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Exploiting Twistor Techniques for One-loop QCD Amplitudes

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1. Introduction

Recently, a "weak-weak" duality between massless gauge theory and a topological string theory propagating in twistor space has been proposed in ref. [1]. This duality implies surprising structure within the S-matrix of gauge theories. In supersymmetric theories this has been exploited to facilitate considerable progress in computing scattering amplitudes. The application of these ideas to QCD has taken longer, however recently progress has been made in developing techniques which can be applied to compute one-loop gluon scattering amplitudes [2,3,4,5]. In this talk we discuss and review the work of ref. [5]. We aim to establish recursion relations in the number of scattering gluons in an one loop amplitude.

2. Twistor Inspired Techniques: Tree calculations

The link to twistor string theory is clearest if we express amplitudes in terms of spinor variables by replacing the massless momentum by $p_{a\dot{a}} = \lambda_a \bar{\lambda}_{\dot{a}}$ where $p_{a\dot{a}} = (\sigma^\mu)_{a\dot{a}} p_\mu$ and use the spinor helicity formalism [6] for the polarisation vectors. The BCFW on-shell recursion relations [7] for tree amplitudes are one of the remarkable formalisms which have arisen from the duality. The recursion relations rely on the analytic structure of the amplitude after it has been continued to a function in the complex plane $A(z)$ by shifting the (spinorial) momentum of two reference legs,

$$\lambda^1_a \longrightarrow \lambda^1_a + z\lambda^2_a\,,\ \ \bar{\lambda}^2_a \longrightarrow \bar{\lambda}^2_a - z\bar{\lambda}^1_a\,. \qquad (2.1)$$

These shifts are equivalent to a shift in the momenta

$$p^1_{a\dot{a}} \to p^1_{a\dot{a}} + z\lambda^1_a\bar{\lambda}^2_{\dot{a}}\,,\ p^2_{a\dot{a}} \to p^2_{a\dot{a}} - z\lambda^1_a\bar{\lambda}^2_{\dot{a}}\,. \quad (2.2)$$

By integrating $A(z)/z$ over a contour at infinity and by assuming $A(z) \to 0$, the unshifted amplitude $A(0)$ can be determined from the residues of the function $A(z)/z$. The poles of this function are at $z = 0$ and at z_i given by the factorisations of $A(z)$ when $P^2(z) = 0$ for some intermediate propagator i/P^2, with the residue given by the product of the two tree amplitudes. A recursion relation is thus obtained which gives the n-point amplitude as a sum over lower point functions [7]

$$A(0) = \sum_i \hat{A}_k(z_i) \times \frac{i}{P_i^2} \times \hat{A}_{n-k+1}(z_i)\,. \quad (2.3)$$

The summation only includes factorisations where the two shifted legs 1 and 2 are on opposite sides of the pole. The tree amplitudes are evaluated at the value of z such that the shifted pole term vanishes, i.e. $P_i(z_i)^2 = 0$.

The technique also extends and in fact the correctness of the MHV formalism [8,9], the other influential output from the twistor duality, can be derived from this approach [10,11]. The BCFW recursion relations differ from the well established Berends-Giele recursion relations [12] in that they are on-shell.

Although the duality relates string theory to $\mathcal{N} = 4$ Super-Yang-Mills theory, the techniques inspired by the duality have much wider applicability. For gluonic tree amplitudes, this is not so surprising since the tree amplitudes for gluonic scattering coincide in QCD and $\mathcal{N} = 4$ SYM. More surprisingly, the techniques may be applied to tree amplitudes with massive particles [13] and to theories including gravity [14,15]. In retrospect, one may regard the duality as having been a tool which has enabled the discovery of the techniques in field theory.

In this talk we discuss developing recursive techniques for one-loop amplitudes.

3. One-Loop QCD Amplitudes

A one-loop amplitude for massless particles can be expanded in the form

$$A = \sum_i c_i I_4^i + \sum_i d_i I_3^i + \sum_i e_i I_2^i + R\,, \qquad (3.1)$$

where I_n^i are scalar integral n-point integrals and R denotes rational terms. Loop amplitudes contain logarithmic (and dilogarithmic) terms which would contain cuts in the complex plane when shifted. Thus the entire amplitude is not suitable for a recursion relation. However, recursion relations may be used on parts of the amplitude:

A) The rational terms

$$R \equiv A - \Big(\sum_i c_i I_4^i + \sum_i d_i I_3^i + \sum_i e_i I_2^i\Big)\,. \quad (3.2)$$

B) The rational coefficients of the integral functions c_i, d_i and e_i.

The two approaches are complementary rather than competing. In both cases, to apply a recursion relation the key is an understanding of the singularity structure in the shifted coefficients $R(z)$, $c_i(z)$, $d_i(z)$ or $e_i(z)$, which can be inferred from the factorisation properties [16] of the full amplitude as $P^2 \to 0$,

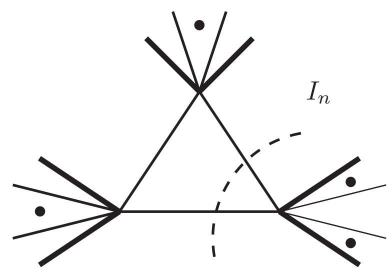
$$A_n^{\text{1-loop}} \overset{P^2 \to 0}{\longrightarrow} \sum_{h=\pm} \Bigg[A_{m+1}^{\text{1-loop}} \frac{i}{P^2} A_{n-m+1}^{\text{tree}} + A_{m+1}^{\text{tree}} \frac{i}{P^2} A_{n-m+1}^{\text{1-loop}} + A_{m+1}^{\text{tree}} \frac{i}{P^2} A_{n-m+1}^{\text{tree}} \mathcal{F}_n \Bigg]\,. \qquad (3.3)$$

For the case of the coefficient $c_i(z)$ we obtain a recursion relation analogous to that for tree amplitudes,

$$c_n(0) \;=\; \sum_{\alpha,h} A^h_{n-m_\alpha+1}(z_\alpha)\, \frac{i}{P_\alpha^2}\, c^{-h}_{m_\alpha+1}(z_\alpha)\,, \qquad (3.4)$$

where $A^h_{n-m_\alpha+1}(z_\alpha)$ and $c^h_{n-m_\alpha+1}(z_\alpha)$ are shifted tree amplitudes and coefficients evaluated at the residue value z_α and h denotes the helicity of the intermediate state.

In order to have a valid bootstrap the shifted coefficient has to vanishes as $|z| \to \infty$; otherwise there would be a dropped boundary term. We can, however, impose criteria to prevent this from happening. Consider an integral and consider the unitarity cut which isolates the cluster on which the recursion will be performed, *i.e.* the one with the two shifted legs.



The dashed line in this figure indicates the cut. The recursion is to be performed with the two shifted legs from the right-most cluster. Then simple criteria for a valid recursion are:

1. The shifted tree amplitude on the side of the cluster undergoing recursion vanishes as $|z| \to \infty$.
2. All loop momentum dependent kinematic poles are unmodified by the shift.

Note that these are sufficient and not necessary conditions.

4. Complication: Spurious Singularities

In addition to physical singularities, pieces of amplitudes also contain *spurious singularities.* A spurious singularity is a singularity that does not appear in the full amplitude but which is present only in some parts of the amplitude. Typical examples are co-planar singularities such as $\frac{1}{\langle 2^+|\not{P}|3^+\rangle}$ which vanishes when $P = \alpha k_2 + \beta k_3$. Such singularities are common in the coefficients of integral functions. These are not singularities of the full amplitude since, on the singularity, the integral functions are not independent but combine to cancel. For example, for six-point kinematics, the product $\langle 2|\not{P}_{234}|5\rangle$ vanishes when $t_{234}t_{612} - s_{34}s_{61} = 0$. At this point the functions $\ln(s_{34}/t_{234})$ and $\ln(s_{61}/t_{612})$ are no longer independent and the combination

$$\frac{a_1}{\langle 2|\not{P}_{234}|5\rangle}\ln(s_{34}/t_{234}) + \frac{a_2}{\langle 2|\not{P}_{234}|5\rangle}\ln(s_{61}/t_{612})\,, \tag{4.1}$$

is non-singular provided that $a_1 = a_2$ evaluated at the singularity. Some spurious singularities can be controlled by the choice of basis functions. For example expressions such as $\ln(r)/(1-r)^3$ will typically appear in amplitudes where r is the ratio of two momentum invariants. These expressions have unphysical singularities at $r = 1$ which cancel when combined with similar singularities in the rational terms. If we consider for a basis integral function the combination $L_2(r) = (\ln(r) - (r - r^{-1})/2)/(1-r)^3$ which is finite as $r \longrightarrow 1$ then both the cut-constructible and rational terms will be individually free of this spurious singularity.

5. Supersymmetric Decomposition of QCD Amplitudes

In general we shall always examine color-decomposed amplitudes. Let $A_n^{[J]}$ denote the leading in color partial amplitude for gluon scattering due to an (adjoint) particle of spin J in the loop. The three choices we are interested in are gluons ($J = 1$), adjoint fermions ($J = 1/2$) and adjoint scalars ($J = 0$). It is considerably easier to calculate the contributions due to supersymmetric matter multiplets together with the complex scalar. The three types of supersymmetric multiplet are the $\mathcal{N} = 4$ multiplet and the $\mathcal{N} = 1$ vector and matter multiplets. We can obtain the amplitudes for QCD from the supersymmetric contributions via

$$\begin{aligned} A_n^{[1]} &= A_n^{\mathcal{N}=4} - 4A_n^{\mathcal{N}=1\ \mathrm{chiral}} + A_n^{[0]}\,, \\ A_n^{[1/2]} &= A_n^{\mathcal{N}=1\ \mathrm{chiral}} - A_n^{[0]}\,. \end{aligned} \tag{5.1}$$

The contribution from massless quark scattering can be obtained from these trivially. When we compute amplitudes in supersymmetric theories we are calculating parts of the QCD amplitude - although the process is incomplete unless we can obtain the non-supersymmetric contribution $A_n^{[0]}$.

For $\mathcal{N} = 4$ SYM, cancellations lead to considerable simplifications in the loop momentum integrals. This is manifest in the "string-based approach" of computing loop amplitudes [17]. As a result, $\mathcal{N} = 4$ one-loop amplitudes can be expressed simply as a sum of scalar box-integral functions [18]. The box-coefficients are "cut-constructible" [18]. That is they may be determined by an analysis of the cuts where the tree amplitudes are the normal four dimensional one. This allows a variety of techniques to be used in evaluating these. Originally an analysis of unitary cuts was used to determine the coefficients firstly for the MHV case [18] and secondly for the remaining six-point amplitudes [19]. Twistor inspired techniques, combined with the application of cut-constructibility have been developed rapidly over the past year [20,21,22]. For theories with less supersymmetry the amplitudes are also cut-constructible and, although more complicated, significant progress has also been made for these theories [23,24,25]. Consequently when we wish to obtain QCD amplitudes, in many cases, the remaining component is that for a scalar circulating in the loop. In a numerical or semi-numerical computation the scalar component is also the simplest component to obtain [26]. This piece is, in principle, cut-constructible provided one performs cuts in exactly in the dimensional regulating parameter [27]; alternately one can split into cut-constructible pieces [19,5,28] plus rational terms and establish recursion relations for the rational [2].

6. Example: Split Helicity Amplitudes

As an example, let us consider the "split helicity" amplitude where the negative helicity gluons in the colour-ordered amplitude are all adjacent.

$$A_n^{1-\mathrm{loop}}(1^-, 2^-, \cdots, r^-, r+1^+, r+2^+, \cdots, n^+)\,.$$

This helicity amplitude has several simplifying features and the tree amplitude in known from the BCFW techniques [29]. The $\mathcal{N} = 4$ component of this amplitude can most easily be obtained using unitarity so we concentrate upon the other two components namely the $\mathcal{N}=1$ chiral component and the scalar component. These two amplitudes have several simplifications: firstly they contain no box integral functions. The QCD amplitudes do contain such functions but they are entirely determined by the $\mathcal{N}=4$ component.

Consider a generic triangle or bubble integral function. Such a function will contain at least one massive corner. The external legs on this corner will be of split helicity. (If the external legs on this corner had the same helicity, then the internal legs would both by necessity be of the opposite helicity. This tree amplitude vanishes in $D=4$ for the scalar and fermionic states and does hence not contribute to the case we are considering.) Let these legs be $a^-, \cdots r^-, (r+1)^+, \cdots b^+$. It can then be shown that the shift

$$\lambda_{r+1} \longrightarrow \lambda_{r+1}+z\lambda_r\,, \quad \bar\lambda_r \longrightarrow \bar\lambda_r - z\bar\lambda_{r+1}\,, \quad (6.1)$$

satisfies the sufficiency conditions for a recursion relation. Starting from the known five and six point functions we can then build the result A) for the $\mathcal{N}=1$ chiral multiple and B) for the cut-constructible part of the scalar contribution for a split helicity partial amplitude. We present here the results for the case with precisely three negative helicity gluons - the NMHV amplitudes.

For the amplitudes with an $\mathcal{N}=1$ chiral multiplet running in the loop the result is,

$$A_n^{\mathcal{N}=1\ \rm chiral}(1^-,2^-,3^-,4^+,5^+,\cdots,n^+) = \frac{A^{\rm tree}}{2}\Big[\mathrm{K}_0[s_{n1}]+\mathrm{K}_0[s_{34}]\Big] - \frac{i}{2}\Bigg[\sum_{r=4}^{n-1}\hat d_{n,r}\,\frac{\mathrm{L}_0[t_{3,r}/t_{2,r}]}{t_{2,r}} + \sum_{r=4}^{n-2}\hat g_{n,r}\frac{\mathrm{L}_0[t_{2,r}/t_{2,r+1}]}{t_{2,r+1}} + \sum_{r=4}^{n-2}\hat h_{n,r}\frac{\mathrm{L}_0[t_{3,r}/t_{3,r+1}]}{t_{3,r+1}}\Bigg] \quad (6.2)$$

where, $\mathrm{L}_0(r) = \ln(r)/(1-r)$, $K_0[s] = -\ln(-s)+2+1/\epsilon$ and we use definitions $K_{a,b} \equiv k_a + k_{a+1}\cdots k_b$, $t_{a,b} = K_{a,b}^2$ and $Pr_{a,b} = \langle a\, a+1\rangle\,\langle a+1\, a+2\rangle \cdots \langle b-1\, b\rangle$. The coefficients are given by

$$\hat d_{n,r} = \frac{\langle 3|K_{3,r}K_{2,r}|1\rangle^2 \langle 3|K_{3,r}\big[k_2, K_{2,r}\big]K_{2,r}|1\rangle}{[2|K_{2,r}|r\rangle[2|K_{2,r}|r+1\rangle\, Pr_{3,r}Pr_{r+1,1}t_{2,r}\,t_{3,r}}\,,$$

$$\hat g_{n,r} = \sum_{j=4}^{r}\frac{\langle 3|K_{3,j}K_{2,j}|1\rangle^2\langle 3|K_{3,j}K_{2,j}\big[k_{r+1},K_{2,r}\big]|1\rangle}{[2|K_{2,j}|j\rangle[2|K_{2,j}|j+1\rangle\, Pr_{3,j}Pr_{j+1,1}t_{3,j}t_{2,j}}\,,$$

$$\hat h_{n,r} = (-1)^n \hat g_{n,n-r+2}\big|_{(123..n)\to(321n..4)}\,.$$

The contribution for a scalar in the loop is [5]

$$A_n^{[0]}(1^-,2^-,3^-,4^+,5^+,\cdots,n^+) = \frac{1}{3}\,A_n^{\mathcal{N}=1\ \rm chiral} - \frac{i}{3}\Bigg[\sum_{r=4}^{n-1}\hat d_{n,r}\frac{L_2[t_{3,r}/t_{2,r}]}{t_{2,r}^3} + \sum_{r=4}^{n-2}\hat g_{n,r}\frac{L_2[t_{2,r}/t_{2,r+1}]}{t_{2,r+1}^3} + \sum_{r=4}^{n-2}\hat h_{n,r}\frac{L_2[t_{3,r}/t_{3,r+1}]}{t_{3,r+1}^3}\Bigg] + \text{rational}\,, \quad (6.3)$$

where,

$$\hat d_{n,r} = \frac{\langle 3|K_{3,r}k_2|1\rangle\langle 3|k_2K_{2,r}|1\rangle\langle 3|K_{3,r}\big[k_2,K_{2,r}\big]K_{2,r}|1\rangle}{[2|K_{2,r}|r\rangle[2|K_{2,r}|r+1\rangle\, Pr_{3,5}Pr_{r+1,1}\,\langle n\,1\rangle}\,,$$

$$\hat g_{n,r} = \sum_{J=4}^{r}\Bigg[\frac{\langle 3|K_{3,J}K_{2,J}P|1\rangle\langle 3|K_{3,J}K_{2,J}\tilde P|1\rangle}{[2|K_{2,J}|J\rangle[2|K_{2,J}|J+1\rangle} \times \frac{\langle 3|K_{3,J}K_{2,J}\big[k_{r+1},K_{2,r}\big]|1\rangle}{Pr_{3,J}Pr_{J+1,1}t_{3,J}\,t_{2,J}}\Bigg]\,,$$

$$\hat h_{n,r} = (-1)^n\hat g_{n,n-r+2}\big|_{(123..n)\to(321n..4)}\,, \quad (6.4)$$

where $P = k_{r+1}K_{r+1,1}$ and $\tilde P = K_{r+1,1}k_{r+1}$. The rational pieces of the scalar contribution remain to be calculated.

The formulas for the split helicity amplitudes with an arbitrary number of negative helicity gluons are given explicitly in ref. [5].

7. Conclusions

The past two years have seen significant progress in the computation of loop amplitudes in gauge theories. Although, these techniques have arisen in the context of highly supersymmetric theories the process of applying them to more general theories such as QCD is underway.

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