \downarrow \) along the nanotube and \( \tau = \pm 1 \) the corner points \( K, K' \) of the graphene Brillouin zone, the dispersion reads

\[
E_{\tau,s} = \pm \hbar v_F \sqrt{(\tau k_\phi - k_g)^2 + k_{||}^2} + \frac{1}{2} g_s \mu_B B,
\]

(1)

where \( g_s = 2 \) and the last term accounts for the usual Zeeman effect. Including explicitly the angle \( \theta \) between the nanotube axis and the applied field and expanding to linear order in \( B \) leads to [16]

\[
E_{\tau,s} = E_0^\pm + \left( \frac{1}{2} g_s \frac{s}{\tau} g_{\text{orb}} \cos(\theta) \right) \mu_B B,
\]

(2)

with \( E_0^\pm = \pm \sqrt{\Delta_g^2 + \epsilon_N^2} \) and

\[
g_{\text{orb}} = \frac{e v_F D}{4 \mu_B \sqrt{1 + \left( \frac{\Delta_g}{\epsilon_N} \right)^2} }.
\]

(3)

Here the result has been written for a nanotube quantum dot with electrons confined to a nanotube segment of length \( L \) where \( \epsilon_N = \hbar v_F N \pi / L \). Experimentally verifying Eq. (3) is the main objective of this work. For the first carriers \( \epsilon_N \ll \Delta_g \), yielding \( g_{\text{orb}} = e v_F D / 4 \mu_B \) which is the value classically expected for an electron in a circular motion of diameter \( D \) and speed \( v_F \) [1]. For increasing electron or hole occupation of the dot (larger \( \epsilon_N \)) \( g_{\text{orb}} \) decreases as shown in Fig. 1(a) with a characteristic scale set by the band gap \( \Delta_g \). This behavior arises from the graphene dispersion as schematically illustrated in the inset and in Fig. 1(b): Because of the linear dispersion, \( v_F \) does not depend on energy (black arrows) and thus the
circumferential component of the velocity (green horizontal arrows), which determines $g_{\text{orb}}$, decreases as electrons with a larger parallel component are added.

To measure $g_{\text{orb}}(N)$ and investigate the relationship of Eq. (3) we have performed level spectroscopy on a nanotube quantum dot at different occupations. Equation (2) contains the well established fourfold degenerate nanotube level structure [17,18]: a factor of 2 from ordinary spin ($s = \pm 1$) and a factor of 2 from the isospin degeneracy ($\tau = \pm 1$) of clockwise, $K$, and anticlockwise, $K'$, orbits [Fig. 1(a) inset]. In real devices the degeneracy is generally split by a combination of spin-orbit coupling [2,5,19] (which favors parallel or antiparallel alignment of orbital and spin magnetic moments) and disorder scattering $\Delta_{\text{SO}}$ (which couples $K$ and $K'$ states). Figures 2(a) and 2(b) show the evolution of the single-particle spectrum upon rotation of the nanotube in a constant field and as a function of field strength for the perpendicular ($B_\perp$) and parallel ($B_\parallel$) orientations. The coupling of the parallel magnetic field to the orbital magnetic moment results in the steep slopes in $B_\parallel$, and $g_{\text{orb}}$ can be determined as indicated. Three parameters thus characterize the level structure: $\Delta_{K_{K'}}$, $\Delta_{\text{SO}}$, and $g_{\text{orb}}$. In Ref. [5], we analyzed the role of $\Delta_{\text{SO}}$ and now we focus on the orbital magnetic moment extracted from the same set of data.

Our experimental setup is as follows: High-quality single-wall carbon nanotubes are grown by chemical vapor deposition from catalyst islands predefined on substrates of highly doped silicon capped with an insulating oxide. Subsequently palladium/gold (10 nm/40 nm) source and drain contact electrodes are defined by electron beam lithography with a spacing of 400 nm that defines the nanotube segment constituting the quantum dot. The electron or hole occupancy of the quantum dot can be tuned by applying a voltage $V_g$ to the conducting backplane of the substrate and we characterize the quantum dot by measuring the two-terminal differential conductance $dI/dV_{sd}$ or transconducance $dI/dV_g$ by standard lock-in techniques; here $V_{sd}$ is the applied source-drain voltage and $I$ the resulting current. The sample is measured at a temperature of $\sim 100$ mK in a dilution refrigerator fitted with a 9 T superconducting magnet and a piezorotator [20] allowing full in-plane rotation of the sample.

Figure 2(d) shows the linear conductance $G$ of the device as a function of $V_g$ revealing a series of peaks characteristic of a quantum dot in the Coulomb blockade regime [21]. In the valleys of low conductance the number of electrons on the quantum dot $n$ is fixed and with increasing $V_g$ a peak emerges when the next charge state $(n+1)$ becomes available for transport. The peak separations, which are extracted in Fig. 2(c), measure the energy, $E_{\text{add}}$, required for adding the next electron. This energy is the combination of the constant electrostatic charging energy and the energy spacing of the quantum levels of the dot. As seen in the figure, $E_{\text{add}}$ is fourfold periodic reflecting the near fourfold degenerate level structure discussed above. In the following, we present the details of the measurements of $g_{\text{orb}}$ in the highlighted quartet.

Figure 3(a) shows the measured transconducance $dI/dV_g$ as a function of applied bias $V_{sd}$ and $V_g$ of the shaded quartet in Figs. 2(c) and 2(d) corresponding to $n_0 \approx 120$ electrons occupying the quantum dot. The diamond shaped zero conductance regions are characteristic for a quantum dot in the Coulomb blockade regime.
with Fig. 2(a) the diamond edge in Fig. 3(a) has split into \( \frac{1}{4} B \) in (b)–(d) yield parallel and perpendicular magnetic field, respectively. The fits with the applied field. (c), (d) As (b) but measured in a dashed lines are a fit to the single-particle model. The leftmost Figs. 2(c) and 2(d). (b) is measured, the lines acquire \( g_{sd} \) rather than the terminology of Figs. 2(a) and 2(b). Since transconductance of the first electron, i.e., the lines terminating at the entering or leaving the bias window \([21]\). This feature eases the identification of the lines in Fig. 3(b) where we enter a sign depending on whether they correspond to levels lying data.

where transport occurs through sequential tunneling of electrons and excited states appear as high-conductance where transport occurs through sequential tunneling of electrons or holes are added to the conduction or valence band confirming the decrease of the orbital magnetic moment for states further away from the band edge as predicted by Eq. (3). This is the main result of our work. The solid line shows a fit to Eq. (3) with the diameter \( D \) and \( \Delta_{g} \) as the free parameters \([24]\). The fit is in reasonable agreement with the measurement and yields a band gap of 23 meV in good agreement with the gap estimate of \( \sim 15 \) meV from the measured stability diagram (see Supplemental Material \([23]\)). Clearly, \( g_{orb} \) decreases as electrons or holes are added to the conduction or valence band confirming the decrease of the orbital magnetic moment for states further away from the band edge as predicted by Eq. (3). This is the main result of our work. The solid line shows a fit to Eq. (3) with the diameter \( D \) and \( \Delta_{g} \) as the free parameters \([24]\). The fit is in reasonable agreement with the measurement and yields a band gap of 23 meV in good agreement with the gap estimate of \( \sim 15 \) meV from the measured stability diagram (see Supplemental Material \([23]\)). The fit, however, yields a diameter of \( D = 5.3 \) nm, which is unrealistically large for nanotubes grown by chemical vapor deposition which are expected to have \( D \leq 3 \) nm. Unexpectedly large diameters are also inferred in other spectroscopic studies \([1,2,25,26]\); e.g., Kuemmeth et al. found a diameter of \( D \approx 5 \) nm \([2]\). Yet other reports find more reasonable
In conclusion, we have investigated the gate dependence of the orbital magnetic moment of quantum states in high-quality carbon nanotube quantum dots. We present low temperature transport measurements of the dependence of quantum state energies on the angle between the nanotube axis and an applied magnetic field. This allows the determination of the nanotube axis from transport measurements alone and an accurate value for the orbital g factor is found by comparison with a single-particle model taking into account both disorder-scattering and spin-orbit interaction. Repeating such measurement over a wide range of gate voltages we find that the orbital magnetic moment decreases with dot occupation in agreement with the expectations from band structure considerations.

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*tsand@fys.ku.dk

[15] In this work, $\Delta_g$ refers to a curvature induced band gap in a nominally metallic nanotube, but in the case of a semiconducting nanotube, the dominating band gap $E_g$ due to a periodic boundary condition around the nanotube circumference should be taken into account.
[16] Including the spin-orbit effect adds a term $\pi \Delta_{SO}/2$ with the effective spin-orbit coupling $\Delta_{SO} = 2(\Delta_0 + \Delta_1 \gamma_0 \gamma_1 / \sqrt{1 + (\gamma_0^2)}/2$), where $\Delta_0$ and $\Delta_1$ are spin-orbit parameters related to the band structure.
[22] In Figs. 3(b)–3(d), an overall shift of the gate-position of the diamond has been subtracted to reveal the relative shifts of the excited states with respect to the ground state.
[24] From the measured 8 shells per 1 V gate voltage and an average level spacing of 3.1 meV measured at large filling when \( \Delta E = \hbar v_F \pi / L = e_N / N \) we estimate 8

\[ e_N = (25 \text{ meV}/V) V_g. \]