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Radiation Reaction and Gravitational Waves at Fourth Post-Minkowskian Order

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We obtain the total impulse in the scattering of nonspinning binaries in general relativity at fourth post-Minkowskian order, i.e., \( \mathcal{O}(G^4) \), including linear, nonlinear, and hereditary radiation-reaction effects. We derive the total radiated spacetime momentum as well as the associated energy flux. The latter can be used to compute gravitational-wave observables for generic (un)bound orbits. We employ the (“in-in”) Schwinger-Keldysh worldline effective field theory framework in combination with modern “multiloop” integration techniques from collider physics. The complete results are in agreement with various partial calculations in the post-Minkowskian and post-Newtonian expansion.

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Introduction.—Waveform models are an essential ingredient in data analysis and characterization of gravitational wave (GW) signals from compact binaries [1]. The level of accuracy plays a critical role, in particular for future detectors such as LISA [2] and ET [3]. In order to benefit the most from the anticipated observational reach [2–9], the modeling of GW sources must therefore continue to develop—both through analytic methodologies [10–18] and numerical simulations [19–21]—parallel with the expected increase in sensitivity with next-generation GW interferometers.

Motivated by the effective-one-body (EOB) formalism [22–27], the boundary-to-bound (B2B) dictionary between unbound and bound observables [28–30], and benefiting from powerful “multiloop” integration tools [31–61], significant progress has been achieved in recent years in our analytic understanding of (classical) gravitational scattering in the post-Minkowskian (PM) expansion in powers of \( G \) (Newton’s constant); both via effective field theory (EFT) [62–78] and amplitude-based [79–98] methodologies. The PM regime incorporates an infinite tower of post-Newtonian (PN) corrections at a given order in \( G \) that may increase the accuracy of phenomenological waveform models [99,100].

However, despite some notable exceptions [26,73–77,79,84,97,98,101], the majority of the PM computations have so far impacted our knowledge of the conservative sector, with potential interactions [66,92] as well as “tail effects” [67,93] known to 4PM order. Yet, until now, complete results had not been obtained at the same level of accuracy. The purpose of this Letter is therefore to report the total change of (mechanical) momentum, a.k.a. the impulse, for the gravitational scattering of nonspinning bodies—including all the hitherto unknown linear, nonlinear and hereditary radiation-reaction dissipative effects—at \( \mathcal{O}(G^4) \), from which we derive the total radiated spacetime momentum and GW energy flux.

Building on pioneering developments in the PN regime [102–109], the derivation proceeds via the EFT approach in a PM scheme [62], extended in [76] to simultaneously incorporate conservative and dissipative effects via the “in-in” Schwinger-Keldysh formalism [110–114]. As discussed in [76], the in-in impulse resembles the “in-out” counterpart used in the conservative sector [66,67], except for its causal structure which entails the use of retarded Green’s functions [76]. After adapting integration tools to our problem, the calculation of the impulse is mapped to a series of “three-loop” mass-independent integrals. As in previous derivations [62,63,66,67], the latter are solved via the methodology of differential equations [32–38]. The relativistic two-body problem is then reduced to obtaining the necessary boundary conditions in the near-static limit. The boundary integrals are computed using the method of regions [45], involving potential (off-shell) and radiation (on-shell) modes [102]. The full solution is thus bootstrapped to all orders in the velocities from the same type of calculations needed in the EFT approach with PN sources [102,109,115–118]. As a nontrivial check, by rewriting retarded Green’s functions as Feynman propagators plus a...
reactive term [76], we recover the value in [66,67] for the Feynman-only (conservative) part. Agreement is also found in the overlap with various PN derivations [26,27,119–124] and partial PM results [98] obtained using the relations in [22].

The B2B dictionary [28–30] allows us to connect scattering data to observables for bound states via analytic continuation. However, similarly to the lack of periastron advance at 3PM [28,29,63], the symmetries of the problem yield a vanishing coefficient for the radiated energy integrated over a period of ellipticlike motion at 4PM, trivially recovered by the B2B map. Nevertheless, since nonlinear radiation-reaction effects do not contribute to the integrated radiated energy at $O(G^4)$, we can then derive the GW flux in an adiabatic approximation [30]. This allows for the computation of radiative observables for generic (un)bound orbits through balance equations, as in the EOB approach [22], thus including an infinite series of velocity corrections.

The EFT in-integrand.—Following the Schwinger-Keldysh formalism [110–114] adapted to the EFT approach in [76,104], the effective action is obtained via a closed-time-path integral involving a doubling of the metric perturbation ($h_{ab}^{\text{ret}}$) as well as the worldline ($x_{a,\pm}$) degrees of freedom, schematically,

$$e^{iS_{\text{eff}}[x_{a,z}]} = \int \mathcal{D}h^+ \mathcal{D}h^- e^{i[S_{\text{EH}}[h^+] + S_{\text{PP}}[h^+,x_{a,z}]]},$$

with $S_{\text{EH}}$ and $S_{\text{PP}}$ the closed-path version of the Einstein-Hilbert and point-particle worldline actions, respectively. We ignore here spin degrees of freedom and finite-size effects (see [64,65]). We also restrict ourselves to the classical regime and therefore the path integral in (1) is computed in the saddle-point approximation—keeping only connected ‘tree-level’ Feynman diagrams of the gravitational field(s)—with the compact objects treated as external nonpropagating sources.

In this scenario, the matrix of (causal) propagators is given by the Keldysh representation:

$$K^{AB}(x-y) = \begin{pmatrix} 0 & -\Delta_{\text{ad}}(x-y) \\ -\Delta_{\text{ret}}(x-y) & 0 \end{pmatrix},$$

with $A,B \in \{+,-\}$ and $\Delta_{\text{ret/ad}}$ the standard retarded and advanced Green’s functions. The impulse, e.g., for particle 1, then follows from [76]

$$\Delta p^\mu_1 = -\eta^{\mu\nu} \int_{-\infty}^{\infty} dt \frac{\delta S_{\text{eff}}[x_{a,\pm}]}{\delta x^\nu_{1,-}(t)} |_{\text{PL}} = \sum_n \Delta^{(n)} p^\mu_1,$$

restricted by Dirac-$\delta$ functions. Following [66,67], the $\ell_{i=1,2,3}$’s are the loop momenta, $n_1$, $\nu_k$ are integers, and $a_i \in \{1,2\}$, with $u_{i,j} = u_{2j} \cdot u_{i,j} = u_{1j}$. In contrast to the conservative part, the $D_{a_i}$’s are now various sets of retarded and advanced propagators, e.g., $\{ (\ell^0 \pm i0)^2 - \ell^2 \cdot \ldots \}$, consistent with causality. The same constraints as before apply on the external data [63], such that the relevant integrals can only depend on $\gamma$. As in our previous calculations [66,67], we conveniently introduce the parameter $x$, defined through the relation $\gamma \equiv (x^2 + 1)/2x [95]$,
and compute these integrals by using dimensional regularization (in $d = 4 - 2\varepsilon$ dimensions) and the method of differential equations [32–38].

The integration problem then resembles the steps already performed for the computation in [66, 67], except for a few notable differences. First of all, as before we use integration-by-parts (IBPs) relations [39–44] and reduce (6) to a basis of master integrals. Because of the fewer number of symmetries of the in-in integrand, the algebraic manipulations become a bit more involved than with Feynman-only propagators. But more importantly, the boundary conditions in the near-static limit $\gamma \approx 1$ must be computed in terms of retarded and advanced Green’s functions. For this purpose, we resort to the method of regions, expanding into potential and radiation modes. The potential-only part was obtained in [66], and recovered here from the full solution. For radiation modes, the same type of integrals appearing in PN derivations [109], combined with leftover integrals over potential-only modes at one and two loops, are sufficient to bootstrap the entire answer. See [61] for more details.

**Total impulse.**—Inputting the values of the boundary master integrals and translating from $x$ to $y$ space, we find

$$c_{\text{tot}}^{(4)} = \frac{3h_{14}m_{1}m_{3}(m_{1}^{2} + m_{3}^{2})}{64(\gamma - 1)^{3/2}} + m_{1}^{2}m_{3}^{2}(m_{1} + m_{2}) \left[ \frac{21h_{22}E_{\gamma}^{2}(\frac{1}{\gamma + 1})}{32(\gamma - 1)\sqrt{\gamma - 1}} + \frac{3h_{3}K_{\gamma}^{2}(\frac{1}{\gamma + 1})}{16(\gamma - 1)^{3/2}} + \frac{3h_{4}E_{\gamma}^{\gamma}(\frac{1}{\gamma + 1})K_{\gamma}^{(\gamma + 1)}}{16(\gamma - 1)^{3/2}} + \frac{\pi^{2}h_{5}}{8\sqrt{\gamma - 1}} \right.$$
See Table I for the list of \( h_i(y) \) polynomials.

The impulse for the second particle follows by exchanging \( 1 \leftrightarrow 2 \) in the masses, incoming velocities, and impact parameter. As expected from the calculations in [66,67], the complete results feature dilogarithms [\( \text{Li}_2(z) \)], and complete elliptic integrals of the first \([K(z)]\) and second \([E(z)]\) kind.

Conservative: As shown explicitly in [76], a (time-symmetric) conservative contribution can be identified by rewriting retarded Green’s functions in terms of Feynman propagators plus a reactive term, and keeping the real part of the Feynman-only piece, i.e., \( \Delta p'_{\text{cons}} = R \Delta p'_f \), with imaginary terms cancelling out against counterparts from the reactive terms. Performing these steps in the full in-in integrand, and associated boundary conditions entering in the total impulse, we readily recover the conservative results in [66,67] including potential and radiation-reaction tail effects.

Dissipative: As discussed in [76], the terms stemming off of the mismatch between Feynman and retarded propagators incorporate dissipative effects. Needless to say, these terms can also be read off directly from the total result by subtracting the conservative part. Following the analysis in [66,67], we disentangle the various pieces according to factors of \( \nu^2 \nu'' \), with \( \nu'' \equiv \sqrt{\gamma^2 - 1} \), which signal the presence of an on-shell mode.

\[
\Delta E_{\text{hyp}} = -\frac{G^4 M^5 \nu^2}{b^4 \Gamma} \left\{ -\frac{15 \pi^2 (\gamma^2 - 1)}{1440 (\gamma^2 - 1)^3} \left( 27 (\gamma^2 - 1) h_{31} + 2 h_{36} \right) + 64 \left( 45 h_{32} - h_{48} \right) + \frac{h_{49}}{1440 \gamma^2 (\gamma^2 - 1)^{5/2}} \right. \\
- \arccosh^2(y) \left( \frac{16 h_{55}}{(\gamma^2 - 1)^2} + \frac{32 h_{54}}{4(\gamma^2 - 1)^{7/2}} \right) - \frac{h_{55} \log(2) \arccosh(y)}{4(\gamma^2 - 1)^2} + \frac{h_{57} \log\left(\frac{\gamma}{\gamma + 1}\right) \arccosh(y)}{4(\gamma^2 - 1)^2} - \frac{h_{58} \log(\gamma) \arccosh(y)}{4(\gamma^2 - 1)^2} \\
+ \arccosh(y) \left( \frac{16 h_{51}}{480 \gamma^8 (\gamma^2 - 1)^3} - \frac{16 h_{52}}{5(\gamma^2 - 1)^{3/2}} \right) - \frac{h_{56} \text{Li}_2\left(\frac{\gamma}{\gamma + 1}\right)}{8(\gamma^2 - 1)^2} + \frac{h_{56} \text{Li}_2\left(\frac{\gamma}{\gamma + 1}\right)}{32(\gamma^2 - 1)^2} + \frac{h_{57} \text{Li}_2\left(\sqrt{\gamma^2 - 1} - y\right)}{2(\gamma^2 - 1)^2} \\
+ \frac{h_{58} \text{Li}_2\left(-\left(y - \sqrt{\gamma^2 - 1}\right)^2\right)}{8(\gamma^2 - 1)^2} + \frac{4(-45 h_{32} + 30 h_{33} - 30 h_{37} + h_{48})}{45(\gamma^2 - 1)^3} + \frac{\pi^2 [54 (\gamma^2 - 1) h_{31} + h_{30} - 4 h_{30}]}{96(\gamma^2 - 1)^2} \\
- \arccosh^2(y) \left( \frac{16 h_{42} - 2 h_{53}}{(\gamma^2 - 1)^2} - \frac{64 (h_{43} + h_{54})}{(\gamma^2 - 1)^{7/2}} \right) + \frac{h_{38} - 490 \gamma (3840 \gamma^2 h_{34} + 4 h_{49})}{352800 \gamma^8 (\gamma^2 - 1)^{5/2}} \right\}.
\]
| h_1  = 515\gamma^6 - 1017\gamma^4 + 377\gamma^2 - 3 | h_{10} = 25\gamma^4 - 30\gamma^3 + 60\gamma^2 + 129\gamma + 76\gamma + 12  |
| h_2  = 380\gamma^2 + 169 | h_{11} = (1 - 5\gamma^2)^2 |
| h_3  = 1200\gamma^2 + 2095\gamma + 834 | h_{12} = 80\gamma^4 - 192\gamma^3 + 152\gamma^2 + 44\gamma + 3 |
| h_4  = 1200\gamma^3 + 2660\gamma^2 + 2929\gamma + 1183 | h_{13} = 15\gamma^2 - (64\gamma^3 + 216\gamma^2 + 258\gamma^2 - 109) |
| h_5  = -25\gamma^6 + 30\gamma^4 + 60\gamma^3 - 129\gamma^2 + 76\gamma - 12 | h_{14} = (4\gamma^2 - 1) (3\gamma^2 - 1) |
| h_6  = 210\gamma^6 - 552\gamma^5 + 339\gamma^4 - 912\gamma^3 + 3148\gamma^2 - 3336\gamma + 1151 | h_{15} = (2\gamma^2 - 3) (2\gamma^2 - 1)^3 |
| h_7  = -\gamma(2\gamma^2 - 3)(15\gamma^2 - 15\gamma + 4) | h_{16} = 8\gamma^6 - 28\gamma^5 + 6\gamma^4 + 3 \gamma^3 |
| h_8  = 420\gamma^6 + 3456\gamma^5 - 1338\gamma^4 - 15822\gamma^3 + 13176\gamma^2 + 9563\gamma - 16658\gamma |
| h_{9} = -22680\gamma^7 + 11340\gamma^9 + 116100\gamma^7 - 34080\gamma^8 + 216185\gamma^7 + 74431\gamma^6 |
| h_{10} = -21894\gamma^7 + 3259\gamma^6 + 9593\gamma^5 + 7953\gamma^4 - 3457\gamma^5 - 3457\gamma^4 |
| h_{11} = +885\gamma^5 + 885\gamma^4 - 210\gamma^3 - 210 | h_{12} = -75\gamma^6 + 1432\gamma^5 + 444\gamma^4 + 366\gamma - 129 |
| h_{12} = 2835\gamma^7 - 10065\gamma^6 + 700\gamma^5 + 13198\gamma^4 + 1818\gamma^3 - 9826\gamma^5 + 5242\gamma^4 |
| h_{13} = 11391\gamma^4 + 18958\gamma^2 + 10643\gamma + 2074 | h_{13} = 16(\gamma^6 + 24\gamma^5 + 226\gamma^4 - 151) |
| h_{14} = \gamma(945\gamma^{10} - 2955\gamma^8 + 4874\gamma^6 - 5014\gamma^4 + 8077\gamma^2 + 5369) | h_{14} = \gamma(4\gamma^8 - 59\gamma^6 + 35\gamma^4 + 60) |
| h_{15} = (2\gamma^2 - 3)(280\gamma^7 - 890\gamma^6 - 610\gamma^5 + 1537\gamma^4 + 380\gamma^3 - 716\gamma^2 - 82\gamma + 85) | h_{15} = -525\gamma^7 + 1065\gamma^5 - 3883\gamma^3 + 1263\gamma |
| h_{16} = (2\gamma^2 - 3)(15\gamma^2 - 30\gamma^2 + 11) | h_{16} = 175\gamma^7 - 150\gamma^6 + 355\gamma^5 + 210\gamma^4 + 185\gamma^3 - 66\gamma^2 - 37\gamma + 6 |
| h_{17} = 315\gamma^8 - 860\gamma^6 + 690\gamma^4 - 960\gamma^3 + 1732\gamma^2 - 1216\gamma + 299 | h_{17} = -175\gamma^5 + 38\gamma^3 + 185\gamma^3 + 37\gamma |
| h_{18} = 315\gamma^6 + 145\gamma^4 + 65\gamma^3 + 21 | h_{18} = \gamma(525\gamma^9 - 1065\gamma^7 - 2773\gamma^5 - 1041) |
| h_{19} = 840\gamma^9 + 1932\gamma^7 + 234\gamma^7 - 17562\gamma^6 + 20405\gamma^5 - 2154\gamma^4 - 11744\gamma^3 |
| h_{20} = 3600\gamma^{10} + 4320\gamma^9 + 23840\gamma^8 + 7824\gamma^7 + 14128\gamma^6 + 16138\gamma^5 + 9872\gamma^4 |
| h_{21} = -4754\gamma^3 + 6384\gamma^7 - 37478\gamma^6 + 13349\gamma^5 - 1471\gamma^4 + 207\gamma^3 - 45 |
| h_{22} = -350\gamma^6 + 1425\gamma^5 - 400\gamma^4 - 1480\gamma^3 + 660\gamma^2 + 124 | h_{23} = -350\gamma^3 + 1263\gamma^2 + 283\gamma^2 - 852 |
| h_{23} = 300\gamma^6 + 210\gamma^5 + 112\gamma^4 + 278\gamma^3 + 2044\gamma^2 + 3692\gamma + 6744\gamma + 1759 |
| h_{24} = 75\gamma^6 - 140\gamma^5 + 283\gamma^4 - 102 | h_{25} = (2\gamma^2 - 3)(210\gamma^6 - 702\gamma^5 + 339\gamma^4 - 576\gamma^3 + 3148\gamma^2 - 3504\gamma + 1151) |
| h_{25} = (2\gamma^2 - 3)(350\gamma^7 - 960\gamma^6 - 705\gamma^5 + 1632\gamma^4 + 432\gamma^3 - 768\gamma^2 - 93\gamma + 96) | h_{26} = \gamma(3 - 2\gamma^2)(35\gamma^2 - 30\gamma^2 + 11) |
| h_{26} = (2\gamma^2 - 3)(35\gamma^2 - 30\gamma^2 + 11) | h_{27} = 15\gamma^2 + 60\gamma - 19 + 8 |
| h_{28} = \gamma(70\gamma^6 - 645\gamma^4 + 768\gamma^2 - 63) | h_{29} = -75\gamma^6 + 90\gamma^4 + 333\gamma^2 + 60 |
| h_{29} = 25\gamma^4 - 30\gamma^3 + 60\gamma^2 + 129\gamma + 76\gamma + 12 |
with $\Delta t$.

However, similarly to the periastron advance at

$\Delta_{\text{hyp}}$, this expression vanishes at 4PM. To obtain

an adiabatic expansion. By writing the PM-expanded

we find, at 4PM order (see[30] for the 3PM term)

\begin{equation}
\frac{b^4 P_{\text{rad}}^{\text{4PM}}}{\pi \Delta_n G^4 M^3 \nu^2} = \frac{37}{30} + \frac{1661 v_{\infty}^2}{560} + \frac{1491 v_{\infty}^3}{400} + \frac{23563 v_{\infty}^4}{10080} - \frac{26757 v_{\infty}^5}{5600} + \frac{70793 v_{\infty}^6}{50680} + \mathcal{O}(v_{\infty}^7). \tag{10}
\end{equation}

Both expressions in (9) and (10) are consistent with the

state of the art in the PN expansion [119–124]. On the other

hand, in the opposite limit, as $\gamma$ gets large,

\begin{equation}
\frac{b^4 \gamma \Delta E_{\text{hyp}}^{\text{4PM}}}{G^4 M^6 \nu^2} \rightarrow \frac{13696}{105} \gamma^2 \nu \log(2\gamma), \tag{11}
\end{equation}

which signals the presence of (logarithmic) mass singularities.

We return to these limits in the conclusions.

**GW energy flux.**—The B2B map allows us to connect the

radiated energy for the scattering process to its counterpart

over a period of an elliptic-like orbit via

$\Delta_{\text{hyp}}(j) = \Delta E_{\text{hyp}}(j) - \Delta E_{\text{hyp}}(-j)$ [30],

where $j = (p_{\infty} b / G M^2 \nu)$, with $p_{\infty} \equiv (M \nu / \Gamma) v_{\infty}$ is the (reduced) angular

momentum. However, similarly to the periastron advance at $\mathcal{O}(G^3)$

[28,29], this expression vanishes at 4PM. To obtain

radiative observables for generic orbits we derived instead

the energy flux. Since nonlinear radiation-reaction terms do

not contribute to the energy loss at this order we can resort

to an adiabatic expansion. By writing the PM-expanded

energy flux in an isotropic gauge as [30]

\begin{equation}
\frac{dE}{dt} = \frac{M}{r} \sum_n \mathcal{F}_E(n)(\gamma) \left(\frac{GM}{r}\right)^{(n+3)}, \tag{12}
\end{equation}

we find at 4PM order (see [30] for the 3PM term)

\begin{equation}
\Delta E(j) = \sum_{n=0}^{\infty} \mathcal{E}_E^{(n)} / j^{n+3}, \quad \xi \equiv (E_1 / E_2),
\end{equation}

$E_a \equiv \sqrt{p_{\infty}^2 + m_a^2}$.

**Conclusions.**—We completed the knowledge of the total

relativistic impulse in the scattering of nonspinning bodies

at 4PM order, including linear, nonlinear, and hereditary

radiation-reaction effects. We also derived the total radiated

spacetime momentum at $\mathcal{O}(G^4)$ and extracted the GW

energy flux, which can then be used to compute observables

for generic (un)bound orbits incorporating an infinite series of velocity corrections. The most intricate part of the calculation involves a series of master integrals with retarded propagators, which we are able to compute to all orders in the velocity through the methodology of differential equations, without resorting to PN resummations. The boundary conditions in the near-static limit are obtained via the method of regions, thus making direct contact with derivations in the PN regime with potential and radiation modes. We find perfect agreement with various partial calculations in the literature. Explicit values can be found in the Supplemental Material and ancillary file.
There are, however, some key aspects of the structure of the impulse at 4PM order which deserve further study. First, concerning the high-energy limit, although nontrivial cancellations occur we find that the \( \Delta^{(4)} p_j \) impulse (in particular the \( e^{(4)}_{ij} \) component) does not transition smoothly, but rather it diverges when \( m_1 \to 0 \) while \( \gamma m_1 m_2 \) is held fixed. Since all the integrals in (6) vanish in dimensional regularization when the velocities obey the null condition (\( u_0^2 = 0 \)), the divergent terms must arise due to the enforcement of timelike worldlines (\( u^0 = 1 \)) in the massive theory from the onset. Moreover, similarly to what happens at \( O(G^3) \) [97], the total radiated energy at \( O(G^4) \) also diverges in the \( \gamma \gg 1 \) limit, this time featuring a factor of \( \log(\gamma) \) with respect to the 3PM case, see (11). We expect this behavior to be tamed in the nonperturbative solution (see, e.g., [127,128] and references therein).

Second, there is the issue of the mass scaling of the impulse [27], and violations thereof, e.g., [117,118]. As we mentioned, some of the radiative contributions affect only the total radiated momentum in the \( \hat{b} \) direction, while conserving energy. Moreover, they are even under time reversal, see (10). To gain intuition about these terms, from the impulse and total recoil, we derived the relative deflection angle (see the Supplemental Material). After expanding in small velocities, we find perfect consistency with the (odd-in-velocity) PN values in [27]. Yet, starting at 5PN order, we encounter conservative-like (even-in-velocity) contributions at \( O(\nu^2) \), beyond the Feynman-only part [129]. In principle, depending on their origin, these terms could be incorporated into a relative Hamiltonian. We will discuss these issues in more detail elsewhere.

In summary, in addition to the natural connections to GW science, e.g., [99,100], the solution of the (classical) relativistic scattering problem at \( O(G^4) \) presented here demonstrates how the worldline EFT approach [15,18] combined with the methodology of differential equations and integration by regions—already successfully implemented both in the conservative [62–67] and dissipative [72–76] sectors—are very powerful tools to tackle the entire two-body dynamics in general relativity within the PM expansion. Complete results at higher PM orders, including spin and tidal effects, are underway.

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[125] At this point, however, we cannot distinguish whether nonlinear radiation-reaction terms are due to either effects at second order in the linear radiation-reaction force or truly nonlinear gravitational corrections.

[126] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.130.101401 for a pdf with expressions for the intermediate values of the impulse as well as the relative scattering angle, and a computer-readable notebook with all the results displayed in the letter.


[129] The deflection angle is however in tension with the two distinct values reported in [117,118].