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Generation of Fragment Angular Momentum in Nuclear Fission

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Abstract. As a fissioning nucleus approaches scission, the angular-momentum bearing modes in the evolving dinuclear complex may be agitated by multiple transfers of individual nucleons. It is discussed how this mechanism populates the various rotational modes at different rates and leads to fragment angular momenta that are preferentially perpendicular to the fission axis but mutually largely uncorrelated. Using the fission simulation code FREYA, it is demonstrated how a measurement of the angular distribution of photons from identified collective transitions in the product nuclei can provide quantitative information on the relative importance of the twisting mode in fission.

1 Introduction

The origin and character of the fission fragment angular momenta are currently topics of intense theoretical and experimental study and neither the general properties of the angular momenta nor the mechanism(s) behind their generation are yet well understood.

The present discussion concentrates on one particular mechanism, namely the exchange of nucleons between the two parts of the scissioning system. This mechanism was found to be the primary cause of the transport phenomena displayed by damped nuclear reactions, including the generation of the angular momenta of the fragments. In those reactions the system retains a binary character throughout. A somewhat similar situation occurs during the late stages of fission, as scission is approached, when a binary geometry develops. It may therefore be expected that the nucleon exchange mechanism may also play a role here and it appears worthwhile to explore the consequences.

2 Rotational modes in a binary system

In a binary system, each of the two partners may carry angular momentum, $S_L$ and $S_H$, where $L$ and $H$ refer to the light and heavy partner, respectively. In addition, there is generally also an angular momentum associated with their relative motion, $L = P \times R$, where $R = R_L - R_H$ is the relative position and $P$ is the relative momentum.

When the system is isolated, as is the case for a nuclear reaction or in fission, the sum of those angular momenta remains constant in the course of time, so $S_L + S_H + L = S_0$, where $S_0$ is the total angular momentum of the system. There are thus six independent internal modes of rotation in the binary system.

If none of those internal rotational modes are agitated, then the system executes a rigid rotation, of which each fragment partner carries the amount $S_\pm = (I_L/I_0)S_0$, where $I_L$ is the moment of inertia of the fragment and the total moment of inertia is $I_0 = I_L + I_H + \mu R^2$. In fission, these contributions are usually relatively small and the fragment spins are dominated by the contributions from the internal modes, $\delta S_L = S_L - S_0 = I_0 \omega_L$.

It is convenient to classify the internal modes as positive or negative modes, in analogy with the treatment of linear momentum in a two-body system,

$$ s_+ \equiv \delta S_L + \delta S_H, \quad s_- \equiv \tilde{I} (\omega_L - \omega_H), \quad (1) $$

where $\tilde{I} \equiv I_L I_H/(I_L + I_H)$ is the reduced moment of inertia pertaining to the relative rotation.

The six internal rotational modes have been given descriptive names [1, 2] and are illustrated in Fig. 1. The positive modes contribute fragment spins that are mutually parallel and directed either perpendicular to $R$ (wriggling) or along $R$ (tilting), while the negative modes contribute fragment spins that are mutually opposite and directed either perpendicular to $R$ (bending) or along $R$ (twisting).

3 Agitation of the rotational modes

Moretto annd Schmitt [2] developed a model in which all the rotational modes are populated statistically at scission. While this provides a conceptually simple and useful reference, it is important to recognize that the different modes are not necessarily being populated to the same degree. To clarify this issue one must consider the time dependence of the modes as the nuclear shape evolves towards scission.

The dynamical evolution of the fragment angular momenta has been studied in detail for the nucleon-exchange mechanism [3, 4] which we shall now focus on.
It is apparent that transfers between high-$\ell$ orbitals generate more angular momentum, both in the two fragments and in the relative fragment motion, than those between low-$\ell$ orbitals. Particularly effective are transfers of the type illustrated on the left in Fig. 2. Such transfers add angular momenta to the fragments that are perpendicular to the dinuclear axis and are mutually parallel. Thus they generate wriggling and the typical spin contribution of such a transfer is of the order of $R_p c$.

Anti-parallel spin contributions require off-center transfers and their typical magnitude is therefore suppressed by a geometric factor $c/R$, where the neck radius $c$ is a measure of the effective transverse extent of the interface between the two systems. Consequently, these modes are expected to build up over a considerably longer time scale than wriggling.

A succession of such individual transfers then leads to a random walk in the six-dimensional space of the fragment spins, each transfer contributing a spin “step” of varying magnitude and direction, depending on the specifics. The resulting diffusive time evolution of the joint fragment spin distribution, $P(S_L,S_H;t)$, can then be described within the formalism for transport theory.

For the present discussion, the most important quantities are the mobility coefficients that govern the rate at which the various rotational modes are being populated as a result of the successive exchanges. Expressions for the mobility coefficients were derived in Ref. [4] based on the Nucleon Exchange Transport model presented in Ref. [3],

\begin{align}
M_{\text{wrig}} &= mN R^2, \\
M_{\text{bend}} &= mN \left( \frac{1}{I_H R_L - I_L R_H} \right)^2 + c^2 \bar{c}\text{ave} \right), \\
M_{\text{twst}} &= mN c^2 \text{ave}. 
\end{align}

The mean rate of nucleon transfers from one fragment to the other is given by $N = \frac{1}{2} \rho \bar{c}^2$ [5] where $\rho$ is the nucleon density in nuclear matter, $\bar{c} = \frac{1}{2} \bar{c}_F$ is the mean nucleon speed, and $c$ is the neck radius. Furthermore, $c^2 \bar{c}_\text{ave} = \frac{1}{2} \bar{c}^2$ is the average value of $c^2$ over the interface between the two parts. $M_{\text{twst}}$ is an order of magnitude smaller than $M_{\text{wrig}}$ because $c^2 \ll R^2$. The first term in $M_{\text{bend}}$ vanishes for symmetric divisions, giving $M_{\text{bend}} = M_{\text{wrig}}$, but $M_{\text{bend}}$ is significantly larger than $M_{\text{wrig}}$ for typical mass divisions (and small neck radii).

In the course of time, as the exchanges continue, the joint spin distribution approaches the statistical equilibrium form employed in Ref. [2]). For each mode $m$, the time scale is given by the relaxation time $\tau_m = I_m / M_m$, where $I_m$ is the moment of inertia for the mode and $M_m$ is its mobility coefficient given above. Thus we generally expect $\tau_{\text{wrig}} < \tau_{\text{bend}} < \tau_{\text{twst}}$ with $\tau_{\text{wrig}} \ll \tau_{\text{twst}}$.

The calculated relaxation times are shown in Fig. 3 as functions of the neck radius $c$, with the relevant range of $c$ indicated. These should be compared with $\tau_{\text{fiss}}$, the time it takes the fissioning system to evolve from the first appearance of a dinuclear geometry to the rupture of the neck. This quantity is not well known [7] and the discussion below assumes that $\tau_{\text{fiss}}$ is in the range of one to several zep- toseconds ($1 \text{zs} = 10^{-21} \text{s}$), as indicated on the figure.
The calculated relaxation times $t_{\text{wrig}}$ for wriggling (bottom curve, green), bending (middle three curves, blue), and twisting (top curve, red), shown as functions of the neck radius $c$ for a tip separation of $d = 4$ fm. For wriggling the result for touching spheres, $d = 0$, is also shown (dashed green). For bending, the solid curve is for the mass division 108:144 (the most probable), while the dashed curves are for 100:152 (lower) and 118:134 (upper) which are each half as probable. Also shown are $c = 1.6$ fm and $c = 2.7$ fm (vertical lines) and $t_{\text{fiss}} = 1$ zs and $t_{\text{fiss}} = 4$ zs (horizontal lines). (Adapted from Ref. [6].)

Figure 3. The calculated relaxation times $t_{\text{wrig}}$ for wriggling (bottom curve, green), bending (middle three curves, blue), and twisting (top curve, red), shown as functions of the neck radius $c$ for a tip separation of $d = 4$ fm. For wriggling the result for touching spheres, $d = 0$, is also shown (dashed green). For bending, the solid curve is for the mass division 108:144 (the most probable), while the dashed curves are for 100:152 (lower) and 118:134 (upper) which are each half as probable. Also shown are $c = 1.6$ fm and $c = 2.7$ fm (vertical lines) and $t_{\text{fiss}} = 1$ zs and $t_{\text{fiss}} = 4$ zs (horizontal lines). (Adapted from Ref. [6].)

### 3.1 Expectations

The calculated $t_{\text{wrig}}$ stays well below the expected range of $t_{\text{fiss}}$ and one should therefore expect that the wriggling mode maintains full equilibrium until the time of scission, which is expected to occur for $c \approx 2$ fm.

By contrast, $t_{\text{twst}}$ is likely similar to or longer than $t_{\text{fiss}}$, so the twisting mode will adjust only slowly as scission is approached. Therefore, for spontaneous fission, where the rotational modes are probably not agitated much as the system emerges from the tunneling, it may not be possible to build up very much twisting before scission occurs. The situation is more complicated for induced fission. For thermal neutron energies the local excitation energy in the barrier region is small and even though the system spends a fairly long time there, the low local temperature will limit the degree of agitation of the rotational modes and, consequently, it may not be possible for the twisting mode to adjust to the ever-increasing temperature as scission is approached. However, as the neutron energy is raised, the local temperature in the saddle region increases correspondingly and the twisting mode is populated more prior to the descent towards scission. Therefore one should expect an ever increasing degree of twisting as the impinging neutron energy is raised, an effect that might be observable.

The bending mode is somewhat intermediate and without a more precise estimate of $t_{\text{fiss}}$, it is not possible to make specific predictions. But if scission occurs at $c \approx 2$ fm and $t_{\text{fiss}}$ is several zeptoseconds then the bending mode is expected to be agitated to an appreciable degree, although likely not fully. If bending is not fully populated, wriggling will dominate and the fragment spins will tend to have parallel directions and their magnitudes will fluctuate in concert. The recent experimental results by Wilson et al. [8] suggest that the two spin magnitudes are in fact fairly independent and they thus put a limit on the possible suppression of the bending mode. It would be very interesting to quantify this by further measurements.

Furthermore, because $t_{\text{bend}}$ depends on the mass asymmetry, the degree of bending at scission should increase with the asymmetry. Because the fragment mass is a readily measurable fission observable, this feature is susceptible to experimental investigation as well.

On the basis of these estimates, we expect the wriggling modes to have reached full equilibrium at scission, while the bending modes may fall somewhat short of that, and though some twisting may be present it is not likely to play a major role.

Finally, total kinetic energy (TKE) gated data may also provide valuable information because small TKE values are associated with elongated scission configurations which take more time to reach. Consequently, if the bending mode is only partially equilibrated, it should have a larger presence in events with small TKE and a smaller presence in events with large TKE. This should be reflected, for example, in the degree of correlation between the two fragment spin magnitudes, something that should also be readily measurable.

### 4 Simulations

The angular momentum features described above have been incorporated into the fission simulation code FREYA [9, 10] in a manner that makes it possible to easily explore a variety of scenarios: At the time of scission, each of the normal modes $m$ is sampled from a Boltzmann distribution with an effective temperature $T_m = c_m T_{\text{scis}}$ where $T_{\text{scis}}$ is the temperature of the dinuclear complex at scission and the coefficient $c_m$ can be adjusted to allow exploration of different degrees of agitation. Thus, for a given value of $c_m$, the distribution of the mode amplitude $s_m$ is

$$P(s_m; c_m) \sim e^{-\frac{s_m^2}{2 T_m}} = e^{-\frac{s_m^2}{2 c_m T_{\text{fiss}}}} ,$$

so the mode is entirely suppressed, e.g. $c_m = 0$, while it has reached full equilibrium for $c_m = 1$.

#### 4.1 Standard FREYA

The standard version of FREYA populates the wriggling and bending modes fully while leaving out tilting and twisting, $(c_{\text{wrig}}, c_{\text{bend}}, c_{\text{twst}}) = (1, 1, 0)$, which can be seen as a rough approximation to the expectations discussed above.

Wriggling or bending each contribute (perpendicular) fragment spins that are highly correlated, being either perfectly aligned (wriggling) or exactly opposite (bending). Nevertheless, the spins resulting from the agitation of both modes are largely uncorrelated [11, 12]. To illustrate this important feature, Fig. 4 shows the magnitude of the fragment spin at the deexcitation stage when only collective rotation remains (i.e. when the yrast line has been reached), as a function of the magnitude of the partner spin. This corresponds to the measurements presented in Ref. [8] and the FREYA results are in very good agreement with those observations.
between the

\[ P_f = \frac{1}{2} \left( 1 - \cos \phi \right) \]

an observable that is accessible via helicity measurements [6]. Because the spins are two-dimensional, both being perpendicular to \( R \), a complete lack of correlation would render \( P(\phi) \) constant. As can be seen, the FREYA distribution is nearly uncorrelated, exhibiting a slight undulation with an amplitude of about 9.2%.

\[ W_0(\theta) = 1 - \frac{1}{2} P_2(\cos \theta) - \frac{1}{2} P_4(\cos \theta) \]  

By contrast, if the emitter spin is perpendicular to the fragment motion, the distribution peaks at 0° and 180°,

\[ W_+ (\theta) = 1 + \frac{1}{2} P_2(\cos \theta) - \frac{1}{2} P_4(\cos \theta) \]

These two idealized distributions are shown in Fig. 6. Also shown in Fig. 6 is the simulation result for the standard scenario, \((c_{\text{wrig}}, c_{\text{bend}}, c_{\text{twist}}) = (1, 1, 0)\). Such simulations include the skewing effect of the emitter motion and take account of the directional changes caused by the hyperbolic Coulomb exit trajectory and the recoils from neutron evaporation and prior photon emissions. Consequently, even though the spins of the primary fragments are perpendicular to the scission axis, the resulting distribution differs from the idealized one. However, it still exhibits a pronounced forward-backward enhancement.

To elucidate the dependence of this observable on the degree of twisting, we have carried out a series of simulations with an ever larger presence of twisting [6].

\[ W(\theta) = 1 - \frac{1}{2} P_2(\cos \theta) - \frac{1}{2} P_4(\cos \theta) \]

The degree of correlation between the fragment spin directions is illustrated in Fig. 5 showing the distribution of the opening angle between the two angular momentum vectors, \( \phi_{JI} \), an observable that is accessible via helicity measurements [6]. Because the spins are two-dimensional, both being perpendicular to \( R \), a complete lack of correlation would render \( P(\phi_{JI}) \) constant. As can be seen, the FREYA distribution is nearly uncorrelated, exhibiting a slight undulation with an amplitude of about 9.2%.

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4.2 Twisting

As described above, the standard version of FREYA does not endow the fission fragments with any twisting at all. However, the discussion in connection with the calculated relaxation times suggests that twisting may in fact be present to a certain degree, reflective of the time scale of the shape dynamics prior to scission.

It appears that the degree of twisting can be determined experimentally by measurements of the type pioneered by Wilhelmy et al. [13]. Such an experiment consists of measuring the angular distribution, \( W(\theta) \), of photons resulting from identified \( E2 \) transitions in the observed fragments. If the angular momentum of the emitting nucleus is perfectly aligned with the direction of motion of the fragment, then its angular distribution is sideways peaked,

\[ W_0(\theta) = 1 - \frac{1}{2} P_2(\cos \theta) - \frac{1}{2} P_4(\cos \theta) \]

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\[ W(\theta) = 1 - \frac{1}{2} P_2(\cos \theta) - \frac{1}{2} P_4(\cos \theta) \]
As the degree of twisting is gradually increased, the angular distribution, being initially close to the idealized perpendicular distribution $W_c(\theta)$ as shown in Fig. 6, changes at an ever increasing rate and eventually, when twisting has grown dominant, $W_c(\theta)$ has become close to the idealized parallel distribution $W_c(\theta)$ [6].

An instructive way to exhibit the sensitivity of $W(\theta)$ to the degree of twisting is to consider the ratio between the yields at $\theta = 0^\circ$ and at $\theta = 90^\circ$. These can be extracted as follows.

Generally, the calculated angular distributions, such as the one shown in Fig. 6, are well represented by fourth-order Legendre fits, $W(\theta) \approx \sum_{n=2}^{4} a_n P_n(\cos \theta)$. Furthermore, the skewing effect of the motion of the emitting fragment can, to a good approximation, be removed by symmetrizing those expansions, i.e., removing the odd-order terms. The ratio between the yields at $\theta = 0^\circ$ and $\theta = 90^\circ$ can therefore be extracted as

$$\frac{W(0^\circ)}{W(90^\circ)} \approx \frac{a_0 + a_2 + a_4}{a_0 - \frac{1}{2}a_2 + \frac{1}{2}a_4}. \quad (8)$$

The result is illustrated in Fig. 7 showing the yield ratio as a function of the relative presence of twisting. There is a pronounced decrease of the ratio as $c_{\text{twst}}$ is increased. It is noteworthy that even the visually small change in $W(\theta)$ resulting from introducing 20% twisting produces a significant decrease in the yield ratio. It is also important that the yield ratio, while thus quite sensitive to $c_{\text{twst}}$, is practically independent of the relative proportion of wriggling and bending: the results for three very different values of $c_{\text{bend}}$ : $c_{\text{wrig}}$ are nearly identical. Therefore its measurement should provide a good indication of the degree of twisting present in the fission fragments.

5 Summary

After first presenting the general classification of the rotational modes in the two-fragment system, we discussed multiple nucleon exchanges as an effective mechanism for building up angular momentum in fission fragments. Using the relaxation times predicted by the Nucleon-Exchange Transport model, we discussed the relative prominence of the various rotational modes at the time of scission, concluding that wriggling is always present, bending is likely present to a significant degree, while twisting plays only a minor role.

The FREYA simulation model, in which wriggling and bending are fully populated while twisting is absent, leads to fragment spins that are perpendicular to the fission axis and essentially independent with respect to both magnitude and direction, features that are in good agreement with the experimental data.

We finally discussed the possibility of determining the degree of twisting experimentally. Simulations with various degrees of twisting suggested that this quantity may indeed be obtained by measuring the forward-to-sideward yield ratio of photons from identified $E2$ transitions. This information would have a crucial bearing on how the fragment spins are being generated in fission.

References