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Errors in positioning of borehole measurements and how they influence seismic inversion

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Abstract
Inversion of seismic data using information from horizontal wells is often hampered by cumulative well-location errors. These errors can have a significant influence on the final subsurface model derived from the data. To achieve a proper data integration and arrive at correct uncertainty estimates, we formulate the problem in a fully probabilistic framework and present a numerical approach for improving subsurface imaging using uncertain well-log data and their uncertain locations as well as uncertain seismic data. The result is improved model error quantification in the seismic inversion process.

KEYWORDS
imaging, inverse problem, uncertainty quantification, modelling, seismic

INTRODUCTION
Quantifying uncertainties in subsurface imaging generated from theoretical models, geological analogs and field measurements, possibly in areas of difficult access, offers great challenges. Geophysical error quantification methods are well-known and described in, for example, Tarantola (2005), but in the application of these methods, measurement location data are mostly considered to be free from errors. In most cases, the assumption of having the precise location of measured data is enough to create a convenient and approximate model, but in certain cases where we rely on remote interpretations of indirect measurements at different scales, errors in the location coordinates of the acquired data may become significant. Hence, depending on the problem, location errors may appear as hidden measurement errors, having consequences for our interpretations, ranging from mild to severe.

Obtaining precise location data is in many cases difficult and, in some cases, impossible. This can be seen in a wide range of engineering applications where physical access is difficult, such as drilling operations, satellite positioning and planetary exploration.

Uncertainty quantification in connection with imaging of the subsurface has long been a topic of study due to its importance in how accurate and reliable our models can be. Computational methods for probabilistic seismic inversion using linearized methods (Buland & Omre, 2003; Hansen et al., 2006) or Markov chain Monte Carlo methods (Mosegaard & Tarantola, 1995; Sambridge & Mosegaard, 2002) have been used, such as in studies carried out by Zhu and Gibson (2018) and Stuart et al. (2019). In such studies, there is a need of keeping several parameters of the problem fixed to reduce the computation cost. Location parameters are often amongst the fixed numbers, and, as we shall see, this may have important consequences for the outcome of the analysis.

In a study carried out by Winkler (2017), the uncertainties in wellbore locations were incorporated and he formulated a probabilistic inverse problem using Bayesian networks. Other studies have also developed methods of assessing the uncertainties in resistivity well logs and the trajectory of the wells...
such as Kullawan et al. (2014). In Eidsvik and Hokstad (2006), seismic data were used in the form of vertical seismic profile travel times to estimate the well positions, the earth model and the seismic velocities.

Unfortunately, joint uncertainty analysis of seismic data, wellbore data and position data is still a poorly developed field. Following the numerical approach presented by Fernandes and Mosegaard (2021) where the uncertainty in well locations was cumulatively propagated throughout the trajectory of the well, we present an integrated seismic inversion formulation. In this fully probabilistic study, the approach of Tarantola and Valette (1982) and Tarantola (2005) is used in a case where positions of the physical data are also accounted for as data. The model parameters to be determined in our study are subsurface acoustic impedances and well trajectory coordinates at which seismic data are constrained to well-log data.

In our formulation, we consider errors in the seismic data, well data and wellbore locations. The cumulated errors in the positions on the well trace are considered in three dimensions, that is, there have three spatial coordinates – depth and two-dimensional horizontal location. The challenge is to correctly integrate uncertainties of the seismic data and all other (dependent) sources of data. The mislocation of a well can interfere with the seismic inversion and introduce errors in subsequent subsurface interpretations. In our study, the variability of the subsurface structure was simulated, from which the uncertainty of the model could be calculated.

This paper presents a complete approach to seismic inversion and is a follow-up to the study of Fernandes and Mosegaard (2021) on uncertain spatial location data. Using a Monte Carlo formulation to compute model realizations and to assess their variability (uncertainty), we aim to provide an improved basis for more realistic interpretations of geological structures.

**THEORETICAL FORMULATION**

Our mathematical formulation is rooted in a probabilistic approach (Bayes, 1763) and based on the formalism of Tarantola and Valette (1982), adapted to the combined geosteering and seismic inverse problem. For unknown elastic parameters \( \mathbf{m} \) and well-position parameters \( \mathbf{r} \), the solution to the inverse problem is given by the joint posterior probability distribution (we ignore normalization constants here and in the following):

\[
\sigma_{m,r} (\mathbf{m}, \mathbf{r}) = \rho_{m,r} (\mathbf{m}, \mathbf{r}) L_{d,c} (\mathbf{m}, \mathbf{r}),
\]

where \( \rho_{m,r} (\mathbf{m}, \mathbf{r}) \) is the joint prior probability distribution of the unknowns \( \mathbf{m} \) and \( \mathbf{r} \) and \( L_{d,c} (\mathbf{m}, \mathbf{r}) \) is a joint likelihood function measuring the fit between the combination of observed seismic- and well-position data \( (d_{\text{obs}}, c_{\text{obs}}) \) and the corresponding computed data \( g_{m,r} (\mathbf{m}, \mathbf{r}) \). Our forward relation is as follows:

\[
\begin{pmatrix} d \\ c \end{pmatrix} \equiv g_{m,r} (\mathbf{m}, \mathbf{r}) = \begin{pmatrix} g_m (\mathbf{m}) \\ g_r (\mathbf{r}) \end{pmatrix},
\]

where \( d \) and \( c \) are the computed seismic data and well location coordinates, respectively, \( \mathbf{m} \) is the physical subsurface parameter (in our case acoustic impedance), and \( \mathbf{r} \) is the true well position. \( g_{m,r} \) is the function that maps true parameters into (noise-free) data.

Our likelihood function can be expressed as follows (Tarantola & Valette, 1982):

\[
L_{d,c} (\mathbf{m}, \mathbf{r}) = \rho_{d,c} (g_{m,r} (\mathbf{m}, \mathbf{r})),
\]

where \( \rho_{d,c} \) is the joint prior/noise distribution of our seismic data and well position data. Since the noise on the seismic...
data and the well position data are statistically independent, we have
\[ \rho_{d,c} (d,c) = \rho_d (d) \rho_c (c) \]  
(4)
and this gives us the following expression for the joint posterior:
\[ \sigma_{m,r} (m,r) = \rho_{m,r} (m,r) L_{d,c} (m,r) = \rho_{m,r} (m,r) L_d (m) L_c (r), \]
(5)
where \( L_d (m) = \rho_d (g_m (m)) \) and \( L_c (r) = \rho_c (g_r (r)) \). Considering that the prior on the subsurface parameters are only available conditioned on the well position, namely as \( \rho_{m|r}(m|r) \), we write \( \rho_{m,r} (m,r) = \rho_{m|r}(m|r) \rho_r (r) \), and get
\[ \sigma_{m,r} (m,r) = L_d (m) L_c (r) \rho_{m|r}(m|r) \rho_r (r). \]
(6)

The subsurface prior

Our conditional prior on the model parameters \( \rho_{m|r}(m|r) \) is given as follows: We use Cartesian coordinates \( (x, y, z) \) to describe positions in the subsurface, and our acoustic impedance is represented by a positive real function \( m(x, y, z) \) over the space. We now chose a family of orthogonal and normalized base functions \( \varphi_1(x, y, z), \varphi_2(x, y, z), \ldots \) and parameters \( m_1, m_2, m_3, \ldots \) such that
\[ m (x,y,z) = \sum_{n=1}^{\infty} m_n \varphi_n (x, y, z). \]
(7)
In practical implementation, we use only a finite number of base functions (e.g. \( M \)) and work with the (least-squares) approximation
\[ m (x,y,z) \approx \sum_{n=1}^{M} m_n \varphi_n (x, y, z). \]
(8)
where the now \( M \) coefficients \( m_n \) are our model parameters and the components of \( m = (m_1, m_2, \ldots, m_M) \). The coefficients \( m_n \) are the scalar products of \( m(x, y, z) \) and \( \varphi_n (x, y, z) \), defined as
\[ m_n = \langle m, \varphi_n \rangle \equiv \int_{\mathcal{M}} m(x,y,z) \varphi_n (x, y, z) \, dx \, dy \, dz, \]
(9)
where \( \mathcal{M} \) is the model parameter space. This follows from the fact that the functions \( \varphi_n \) are orthogonal \( (\langle \varphi_k, \varphi_l \rangle = 0 \) for \( k \neq l \)) and normalized \( (\langle \varphi_k, \varphi_k \rangle = 1 \) for all \( k \)). We now define \( \rho_{m|r}(m|r) \) as a Gaussian over the \( m \)-space with zero mean and covariance \( C_\rho \), conditioned on the linear subspace
\[ \mathcal{W} = \left\{ m \mid \sum_{n=1}^{M} m_n \varphi_n (r_k) \approx m (r_k) \text{ for } k = 1, \ldots, K \right\}, \]
(10)
where \( r_k \) is the position of the \( k \)-th the well point. This means that realizations of \( \rho_{m|r}(m|r) \) are weighted sums of our kernels, all fitting the given values at the well points. Details of how to compute realizations of \( \rho_{m|r}(m|r) \) can be found in the Appendix.

The well location prior

In this study, we assume that the prior information about the well positions \( \rho_r (r) \) is uniform (constant), and hence that all information about the well trace is obtained from drilling data and their estimated position uncertainties. All this information is given by \( L_c (r) \). This leads to
\[ \sigma_{m,r} (m,r) = L_d (m) L_c (r) \rho_{m|r}(m|r). \]
(11)

The seismic likelihood

The seismic likelihood function is given by
\[ L_d (m) = \rho_d (g_m (m)), \]
(12)
and in this study we assume that the seismic noise is Gaussian, leading to
\[ L_d (m) = \mathcal{K} \exp \left( -\frac{1}{2} \left( d_{\text{obs}} - g_m (m) \right)^T C_n^{-1} \left( d_{\text{obs}} - g_m (m) \right) \right). \]
(13)
where \( d_{\text{obs}} \) is the vector of all the observed data, \( g_m \) is the function mapping the model parameters into seismic data, \( C_n \) is the covariance matrix of the seismic noise and \( \mathcal{K} \) is a normalization constant. In our numerical example below, the ‘observed’ data \( d_{\text{obs}} \) are synthetic, the unknown model parameters of \( m \) are acoustic impedances and \( g_m \) is convolution with a known wavelet.

The well location likelihood

Following Fernandes and Mosegaard (2021), if the well position measurements are \( c = (c_1, \ldots, c_K) \), where \( c_i \) is the
Location of the $i$th measurement starting from the surface, the uncertainty of $c$ is

$$\rho_c(c) = \prod_{j=1}^{K-1} \rho_c(c_{i+1}|c_i),$$ \hspace{1cm} \text{(14)}$$

expressing the accumulation of uncertainty, since the position and uncertainty of point $i+1$ depends on the position and uncertainty of the point $i$.

The well position likelihood function is then

$$L_c(r) = \prod_{i=1}^{K-1} \rho_c(h_{i+1}(r_i)),$$ \hspace{1cm} \text{(15)}$$

where $r_i$ and $r_{i+1}$ are connected through $r_{i+1} = h_{i+1}(r_i)$.

**NUMERICAL METHOD**

We follow the extended Metropolis sampling strategy outlined in Mosegaard and Tarantola (1995). A flow diagram of the algorithm is shown in Figure 1. The characteristic feature of this method is that an algorithm sampling the prior (when run independently) is used to randomly propose perturbations of the current model $x = (m, r)$. Accepting the proposed models using a likelihood-ratio acceptance probability will then ensure correct (asymptotic) sampling of the posterior distribution.

The challenges in our implementation come from the interdependencies between the well data, the well location measurements and the seismic data. As explained above, well location measurements at one point during the drilling are always conditioned on the (uncertain) location of the previous, shallower point. The seismic data, on the other hand, depend on the (unknown) subsurface parameters. In the well, the subsurface parameters are known with higher precision than in the surroundings, but the location of the well points is uncertain.

To satisfy all these interrelated, soft constraints, we proceed in the following way (see Fig. 2):

In each iteration, choose between perturbing the location of a single well point $r^{(i-1)} \rightarrow r^*$ (with probability $\alpha$), or perturbing the subsurface model $m^{(i-1)} \rightarrow m^*$ (with probability $1 - \alpha$).

If a well point is chosen for perturbation:

1. Perturb the well trace $r^{(i-1)} \rightarrow r^*$ by changing the location of a random well point using one step of a random walk in the ($x, y, z$)-space. In this case, we use a spatially isotropic Gaussian perturbation centred at the well point. This walk will, if unimpeded, sample the constant prior $\rho_c(r)$.

2. Compute the acceptance probability

$$P_{\text{acc}} = \min \left(1, \frac{P(n|r^*) \, P(r^*|p)}{P(n|r^{(i-1)}) \, P(r^{(i-1)}|p)} \right)$$ \hspace{1cm} \text{(16)}$$

where $p$ is the location of the previous point and $n$ is the location of the next point (see Fig. 2).

1. Generate a random number $u \in [0, 1]$
2. If $u < P_{\text{acc}}$
3. Accept the perturbed well point location: $r^{(i)} = r^*$.
4. Adjust the subsurface model parameters $m \rightarrow m + \Delta m$ to fit the impedance in the new well point location. The adjustment is a linear combination of the kernel functions and is designed to have minimal norm $||\Delta m||$; see the Appendix for details.

5. Otherwise, reject $r^*$ and set $r^{(i)} = r^{(i-1)}$.

If a subsurface parameter (not coinciding with a well point) is chosen for perturbation:

1. Perturb the model $m^{(i-1)} \rightarrow m^*$ by changing a random model parameter with a Gaussian random number with zero mean and variance $\sigma^2_{\text{mod}}$.
2. Compute the resulting change in the impedance model, under the constraint that it remains unchanged at all the well points.
3. Compute the acceptance probability

$$P_{\text{acc}} = \min \left(1, \frac{L_d(m^*)}{L_d(m^{(i-1)})} \right).$$ \hspace{1cm} \text{(17)}$$

4. Generate random number $u \in [0, 1]$
5. If $u < P_{\text{acc}}$: accept the perturbed model: $m^{(i)} = m^*$.
6. Otherwise, reject $m^*$ and repeat $m^{(i-1)}; \, m^{(i)} = m^{(i-1)}$.

**RESULTS**

We use an acoustic impedance model and corresponding velocity model from a geographic area of size 8 km $\times$ 8 km, with a depth of 2.3 km, and a maximum well depth of 2 km (Fig. 3). The models considered in this case were taken from the North Sea F3 Demo 2016 training v6 dataset, Offshore Netherlands (http://terrannubis.com/datainfo/F3-Demo-2016).

To represent our impedance volume, we use local base functions centred at the grid points, all having the same shape, given by a three-dimensional (3D) kernel function (see Fig. 4). We assume that the impedance is a spatially homogeneous Gaussian random process, and under this assumption
FIGURE 2 Flow diagram of random perturbations of unknown parameters in our algorithm. $x = (m, r)$ is the joint set of unknowns, where $m$ and $r$ are the subsurface parameters and the well location parameters, respectively.

FIGURE 3 Acoustic impedance model

the impedance can be viewed as a convolution of 3D white noise with the kernel. The kernel is found as the inverse Fourier transform of a smoothed, 3D wavenumber spectrum (see, e.g., Dudgeon & Mersereau, 1995) of the impedance volume, providing a local function which, when convolved with white noise, gives an impedance volume with the same 3D power spectrum as the given impedance volume. The smoothing is adjusted to keep the approximation error below a given threshold, here 1%. In practical applications of our method, the kernel should be estimated from available well information.

We carried out a synthetic study where a 3D seismic data cube was computed by deriving the reflectivity, as a function of two-way time, from the acoustic impedance and velocity model at each point in the cube, and convolving each trace with a 40-Hz Ricker wavelet (Fig. 5). For the well log, we simulated 50 points along a well trajectory where geological data (here acoustic impedances) were collected during the drilling
operations. Errors in the wellbore locations were considered to be cumulative, and each increment introduces a Gaussian error on each location coordinate.

We ran our Markov chain Monte Carlo sampling algorithm (Mosegaard & Tarantola, 1995) using the scheme for perturbing the impedance model and the well trajectory described above. As the initial model, we used the mean model found by Monte Carlo inversion with a fixed well trajectory (the ‘projected’ trajectory). We ran three independent sequences of random sampling, each with 3000 iterations, with a 50% probability of perturbing a trajectory point and a 50% probability of perturbing the impedance model. With 3000 iterations, the sampling reached an equilibrium where the output (e.g. the logarithms of the likelihoods of the sampled models) fluctuated around an approximately constant level.

Acoustic impedance sample models from the posterior distribution can be seen in Figure 6 alongside the initial acoustic impedance model and the mean well trajectory. The variability of the impedance model and the well trace reflects their uncertainties in a way that is consistent with all a priori assumptions and uncertainties in location data and seismic data. These variabilities generate synthetic seismic data that all fit the (simulated) observed data ‘within their error bars’.

**DISCUSSION**

Collecting and measuring data at locations that are difficult to access presents challenges to the accuracy of the data. This paper presented a method to quantify the total errors in locations and measurements, incorporating them into seismic inversion, with the aim of providing a more reliable model of subsurface parameters and of the well trajectory.

Every time additional uncertainties are added to the solution of an inverse problem, the resulting solutions are more poorly resolved. This is also the case in this study. On the other hand, assuming that the borehole data were error-free would result in smaller apparent uncertainties in the inversion result, and this would introduce spurious artefacts: Assuming that well data are too precise (in location and rock property measurements) puts too much weight on these data, compared to the seismic data. As a result, actual (neglected) errors in borehole data will give rise to unrealistic geological structures and properties. In the method proposed in this paper, we aim at a more proper evaluation of uncertainties, giving a correct weighting of all data.

Knowing that the measurements themselves can vary not only from errors inherently in the data collection processes but also from the location accuracy, can change the way we deal with the problem. As a perspective, the growing position
errors can be approached from earlier stages, even on the fly, and interpretations made at more uncertain locations can be considered with more care.

**CONCLUSION**

Seismic inversion with well-calibration was expanded to a more realistic and complete case by including uncertain borehole measurements and locations in the seismic inversion. This error quantification allowed us to provide a more reliable estimate of the uncertainties inherent in seismic inversion assisted with data from horizontal wells.

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**DATA AVAILABILITY STATEMENT**

The data that supported the findings of this study are openly available in Terra Nubis at https://terranubis.com/datainfo/F3-Demo-2016, reference: North Sea F3 Demo 2016 training v6 dataset, Offshore Netherlands.

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ASSume that the subsurface model $m(x, y, z)$, with parameters $\mathbf{m} = (m_1, m_2, \ldots, m_M)$, is fixed in $K$ well point positions $\mathbf{r} = (\mathbf{r}_1, \ldots, \mathbf{r}_K)$, and that we wish to perturb $m(x, y, z)$ with the amount $\delta m$ at a point $\mathbf{r}_{K+1}$, not coinciding with any of the points $\mathbf{r}_1, \ldots, \mathbf{r}_K$. If the perturbed model is $m'(x, y, z)$, its parameters $\mathbf{m}' = (m'_1, m'_2, \ldots, m'_M)$ must satisfy

$$\sum_{n=1}^{M} \left( m'_n - m_n \right) \varphi_n (\mathbf{r}_k) = 0 \quad \text{for} \quad k = 1, \ldots, K.$$  

(A.1)

and

$$\sum_{n=1}^{M} \left( m'_n - m_n \right) \varphi_n (\mathbf{r}_{K+1}) = \delta m$$

(A.2)

Defining a $(K + 1) \times M$ matrix $\mathbf{F}$ with components $F_{ij} = \varphi_j (\mathbf{r}_i)$, the above equation can be expressed as

$$\mathbf{F} \Delta \mathbf{m} = \mathbf{a}.$$  

(A.3)

where $\Delta \mathbf{m} = \mathbf{m}' - \mathbf{m}$, and where

$$\mathbf{a} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \delta m \end{pmatrix}$$

(A.4)

is a vector with $K + 1$ components. The system of equations (2) is, in general, underdetermined and has infinitely many solutions, but the least-squares solution for $\Delta \mathbf{m}$ is

$$\Delta \mathbf{m}_{LS} = \mathbf{F}^T (\mathbf{F} \mathbf{F}^T)^{-1} \mathbf{a}.$$  

(A.5)

Knowing $\mathbf{m}$ and $\Delta \mathbf{m}_{LS}$, we can compute $\mathbf{m}' = \mathbf{m} + \Delta \mathbf{m}_{LS}$.

The above procedure can be used, both when perturbing a point that is not a well point, but also when a well point location is perturbed. In the latter case, the original position of the perturbed point is erased from $\mathbf{r} = (\mathbf{r}_1, \ldots, \mathbf{r}_K)$, which is then reduced to $K - 1$ components, and at the new position of the perturbed point, there is a change $\delta m$ in the model. At this point, the original model value is replaced by the value carried with the perturbed well point, and parameters of the surrounding model are updated to preserve the variability given by the kernel function.