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Subaru High-z Exploration of Low-Luminosity Quasars (SHELLQs) – XV. Constraining the cosmic reionization at $5.5 < z < 7$

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**ABSTRACT**

Revealing the cosmic hydrogen reionization history is one of the main goals of the modern cosmology. $z > 5$ quasars (QSOs) have been used as back-lights to investigate the evolution of the intervening intergalactic medium (IGM) during the cosmic reionization since their first discovery. However, due to the small population of luminous QSOs (~130 QSOs known to date), a tight constraint on the reionization history has not yet been placed. In this work, we aim to tighten the constraint using the 93 QSOs ($5.5 < z < 7.1$) recently discovered in the Subaru High-z Exploration of Low-Luminosity Quasars (SHELLQS) project. This is the largest QSO sample used to constrain the epoch of reionization. We measure the mean IGM Ly$\alpha$ transmission and the QSO near-zone size using the UV spectra of these QSOs. The mean IGM Ly$\alpha$ transmission rises above zero at $z \lesssim 6$, indicating the end of the reionization. The near-zone sizes of the SHELLQs QSOs are consistent with sizes spanned by QSOs of lifetime $t_\star \sim 1$–100 Myr in simulations. Due to the scatter created by the low signal-to-noise spectra and large Ly$\alpha$ redshift uncertainty, we cannot conclude whether the redshift evolution of the near-zone size is affected by the reionization effect.

**Key words:** (galaxies:) intergalactic medium – (galaxies:) quasars:absorption lines – (galaxies:) quasars: general – (cosmology:) observations – (cosmology:) dark ages, reionization, first stars

1 INTRODUCTION

The epoch of reionization (EoR) is the last phase change of the Universe. During the EoR, the intergalactic neutral hydrogen turns into hot plasma through photoionization by the UV or X-ray radiation of earlier light sources (Barkana & Loeb 2001). Ionization prevents hydrogen from collapsing into dense systems, leading to a decline in galaxy formation (e.g. Susa & Umemura 2000). Thus, to understand the galaxy formation in the early Universe, it is crucial to study the ionization history of the intergalactic hydrogen, more generally, the intergalactic medium (IGM). The ionization of the IGM competes with the recombination. The former can be constrained by observing the physical properties of the ionizing sources, while the later (IGM recombination) can be constrained by observing the physical properties of the IGM.

The amount of ionizing photons as ionizing sources that contribute to reionization scales with its luminosity function (LF) and ionizing photon escape fraction ($f_{\text{esc}}$) (Madau, Haardt & Rees 1999). Common objects studied for this purpose are (faint) star-forming galaxies

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(SFGs) and quasars (QSOs) due to their abundance and/or high $f_{\text{esc}}$ (e.g. Pawlik, Schaye & van der Werf 2009). In recent observations (e.g. Onoue et al. 2017; Matsuoka et al. 2018c; Kim et al. 2020), QSOs have been ruled out as the major ionizing source by extending the QSO LF to faint ends at $z \sim 6$, and using reasonable assumptions of $f_{\text{esc}}$ for QSOs. Hence, high-$z$ SFGs become more possible contributors over QSOs although the exact number of such SFGs and the evolution of their $f_{\text{esc}}$ are still under debate (e.g. Maseda et al. 2020).

The ionization state of the IGM during the EoR (see McQuinn 2016 and Kakihuchi et al. 2016 for a summary of the evolution of the IGM neutral fraction) is better studied than ionizing sources because of the difficulty in directly observing these low-luminosity sources at high $z$. The first direct evidence of the IGM opacity evolution during the EoR is the rapid decline of the mean Ly $\alpha$ transmission (or transmitted flux ratio) at $z \sim 6$ in the UV continua of QSOs, which indicates the end stage of the EoR (Becker et al. 2001). More studies on the mean Ly $\alpha$ transmission using more lines of sight (LOs) to high-$z$ QSOs also suggest the end of the EoR at $z \sim 5$ or $z \sim 6$ (Becker, Bolton & Lidz 2015a; Barnett et al. 2017; Bosman et al. 2018; Eilers, Davies & Hennawi 2018; Lu et al. 2020; Yang et al. 2020). Other studies on the IGM opacity such as the Ly $\alpha$ transmission on LOSs towards gamma-ray burst (GRB) afterglows (Hartoog et al. 2015), the damping wings of QSOs and GRB afterglows (e.g. Tonati et al. 2006; Mortlock et al. 2011; Greig, Mesinger & Bañados 2019; Wang et al. 2020, also see Miralda-Escudé 1998; Madau & Rees 2000 for theoretical basis), and the dark pixels analyses (e.g. McGreer, Mesinger & D’Odorico 2015) also show rapid increase of the IGM opacity at $z \gtrsim 5$.

Indirect probes of the IGM opacity such as the Ly $\alpha$ LF, the decline of the Ly $\alpha$ emitter fraction towards $z \gtrsim 5$ or 6, the clustering of Ly $\alpha$ emitter and the Ly $\alpha$ equivalent width also suggest the IGM reionization at $z \gtrsim 5$ (e.g. Kakihuchi et al. 2016; Mason et al. 2018; Jung et al. 2020, and reference therein). Towards higher redshifts, the mid-point of the EoR is constrained to be at $z = 7.7 \pm 0.7$ by the Thomson optical depth of cosmic microwave background photon by Planck Collaboration VI (2020) measurement.

Early results from the H$\text{I}$ 21-cm intensity mapping (Ghara et al. 2020, and references therein) pushed the upper limit of the IGM spin temperature up to $z = 20$–25 (Gehlot et al. 2019), and the upper limit of the IGM ionized fraction, $x_{\text{HI}} \lesssim 0.13$, at $z = 9.1$ (Ghara et al. 2020), although current 21-cm mappings are still suffering from large foreground noise.

The global IGM ionization history has been mapped beyond the mid-point of reionization. However, higher order terms such as the geometry and topology, i.e. spatial fluctuation, of the IGM ionization state have not been well resolved. Such higher order terms are critical and being extensively investigated recently (e.g. Greig et al. 2020; Meiksin 2020; Yang et al. 2020) because only models with correct physical parameter set-up can reproduce reionization to high-order terms (Fan et al. 2006), while the global ionization history is easy to be mimicked by altering the model parameters or physics. In practice, the observed spatial fluctuation of the Ly $\alpha$ optical depths through QSO LOSs (Becker et al. 2015b; Bosman et al. 2018; Eilers et al. 2018; Yang et al. 2020) is one of the current biggest hint to the IGM topology. However, barely a current model can reproduce such a feature without invoking extreme physics. For example, models proposing a late reionization (e.g. Keating et al. 2020) fail to reproduce an observed long opaque trough at $z \sim 5.2$ and have a significantly uncertain temperature of the IGM. Models proposing large temperature fluctuations (e.g. D’Aloisio, McQuinn & Trac 2015) predict a more redshift-space extended and later reionization than observations. Models proposing rare bright sources which contribute similarly as galaxies to the UV background (e.g. Chardin, Puchwein & Haehnelt 2017; Galliongo et al. 2019) require a higher spatial density of intermediate-luminosity QSO than that observed in QSO surveys. More models are proposed recently (e.g. Becker et al. 2018; D’Aloisio et al. 2018; Meiksin 2020), dedicated to depicting a more accurate reionization picture.

In observation, to date, only $\sim 130$ $z > 6$ QSOs are used on constraining the (fluctuation of) IGM opacity (Barnett et al. 2017; Bosman et al. 2018; Eilers et al. 2018; Yang et al. 2020). Extending the observation to higher redshift or with a larger sample size is important in strengthening the constraints. Therefore, in this work, we present the constraints on the IGM ionization state with 93 newly found QSOs from the Subaru High-$z$ Exploration of Low-Luminosity Quasars (SHELLQs) project (Matsuoka et al. 2016, hereafter SQI). The SHELLQs project (SQI) is designed for finding low-luminosity QSOs at high $z$ from the ongoing Subaru Hyper Suprime-Cam Subaru Strategic Program survey data (Miyazaki et al. 2012, 2018; Aihara et al. 2018a,b). The larger population of low-luminosity QSOs that can be found compared to the high-luminosity ones will significantly enhance the sample size in the near future. Besides, with the higher spatial density of low-luminosity QSOs, smaller scale IGM fluctuations can be measured. It is also of interest to compare the IGM properties around the low-luminosity QSOs to those around the more luminous ones. Because more luminous QSOs may reside in more massive haloes (see Section 2.2 of Inayoshi, Visbal & Haiman 2020, and reference therein), the IGM around them may have different reionization history (Yu & Lu 2005).

Previously, Ishimoto et al. (2020) presented a pioneering IGM measurement from SHELLQs. They measured the near-zone sizes of 11 SHELLQs QSOs, which were found to have shallow redshift evolution. In this paper, with the rest-frame UV spectra of all the 93 SHELLQs QSOs obtained by the SHELLQs team (SQI; Matsuoka et al. 2018a, hereafter SQI; Matsuoka et al. 2018b, hereafter SQIV; Matsuoka et al. 2019a, hereafter SQVII; Matsuoka et al. 2019b, hereafter SQX), we measure the IGM LOS Ly $\alpha$ transmissions and the QSO near-zone sizes, which are common practices among the EoR studies using QSO UV spectra (e.g. Fan et al. 2006). Detailed descriptions and discussions of these measurements can be found in their corresponding sections. We note that we do not combine the sample in this work with that from other works to improve statistical error, given different data quality and source properties.

This paper is structured as follows: In Section 2, we summarize the observations of SHELLQs. Then we perform continuum fitting on our UV spectra using power-law plus emission lines in Section 3.1 and principal component spectra (PCS) of QSOs in Appendix. B. After that, we measure the IGM Ly $\alpha$ opacity in Section 3.2 and the QSO near-zone sizes in Section 3.3, followed by the result and discussion in Section 4. And finally, we conclude the paper in Section 5. Throughout this paper, we adopt cosmological parameters of $\Omega_{\text{m}} = 0.31$, $\Omega_{\Lambda} = 0.69$, and $h = 0.68$ (Planck Collaboration VI 2020) unless otherwise mentioned.

## 2 Data

We use the UV spectra of the 93 QSOs from SHELLQs. These QSOs are within $5.66 \leq z_{\text{Ly} \alpha} \leq 7.07$, with $-20.91 \lesssim M_{\text{I}} \lesssim -26.18$. The resolutions of the UV spectra is $R \sim 1200$ at $\sim 9000$ Å (observed frame). The signal-to-noise ratios (SNRs) of the QSOs are $\approx 5$ per pixel at 1275–1285 Å (rest frame). 10 of the QSOs have $z_{\text{MgII}}$ and eight of them have $z_{\text{CII}}$. We use $z_{\text{CII}}$ or $z_{\text{MgII}}$ instead of $z_{\text{Ly} \alpha}$ when available because of their smaller redshift uncertainties. We note that
we use $z_{\text{CII}}$ for five-eighths of these QSOs which have both $z_{\text{MgII}}$ and $z_{\text{CII}}$. The information of these QSOs is shown in Table A1. We summarize the observations of the UV spectra, the $z_{\text{MgII}}$, and the $z_{\text{CII}}$ in the following subsections. More detailed descriptions of the observations can be found in corresponding references.

### 2.1 UV spectra

The UV spectra are obtained either with the Gran Telescopio Canarias (GTC) (SQI, SQII, SQIV, SQV) or with the Subaru telescope (SQI, SQII, SQIV, SQVII, SQX). 45 of the UV spectra are taken with Optical System for Imaging and low-resolution Integrated Spectroscopy (OSIRIS; Cepa et al. 2000) mounted on the GTC, with the R2500grism and 1.0-arcsec-wide longslits. The spectral coverage is $\lambda_{\text{observed}} = 7400$–10 300 Å with $R \sim 1500$. The rest of the spectra are obtained with the Faint Object Camera and Spectrograph (FOCAS; Kashikawa et al. 2002) mounted on the Subaru telescope in the multi-object spectrograph mode with the VPH1900 grism and SO58 order-sorting filter, 1.0-arcsec-wide slitlets. The spectral coverage is $\lambda_{\text{observed}} = 7500$–10 500 Å, $R \sim 1200$ (But for QSO J1400+0106, it was observed with 2.0-arcsec-wide longslit). After performing data reduction, all the 1D UV spectra are binned by 1.5 Å per pixel (SQI, SQII, SQIV, SQVII, SQX).

### 2.2 Mg II redshift

The NIR spectra are obtained with the Very Large Telescope (VLT), the Gemini-North telescope, and the Subaru telescope. The NIR spectra are further reduced to measure the Mg II emissions of 10 of the SHELLQs QSOs (SQVI, SQVII; Onoue et al. 2021, hereafter SQXIV). In SQVI, the VLT/X-Shooter observation of three spectra was carried out with VIS (0.55–1.02 μm) and NIR (1.20–2.48 μm) arms. The slit widths for both VIS and NIR arms were 0.9 arcsec. The spectral resolution of VIS and NIR is $R \sim 7410$ and $R \sim 5410$, respectively. The observation of three spectra with the Gemini Near-Infrared Spectrograph (GNIRS; Elias et al. 2006) mounted on the Gemini-north telescope was carried out with 0.675 arcsec slit or 1.0 arcsec slit. The spectral coverage is $\lambda_{\text{observed}} = 0.9$–2.5 μm and $R \sim 760$ or 510. SQVII observed a spectrum with Gemini/GNIRS in queue mode, and the cross-dispersed mode with 321/mm grating, 1.0-arcsec-width slit. The spectral coverage is $\lambda_{\text{observed}} = 0.85$–2.5 μm and $R \sim 500$; they also obtain the K-band spectrum with the Multi-Object Infrared Camera and Spectrograph (MOIRCS; Ichikawa et al. 2006) on the Subaru telescope in the multi-object spectrograph mode with the VPH-K grism. The slit width was 0.8 arcsec, with spectral coverage of $\lambda_{\text{observed}} = 1.8$–2.5 μm and $R \sim 1700$. In SQXIV (detailed observational set-up therein), three K-band spectra were obtained using Subaru/MOIRCS as well.

### 2.3 [C II] redshift

Eight of the SHELLQs QSOs have [C II] redshift measured using Atacama Large Millimeter Array (ALMA) (Izumi et al. 2018, hereafter SQII; Izumi et al. 2019, hereafter SQVIII). Four of the [C II] emissions from the SHELLQs were observed during ALMA Cycle 4 at band 6 (SQIII). The total bandwidth of the observations is ∼7.5 GHz, divided into four spectral windows of width 1.875 GHz. The final velocity resolution is ∼50 km s$^{-1}$ after data reduction. Another three of the [C II] emissions from the SHELLQs were observed during ALMA Cycle 5 at band 6 (SQVIII). The total bandwidths of the observations are ∼7.5 and ∼6.5 GHz, respectively, divided into four or three spectral windows of width 1.875 GHz. The final velocity resolution is ∼100 km s$^{-1}$ after data reduction. Further descriptions about the last [C II] emission measurement will be found in Izumi et al. (in preparation). We adopt the [C II] redshifts for six QSOs which have both Mg II and [C II] observations because Mg II emission arises within the QSO broad-line region, which may suffer from velocity shift from the QSO redshift.

### 2.4 Spectral quality

One of our aims is to investigate the Lyα transmission in the spectra of our sources to understand the IGM evolution over time. The transmission measurement is affected by the noise level of the spectrum and the prediction of the QSO intrinsic continuum. We measure the spectral noise of the UV spectra by first cutting off the noisy spectral edge. The final spectral range we used is shown in Fig. 1. We estimate the median SNR per pixel of each UV spectrum at $\lambda_{\text{rest}} \sim 1250$–1280 Å (Table A1 and Fig. 2). The wavelength windows for SNR estimation of some spectra are slightly adjusted to avoid strong sky emissions. The median SNR of the SHELLQs QSOs is 1.22 ± 0.16 per pixel. The SNRs are low due to the short exposure time of the spectra (see Section 2 for data description and references therein).

### 3 ANALYSIS

In this section, we describe the analysis that was performed on the UV spectra of the SHELLQs QSOs. The most common way to study the EoR with QSOs is to measure the reddened Lyα 1216 Å absorption due to the foreground neutral hydrogen in the QSO UV continua (e.g. Fan et al. 2006; Eilers et al. 2018). The redshift distribution of the Lyα absorption strength at $z \geq 5$ traces the evolution of the IGM during the EoR. The Lyα absorption strength (or the Lyα transmission) is measured by dividing the observed flux of the QSO by its intrinsic continuum. Thus, the measurement requires a determined QSO intrinsic continuum (Section 3.1). At the very end of the reionization ($z \sim 6$), due to the increasing amount of the IGM being ionized, the IGM neutral fraction could be measured through the mean Lyα transmission (Section 3.2). Towards higher $z$ ($z \sim 6$), the high neutral fraction ($x_{\text{HI}} > 10^{-4}$) caused the saturation of the Lyα transmission. Thus, the size evolution of the QSO HII region, which is a more sensitive probe of IGM evolution, should be used. (Section 3.3).

#### 3.1 Continuum fitting

To calculate the transmitted flux ratio blueward of the Lyα emission, the intrinsic continuum should be first determined. There are two main ways to fit and predict the QSO’s intrinsic continuum, i.e. the power-law (plus Lyα + Nv emission line Gaussians) continuum fitting (e.g. Becker et al. 2015b; Bosman et al. 2018) and the principal component analysis (PCA) (e.g. Eilers et al. 2018; Ishimoto et al. 2020). The PCA method is performed by fitting a linear combination of a set of PCs to the unabsorbed part (the red part) of the observed high-$z$ QSOs. The PCs are extracted from a larger sample of low-$z$ QSOs which are free from foreground neutral hydrogen absorption. Recent works such as Eilers et al. (2018) adopt the PCs provided by Pâris et al. (2011), which were extracted from 78 $z \sim 2.9$ QSOs and covered $\lambda_{\text{rest}}$ frame 1000–2000 Å. In general, PCA can better predict the intrinsic flux due to the emission lines.

However, we do not have a large wavelength coverage and high SNRs for a reliable PCA analyses. Therefore, we adopt a more robust method, i.e. a power-law plus Lyα and Nv emission line Gaussians...
fitting (hereafter PWLG). This method is more robust for our data and returns a more stable flux measurement (we found that PCA predicts unrealistic continua in some cases). We discuss the systematic offset of the transmission caused by different continuum measurements with these two methods in Appendix B.

### 3.1.1 Power-law plus Lyα emission as the continuum

We first fit a PWL continuum, $F_\lambda = F_0 \lambda^{-\alpha}$, to the QSO spectrum at the wavelength windows redder than the Lyα emission line and avoid absorptions, noisy regions, and emission lines such as Lyα, N\textsc{v}, and S\textsc{ii}.h. The typical wavelength range for the PWL fitting is 1250 Å $< \lambda_{\text{restframe}} < 1300$ Å, but case by case. The fitting range of PWL index, $\alpha_z$, is fixed at [−1.5, 2.0], which is a common range of PWL index of QSOs (e.g. Lusso et al. 2015). After fitting the PWL continuum and subtracting the observed flux by the continuum, in order to obtain a better prediction of the QSO continuum around the QSO H\textsc{ii} region, we then fit the broad and narrow Lyα emissions and the N\textsc{v} emission with three Gaussians. The intrinsic continuum adopted for the following analysis is then the fitted PWL continuum plus the Lyα and N\textsc{v} emission line Gaussians. We call this PWLG hereafter. Fig. B1 shows some examples of the spectra and their fitted continua.

### 3.2 The IGM opacity

After the intrinsic continuum, $f_{\text{in}}$, is determined, the Lyα transmitted flux ratio (transmission) could be measured by dividing the observed flux, $f_{\text{obs}}$, by $f_{\text{in}}$. At $z_{\text{HI}} < 10^{-4}$, the ionization state of the IGM during the EOR could be expressed by the volume-averaged transmission, $T$, and the CDF of effective Lyα optical depth, $\tau_{\text{eff}}$. The redshift evolution of $T$ indicates the global evolution of the IGM ionization fraction. The CDF of $\tau_{\text{eff}}$ in a redshift bin reflects the large-scale ($\gtrsim 50$ cMpc h$^{-1}$) spatial fluctuations of $\tau_{\text{eff}}$ at that redshift.

#### 3.2.1 Lyα transmission and optical depth

The $T$ in a redshift bin is measured as follows (Fan et al. 2001):

$$T = \left( \frac{f_{\text{obs},v}}{f_{\text{in},v}} \right),$$

(1)

where $v$ denotes the observed frequencies within a corresponding redshift bin. In this work, we adopt a bin size of 50 cMpc h$^{-1}$ ($\Delta z \sim 0.15$ at $z \sim 6$) (Becker et al. 2015b; Bosman et al. 2018), which is commonly used for mean IGM transmission (e.g. Fan et al. 2006; Becker et al. 2015b; Eilers et al. 2018). For each LOS, we measure the inverse-variance-weighted $T$ in each redshift bin in the wavelength window 1100–1176 Å (rest frame), which is in between the Lyβ and Lyα emissions. Fig. 1 shows the redshift bins of $T$ in each

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Figure 1. Spectral coverage of the 93 SHELLQs QSOs rest-frame UV spectra sorted by the QSO $z_{\text{Ly}\alpha}$. The vertical axis is the number of QSO. The green crosses are the beginning/ending wavelength of each spectrum. The red crosses mark the Lyα 1216 Å in the observed frame of each spectrum. The blue crosses are the beginning/ending wavelength of the wavelength window in each spectrum for measuring LOS transmission. The horizontal grey bars of alternating darkness are 50 cMpc h$^{-1}$ bins in each spectrum for measuring transmissions in individual LOS. The seven orange lines are $\Delta z = 0.2$ bins for measuring the spatial fluctuation of optical depth.

Figure 2. The SNR distribution of the SHELLQs QSOs UV spectra in this work. The SNR of each spectrum is measured using the median SNR at $\lambda_{\text{restframe}} \sim 1280$ Å per 1.5 Å pixel. On the scatter plot, different colours correspond to different QSO luminosities as shown in the legend.
The Lyα transmission as a function of redshift measured through LOSs of QSOs. The grey dots with error bars are the transmissions through the LOS of each SHELLQs QSO, with a bin size of 50 cMpc h\(^{-1}\). The red dots with error bars are the inverse-variance-weighted mean transmission of individual SHELLQs QSO transmission in a bin size of \(\Delta z = 0.25\); the error bars are calculated using the error propagation formula of weighted mean. The green and blue dots with error bars are transmissions taken from Eilers et al. (2018) and Bosman et al. (2018), respectively. All error bars are quoted \(\pm 2\sigma\).

![Figure 3](https://academic.oup.com/mnras/article-lookup/10.1093/mnras/stac707)

Table 1. The mean IGM Lyα transmissions and optical depths in the SHELLQs LOSs. The bin size is \(\Delta z = 0.25\). Errors are quoted 2\(\sigma\).

<table>
<thead>
<tr>
<th>(\Delta z)</th>
<th>(\mathcal{T})</th>
<th>(\tau_{\text{eff}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.25</td>
<td>-0.0055 ± 0.0726</td>
<td>&gt;2.6232</td>
</tr>
<tr>
<td>5.50</td>
<td>0.0571 ± 0.0212</td>
<td>2.862 (\pm) 0.4038</td>
</tr>
<tr>
<td>5.75</td>
<td>0.0291 ± 0.0147</td>
<td>3.5386 (\pm) 0.4104</td>
</tr>
<tr>
<td>6.00</td>
<td>0.0103 ± 0.0241</td>
<td>&gt;3.7246</td>
</tr>
<tr>
<td>6.25</td>
<td>0.0169 ± 0.0319</td>
<td>&gt;3.4444</td>
</tr>
<tr>
<td>6.50</td>
<td>-0.0279 ± 0.0207</td>
<td>&gt;3.8761</td>
</tr>
<tr>
<td>6.75</td>
<td>-0.0170 ± 0.0525</td>
<td>&gt;2.9462</td>
</tr>
</tbody>
</table>

SHELLQs spectrum. The resulting \(\mathcal{T}\) are shown in Fig. 3 and Table 1; 2\(\sigma\) error is quoted (we do not include continuum error as done by Becker et al. 2015b and Eilers et al. 2018, and we caution the readers that the transmission error in this work might be larger if we include a larger continuum error due to the lower SNR of our spectra). Note that Table 1 only shows the mean \(\mathcal{T}\) of all SHELLQs LOSs in a redshift bin size of 0.25. We use a bin size of 0.25 for the mean \(\mathcal{T}\) of all SHELLQs LOSs to reduce the error caused by low SNR of our UV spectra and to compare to the result of Eilers et al. (2018).

The Lyα optical depth, \(\tau_{\text{eff}}\), in each bin, is then derived using

\[
\tau_{\text{eff}} = -\ln\mathcal{T}.
\]

If the \(\mathcal{T}\) is lower than the 2\(\sigma\) detection limit, we adopt the 2\(\sigma\) detection limit as the lower limit of \(\tau_{\text{eff}}\). The result is shown in Fig. 4 and Table 1.

3.2.2 Spatial fluctuations of \(\tau_{\text{eff}}\)

We measure the CDFs of \(\tau_{\text{eff}}\) at 5.3 < \(z\) < 6.7 with a bin size of \(\Delta z = 0.2\) following Becker et al. (2015b), Bosman et al. (2018), and Eilers et al. (2018). The LOSs used in each redshift bin of CDF are shown in Fig. 1. To be more specific, in each \(\Delta z = 0.2\) CDF bin, we measure the CDF of \(\tau_{\text{eff}}\) measured using the 50 cMpc h\(^{-1}\) binning in Section 3.2.1. The uncertainties of CDFs are derived by the bootstrap method with 10000 repetitions. The result is shown in Fig. 5.

Fig. 6 shows the CDFs using the full sample versus that using only non-detections. It is shown that the CDFs using the full sample have a small difference compared to that using only non-detections. Because the \(\tau_{\text{eff}}\)s are dominated by non-detections in our sample especially when it approaches higher redshift, we caution that the CDFs measured in this work are only lower limits derived from the low SNR (0.75 < \(\tau_{\text{eff}}\) < 4) sample. The effect due to the detection limit should be taken into account when comparing our result to other observations or simulations. It is also of interest to inspect the highly transparent sightlines to check whether there are any transmission spikes, which can arise from ionized patches in the mostly neutral IGM, especially given our large sample size. However, after inspecting the spectra of the \(\tau_{\text{eff}}\) bins which have detections, due to the large spectral noise, we cannot identify any clear example of highly transparent sightlines.

3.3 Near-zone size \(R_{\text{NZ}}\)

At \(z > 6\), the mean Ly α absorption is saturated due to the high neutral fraction (\(x_{\text{HI}} > 10^{-4}\)), thus probes which are more sensitive to neutral fraction are needed. The size of the H\(_2\) region carved out by QSO was proposed to be such indicator of \(x_{\text{HI}}\) due to its dependency on the proximate \(x_{\text{HI}}\) of the QSO (Cen & Haiman 2000). Assuming a steady QSO emissivity and negligible recombinations, the radius of...
Figure 5. Cumulative distribution functions (CDFs) of Ly $\alpha$ optical depths at $z = 5.3-6.7$ with a bin size of $\Delta z = 0.2$. The numbers in the parentheses represent the number of LOSs in each bin. The dashed lines are $1\sigma$ bootstrapped errors. The different line colours correspond to CDFs in different redshift bins shown in the legend.

Figure 6. The comparison between CDFs of Ly $\alpha$ optical depths using all the $\tau_{\text{eff}}$ measurements (solid lines) and that using only upper limits of $\tau_{\text{eff}}$ (dashed lines). The different line colours correspond to CDFs in different redshift bins shown in the legend.

H $\alpha$ region, $R$, in a uniform partially ionized IGM is written as

$$R = \left( \frac{3\dot{N}_{\text{ion}}t_{\text{q}}}{4\pi n_{\text{HI}}^2} \right)^{1/3},$$

where $\dot{N}_{\text{ion}}$ and $t_{\text{q}}$ are the ionizing photon production rate (which is related to the QSO luminosity) and the lifetime of the QSO, respectively. $n_{\text{HI}}$ is the number density of hydrogen within $R$. Note that $R$ also scale with QSO properties; some recent works (e.g. Chen & Gnedin 2020; Davies, Hennawi & Eilers 2020) show that $R$ is more dependent on $t_{\text{q}}$ than $n_{\text{HI}}$ and cannot reflect the H $\alpha$ region size. In observation, the ionized region around a QSO is then called the QSO near zone, to be distinguished from H $\alpha$ region.

In observation, the near-zone size can be measured by defining it as the proper distance from the QSO to where $T$ first drops down to 0.1 (e.g. Fan et al. 2006; Carilli et al. 2010). The radius measured using this definition is usually called near-zone size and denoted as $R_{\text{NZ}}$.

In practice, a spectrum is first smoothed with a 20 Å boxcar function, then the size is calculated using $R_{\text{NZ}} = D_{\text{QSO}} - D_{\text{QSO}}(1 + z_{\text{q}})$, where $D_{\text{QSO}}$, $D_{\text{GP}}$, and $z_{\text{q}}$ are the comoving distance to the QSO, the comoving distance to where $T$ first drops down to 0.1 (the start of the GP trough), and the QSO redshift, respectively (Fan et al. 2006).

In this work, we adopt the beginning of the first three continuous pixels with $T \leq 0.1$ as where $T$ first drops down to 0.1.

We consider the spectral uncertainty and the redshift error during the error estimation. The uncertainties of $R_{\text{NZ}}$ due to the Ly $\alpha$, Mg II, or [Ca II] redshift errors are $\Delta R_{\text{NZ}} \approx 1.43$, 0.39, and 0.14 pMpc, respectively (Eilers et al. 2017).

Since $R_{\text{NZ}}$ depends on the QSO emissivity, all the measured $R_{\text{NZ}}$ should be corrected to the radius under the same QSO luminosity when discussing the effect of $R_{\text{NZ}}$ (Fan et al. 2006). Following Eilers et al. (2017), we correct the measured $R_{\text{NZ}}$ to a radius, $R_{\text{NZ, corrected}}$, measure at $M_{1450} = -27$ mag using

$$R_{\text{NZ, corrected}} = R_{\text{NZ}} \cdot 10^{0.4(M_{1450}-M_{1450,0})/2.5},$$

assuming the QSO is embedded in a highly ionized IGM ($\Omega_{\text{HI}} = 1.5 \times 10^{-4}$, $\Gamma_{\text{bkg}} = 2 \times 10^{-13}$) and with a QSO lifetime of $10^7$ yr (Eilers et al. 2017). The result is shown in Table A1 and Fig. 7. In Fig. 7, we only show the $R_{\text{NZ, corrected}}$ of SNR > 3 QSOs and use them for further analysis, because it requires such an SNR to determine
the transmission at $T = 0.1$ level to 1σ precision (in Appendix D we perform a test to determine the SNR threshold).

4 RESULT AND DISCUSSION

4.1 The evolution of the mean IGM opacity at $5.5 < z < 7.0$

Fig. 3 shows the inverse-variance-weighted $T$ measure through individual LOS to the SHELLQs QSOs in a bin size of 50 cMpc h$^{-1}$, and the mean $T$ over all the LOSs of the SHELLQs QSOs in a bin size of $Δz = 0.25$ (note that this is not the same as the $Δz = 0.2$ bin size for the spatial fluctuation measurement). The individual observational errors are dominated by spectral noises. Nevertheless, the mean $T$ error is much smaller due to the large sample size. At $z \sim 5.25$, we obtain a negative mean $T$ due to a negative individual $T$ in the bin. Besides, since we only have a few sightlines in the $z \sim 5.25$ bin, the uncertainty of it is large. We can only estimate the lower limit of its $τ_{\text{eff}}$. At $5.50 < z < 6.25$, the mean $T$ using the SHELLQs QSOs is consistent with the $T$ measured using most of the known high-$z$ QSOs ($\sim$130 LOSs) (Eilers et al. 2018; Yang et al. 2020), within 2σ. The median $T$ in this work at this redshift range is 0.011 ± 0.023 and 0.016 ± 0.017 higher than that measured in Eilers et al. (2018) and Yang et al. (2020), respectively. As discussed in previous works (e.g. Eilers et al. 2018; Bosman et al. 2020; Yang et al. 2020), the major artificial factors (the factor that is not intrinsic propriety such as the LOS variance) that contribute to the transmission uncertainty are the continuum uncertainty and the spectral quality.

In terms of the $T$ offset caused by different continuum fitting methods [here we only compare PWL to PCA, discussion on other methods (e.g. Reiman et al. 2020; Durovčiková et al. 2020) is not the scope of this paper], Bosman et al. (2020) found that the reconstructed continuum is biased by $-9.58 \pm 8.22$ per cent using PWL (PCA). Such a bias would result in higher (lower) transmission and lower (higher) optical depth measured by PWL (PCA). In our measurement, the median $T$ measured using PWLG is 0.0029 ± 0.0239 higher than that measured using PCA (Appendix B), which means there is almost no difference except for the uncertainty. We measured a smaller difference because of the broad Lyα emission we fitted. This could enhance the estimated continuum level outside the QSO near zone. Another reason may be the lack of information for PCA fitting in our sample. Since the SNR is low and the spectral wavelength coverage is short, minor continuum features such as the ion emission may not be well resolved by the PCA fitting. Yang et al. (2020) also suggested that the continuum fitting effect is negligible (comparing to other factors) by comparing their $T$ measured using PWL to Fan et al. (2006), who use PWL as well, and Eilers et al. (2018), who used PCA fitting. They reached the conclusion by finding that their result is more consistent with Eilers et al. (2018) than with Fan et al. (2006). The scatter of the transmission offset between using PCA and PWLG in this work is mainly due to some bad fittings that were predicted by PCA. Because of the large spectral noise of the SHELLQs, the weight factor of the higher order PCS were enhanced in order to fit the noisy spectral shape, which results in a non-physical continuum at the blue side of the spectra. Further studies on the $T$ bias due to the spectral quality would help improve the discussion on the $T$ difference.

In this paper, we leave the further discussion on the bias on $T$ due to the SHELLQs spectral quality and data reduction for a future paper when the higher SNR and spectral resolution data are available. The purpose of this paper is solely to show the current constraining power with our large sample size and to learn about the near-zone size of fainter QSOs, not to draw a conclusive constraint from the current data with limited quality. In principle, further discussions are needed on the effect caused by (1) different SNR (2) the masking procedure of foreground absorber as were suggested by Eilers et al. (2018) (3) the spectral zero-point offset. Here, we only discuss them briefly. For the SNR effect, lower SNR data can only provide poor constraint on the lower limit of $τ_{\text{eff}}$ measurements. Besides, due to the low SNR, it is difficult to identify foreground emissions or absorptions and further study the transmission spikes or mask any foreground absorptions. Without masking out foreground absorbers such as damped Lyα systems due to our low SNR and wavelength coverage, the transmission can be underestimated, although some studies (e.g. Becker et al. 2015b) found that the influence is minor. As for zero-point offset, a negative (positive) zero-point offset of the spectrum can result in an underestimated (overestimated) transmission. We examine the offset by visually inspecting the flux distribution in the spectral range where we measure the transmission for each QSO (in Appendix C we attach the distribution of each sightline). We found that approximately seven out of our 93 spectra have minor negative zero-point offsets, which causes $<0.5$ negative offset of their peak transmission in their transmitted flux distribution. We do not remove these QSOs as the effect should be minor to the overall mean transmission.

Fig. 4 shows the mean $τ_{\text{eff}}$ converted from the mean $T$ using the SHELLQs QSOs. At $z > 6$ we can only constrain the lower limit of the $τ_{\text{eff}}$ up to $τ_{\text{eff}} \lesssim 4$ due to the limitation of the SNR. The CDFs of $τ_{\text{eff}}$ (Fig. 5), which are derived using the distribution of $τ_{\text{eff}}$ measured through individual LOSs, indeed significantly suffer from the large spectral noise. The apparent redshift evolution of the CDF in this work is mainly due to the enhanced SNR at the high-$z$ bins. The detected transmitted flux at the low-$z$ bins has a negligible contribution to their CDF shape. As Bosman et al. (2018) discussed, spectra with low SNR would not be helpful for constraining the $τ_{\text{eff}}$ fluctuation at high $z$ because they cannot push the lower limit of $τ_{\text{eff}}$ to the high opacity end. Thus, we do not further compare our result to the recent CDF measurements (Bosman et al. 2018; Eilers et al. 2018; Yang et al. 2020). These measurements have much higher SNR ($SNR \gtrsim 7, τ_{\text{eff, lower limit}} \gtrsim 4$) than the SHELLQs spectra ($0.75 < τ_{\text{eff, lower limit}} < 4$).

We do not further convert the measured optical depth and the corresponding fluctuation into $x_{\text{HI}}$ due to the current data quality. Although it is possible to simply calculate the $x_{\text{HI}}$ assuming a uniform distributed IGM (equation 2 in Fan et al. 2006), the calculated $x_{\text{HI}}$ would be a few orders of magnitude lower than the true value assuming a real density fluctuation of the IGM. It requires simulation which takes the density fluctuation of the IGM into account to derive correct $x_{\text{HI}}$ (Becker et al. 2015a), which is beyond the scope of this paper. Nevertheless, the $x_{\text{HI}}$ at $5.5 < z < 6.25$ may be $\gtrsim 2 \times 10^{-5}$ as measured by Fan et al. (2006) and Yang et al. (2020), since our measured $τ_{\text{eff}}$ is consistent with their measurement at this redshift range.

4.2 $R_{NZ}$, corrected and the inferred QSO lifetime

Fig. 7 shows the QSO near-zone sizes versus redshift after correcting to M1450 = $-27$ ($R_{NZ,corrected}$). The red dots with error bars are the nine SNR > 3 QSOs in the SHELLQs sample (J1417+0117, J1423−0018, J1254−0014, J1347−0157, J1152+0055, J0213−0626, J2210+0304, J2356+0017, J1243+0100). We fit the redshift evolution of $R_{NZ,corrected}$ to these nine QSOs with a
power-law function and bootstrap resampling, and obtain

$$R_{\text{NZ, corrected}} = (9.68 \pm 0.09) \times \left( \frac{1 + z}{7} \right)^{-10.27 \pm 0.27}. \quad (5)$$

We will use this fitted trend for the discussions later on, albeit it can suffer from small sample size and large scatter of the data points. We also fit the trend to all 93 SHELLQs QSOs, which gives

$$R_{\text{NZ, corrected}} = (6.55 \pm 0.09) \times \left( \frac{1 + z}{7} \right)^{-8.23 \pm 0.46}. \quad (6)$$

Comparing to the trend fitted using the nine SNR > 3 QSOs, the trend using the full sample is shallower at $z < 6.5$, and similar at $z > 6.5$. We do not adapt the trend using the full sample because a lower SNR can cause an underestimated $R_{\text{NZ, corrected}}$, although larger sample size can reduce scatter.

Next, we discuss the comparison between our result and other observations (Fan et al. 2006; Carilli et al. 2010; Eilers et al. 2017; Mazzucchelli et al. 2017). The trends of other observations are also shown in Fig. 7. Besides, we show the redshift distribution of QSOs in different observations in Fig. 8. Compared to that in Carilli et al. (2010), who use 27 QSOs at $5.7 < z < 6.5$, we find a similar slope at $z \sim 6$, but with $-1$–2 pMpc larger $R_{\text{NZ, corrected}}$. Because of the high redshift of our QSOs, we are able to extend the measurement to $z > 6.5$ and find a shallower trend. Compared to Eilers et al. (2017) and Mazzucchelli et al. (2017), we find a much steeper redshift evolution. Besides, we find smaller $R_{\text{NZ, corrected}}$ at $z > 6.5$. Our result is more consistent with that of Carilli et al. (2010).

Eilers et al. (2017) have performed a careful check using a subsample which is overlapped with the sample in Carilli et al. (2010) and concluded that the major cause why there are distinct measured trends of the $R_{\text{NZ, corrected}}$ evolution is unclear. The large $R_{\text{NZ, corrected}}$ of the $z < 6$ QSOs measured by previous work (Carilli et al. 2010, and reference therein) are the drivers of their steep slope. There are several main reasons that could cause the large $R_{\text{NZ, corrected}}$ – the definition of $R_{\text{NZ, corrected}}$, the spectral quality, and the sample size. Previous work adjusted the definition of some measured $R_{\text{NZ, corrected}}$ to the second or third drop of the transmission when the absorption profile is bumpy, while Eilers et al. (2017) retained the first drop. Eilers et al. (2017) obtained higher quality data compared to previous work. Eilers et al. (2017) included QSOs with weak emission line feature in their analysis, while they are excluded by previous work. Eilers et al. (2017) measured the trend with larger sample size.

In this work, we do not alter the definition of $R_{\text{NZ, corrected}}$ or exclude QSOs according to their line profile. The causes of the discrepancy thus could be the data quality and the intrinsic property of the sample. We found the data quality is the major cause as described in the following paragraphs.

In terms of data quality, since the SNRs of our UV spectra are low and the redshift uncertainties are larger, we found a larger scatter of the $R_{\text{NZ, corrected}}$ distribution. Besides, the unsuccessful sky subtraction or oversubtraction could alter the measured $R_{\text{NZ, corrected}}$. For example, under the visual inspection of the QSO J1423–0028, which has $R_{\text{NZ, corrected}} \sim 12$ pMpc, we found that there is weak enhanced flux at wavelengths that overlap with strong sky emission wavelengths right shorter than the QSO Ly α emission wavelength. It is possible that the sky emission is not fully subtracted, which creates fake Ly α transmission and producing a large $R_{\text{NZ, corrected}}$. We did not redo the data reduction in this work for a result less affected by the sky emission due to the limited improvement and future plans for obtaining higher SNR and resolution spectra of these QSOs. However, we compare our measurement to the $R_{\text{NZ, corrected}}$ measured by Ishimoto et al. (2020), who used the 11 SHELLQs QSOs that have NIR spectra (Fig. 9). The NIR spectra of these QSOs have higher resolution and longer exposure time. Note that we normalized their measured $R_{\text{NZ}}$ to $M_{1450} = -27$. We find the median $R_{\text{NZ, corrected}}$ measured using the UV spectra is $0.23 \pm 1.35$ pMpc larger than the result measured using the NIR spectra. Our results have good consistency except for the three QSOs – J1202–0057 ($z =\ldots$)
the observed wavelengths of J1202−0057’s proximity zone, which makes it hard to determine the correct flux level from the QSO. J0859+0022 suffer from the sky emission as well, while the determination of continuum level also contributes to its larger $R_{NZ,\text{corrected}}$. The UV spectrum of J0921+0007 is less affected by the skyline, the bumpy absorption structure and the data quality is the cause.

In terms of the QSO intrinsic properties, the exclusion of the QSOs with lineless UV spectra should not affect the result much. In addition to the evidence that the lineless QSOs are not significantly different from others (Diamond-Stanic et al. 2009), we checked the correlation between the QSO Ly $\alpha$ equivalent width (EW) and $R_{NZ,\text{corrected}}$ and found no strong correlation (Appendix E). Thus, the exclusion of the lineless QSOs will only affect the measured $R_{NZ,\text{corrected}}$ trend by reducing the sample size. However, further study on checking the correction between higher UV ionization line EW and $R_{NZ,\text{corrected}}$ should be performed since there are many factors affecting the observed QSO Ly $\alpha$ EW, such as foreground self-shaded cloud. The redshift distribution difference between each sample could also contribute to different fitted trend. In Fig. 8, we plot the redshift distributions of QSOs measured in this work and literature (Fan et al. 2006; Carilli et al. 2010; Eilers et al. 2017; Mazzucchelli et al. 2017). Studies using QSOs with lower median redshift (Fan et al. 2006; Carilli et al. 2010) measured steeper trend than those using QSOs with higher median redshift (Eilers et al. 2017; Mazzucchelli et al. 2017).

Now, to figure out the cause of $R_{NZ,\text{corrected}}$ evolution, we compare the observed $R_{NZ,\text{corrected}}$ to simulations (Wyithe, Bolton & Haehnelt 2008; Keating et al. 2015; Eilers et al. 2017; Chen & Gnedin 2020; Davies et al. 2020). We overplot the Keating et al. (2015), Davies et al. (2020), and Chen & Gnedin (2020) simulations in Fig. 7.

The simulations all use similar radiative transfer simulation with that of Bolton & Haehnelt (2007). However, they assumed different background reionization models and QSO properties, which affect the evolutionary trend and size of $R_{NZ,\text{corrected}}$. Wyithe et al. (2008) assumed a steep rise of $\Gamma_{\text{big}}$ and QSOs reside in dense, biased region and found their result consistent with the Fan et al. (2006), whose work observed steep slope and large $R_{NZ,\text{corrected}}$ at $5.7<z<6.4$. Keating et al. (2015) used high spatial resolution simulation which was able to resolve the Lyman Limit Systems (LLSs). They assumed a highly ionized IGM ($\log \Gamma_{\text{big}} = -12.8$) at $5.5<z<7$, a Case B recombination coefficient, $t_{\text{rec}} = 1\text{Myr}$, and different host halo masses of QSOs. They found a weak dependency of $R_{NZ,\text{corrected}}$ on the host halo mass, and $R_{NZ,\text{corrected}}$ consistent with Carilli et al. (2010) while the slope is shallower. Eilers17 and Davies20s’ simulation results are consistent with Eilers17’s observation. Their simulations are inconsistent with Fan06 and Carilli10s’ observation. Eilers et al. (2017) suggest that the consistency between their observation and simulation indicates that the observed evolution of $R_{NZ,\text{corrected}}$ is due to cosmic expansion, rather than any reionization effect. Chen & Gnedin (2020) also used simulation which is able to resolve the LLSs, found their result consistent with Eilers et al. (2017) observation. Both Eilers et al. (2017) and Chen & Gnedin (2020) found the observed $R_{NZ,\text{corrected}}$ to be consistent with that spanned by QSOs with $t_{\text{rec}} \approx 1\text{Myr}$.

Now we compare our observed $R_{NZ,\text{corrected}}$ to the simulations. At $z < 6.5$, Wyithe et al. (2008), Keating et al. (2015), and the $R_{NZ,\text{corrected}}$ of QSOs with $t_{\text{rec}} \approx 100\text{Myr}$ in Davies et al. (2020) are consistent with the median $R_{NZ,\text{corrected}}$ of our observation ($5.78 \pm 1.95\text{pMpc}$) within 1$\sigma$. Due to the large scatter of our data, we cannot distinguish between the evolution due to the reionization effect and the evolution simply due to cosmic expansion. However, we can constrain the median $t_{\text{rec}}$ of our sample to be $t_{\text{rec}} \approx 1-100\text{Myr}$, according to the assumed $t_{\text{rec}}$ of the simulations. Physical causes of unexpectedly large or small $R_{NZ,\text{corrected}}$ Scattered far above or below the median can be examined once higher SNR spectra and higher ionization line redshifts of these QSOs are obtained. At $z > 6.5$, the $R_{NZ,\text{corrected}}$ with $t_{\text{rec}} = 0.03$ or 1 Myr in Chen & Gnedin (2020) are consistent with the median of our data ($3.07 \pm 1.12\text{pMpc}$) within 1$\sigma$; the $R_{NZ,\text{corrected}}$ with $t_{\text{rec}} = 30\text{Myr}$ in Chen & Gnedin (2020) is consistent with the 95 percentage distribution of our data. Our QSOs seem to have a smaller $t_{\text{rec}}$ at $z > 6.5$ compared to that at $z \approx 6.5$. We visually inspected the $z > 6.5$ SHELLQs with $R_{NZ,\text{corrected}} < 2$. These QSOs have either broad absorption lines near Lyα emissions (e.g. J1205−0000), or (mostly) weak Lyα emissions or Lyα damping wings, which are very likely due to the foreground Lyα absorber or high IGM neutral fraction. It is of interest to study the fraction of these weak Lyα emission QSOs since it is crucial for us to understand $t_{\text{rec}}$ and LLSs at high $z$ (Chen & Gnedin 2020; Ishimoto et al. 2020). However, we did not carry out such analysis because of the low spectral quality and the scope of this paper. Overall, our result suggests that the median QSO age is consistent with $t_{\text{rec}} \approx 1\text{Myr}$. The steeper evolutionary trend of our $R_{NZ,\text{corrected}}$ at $5.5 < z < 7.1$ comparing to the combined trend of Davies et al. (2020) and Chen & Gnedin (2020) is possibly due to the increased Lyα transmission at $z \approx 5.5$. Further discussion about the small $R_{NZ,\text{corrected}}$ and $t_{\text{rec}}$ of the SHELLQs QSOs using the NIR spectra of SHELLQs can be found in Ishimoto et al. (2020).

5 CONCLUSION

In this paper, we constrained the reionization history using 93 QSOs at $5.6 < z < 7.1$ discovered by the SHELLQs project. We use the UV spectra of these 93 QSOs obtained in SQI, SQII, SQIV, SQVII, SQX. Although these spectra have low spectral resolution ($R \approx 1200$) and low SNR (SNR $\sim 1.22$ per 1.5 Å pixel), the sample size and its redshift coverage in this work are the largest compared to those from other studies (e.g. Eilers et al. 2018; Yang et al. 2020). Thus, we are able to witness a more completed reionization history in redshift space. By measuring the redshifted Ly $\alpha$ absorptions in the QSO spectra due to foreground neutral hydrogen, we constrained the redshift evolution of the mean IGM Ly $\alpha$ opacity. Besides, we also measured the proximity zone sizes of these faint QSOs.

For the mean IGM Ly $\alpha$ opacity, we found $\tau_{\text{eff}} \sim 2.86 (3.54)$ at $z = 5.5 (7.57)$, which is consistent with other IGM opacity measurements (e.g. Eilers et al. 2018; Yang et al. 2020). At $z \gtrsim 6$, the Ly $\alpha$ transmission started to be consistent with zero, indicating highly opaque IGM at the corresponding redshift, i.e. the reionization is ending at $z \sim 6$. Due to the low SNR of our spectra, we only obtained lower limits of $\tau_{\text{eff}} (\tau_{\text{eff}} \gtrsim 3.5)$. The measurement on the LOS $\tau_{\text{eff}}$ fluctuation is limited by the SNR limit as well. To compare with the $\tau_{\text{eff}}$ fluctuations in different reionization models (e.g. Becker et al. 2018; Keating et al. 2020), a $\tau_{\text{eff}} \gtrsim 2$−$8$ is required.

For our QSO proximity zone measurement of RNZ, our sample size covers the widest redshift range ($6.0 < z < 7.2$). We found a redshift evolutionary trend similar to that measured by Carilli et al. (2010), steeper than that measured by Eilers et al. (2017) and Mazzucchelli et al. (2017). The difference in the measured trend could be possibly due to our sky-emission-contaminated sample at $z < 6$ and the inclusion of QSOs with damping wings or foreground absorbers at $z \gtrsim 6$. Due to the large scatter of our RNZ, we cannot tell whether the evolutionary trend is due to cosmic expansion or any
reionization effect when comparing to simulations. The $R_{NZ}$ corrected size is consistent with the size spanned by QSOs which have $t_{q} \sim 1–100$ Myr in simulations (Wyithe et al. 2008; Keating et al. 2015; Chen & Gnedin 2020; Davies et al. 2020).

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DATA AVAILABILITY

The data used in this article are available upon request to the corresponding author.

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APPENDIX A: INFORMATION OF THE QSOS IN THIS WORK

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Mortlock D. J. et al., 2011, Nature, 474, 616
Table A1. Information of the QSOs used in this work.

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<th>Object</th>
<th>Redshift</th>
<th>Redshift line</th>
<th>Redshift reference</th>
<th>$M_{1580}$ (mag)</th>
<th>$M_{1580}$ reference</th>
<th>Instrument$^a$</th>
<th>Instrument reference</th>
<th>SNR (per pixel)</th>
<th>$R_{NZ, corrected}$ (pMpc)</th>
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<td>SQX</td>
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$^a$ Instrument: F = FUSE, S = STIS, SII = MIKE, HR = HIRES.
### Table A1 – continued

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<th>M1450 reference</th>
<th>Instrumenta</th>
<th>Instrument reference</th>
<th>SNR (per pixel)</th>
<th>R_{NZ, corrected} (pMpc)</th>
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</tbody>
</table>

Note. *F:FOCAS/Subaru; O:OSIRIS/GTC.

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**APPENDIX B: CONTINUUM PREDICTION USING PRINCIPLE COMPONENT ANALYSIS**

In this section, we aim to compare the Ly α transmission and R_{NZ, corrected} measured using PWLG to PCA. Because some literature (e.g. Fan et al. 2006; Yang et al. 2020) use PWL fitting, while some (e.g. Bahados et al. 2018; Davies et al. 2018; Eilers et al. 2018; Ishimoto et al. 2020) use PCA, the measured reionization history could have a potential bias due to the fitting method (Bosman et al. 2020).

We here only briefly describe how we fit the continuum using PCA (details and mathematical background about reconstructing QSO continua with PCA could be found in Páris et al. 2011). To predict the intrinsic QSO continuum bluer than the Ly α emission with PCA, a set of PCS should be first determined. Then a linear regression of the continuum redder to the Ly α emission using the PCS as the input models is performed. The linear combination of the PCS blueward to the Ly α emission is then called the predicted intrinsic continuum of the QSO. In this work, we use the PCS by Páris et al. (2011), which were extracted from 78 SDSS QSOs at z ~ 2.9. We use six PCS to fit the SHELLQs QSO continua instead of all 10 continua provided by P11, because fitting with higher order PCS would be too sensitive to spectral noise and therefore predict unrealistic continuum (Eilers et al. 2017, SQXI).

Fig. B1 shows examples of continuum fitting using PWL and PCA. In Figs B2 and B3, we plot the transmission and R_{NZ, corrected}, respectively, measured using PCA versus using PWL. It is clearly seen that, although with scatter, the results measured using PWL and PCA are consistent.
APPENDIX C: FLUX DISTRIBUTIONS AND ZERO-FLUX OFFSETS

The Lyα transmission measurement can be biased if the spectrum is not well calibrated, especially when the transmitted flux is small. For a well-calibrated spectrum, assuming no transmitted flux (the IGM is highly neutral), the probability density function (PDF) of the measured flux should be a Gaussian centred at 0. If there is transmitted flux, the left wing of the distribution will remain a Gaussian centring at 0, while the right wing will deviate due to the positive transmission. If the spectrum is not well calibrated, the centre of the PDF will be biased (which we say the spectrum has a zero-flux offset), or the left wing of

Figure B1. Four examples of continuum fitting in this work. The black, green, red, and blue lines are the smoothed QSO spectra, the spectral noise, the fitted PWLG continua, and the fitted PCA continua, respectively.

Figure B2. Lyα transmission measured using PCA continuum versus that measured using PWLG continuum. Each data point is the mean transmission in a 50 cMpc$^{-1}$ bin on an LOS. Colours coded by the redshift of the bin, as shown in the legend.

Figure B3. $R_{NZ,corrected}$ measured using PCA continuum versus that measured using PWLG continuum.
the PDF will have a non-Gaussian distribution. In Figs C1, C2, and C3, we plot the PDF of the QSO flux at the wavelength range where we measure Lyα transmission for each SHELLQs QSO, to check if there is any zero-flux offset. Approximately seven out of our 93 spectra have minor negative zero-point offsets (J0923+0402, J0206−0225, J1429−0008, J1406−0144, J2255+0251, J0001+0000, J0112+0110), which can result in a slightly underestimated mean transmission.

Figure C1. The PDF of the QSO flux at the wavelength range where we measure Lyα transmission for each SHELLQs QSO. The fluxes are in unit of \(10^{-20}\) erg s\(^{-1}\) cm\(^{-2}\) \(\text{Å}\)^\(^{-1}\). The grey lines are plotted centred at flux = 0 for visual comparison to the PDFs’ centres.
Figure C2. continued.
APPENDIX D: THE MINIMUM SNR REQUIREMENT FOR NEAR-ZONE SIZE MEASUREMENT

To measure the QSO near-zone size ($R_{NZ}$), the transmission = 0.1 level needs to be well determined. Lower SNR can hamper the transmission level determination and therefore result in an underestimated $R_{NZ}$. Here, we preform a simple test to evaluate what SNR is required to measure an unbiased near-zone size. In the test, we use real high-SNR, $z > 6$ QSOs spectra from the open-access database IGMspec (Prochaska 2017) as the real spectra from QSOs, add spectral noise to the spectra, and compare the noise-added $R_{NZ,\text{corrected}}$ ($R_{NZ,\text{corrected, mock}}$) to their real $R_{NZ,\text{corrected}}$ ($R_{NZ,\text{corrected, real}}$).

| Figure C3. continued. |
APPENDIX E: THE RELATION BETWEEN QSO LYA EQUIVALENT WIDTH AND THEIR NEAR-ZONE SIZES

In some earlier works (e.g. Fan et al. 2006; Carilli et al. 2010), QSOs with weak emission line features are excluded from the QSO near-zone size analysis, because these QSO may have different intrinsic spectra, which may lead to erroneous result if the measurement is done using the same continuum fitting method. In this work, to check whether QSO emission line feature (especially around the QSO Ly\(\alpha\) emission) affects the measured \(R_{\text{NZ, corrected}}\) trend, we naively plot \(R_{\text{NZ, corrected}}\) of our QSOs as a function of their EWs of the Ly\(\alpha\) emission line, EW(Ly\(\alpha\)). The result is shown in Fig. E1. No clear correlation between \(R_{\text{NZ, corrected}}\) and EW(Ly\(\alpha\)) is found in Fig. E1.

![Figure D1](image-url)

**Figure D1.** The median fractional difference in the near-zone size between \(R_{\text{NZ, corrected real}}\) and \(R_{\text{NZ, corrected mock}}\) as a function of SNR. The black dots with error bars are the medians of all 34 QSOs. The orange dots with error bars are the medians of QSOs which have \(R_{\text{NZ, corrected real}} < 5\). The blue dots with error bars are the medians of QSOs which have \(R_{\text{NZ, corrected real}} > 5\).

In detail, we first select \(z > 6\) QSO spectra from IMMSpec which have M1450 measurements. Only the 34 QSOs from Eilers et al. (2017) match our criteria. We assume that the spectra of these QSOs are the real spectra whose spectral noises are negligible. We degrade the spectral resolutions of these spectra to \(R = 1200\), the typical spectral resolution of our SHELLQs spectra. We measure the near-zone sizes of these degraded spectra and use them as the real near-zone sizes (\(R_{\text{NZ, corrected real}}\)). After that, we add Gaussian noise to these spectra and measure their noise-added near-zone sizes (\(R_{\text{NZ, corrected mock}}\)). We measure the noise-added near-zone sizes for 30 different assumed noise levels (SNR = [0.1, 5.9] (per 1.5 \(\AA\) pixel) with an increment of 0.2). In Fig. D1, for each assumed SNR, we plot the median near-zone size fractional difference (\(\frac{R_{\text{NZ, corrected mock}} - R_{\text{NZ, corrected real}}}{R_{\text{NZ, corrected real}}}\)) of the 34 QSOs. It can be seen that at SNR < 3, as the SNR increases, the level of underestimate decreases. The level of underestimation saturates at SNR > 3, which means that to measure an unbiased \(R_{\text{NZ, corrected real}}\), we need SNR > 3.

![Figure E1](image-url)

**Figure E1.** \(R_{\text{NZ, corrected}}\) versus EW(Ly\(\alpha\)) of the SHELLQs QSOs. The different colours of the data points correspond to different QSO redshifts shown in the legend.

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