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Prices, Wages and Fertility in Pre-Industrial England

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Abstract A two-sector Malthusian model is formulated in terms of a cointegrated vector autoregressive (CVAR) model on error correction form. The model allows for both agricultural product wages and relative prices to affect fertility. The model is estimated using new data for the pre-industrial period in England, and the analysis reveals a strong, positive effect of agricultural wages as well as a small and, surprisingly, positive effect of real agricultural prices on fertility. Furthermore, it is demonstrated that there is constant returns to scale with respect to labour in the manufacturing sector and strongly decreasing returns to scale in the agricultural sector.

JEL Classification Codes C32, N3, O1.

Keywords: Malthus, cointegration, pre-industrial England.

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1 Introduction

In recent years, unified growth theory has generated increased interest in the Malthusian model (see for example Galor and Weil (2000)). According to most unified growth theories, all countries start off from a Malthusian regime that is later exchanged for a regime of modern economic growth. Some unified growth theories, however, such as Strulik and Weisdorf (2008), claim that Malthusian forces persist through the modern growth era. While the standard Malthusian model is concerned with the effect of income on fertility, Strulik and Weisdorf argue that the price of children (i.e. the price of food) relative to other goods plays a key role for the demand for children. The idea that prices could have a negative effect on fertility has originally been put forth by Weisdorf (2008).

I present a simple way to analyse the effect of income and prices on fertility. I extend the model in Weisdorf (2008) and formulate a cointegrated vector autoregressive (CVAR) model, inspired by Møller (2008), to analyse the interplay between birth rates, agricultural product wages and agricultural prices divided by industrial prices. For the sake of brevity, I hereafter refer to product wages as "wages" and relative prices as "prices". Likewise, the terms "industrial" and "manufacturing" are used interchangeably. I employ the model on the new price data from Broadberry et al. (2011) and the wage data from Clark (2005, 2007). The main goal of this analysis is to determine which variable — wages or prices — plays the bigger role for fertility.

The existing literature has mainly focused on estimation of one-sector Malthusian models without price effects. Galloway (1988) finds the existence of a preventive check in five European countries (including England) before and during the eighteenth century. Lee and Anderson (2002) and Møller and Sharp (2008) focus on England and find that the positive effect of wages on fertility persists throughout the eighteenth century. Nicolini (2007), on the other hand, finds that the preventive check in England is quite strong from 1540 to 1740 but changes dramatically thereafter. Using the wage series from Clark (2005), Crafts and Mills (2009) perform similar analyses as Lee and Anderson (2002) and Nicolini (2007), and conclude that the preventive check breaks down already in the middle of the seventeenth century.

This article shows that wages have a positive effect on fertility beyond the first half of the eighteenth century, and that prices matter comparatively little. Contrary to the theoretical prediction, the analysis also shows that the effect of the prices on fertility is positive.

Compared to the one-sector cointegration analysis conducted in Møller and Sharp (2008), the main advantage of the present two-sector model is the inclusion of the potential price effect on fertility. While the model needs to be applied to data from several countries and time periods, the results reported in this paper do not lend strong support to the unified growth theory in Strulik and Weisdorf (2008).

2 The Model

The Malthusian model used in this paper is a modification of Weisdorf (2008). Weisdorf's model is an overlapping generations (OLG) model that describes an economy with a manufacturing and an agricultural sector in which agents live for two periods. The agents have preferences over children and manufactured goods, and each generation gives birth to a certain number, n, of children and consume a certain number, m, of manufactured goods. The price of raising a child is one unit of the agricultural good.

The agents are assumed to have a hierarchy of demands. As adults, they are primarily concerned with having offspring and secondarily with consuming manufactured goods. Weisdorf models this by setting the income elasticity of demand of children to zero and describes the adult individuals' preferences with a quasi-linear utility function, u, of the form $u(n,m) = \varphi \ln n + \gamma m$.[1] The adult budget constraint is given by $P_A b + P_M m = W$ where P_A is the price of food consumed by a child, P_M is the price of manufactured goods, and W is the nominal wage. Utility maximisation implies that the optimal lifetime demand for children is $n = \left(\frac{\gamma}{\phi}\frac{P_A}{P_M}\right)^{-1}$, i.e. a decreasing function of the price. If preferences are instead described by the the standard Cobb-Douglas utility function, $u(n,m) = \varphi \ln b + \gamma \ln m$, the demand for children would be $\frac{\phi}{\phi+\gamma}\frac{W}{P_A}$, creating the well-known positive income effect from the Malthusian model. In the present model, there are two kinds of preventive checks: the usual Malthusian positive effect of wages on fertility and a negative effect of prices on fertility.

Furthermore, I modify the OLG model to better exploit availability of yearly data by using a period length of one year, introducing various autoregressive components, and allowing for slow adjustment of the prices. I first introduce the stylised theory model and then augment it

[1] Weisdorf (2008) sets $\varphi = \gamma = 1$.

to produce the statistical model that I take to the data.

2.1 The Theory Model

The response of fertility to changes in wages and prices is delayed by the time lag between the decision to have a child and the time at which pregnancy is achieved plus the natural delay caused by the nine months waiting time from conception to birth. One way in which a couple could regulate fertility was by delaying or postponing their marriage, since marriage marked the intention to start a family and have children. Using historical English data from Wrigley et al. (1997), Klemp and Weisdorf (2011) document that the average time from marriage to the first birth in couples where marriage seems to mark the onset of intercourse was around one and a half years. Therefore, it is reasonable to assume at least one year of response time of fertility to changes in wages and prices. Here I will assume a lag of one year, and later I will allow for more lags in the birth relation. It is assumed in the model that the skill-corrected agricultural and industrial nominal wages are equal in the long run. I thus let the nominal agricultural wages represent the nominal wage level, whereby the fertility decision is determined by the agricultural product wages and the real prices. Formally, the population growth factor $1+b$ in period t, where b is the birth rate, is given by the equation

$$1 + b_t = f(W_{a,t-1}, P_{t-1}), \tag{1}$$

where W_a is the agricultural product wage and P is the agricultural prices divided by the manufacturing prices. I assume that the yearly population growth factor exhibits constant wage and price elasticities. Letting $e(h, y)$ denote the elasticity of a function h with respect to the yth argument, this translates to

$$e(f, 1) = \eta_W, \tag{2}$$

$$e(f, 2) = -\eta_P. \tag{3}$$

It is straightforward to show that the elasticity of the birth rate with respect to the wage is $\epsilon_W = (1+b^{-1})\eta_W$ and that the elasticity of births with respect to the prices is $\epsilon_P = -(1+b^{-1})\eta_P$.

The production technology in each sector is described by the Cobb-Douglas function. Total production in the agricultural sector in period t is $X_{a,t}L_{a,t}^{1-\lambda}$, where X_a is the stock of agricultural technology, L_a is the labour force in agriculture, and λ is a production scale parameter. Likewise, total production in the industrial sector in period t is $X_{m,t}L_{m,t}^{1-\mu}$, where X_m is the stock of manufacturing technology, L_m is the labour force in manufacturing, and μ is a production scale parameter.[2] By profit maximisation, the product wage in period t in the agricultural sector, $W_{a,t}$, is

$$W_{a,t} = X_{a,t}L_{a,t}^{-\lambda}. \tag{4}$$

Likewise, the product wage in period t in the manufacturing sector, $W_{a,t}$, is

$$W_{m,t} = X_{m,t}L_{m,t}^{-\mu}. \tag{5}$$

Assuming that individuals in any period, t, are employed in only one sector, the total labour force, L_t, is the sum of the labour force in the manufacturing sector, $L_{m.t}$, and in the agricultural sector, $L_{a,t}$, that is

$$L_t = L_{m,t} + L_{a,t}. \tag{6}$$

Each agent consumes one unit of agricultural goods per year. The equilibrium on the food market is therefore

$$L_t = X_{a,t}L_{a,t}^{1-\lambda}. \tag{7}$$

Note that in contrast to Weisdorf (2008), who assumes that individuals only demand food during the first half of life, I assume that individuals start to demand food following the year of birth. This assumption is appropriate given the short period.

Because production in the manufacturing sector is more skill-intensive than production in the agricultural sector, I allow for a wage skill premium, S, to labour in the manufacturing sector. In the long-run equilibrium, the prices adjust so that the skill-corrected real income of

[2]Weisdorf (2008) assumes $\mu = 0$.

both sectors equalises, i.e.

$$P_t = W_{m,t}/(SW_{a,t}), \tag{8}$$

where P is the agricultural prices divided by the industrial prices. Given that shocks to the variables bring the economy out of the long-run equilibrium and that the economy does not instantaneously recover from these shocks, I will allow for gradual adjustment of the prices in the empirical model.

Finally, migration is ignored, and the death rate is taken as exogenously given, whereby population evolves through the relation

$$L_t = L_{t-1}(1 + b_{t-1} - d). \tag{9}$$

An underlying assumption of this equation is that the labour force and the total population coincide. Although children enter the labour force after a lag of 10-20 years, there is indeed an immediate effect of child birth on labour supply because parents need to work harder to support a larger family. Children, in other words, indirectly enter the labour force. As the children grow up and start to earn their own income, they instead become directly part of the aggregate labour force. Of interest here is the long-run effect, which is in any case that population growth leads to growth in the labour force. Given the fact that yearly data is available, it would entail a great loss of information to resolve the problem by aggregating the data into longer time periods. Instead I will follow Møller and Sharp (2008), who used a similar line of reasoning, and allow for a gradual dynamic adjustment of the labour force.

Equations (1) to (9) characterise the economy and complete the stylised theory model. The next step is to linearise the model to formulate it in terms of a CVAR model. I let lowercase letters denote the logarithm of the corresponding capitalised variable. Using Equations (2) and (3), a log-linear approximation of Equation (1) around the long-run values $w_{a,t-1} = \bar{w}_a$ and $p_{t-1} = \bar{p}$, where $b_t = \bar{b}$, is given by

$$b_t = \eta_W w_{a,t-1} - \eta_P p_{t-1} + c_b, \tag{10}$$

where $c_b = \bar{b} - \eta_W \bar{w}_a + \eta_P \bar{p}$ is a constant. Equations (4) and (5) are log-linearised by

$$w_{a,t} = x_{a,t} - \lambda l_{a,t}, \tag{11}$$

$$w_{m,t} = x_{m,t} - \mu l_{m,t}. \tag{12}$$

Using that the sises of the agricultural and manufacturing sectors are almost equal during the early eighteenth century (Crafts, 1985), a log-linear approximation of Equation (6) is

$$l_t = (1/2)(l_{m,t} + l_{a,t}) + \log(2). \tag{13}$$

Equations (7) to (9) are linearised by

$$l_t = x_{a,t} + (1 - \lambda) l_{a,t}, \tag{14}$$

$$p_t = w_{m,t} - w_{a,t} - s, \tag{15}$$

$$l_t = l_{t-1} + b_{t-1} - d. \tag{16}$$

The complete linearised model consists of Equations (10) to (16).

2.2 The CVAR Model

The next step is to formulate the theory model as a CVAR model in the four observable variables b_t, p_t, $w_{m,t}$ and $w_{a,t}$. To do so, I need to model the evolution of the logarithms of the unobservable labour demand terms, $x_{m,t}$ and $x_{a,t}$. I assume that both terms grow linearly:

$$x_{a,t} = x_{a,t-1} + g_a, \tag{17}$$

$$x_{m,t} = x_{m,t-1} + g_m. \tag{18}$$

This means that the there is exponential growth in the labour demand terms. A similar formulation is made by Lee and Anderson (2002) who assume labour demand to grow exponentially but also allow the growth rate to accelerate or decelerate stochastically over time. In the present two-sector model, the possibility of intersectoral labour mobility implies that the growth rate of the manufacturing wages depends on the growth rate of the agricultural labour demand if $\mu \neq 0$

(see the definition of c_m below). Thus it is much simpler to introduce a stochastic element only in the growth rate of the wages, as I do here, rather than in the labour demand terms. This amounts to assuming that there are stochastic trends in the wages, and it makes it possible to remove the labour force levels from the model. Furthermore, I add error terms to the crude birth rate and the prices.

To illustrate the correspondence between the theory model and the CVAR model, I first present what I term the "simple" CVAR model that includes no new assumptions. I then augment the model to produce the "empirical" CVAR model that I will take to the data.

Using Equations (10) to (16) and the assumptions stated above, the autoregressive process is given by the equations:

$$b_t = \eta_W w_{a,t-1} - \eta_P p_{t-1} + c_b + \varepsilon_{b,t}, \tag{19}$$

$$p_t = w_{m,t} - w_{a,t} - s + \varepsilon_{p,t}, \tag{20}$$

$$w_{m,t} = w_{m,t-1} - \hat{\mu} b_{t-1} + c_m + \varepsilon_{m,t}, \tag{21}$$

$$w_{a,t} = w_{a,t-1} - \hat{\lambda} b_{t-1} + c_a + \varepsilon_{a,t}, \tag{22}$$

where $\hat{\mu} = \mu(1-2\lambda)/(1-\lambda)$ and $\hat{\lambda} = \lambda/(1-\lambda)$ are the equilibrium scale effects, $c_m = \hat{\mu} d - \frac{\mu}{1-\gamma} g_a + g_m + 2\log(2)\mu$ and $c_a = \hat{\lambda} d + \frac{g_a}{1-\lambda}$ are constants, and $\varepsilon_{b,t}$, $\varepsilon_{p,t}$, $\varepsilon_{m,t}$, $\varepsilon_{a,t}$ are normally distributed error terms with zero means and constant variances. The process can be written on the error correction form,

$$\Delta y_t = \Pi y_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta y_{t-i} + \Phi D_t + \varepsilon_t, \tag{23}$$

where y_t is a j-dimensional vector containing the variables of interest, D_t is a j-dimensional vector of deterministic terms, and ε_t is a j-dimensional vector of independent, multivariate normal error terms with zero mean and a constant non-singular variance matrix. In the present case, where $j = 4$, $y_t = (b_t, p_t, w_{m,t}, w_{a,t})'$, $k = 1$ and $D_t = 1$, the matrices are given by

$$\Pi = \begin{pmatrix} -1 & -\eta_P & 0 & \eta_W \\ \hat{\lambda} - \hat{\mu} & -1 & 1 & -1 \\ -\hat{\lambda} & 0 & 0 & 0 \\ -\hat{\mu} & 0 & 0 & 0 \end{pmatrix} \text{ and } \Phi = \begin{pmatrix} c_b \\ c_m - c_a - s \\ c_m \\ c_a \end{pmatrix}. \tag{24}$$

If the conditions of Theorem 4.2 of Johansen (1995) (Granger's Representation Theorem) are satisfied, then y_t is a cointegrated $I(1)$ process, and the model can be analysed with the Johansen maximum likelihood procedure (ibid.). The first condition is that the roots of the characteristic polynomial is either one or outside the unit disc and that at least one root is equal to one. The characteristic polynomial is in the present case given by

$$C(z) = (1 - z)\left(z^2\left((\hat{\lambda} - \hat{\mu})\eta_P + \hat{\lambda}\eta_W\right) + 1 - z\right). \tag{25}$$

Assuming $0 < (\hat{\lambda} - \hat{\mu})\eta_P + \hat{\lambda}\eta_W < 1$, it follows that $C(z) = 0$ when either $z = 1$ or $|z| > 1$, and thus the first condition is fulfilled. The second condition is that $| - \alpha'_\perp \Gamma \beta_\perp| \neq 0$, where $\Gamma = I - \sum_{i=1}^{k} \Gamma_i$. Assuming $\frac{\hat{\lambda}}{\hat{\mu}} \frac{\eta_P + \eta_W}{\eta_P} \neq 1$, it follows that $| - \alpha'_\perp \Gamma \beta_\perp| = 1 - \frac{\hat{\lambda}}{\hat{\mu}} \frac{\eta_P + \eta_W}{\eta_P} \neq 1$. Both of these assumptions are reasonable and also empirically valid. Because $\text{rank}(\Pi) = 3 < j$, it follows from the Theorem that there exists $p \times \text{rank}(\Pi)$ matrices α and β with the same rank as Π such that $\Pi = \alpha\beta'$, where β is the cointegration vectors and α is the adjustment coefficients. These matrices are unique up to a given normalisation. In the present case the matrices are given by

$$\alpha = \begin{pmatrix} -1 & -\eta_P & 0 \\ \hat{\lambda} - \hat{\mu} & -1 & 1 \\ -\hat{\mu} & 0 & 0 \\ -\hat{\lambda} & 0 & 0 \end{pmatrix} \text{ and } \beta' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\frac{\eta_W}{\eta_P} \\ 0 & 0 & 1 & -\frac{\eta_W + \eta_P}{\eta_P} \end{pmatrix}. \tag{26}$$

The first cointegration vector shows that the crude birth rate is stationary. The second and third cointegration vectors show that the prices cointegrate with the wages in either sector.

The next step is to add some flexibility to the model with regards to the sluggishness of the system and the number of relevant lags and to account for unmodelled mechanisms that translate into autoregressive dynamics. This results in the empirical model, given by the equations

$$b_t = \sum_{i=1}^{k} \eta_{w,i} w_{a,t-i} - \sum_{i=1}^{k} \eta_{p,i} p_{t-i} + \sum_{i=1}^{k} q_i b_{t-i} + c_b + \varepsilon_{b,t}, \tag{27}$$

$$p_t = \sum_{i=0}^{k} r_i w_{m,t-i} - \sum_{i=0}^{k} u_i w_{a,t-i} + \sum_{i=1}^{k} v_i p_{t-i} - s + \varepsilon_{p,t}, \tag{28}$$

$$w_{m,t} = w_{m,t-1} - \sum_{i=1}^{k} \hat{\mu}_i b_{t-i} + c_m + \varepsilon_{m,t}, \tag{29}$$

$$w_{a,t} = w_{a,t-1} - \sum_{i=1}^{k} \hat{\lambda}_i b_{t-i} + c_a + \varepsilon_{a,t}. \tag{30}$$

This implies the following matrices:

$$\Pi = \begin{pmatrix} -(1-q) & -\eta_P & 0 & \eta_W \\ u_0\hat{\lambda} - r_0\hat{\mu} & -(1-v) & r & -u \\ -\hat{\mu} & 0 & 0 & 0 \\ -\hat{\lambda} & 0 & 0 & 0 \end{pmatrix}, \tag{31}$$

where now the parameters of interest are redefined as $\eta_P = \sum_{i=1}^{k} \eta_{p,i}$, $\eta_W = \sum_{i=1}^{k} \eta_{w,i}$, $\hat{\lambda} = \sum_{i=1}^{k} \hat{\lambda}_i$, $\hat{\mu} = \sum_{i=1}^{k} \hat{\mu}_i$, and where $q = \sum_{i=1}^{k} q_i$, $v = \sum_{i=1}^{k} v_i$, $r = \sum_{i=0}^{k} r_i$, and $u = \sum_{i=0}^{k} u_i$,

$$\Gamma_i = \begin{pmatrix} -\sum_{i=2}^{k} q_i & \sum_{i=2}^{k} \eta_{p,i} & 0 & -\sum_{i=2}^{k} \eta_{w,i} \\ r_0 \sum_{i=2}^{k} \hat{\mu}_i - u_0 \sum_{i=2}^{k} \hat{\lambda}_i & -\sum_{i=2}^{k} v_i & -\sum_{i=2}^{k} r_i & \sum_{i=2}^{k} u_i \\ \sum_{i=2}^{k} \hat{\mu}_i & 0 & 0 & 0 \\ \sum_{i=2}^{k} \hat{\lambda}_i & 0 & 0 & 0 \end{pmatrix}, \tag{32}$$

and

$$\Phi = \begin{pmatrix} c_b \\ r_0 c_m - u_0 c_a - s \\ c_m \\ c_a \end{pmatrix}. \tag{33}$$

Again, the rank of Π is three. The corresponding α and β matrices are given by

$$\alpha = \begin{pmatrix} -(1-q) & -\eta_P & 0 \\ u_0\hat{\lambda} - r_0\hat{\mu} & -(1-v) & r \\ -\hat{\mu} & 0 & 0 \\ -\hat{\lambda} & 0 & 0 \end{pmatrix} \text{ and } \beta' \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\frac{\eta_W}{\eta_P} \\ 0 & 0 & 1 & -\frac{(1-v)\eta_W + u\eta_P}{r\eta_P} \end{pmatrix}. \tag{34}$$

The empirical and the simple CVAR models have the same fundamental structures: the crude birth rate is stationary and the price cointegrates with the wages in agriculture and in manufacturing. The main difference between the empirical and the simple model is that the empirical model allows more flexibility in the coefficients in α and that it removes the restrictions between $\alpha_{1,2}$, $\beta'_{2,4}$ and $\beta'_{3,3}$ and the restrictions between $\alpha_{2,1}$, $\alpha_{3,1}$ and $\alpha_{4,1}$.

3 Data

Data on the annual birth rate comes from the Anglican church books and is collected by the Cambridge Group for Population and Social Structure as documented in Wrigley and Schofield (1981). The data covers the years 1541–1914 and is reported as the crude birth rate, i.e. births per 1,000 individuals. The birth rate variable used below is thus the crude birth rate divided by 1,000. The data is subject to methodological criticism related primarily to the back projection method used by Wrigley and Schofield to estimate the population size, which is needed to calculate the crude birth rate (Lee, 1985). Lee concludes, however, that Wrigley and Schofield's estimates are robust. It is important to note that although the population size is estimated on a five-year basis, the crude birth rate is based on the actual number of recorded births in the observed parishes on a yearly basis. Therefore, the crude birth series contain yearly variation. Furthermore, the number of births per year has much more variation than the population size, which means that knowledge of the exact population size is of little importance.

I use the new estimates of the industrial and agricultural prices for the years 1700–1870 presented by Broadberry et al. (2011). Nominal wages for industrial and agricultural labour are provided by Clark (2005, 2007). Wages to manufacturing labour are composed of wages to 29 different types of workers, and wages to agricultural labour are composed of day wages of various kinds of agricultural workers and payment for threshing services.

All necessary data is available for the period 1700–1869. The baseline results, presented in the next section, are based on the period 1701–1759. The plots of the data in this period, provided by Figure 1, indicate that all four time series are difference-stationary. Three key periods characterised by low birth rates, high prices and low wages in agriculture strike out in the plots. The first period is in 1709 and is associated with the The Great Frost which was an extraordinarily cold winter in 1708/09 — the coldest winter since 1500 (Luterbacher et al., 2004). The second period is in 1728–1729 and is associated with the English subsistence crisis around 1727–1728, caused by harvest failure and epidemic disease that greatly affected child mortality and the longevity of surviving children (Appleby, 1980; Klemp and Weisdorf, forthcoming). The third period is 1740–1742 and is associated with the spread of epidemic disease following the severe winter of 1739/40 (Post, 1984).

4 Estimation

To estimate the CVAR model, I first find a well-specified, unrestricted VAR model using the general-to-specific methodology. I then impose the reduced-rank restriction and the restrictions implied by the theory on the α and β matrices and estimate the model. Estimations are performed with PcGive version 13.1 and Autometrics version 1.5e in OxMetrics version 6.10 (Doornik and Hendry, 2009; Doornik, 2009) and CATS version 2 in RATS version 6.30 (Dennis, 2006). It is not possible to impose the restrictions on the Γ_i matrices, i.e. on the short-run dynamics. This is not problematic as the short run effects do not matter asymptotically.

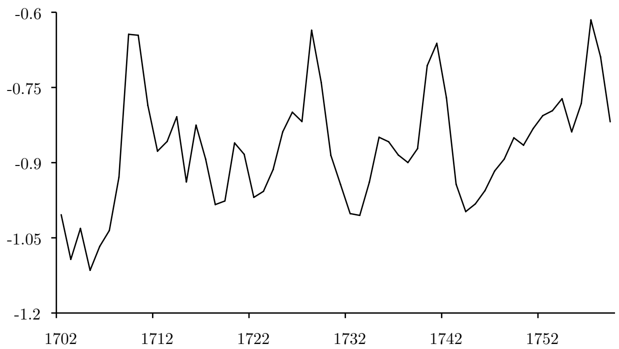
4.1 The Unrestricted VAR Model

I use the automated general-to-specific model-selection procedure, Autometrics, which is implemented in PcGive. Autometrics takes a General Unrestricted Model (GUM) as given and simplifies it, following multiple simplification paths. Each simplified model is required to pass various model specification tests, and formal criteria are used to pick one of possibly many simplified models. The procedure finds and includes significant impulse dummies.

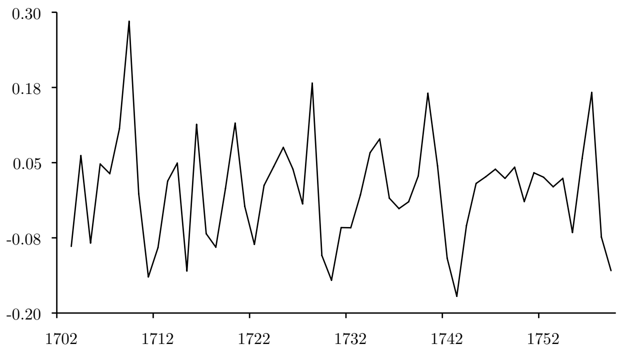
In accordance with Møller and Sharp (2008), I initially restrict the estimation to the period before 1760, which is usually considered within the pre-industrial period. Initial investigations

(a) Crude birth rate

(b) Differenced crude birth rate

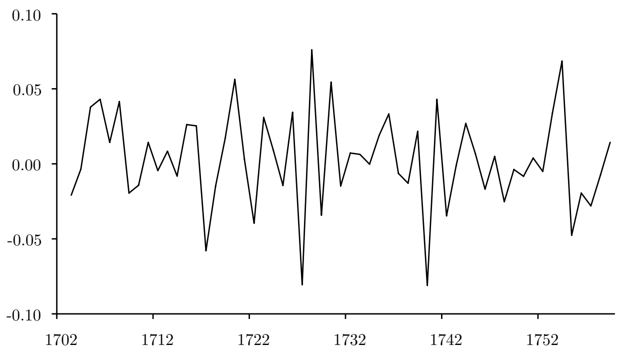


(c) Relative prices

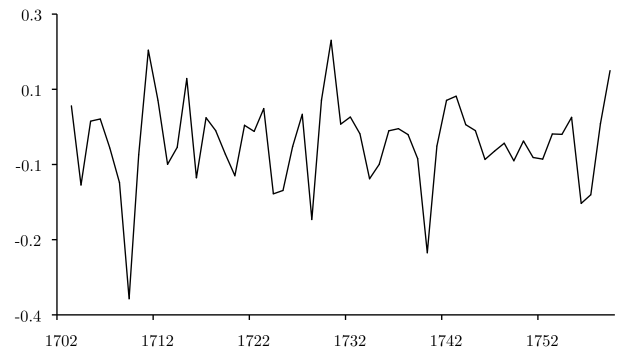


(d) Differenced relative prices

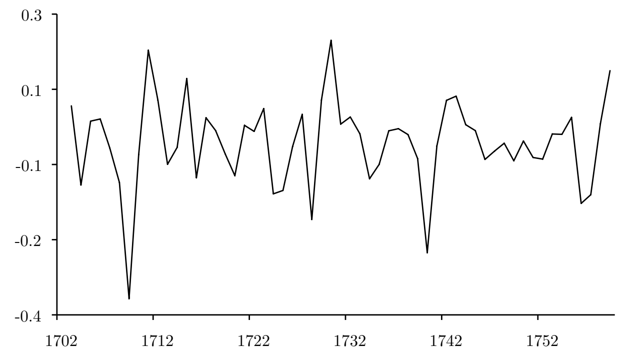
(e) Log of product wages in industry



(f) Differenced log of product wages in industry



(g) Log of product wages in agriculture



(h) Differences log of product wages in agriculture

Figure 1: Levels and differences of the data. Source: Wrigley and Schofield (1981), Broadberry et al. (2011), Clark (2005, 2007) and own calculations.

reveal that the year 1700 produces a large residual, and therefore I exclude it from the baseline sample. The results reported below are thus based on the sample years 1701–1759. I later re-introduce the observation from the year 1700 and include years after 1759 as robustness checks.

I formulate a GUM with three lags and a constant and run Autometrics with the standard settings and outlier detection by large residuals.[3] This procedure results in a VAR with one lag on $n_t, p_t, w_{m,t}$, two lags on $w_{a,t}$, impulse dummies in 1709 and 1740, and a transitory impulse dummy in 1727–1728 corresponding exactly to the three periods described earlier. To perform the cointegration analysis I need to include the same number of lags on all variables. Because each additional lag in the model introduces 16 additional explanatory variables, I exclude the second lag on $w_{a,t}$.[4] I formulate a new GUM with one lag and a constant and re-run Autometrics. The resulting model has one lag on each variable, an impulse dummy in 1709 and 1740 and a transitory impulse dummy in 1727–1728. This model is the baseline unrestricted VAR model. When imposing the reduced-rank restriction, I will allow for a trend and breaks in the cointegrating relations in 1709 and 1740

4.2 The Restricted Model

Before I impose the restrictions, I perform the I(1) and I(2) rank tests implemented in CATS. I include a trend in the cointegrating relations (the CIDRIFT option) and simulate the critical values of the rank test distribution of the model with the dummies and the break using a random walk length of 5000 and 5000 replications. The I(1) test indicates a rank of two ($p = 0.306$) or three ($p = 0.420$). The I(2) test requires a model with a least two lags on all variables. Furthermore, it is not possible to simulate the critical values of the I(2) test. I therefore include a second lag and perform the test without dummies and breaks. The test quite clearly indicates that there are no I(2) trends in the system and that the rank is either two ($p = 0.336$) or three ($p = 0.537$). Based on these criteria it seems reasonable to impose the restriction rank$(\Pi) = 3$.

I impose the identifying restrictions on α and β, remove insignificant coefficients and esti-

[3] I use the large residual detection of outliers instead of the dummy saturation procedure because the former is able to detect the temporary effect of the famine in 1727–1728.

[4] A VAR-model with too few lags tend to produce autocorrelated residuals (see for example Juselius (2006) p. 72). In the present case autocorrelation is not a problem, indicating that a lag length of 1 is suitable.

Δb_t	-0.655*	1.397×10^{-3}	0
	(0.097)	$(0.252 \times 10^{-3})^*$	(–)
Δp_t	17.660	-0.428*	0.481*
	(4.325)*	(0.106)	(0.122)
$\Delta w_{m,t}$	0	0	0
	(–)	(–)	(–)
$\Delta w_{a,t}$	-15.532	0	0
	(4.419)*	(–)	(–)

Table 1: Estimated α matrix

mate the final model. The break in 1740 and the trend were found to be insignificant in the cointegrating relations. The break in 1709 was only significant in the first cointegrating relation, corresponding to a shift in the birth rate. Furthermore, the manufacturing scale effect was insignificant. The restrictions are accepted for the baseline sample with $p = 0.200$. The estimates of α, β' and Φ are given in Table 1, Table 2, and Table 3. Note that there is no standard errors to report for the restricted coefficients. An asterisk indicates statistical significance at the 5 percent level. The implied estimates of the parameters of interest are $\eta_P = -1.397 \times 10^{-3}$, $\eta_W = 10.015 \times 10^{-3}$, $\hat{\mu} = 0.000$ (imposed and accepted), and $\hat{\lambda} = 15.532$. The estimates of the autoregressive parameters are $q = 0.345$, $v = 0.572$, $r = 0.481$, and $u = 0.448$. The negative estimate of η_P means that higher prices lead to a higher birth rate in the following years. This surprising result is robust to changes to the model and the sample, as will be explained further below. However, the effect is small: the elasticity of the population growth factor with respect to prices is less than one seventh of the elasticity with respect to the product wage. It is convenient to obtain the elasticity of the birth rate with respect to prices and wages. If wages and prices are in their long-run equilibrium, $w_{a,t} = \bar{w}_a$ and $p_t = \bar{p}$, it follows that the long-run birth rate is given by $c_b = \bar{b}$. Table 3 thus reports that $\bar{b} = 0.040$.[5] Using this in the formulas on page 4 yields the miniscule point elasticity $\epsilon_P = 0.036$, which means that an increase in prices of one percent increases the birth rate by just about 0.036 percent. The point elasticity of wages is $\epsilon_W = 0.260$ meaning that a one percent increase in wages, increases the birth rate by about 0.260 percent. This estimate is in line with the size of estimates for the mid-sixteenth to the mid-nineteenth centuries period reported in Lee and Anderson (2002), ranging from 0.123 to

[5]For comparison, the mean birth rate in the baseline sample is 0.033.

b_t	p_t	$w_{m,t}$	$w_{a,t}$	B_{1709}
1	0	0	0	-2.516×10^{-3}
(–)	(–)	(–)	(–)	$(0.736 \times 10^{-3})^*$
0	1	0	7.171	0
(–)	(–)	(–)	$(0.839)^*$	(–)
0	0	1	5.456	0
(–)	(–)	(–)	$(0.741)^*$	(–)

Table 2: Estimated β' matrix

0.14.

It is straightforward to back out the estimates of μ and λ. Because $\hat{\mu} = 0$, it follows directly from the definition of $\hat{\mu}$ that $\mu = 0$, which means that there is constant returns to scale in the manufacturing sector. Likewise, given the definition of $\hat{\lambda}$, the value of λ is found to be $\lambda = 0.940$, which means that there is strongly decreasing returns to scale in the agricultural sector. This result is interesting in light of Møller and Sharp (2008), who found constant returns to scale on the aggregate level. It is realistic that the manufacturing sector is much closer to having constant returns to scale than the agricultural sector, because it is much less dependant on land, which is an approximately fixed factor. Because $\mu = 0$, the third coordinate of Φ provides an estimate of g_m. The growth rate of manufacturing technology during the pre-industrial period under investigation is found to be a mere 0.5 percent per year.

Overall, the model appears to describe the data quite well: starting with a parsimonious unrestricted VAR model, the restrictions are accepted on a relatively long time-series sample in the relevant period. The estimates are all in line with the *a priori*, except for the positive effect of prices, and the coefficient sizes are reasonable.

4.3 Robustness

Next I examine the robustness of the results to changes in the sample, different model selection settings, and inclusion of the manufacturing product wage in the fertility equation.

The restrictions are accepted and the conclusions unchanged when 1700 is included in the sample and even when the sample is extended to the period 1700–1769 ($p = 0.056$). The p-value fall below 0.05 after 1770 and continues to decrease as more observations are included, but it

n_t	0.040
p_t	-0.758
$w_{m,t}$	0.005
$w_{a,t}$	0.488

Table 3: Estimated Φ matrix

may be possible to accept the restrictions on an even longer sample by including dummies and breaks in the period after 1759.

I perform the analysis on the baseline sample using variations in the settings of Autometrics. In all models I allow for a break in 1709. First, I try different settings related to the number of included regressors. I use the "huge" model size setting, which generates a model that includes impulse dummies in 1709, 1717, 1731, 1740, 1754 and 1757 and a transitory impulse dummy in 1727–1728. The qualitative conclusions remain the same (the restrictions are accepted with $p = 0.057$), although the both effects are now smaller ($\eta_P = 0.623 \times 10^{-3}$ and $\eta_W = 7.975 \times 10^{-3}$). I then use the "tiny" model size setting, and again the qualitative conclusions are unchanged (the restrictions are accepted with $p = 0.157$) with smaller effects ($\eta_P = 0.358 \times 10^{-3}$ and $\eta_W = 9.248 \times 10^{-3}$).

Using outlier detection by dummy saturation with the default model size setting results in a model with impulse dummies in 1709, 1727, 1728, 1730, 1740 and 1757, the p-value drops to $p = 0.020$, but the estimates remain almost unchanged ($\eta_P = 1.217 \times 10^{-3}$ and $\eta_W = 9.178 \times 10^{-3}$). Changing the start of the sample to 1702 leads to acceptance of the restrictions ($p = 0.057$).

I also included the manufacturing product wage in the fertility equation (Equation 1) completely analogously to the inclusion of the agricultural product wage. The only difference in terms of the imposed restrictions on the matrices is that now $\alpha_{1,3}$ is unrestricted and captures the elasticity of the population growth factor with respect to the manufacturing product wage.[6] Again, the same conclusions result. The effect of the agricultural wages is still significant, but the effect of both prices and manufacturing wages are insignificant, possibly due to multicollinearity, the effect of manufacturing wages more so than the effect of prices.

[6]Furthermore, the interpretation of the coefficients in the β matrix changes.

The overall impression from these robustness checks is that the conclusions are not dependant on idiosyncratic factors.

5 Conclusion

I have formulated a general version of the two-sector Malthusian model in Weisdorf (2008). I derived the related CVAR model and added flexibility to allow for sluggishness in the system. Using new English data for the pre-industrial period, I generated two main findings. First, I found a small but positive effect of prices on fertility. Second, I confirmed the existence of a conventional Malthusian preventive check beyond the first half of the eighteenth century. The analysis demonstrated that the there is constant returns to scale in the manufacturing sector and decreasing returns to scale in the agricultural sector. Furthermore, the growth rate in manufacturing technology is 0.5 percent per year.

The absence of a negative effect of prices on fertility does not support the unified growth theory in Strulik and Weisdorf (2008). It is important to note that these first results are based only on data from one specific country (albeit an important one in relation to the Industrial Revolution) in one specific period. It is therefore unknown whether the results can be generalised. The analysis presented here can easily be performed on data from other countries or periods.

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