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Abstract

Blockchain Sharding is a blockchain performance enhancement approach. By splitting a blockchain into several parallel-run committees (shards), it helps increase transaction throughput, reduce computational resources required, and increase reward expectation for participants. Recently, several flexible sharding methods that can tolerate up to $n/2$ Byzantine nodes ($n/2$ security level) have been proposed. However, these methods suffer from three main drawbacks. First, in a non-sharding blockchain, nodes can have different weight (power or stake) to create a consensus, and as such an adversary needs to control half of the overall weight in order to manipulate the system ($p/2$ security level). In blockchain sharding, all nodes carry the same weight. Thus, it is only under the assumption that honest participants create as many nodes as they should that a $n/2$ security level blockchain sharding reaches the $p/2$ security level. Second, when some nodes leave the system, other nodes need to be reassigned, frequently, from shard to shard in order to maintain the security level. This has an adverse effect on system performance. Third, while some $n/2$ approaches can maintain data integrity with up to $n/2$ Byzantine nodes, their systems can halt with a smaller number of Byzantine nodes. In this paper, we present a $p/2$ security level blockchain sharding approach that does not require honest participants to create multiple nodes, requires less node reassignment when some nodes leave the system, and can prevent the system from halting. Our experiments show that our new approach outperforms existing blockchain sharding approaches in terms of security, transaction throughput and flexibility.

Keywords: Blockchain, Distributed Ledger, Blockchain Security, Blockchain Sharding, Blockchain Performance

1. Introduction

Different kinds of blockchain, e.g. Nakamoto Blockchain [1] and Ethereum [2], have been proposed in the past ten years. While blockchains have been designed initially to handle cryptocurrencies, they have since shown promise for more sophisticated usage, such as powering Decentralised Autonomous Organisations (DAO) or Decentralised Autonomous Companies (DAC), where anonymous participants can carry out tasks together without centralised control. Various mechanisms have been proposed to ensure the integrity of such decentralised work as well as the incentives for the participants. However, blockchains still suffer from both security and performance problems which significantly limit their applicability in practice.

The fairness and decentralisation of blockchain-based systems depend on how participants reach public consensus. This is usually done by a strength competition known as mining. In a given time window, participants synchronise on new transactions, and then verify and approve the first legitimate block that reaches a threshold strength. If a block is approved, participants will compete to create a new block with the threshold strength on top of this block, and rewards are then given to the creators of the approved block as incentive. This procedure, however, overlooks the heterogeneous nature of devices used in a blockchain, causing a vicious circle between reward rate deprivation and arms race for stronger computational capability as well as broader network bandwidth. This can ultimately result in a centralised system where some participants are always winners of competitions, while others leave the system. Besides, as blockchains seek to improve throughput, they may choose extending block size or employing mining pools. Extending block size may force less powerful devices to leave the system as they do not have the capacity needed to constantly download and verify large blocks from the network. This drives a blockchain system gradually towards a centralised one. A mining pool assembles less powerful participants and uses them collectively to create blocks. But as participants in a mining pool do not know how their computation power is used, and this raises concerns over system security.
Various approaches have been explored to solve the security and performance problems associated with blockchains. These approaches can be commonly categorised as off-chains [3, 4], lightweight blocks [5, 6, 7], weighted models [8, 9, 10], directed acyclic graphs (IOTA) [11, 12], and blockchain sharding [13, 14, 15, 16]. Among these, blockchain sharding is promising as it offers a good balance between security and performance. Blockchain sharding works by splitting a blockchain into several parallel-run committees (shards), thereby increasing transaction throughput, reducing computational capacity required, increasing reward expectation for participants, yet still maintaining the security level required. Recently, several flexible sharding methods that can tolerate up to $n/2$ Byzantine nodes ($n/2$ security level) have been proposed. However, these methods suffer from three main drawbacks. First, in a non-sharding blockchain, nodes can have different weight (power or stake) to create a consensus, and as such an adversary needs to control half of the overall weight in order to manipulate the system ($p/2$ security level). In blockchain sharding, all nodes carry the same weight. Thus, it is only under the assumption that honest participants create as many nodes as they should that a $n/2$ security level blockchain sharding reaches the $p/2$ security level. Second, when some nodes leave the system, other nodes need to be reassigned, frequently, from shard to shard in order to maintain the security level. This has an adverse effect on system performance. Third, while some $n/2$ approaches can hold data integrity with up to $n/2$ Byzantine nodes, their systems can halt with a smaller number of such nodes.

In this paper, we propose a $p/2$ security level blockchain sharding approach that does not require honest participants to create multiple nodes, requires less node reassignment when some nodes leave the system, and can prevent system halting. The new approach combines the Multiple Winners Proof of Work consensus protocol (MWPoW) [17] with the flexibility of $n/2$ blockchain sharding, where chains are like shards in previous $n/2$ approaches but they can dynamically merge or split depending on the workload. We call our new approach Multichain MWPoW Sharing Approach or Multichain MWPoW for short, which has the following novelties:

- **Increased Byzantine Resilience.** To the best of our knowledge, Multichain MWPoW is the first sharding approach that can withstand up to 50% of the total power being adversary (Byzantine) without assuming that honest nodes have to create as many nodes as necessary. There is less chance for a global halting to occur too, as adversaries cannot deliberately plan it like in the scenario discussed in Section 4.2. On the other hand, if a global halting does happen by accident, it can still be resolved like a local halting.

- **More Flexibility.** A chain (shard) can be split and merged based on its data flow. Every chain can have a different number of participants. In contrast, it is less flexible in other sharding approaches. The number of shards in [16, 18] are fixed. In [19], the number can change, but nodes need to be equally divided among shards, and when a node leaves a shard, the shard will need to be cancelled and the organisation of a new shard will be required. Our approach is therefore more efficient.

- **Increased Transaction Throughput.** Multichain MWPoW allows fewer number of nodes per chain and the chains are more stable, hence more shards can be created to process transactions in parallel with the same level of security guarantee. Furthermore, as less time is spent on dealing with halting and attacks by adversaries are harder to engineer, Multichain MWPoW is able to devote more time to processing transactions.

- **Faster Transaction Confirmation.** There are no levels of election in Multichain MWPoW. A transaction is confirmed when the governing chain has confirmed it, and there is only one governing chain per transaction. As such the confirmation time for a transaction is significantly reduced.

Our experiments show that Multichain MWPoW outperforms existing blockchain sharding approaches in terms of security, transaction throughput and flexibility.

In the rest of the paper, we first describe the blockchain sharding idea and provide a brief review of some of relevant sharding blockchain approaches in Section 2. We propose our Multichain MWPoW sharding approach in Section 3, detailing its consensus protocol and operation procedure. We then offer a complete security analysis for a number of popular sharding solutions and compare it to that of Multichain MWPoW in Section 4. We report our performance study in Section 5. Finally, we draw conclusions in Section 6.
2. Blockchain Sharding and Related Work

A blockchain system contains chains of blocks, each embedding the information (transactions) of a specific period. A blockchain is periodically updated by its participants who create new blocks and attach them to the existing chains over time. The rate at which a new block can be generated within a fixed time interval is controlled by the system and there is also a pre-set security requirement that honest participants must have more than 50% of the total calculation power within the system at any time. This ensures that honest participants can collectively create a longer chain of blocks than anyone else, allowing new participants to follow the correct records by simply staying with the longest chain (or the mainchain). As such, the need for centralised record keeping is removed.

Blockchains were initially proposed to deal with cryptocurrency transactions. In that setting, the participants only need to check whether the sender of a transaction has spent the fund or not to avoid double-spending: no one should be able to send the same money to more than one receiver at the same time. However, when we use blockchains to power other decentralised applications, additional performance and security issues have to be considered. Among the proposed approaches to addressing these issues, blockchain sharding has received much recent attention.

2.1. Blockchain Sharding

“If a tree falls in the forest and no one is around to hear it, does it make a sound?” The quote questions the relevance of unobserved events – if nobody hears the tree fell, whether it has made a sound or not is of no consequence [3]. Blockchain sharding is based on this observation, as illustrated in Figure 1: if only a few participants care about a particular transaction, it is not necessary for all other participants in the blockchain network to know about that transaction [3].

![Figure 1: The philosophy of blockchain sharding](image)

Note that the fact that a tree has fallen, and the correct time of its fall has been recognised by most people around the tree assumes that these people have not colluded. This assumption is reasonable if a sufficient number of people are randomly assigned to and evenly distributed in sub-areas of the forest, and if we relocate people from time to time to prevent accumulation of potential collusion in any particular sub-area. Note also that this proposal can operate securely only when (1) people assigned to a sub-area have the right to record information about this sub-area; (2) no one in the sub-area can control its reporting; (3) the assignment follows a globally recognised rule, not dictated by some specific group or individual; and (4) only qualified people can be assigned and the qualification period is not shorter than the time in which one continuously stays in a sub-area, so as to avoid instability due to people frequently quitting a sub-area and starting over again.

Blockchain sharding follows from these observations. As long as random and distributed assignment of nodes is secure and follows the principle of proportionality, taking control of a shard would be as hard as taking control of the whole system. When the security level is maintained for the whole blockchain, we will have more than half of the total population (or half of the total calculation power) from honest people. If this power can be securely and proportionally distributed into the shards, then local verification of transaction(s) (i.e., within a shard) will be as good as a global one (i.e., over the whole system), This helps improve the performance of a blockchain system.

2.2. Blockchain Sharding Approaches

Elastico was the first consensus protocol proposed for blockchain sharding [13]. While it is a step forward in this direction, it has the following weaknesses. First, after every iteration, all shards need to be rebuilt and node identities need to be reset. Second, because it demands a significant amount of time to fill up all the shards by solving enough Proof of Work (PoW) [20], latency grows linearly with network size. Third, an adversary may calculate PoW in advance so that he or she can mislead the process of assigning nodes to shards. Fourth, as shards of small size...
(around 100 members) are needed in order to restrict the running of Practical Byzantine Fault Tolerance (PBFT) [21] in each shard, it increases failure probability (which can be over 0.97 after just six iterations) [14]. Furthermore, even though Elastico only requires each participant to verify a subset of transactions, they must still synchronise every block from every shard. All these performance and security limitations mean that Elastico is not a practical sharding solution. RSCoin [15] is a sharding protocol designed to scale up support for centrally-banked cryptocurrencies. It is an approach that attempts to transparentise today’s banking systems by combining a distributed network with a centralised monetary supply. However, this blockchain protocol relies on a trusted source and it is not Byzantine fault-tolerant because each shard is executed on a two-phase commit protocol.

Some improvements have been suggested to overcome these weaknesses. OmniLedger [14] solved the problems suffered by the Elastico protocol, but it can only tolerate up to \( n/4 \) adversary nodes where \( n \) is the total number of nodes in a blockchain. RapidChain [16] increased this tolerance to \( n/3 \), but every shard must use a fixed size instead of a Binominal (or approximately, Poisson) random variable. As such, the shard size can still be a significant issue that will limit improvement on blockchain throughput.

Xu and Huang [18] have recently proposed a new approach that takes blockchain sharding to a \( n/2 \) security level or can tolerate up to \( n/2 \) adversary nodes. This approach classifies nodes into different classes and maintains an equal number of nodes from each class in every shard. This approach does not only increase security level, but also substantially shrinks shard size. However, such a blockchain can grind to a halt by an adversary with less than \( n/2 \) nodes. That is, a blockchain stops to generate new blocks even though records in the blockchain are all correct. There is an extension to this approach [19], which dynamically alters the number of classes as well as the size of shards to bound the probability of global halting and can recover the system from halting. However, the number of shards allowed in this extension can be reduced drastically, which can destabilise throughput. Also, for both solutions, every time new nodes are added or existing nodes leave the blockchain, a global node membership adjustment will take place. These drawbacks increase the frequency in data synchronisation and uncertainty in system stability and throughput. The solution we propose in this paper overcomes these limitations.

### 2.3. Intra-shard Consensus Protocols

For nodes in a shard of size \( m \) to reach a consensus, two common types of protocol may be used: partially synchronous consensus protocols (like PBFT [21]) or synchronous consensus protocols (like the protocol proposed by Ren et al. [22]). They allow up to \( t < m/3 \) and \( t < m/2 \) adversary nodes, respectively. In blockchain sharding, a node will be selected as the block publisher in every epoch by some mechanism. This block publisher will propose a block for other nodes to vote whether it should be accepted into the mainchain of the shard. Partially synchronous consensus protocols are more practical in operation, but as there can be multiple candidates (blocks) in one epoch due to asynchronous communication, they can tolerate less adversary nodes. Synchronous protocols, on the other hand, will have only one candidate block to work with in every epoch of mining. As such, every node will either support the block or not vote for it. If the block does not receive a threshold number of votes, it will be abandoned and a new block will be proposed by the new block publisher in the next epoch. If a block publisher publishes a faulty block, the adversary nodes support it, and other nodes do not vote for it, then clearly the maximum number of adversary nodes that a synchronous protocol can tolerate is \( t < m/2 \). MWPoW, which is part of the Multichain MWPoW sharding approach we propose, is an asynchronous protocol, yet it allows \( t < m/2 \) adversary nodes owing to the Proof of Work mechanism [17].

### 3. Multichain MWPoW: A \( p/2 \) Security Level Sharding

Blockchain sharding approaches use plurality voting instead of strength competition to generate consensus. That is, they trust a statement voted by most people, rather than by most strength/influence, inside a shard. Thus, while an adversary in a classical blockchain needs more than 50% of the total strength to interference the mainchain (referred to as \( p/2 \) security level), an adversary in a sharded blockchain only needs to control more than 1/3 or 1/2 of the total number of nodes to force faulty information into the mainchain (referred to as \( n/3 \) or \( n/2 \) security level)\(^{1}\). Thus, in

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\(^{1}\)In an \( n/2 \) or \( n/3 \) sharding, a node only needs to have certain (threshold) strength to join the system, even though it may have more strength than required.
order to ensure the equivalent $p/2$ security level in blockchain sharding, honest participants need to create multiple nodes within the system to fully represent their voting strength. However, there is a cost for a participant to maintain many nodes, since their nodes can be assigned to different shards, requiring increased workload in synchronising and processing data. Due to this cost, an honest participant, especially those using a device with limited computational power, is likely to create just one or a small number of nodes, leaving room for an adversary to create many nodes to gain control over the system.

In this section, we propose a $p/2$ blockchain sharding approach that (1) requires less frequent data synchronisation and membership adjustment in comparison to [18, 19]; (2) significantly reduces halting happening [18]; (3) causes less loss in transaction throughput than [19] when recovering from halting; and (4) lifts an $n/2$ Byzantine node resistant blockchain into a $p/2$ Adversary (Byzantine) power resistant level. In the following we describe this approach in detail.

3.1. Model Description

We assume that participants or nodes can have different strengths when voting a consensus. In PoW-based systems, this strength represents the calculation power that a node has, whereas in PoS-based systems, it represents the amount of stock that a node has. In this paper, we refer to strength as calculation power. Our model has three main components.

1. **Node classification.** Suppose that we have $n$ nodes $L = \{L_0, \ldots, L_n\}$ in the system and we list these nodes in order of their strengths. Let $CP_x$ represent the strength of $L_x$ and $S_g$ be a pre-defined number of groups of nodes such that every group has a lower strength boundary $bl(i)$ than others in the $CP$ list. That is,

$$bl(i) = CP_{\lfloor \frac{i}{S_g}\rfloor}, i \in [0, S_g)$$

Every shard must have at least one node from every group.

2. **Block evaluation.** We assume that when an adversary proposes a block containing faulty information, the honest nodes will not vote for it. Let $AP$ be the overall strength of adversary nodes. The chance for a block in shard $j$ to be controlled by the adversary is

$$Pr(j) = \prod_{i=0}^{i<S_g} \left( \frac{AP \times DG(i) \times bl(i)}{N_g(S(i,j))} \right)$$

where $N_g(S(i,j))$ is the number of nodes in group $i$ which are currently located in shard $j$ and have voted for the block; $DG(i) = 1$ if at least one node from group $i$ in shard $j$ has voted for this block, otherwise $DG(i) = 0$; and

$$it = \sum_{i=0}^{i<S_g} DG(i) \times bl(i)$$

In order to maintain the system at a $p/2$ security level, we consider

$$AP = \frac{\sum_{i<n} CP_x}{2}$$

Equation 2 over-estimates the probability because we assume every node in any group has the same strength ($bl(i)$). Figure 2 gives an illustration of Equation 2. We can accept this block safely if (1) more than half of the strength in shard $j$ has voted for it (the majority principle), and (2) the chance for the block to be a wrong one is lower than the security threshold. Nodes can still mine on blocks that are insecure, but transactions contained in these blocks would only be accepted when the blocks or the branches stemmed from them reached the security threshold.

3. **Shard merge.** From time to time, some shards may merge. Shard $j$ will be merged with another when:

(a) $\text{Max}(Pr(j)) > Th$ where $Th$ is a security threshold (e.g. $10^{-6}$). Note that $\text{Max}(Pr(j)) = Pr(j)$ when $\forall i \in [0, S_g), DG(i) = 1$. It is obvious that shard $j$ should be merged with others when all the nodes inside it have voted for a block but yet the likelihood for this block to be faulty (i.e. proposed by the adversary) is still larger than the required security threshold.
Figure 2: An explanation of Equation 2. Here, $S_g = 4$. The heights of the bars represent the strengths of the nodes. If a block has supports from every group, then $n = \sum_{i=0}^{3} bl(i)$. The adversary would create $AP/\bar{tt}$ number of nodes in every group, as this would be the best strategy for the adversary to place his or her nodes in the system (see Section 4.2).

(b) When at least five continuous blocks in the mainchain of shard $j$ have not reached the security threshold. In this case, we say a local halting has occurred.

(c) When there is no node from a group currently located in shard $j$.

To reduce chances of local halting, we want every possible $AP/\bar{tt}$ to be as small as possible. Therefore, when adding new nodes, the system should prioritise those shards whose strength is close to the average node strength. It should post penalty or delay adding nodes that would raise $AP/\bar{tt}$. Restrictions should also be placed to avoid an extremely unbalanced power distribution inside the system.

3.2. MWPoW Intra-shard Protocol

For intra-shard consensus, our Multichain MWPoW implements a revised version of Multiple Winners Proof of Work (MWPoW) protocol [17], which is an asynchronous consensus protocol, but allows up to $f < n/2$ adversary nodes in an $n$-node system. This enables Multichain MWPoW to operate a sharded blockchain at a $p/2$ security level. In the rest of the paper, we refer a shard as a chain. To avoid the double-spending problem, transactions are governed by different chains in our approach: new transactions can only be conducted on the governing chains of their INPUT transactions. If a user wants to transfer a transaction to another chain, they need to conduct a cross-chain operation. For ease of reference, a brief introduction of the MWPoW protocol is given in Appendix A and the reader is referred to [17] for details.

3.3. Multichain MWPoW Operation Outline

Suppose that we have several chains in a Multichain MWPoW system. The block interval time of every chain is set to be the same. Thus, all chains generate blocks in an approximately same time window. Every chain is given a Chain ID which is formatted as $C + digits$. Chain ID is given and changed based on split/merge activities. The split of a chain is binary at any time. The two new chains following the split will have new chain ID by appending a “0” or “1” at the end of their parent ID. When two chains are merged, if they stem from the same branch, the ID after merging is the old ID of that branch. If two chains are merged into one and they do not stem from the same branch, then the ID after the merge is the smaller of the two. When a chain is merged/split, the new chain(s) become the Offspring chain(s) of the current chain and this chain becomes a history chain of its offspring chain(s).
There are three types of blocks in every chain: Ordinary block (Ob), Power-assignment block (Pab), and Fuel-up block (Fub), which take turns to be repeated written into the chain following the sequence of Ob, Pab, Fub, Ob, etc. The block in every chain records a list of valid registered miners inside its chain.²

- An Ob records the transactions and the Shares of the MWPoW protocol.
- A Pab records the transactions, the Shares of the MWPoW protocol and an assignment box which contains assignment requests. There are two sections in an assignment box: a New participant section and a Re-assignment section. Inside each section, there is NC number of subsections each corresponding to a chain in the system, NC is the number of chains. The owners of assignment requests are being assigned to the chains, respectively.
- Entrance ticket is a dataset that contains an assignment request and a Merkle branch. A Fub of chain X in block height H records the Entrance tickets which the Merkle branch inside proves the following conditions: (1) the assignment request has been written to a Pab of a chain; (2) this Pab assigns the owner of the assignment request to chain X; (3) this Pab is at block height $H - Ti$.

**Joining the system**

When a node joins the system, it first creates an assignment request, which contains the overall calculation power it will put into the system for $Ti$ block intervals, its public identity key and a PoW proving the calculation power. The node can submit the assignment request to any chain X, the assignment request should contain a hash of the latest Fub of chain X, proving that the PoW has not been pre-calculated. It then sends the assignment request to nodes in chain X. The node creates and sends the Entrance ticket to chain Y after a Pab of chain X assigns it to chain Y. When the Entrance ticket is recorded by a Fub of chain Y, the node has gained a membership in chain Y starting from next block height.

**Reassignment**

Lifelength is a pre-defined number which refers to the time (continuous iterations of mining game) that a miner can play in a chain after being assigned to this chain. There is a pre-defined Lifelength $Ti$ and $Ti \mod 4 = 0$. When a node has stayed in a chain X for $Ti$ block intervals, it will then be assigned to a new chain by the Pab in chain X. The node does not need to create a new assignment request.

**Mining**

When a Fub is published with Bob’s entrance ticket in it, a TR (Try Range) which is a number interval between $[0, 2^{256}]$ is assigned to Bob, and Bob has joined the chain after this block (Bob is consider registered). Bob can then start mining according to the rules of MWPoW.

**Security designs**

In this section we discuss the designs that fulfill the security model discussed in Section 3.1. Nodes only hear the blocks of the chain they are assigned to as well as the block header of the announced blocks from other chains. When a block is announced (reached the Acceptance difficulty), miners in all chains should download the block header of this block. They do not know the overall number of nodes in the system. However, the block header records the number of nodes in the chain. Therefore, the number of nodes in the system can be derived by summing up the number of participants indicated in the block headers of the latest accepted blocks in every chain. The block header also records an integer array $BL_{candidate}$ of size $Sg$, $BL_{candidate}(i) = RCP_{\lfloor ox(NPC[Sg]) \rfloor}, i \in [0, Sg)$ (5)

²In the original MWPoW, the participant list is not written in the block, which can be derived by counting the Entrance tickets and Shares from the start of the system. Including participant list does not increase bandwidth demand significantly because the block is encoded using Graphene[5]. Nodes do not need to swap any clear text of the participant list unless a discrepancy is detected.
Rank the calculation powers of the nodes inside a chain into an ascending list \( RCP_{0..NPC-1} \), where \( NPC \) is the number of registered participants inside this chain. Let \( NC \) be the number of chains,

\[
bl(i) = \min(Bl_{candidate}(i,j)), i \in [0, SG), j \in [0, NC)
\]

where \( Bl_{candidate}(i,j) \) refers to \( Bl_{candidate}(i) \) of the latest accepted block in chain \( j \). Miners of chain \( j \) then classify the nodes inside the chain according to the derived \( bl \). Every time the group boundary is determined, miners should examine if some restrictions are met. These restrictions include:

1. \( 2 \times \text{Threshold} \cdot \text{Chainpower} \geq \text{Chainpower} \).
2. There is at least one node from every group in this chain.
3. \( \alpha(Pr(j)) \leq \text{Threshold} \), where \( \text{Threshold} \) is a predefined security threshold.

\( \text{Chainpower} \) is the amount of overall registered power (in PoW difficulty form) inside chain \( j \); \( \text{Threshold} \cdot \text{Chainpower} \) is the sum all values in \( RCP_{0..\lfloor 2 \times NPC - 1 \rfloor} \). \( Pr(j) \) is given in Equation 2 and \( \alpha(Pr(j)) \) is \( Pr(j) \) calculated by assuming all the nodes inside chain \( j \) have voted for a block. If these restrictions are not met, then chain \( j \) should be merged with others.

Apart from the rules of MWPoW regarding final acceptance of a block, a block reaches Acceptance Difficulty is accepted when the chance for the block to be incorrect is lower than the required security threshold. This chance is calculated using Equation 2. If there are two blocks which reach the Acceptance Difficulty during the same epoch in a chain and the chance for them to be incorrect are lower than the security threshold, then the one (say Alice) with more Support Rate is accepted if the differences between the Support Rates of the two blocks is more than a specific value. This value is defined as one that an adversary can gain with a pre-defined security threshold probability. The chance for the adversary to control this particular value of Support Rate difference can also be calculated using Equation 2 by enumerating some voters of Alice and assuming the enumerated voters are controlled by the adversary (only use the enumerated votes to calculate \( DG(i) \) and \( NgS(i,j) \)). The enumerated adversary voters together should contribute the amount of difference between the Support Rate of Alice and the other block. The chance for the enumerated voters to be adversary should be lower than the security threshold probability.

Algorithm 1 shows the working procedure for nodes in Multichain MWPoW. Figure 4 shows the block structure of Multichain MWPoW. Global block header is a Merkle root of the hash of all the latest accepted blocks of all chains. When one transfers a transaction between chains, the transaction is written into the Crosschain section. Same as MWPoW, a block is simplified using the simplification algorithm Graphene [5] which combines Bloom filter [23] and IBLT [24]. Figure 5 shows an example of the working procedure of Multichain MWPoW. The full operation details of Multichain MWPoW is given in Section 3.4.
Bob’s Assignment request

Phase 1:
Bob (a new participant) creates an Assignment request based on a Fub of a chain (C4) and send it to C4 after creation.

Phase 2:
The following Pab in this chain (C4) after Ti iterations embeds Bob’s Assignment request and assigns Bob to a chain (C7).

Phase 3:
Bob submits a Entrance ticket to the chain assigned (C7), this Entrance ticket indicates he was assigned by C4.

Phase 4:
The assigned chain (C7) does the verification then embed Bob’s Entrance ticket to a Fub, a Try Range is assigned.

Phase 5:
Bob is reassigned when it has been in C7 for Ti continuous rounds. Bob’s Assignment request will be embedded to the next Pab in C7 and it be assigned to a chain. The process then goes back to Phase 3.

Timeline of interaction with C4:
1. Bob starts creating a Assignment request using the hash of this Fub as HashPrevblock
2. Bob starts sending the Assignment request
3. The Assignment request is embedded to this Pab of C4
4. The exact assignment of C4 is determined
5. Ti iterations passed

Timeline of interaction with C7:
1. Bob starts creating a Assignment request using the hash of this Fub as HashPrevblock
2. Bob starts sending the Assignment request
3. The Entrance ticket is embedded to the Fub in C7, a Try Range is assigned to Bob
4. Ti iterations passed
5. Bob starts sending the Entrance ticket to C7
6. The Entrance ticket is embedded to the Fub in C7, a Try Range is assigned to Bob
7. Bob is reassigned by C7
8. Bob starts mining
Algorithm 1: Operating procedure for nodes

**Register power.** A miner can create an Assignment request based on a Fub of a chain (HashPrevblock should be the hash of that Fub). After the Assignment request is constructed (usually after $T_i$ iterations of the game as the hash difficulty of this Assignment request must reflect $T_i$ times of its $CP$), the participant sends the Assignment request to that chain.

**Wait for the power assignment.** In every four iterations (whenever a Pab is created), qualified Assignment requests of new participants are selected by miners in a chain. A random assignment protocol is used to place all the selected Assignment requests into the assignment box of the new Pab.

**Register with the chain assigned to.** After a Pab Alice, which contains the participant’s Assignment request, is announced (reaches Acceptance Difficulty), the participant creates an Entrance ticket, which contains that Assignment request and a Merkle branch. The Merkle branch should prove that this Assignment request has been assigned to a specific chain by Alice. Finally, the participant should submit this Entrance ticket to the chain assigned by Alice.

**Get a TR.** Miners in the assigned chain check if the Entrance ticket they have received is valid. They also check if the Pab (Alice), which has made this assignment, is the latest accepted Pab in its chain. A Try Range is given to the participant at the next Fub, then the original rules of MWPoW will apply.

**An episode of mining game**
- Miners try to create a block and find a Nonce that fulfils $ED$ in their TR. If a miner’s block has reached $ED$ and miners approve this block, they will try to find a Nonce of $AD$ in their TR.

**The episode ends when a Share of AD is broadcast;**

**Split and merge chains.** When a chain violates chain limits (see the appendix), the latest block will indicate if the chain should be split or merged. The miner then enters the split or merged chain following the rule of split/merge.

**Re-assignment.** When a miner has been inside a chain for $T_i$ rounds of a game, it is re-assigned to another chain.

**Expel.** The same as MWPoW, if a miner has not sent at least two valid Shares per iteration that are successfully embedded in the next block, it is expelled.

### 3.4. Multichain MWPoW Operation details

In this section we describe the operations of proposed Multichain MWPoW in more detail.

#### 3.4.1. Global block header and dispute resolution

Every block embeds a global block header; the hash of the global block header is written into the block header of every block. Global block header records the hashes of the latest finally accepted block of all the chains. We allow these hashes to be the second latest one because the block generation among chains goes not completely synchronised.

Because nodes only synchronise information with its chain and the block headers of the announced blocks of other chains, nodes are unable to determine if a block of another chain is genuine and can be finally accepted. To address this issue, we propose a mechanism:

1. When a block is announced or finally accepted, relevant miners should broadcast this information to miners of other chains.
2. Miners should periodically ask several miners in other chains to see if the announced blocks have been finally accepted.
3. If a conflict is known to a node, this node should synchronise with the participant list in the last block before the suspicious one. It should then determine the genuine Shares and calculate the support rate of the blocks.
4. A block carrying a wrong finally accepted block hash in its global block header should be rejected. Miners should not mine on this block in any circumstances.

Because the calculation power is distributed in chains, it is easier for Byzantines to over-write specific blocks in a chain using a greater calculation power. To prevent unregistered nodes affecting the generation of blocks, we rule that:
1. The Shares sent to the network should be signed by the Identity Keys which have been claimed in the Assignment requests;
2. The Nonces should be within the Try Range that is associated with the Identity Keys.

Under this mechanism, the first block of a fraud chain of blocks can be determined as invalid because Byzantine cannot provide the correct Shares which are signed by previous participants.

When a new block of a chain which fulfils the Entrance difficulty comes out, nodes of that chain should check the global block header of that block before contributing Shares for it. In this way, when a block is announced, at least a certain amount of calculation power agrees with the global block header attached. Because nodes synchronise all the block headers of the announced blocks of all the chains, they can see the differences between the Merkle root of all the global block headers. A node will request and verify the relevant global block headers if it cannot construct the same Merkle root of the global block header. Figure 6 is an example of a global block header, where NC is the Chain ID, and LASH is the hash of the latest finally accepted the block.

<table>
<thead>
<tr>
<th>NC</th>
<th>LASH</th>
</tr>
</thead>
<tbody>
<tr>
<td>C2</td>
<td>EA232341AEAEFEN2EKRWF23EWRK2L</td>
</tr>
<tr>
<td>C6</td>
<td>FB1113A122F1AQFWSLEEF23ERK1LR4</td>
</tr>
<tr>
<td>C7</td>
<td>CAA313A152F1AQF1AWLEWA3WFETQ</td>
</tr>
</tbody>
</table>

Figure 6: Global block header

In summary, if a Byzantine attempts to change a finally accepted block of a chain, it must place enough power inside this chain through the normal procedure. If the power is not registered before, it cannot generate valid Shares, nodes inside the chain will not recognise an invalid block which has reached the Acceptance Difficulty. When nodes of other chains ask which block has been finally accepted, or the honest miners inside a chain has received a fraud block of that chain from the network, the registered honest power inside that chain will appoint another block to the network. When a conflict of finally accepted block has occurred, the Byzantine’s block cannot pass the verification of other chains.

3.4.2. Duty Range, Chain split and merge

TransOnhold is a number added to the block header. This number stands for the number of transactions/Assignment requests received by the creator of the block. These transactions/Assignment requests should be legal and within the Duty Ranges. In the meantime, they should have not yet been written into a block in the main-chain of this chain. Duty range is a range of transactions/Assignment requests/entrance tickets, which should be processed by a chain. Duty ranges for a chain include all the Assignment requests, transactions, and Entrance tickets which:

- The HashPrevBlock of the Assignment requests is the hash of a block inside this chain.
- The HashPrevBlock of the Assignment requests indicates a block in the history chain of the current chain. The hash of this Assignment request is within a specific range.
- All the Input transactions of the transactions have been committed to any block of this chain.
- All the Input transactions of the transactions have been embedded in the history chains of the current chain. The hash of these Input transactions is within a specific range.
- The Entrance tickets which have indicated their creators are assigned to this chain.
- The Entrance tickets, which have indicated their creators, are assigned to the history chains of this chain, and the hash of the Entrance tickets are within a range.

The valid Assignment requests, transactions and Entrance tickets of a chain comply with the following:

- They are under the government of this chain (inside the Duty Range).
- The INPUT transactions are not used before.
When $TransOnhold$ indicated in the latest finally accepted block of a chain is more substantial than $2 \times K$, then this block is split into two due to the next block’s height. However, a chain cannot be split when either of the split chains will not meet the chain restrictions stated in Section 3.3. When a chain $C1$ is split:

- **Duty Ranges.** According to the hash of the transactions written in the blocks of chain $C1$, if the hashes of the transactions are within the range of $0$ to $2^{255}$ then these transactions are governed by chain $C2$. Otherwise, the transactions are governed by chain $C3$. The duty ranges inherited from chain $C1$ are also equally split into two. Chain $C2$ will take the duty ranges of $C1$ with lower half hashes while the chain $C3$ will take the upper half. The rule also applies to the Entrance tickets on hold. If the hashes of which are within $2^{255}$, then the Entrance tickets are processed by $C2$. Otherwise, they are processed by $C3$.

- **Participants.** Rank all the participants by the amount of their Calculation Power Claim in ascending order, a participant is relocated to $C2$ if $Pindex \mod 2 = 0$ where $Pindex$ is the index number of this participant inside the ranked participant sequence, otherwise this participant is relocated to $C3$.

Figure 7 shows an example of chain split and merge, where the system starts from one chain $C1$, orange squares are blocks in the chains that currently exist while gray squares are the blocks in history chains.

When merging, a chain will be merged with another that is closest to it in Chain ID. If there are two chain candidates, select the one with a smaller Chain ID. The duty ranges of the chains are also merged. Assume chain $C5$ is merged into chain $C3$ after a block Alice in $C5$ is announced. When Alice is announced, the miners in $C3$ are aware of this merging because they synchronise the block header of all the announced blocks. The miners in $C3$ then synchronise the data in $C5$ between the block interval of the last finally accepted block of $C5$ and Alice. The miners use this data to verify Alice. If they believe $C5$ should be merged with $C3$ according to the rules, they will mine on the merged chain. When a safe number of nodes in both $C3$ and $C5$ has approved this merge through mining in the merged branch, then the merge is completed. This safe number can be calculated using Equation 19. When chain $C5$ has not generated a finally accepted block for five continuous block intervals, the chains into which chain $C5$ is possible to merge should synchronise the data from $C5$ and determine if they should merge with $C5$. The merged chain starts at the next block height of the highest block height in its history chains. When a chain $C3$ seeking to merge to $C5$, $C5$ is also seeking to merge; if $C5$ is trying to merge with another chain $C6$, then three or more chains merge into one at the same time. Figure 8 shows an example of chain merge and split, where grey squares are abandoned blocks. If a sufficient number of nodes in $C3$ and $C5$ agree on merging in block height 14, then their other branches are abandoned.

### 3.4.3. Crosschain operation

Because every chain can confirm the situation of blocks in other chains (has been / not yet finally accepted), we take advantage of that to conduct crosschain operations. When a user wants to transfer a transaction to another chain, it first sends the cross-chain-request to the chain that governs the transaction (Origin chain). If this cross-chain-request is written into the crosschain section of a finally accepted block afterward, the user then sends a cross-chain-confirm to the transfer destination chain. The cross-chain-confirm is a Merkle branch that can prove the cross-chain-request.
has been written into the crosschain section. The destination chain should write the cross-chain-confirm into its
cross-chain section, and then the transaction is transferred. The difference in block height between the cross-chain-
request and the cross-chain-confirm embedded the blocks should be less than three. If the cross-chain-confirm cannot
be written into the destination chain in time, the user will ask the origin chain to cancel the cross-chain request.
The miners in the original chain will acquire the cross-chain section of relevant blocks of that destination chain and
determine if the transfer should be cancelled. If the user does not send the cancel request, the transaction is being
transferred to the destination chain. Figure 9 shows an overview of the cross-chain operation. New transactions of
the destination chain can refer to the cross-chain-confirms written in the cross-section of this destination chain as the
INPUT transactions.

3.4.4. Power assignment block

In this section, we show the procedures of forming a Pab for a chain C5. There are two parts in forming a Pab: Periodical power re-assignment and New power adding. The structure of the Assignment Box and Assignment request
is shown in Figure 10 and Figure 11 respectively.

Periodical power reassignment Select the nodes which were added to C5 at the block height BH − Ti, where BH
is the current block height. Place the selected nodes’ Assignment request into a list PSL by ascending order of
calculation power claim indicated in their Assignment requests. Create a new sequence RPS L,

\[ RPS L_i = Hash(MGBH \oplus Hash(PS L_i)) \]  

(7)

where MGBH is the Merkle root of the global block header indicated in the latest block of C5. Link PSL_i with
RPS L_i and rank RNAJ by alphabetical order. After that, a new index of PSL can be reached. Let NC be the number
of sub-sections in the “Re-assignment section” of the Assignment box. Acs(i) represents sub-section i,

\[ Acs(i) = \bigcup_{(\text{hash}(PS L_j) + j) \mod NC = i} PS L_j \]  

(8)

Nodes in Acs(i) are assigned to chain i. If chain i becomes a history chain right after, nodes in Acs(i) are assigned to
its OﬀSpring chains according to the Duty Range.

Adding New Power Miners in C5 take the Assignment requests from all unassigned Assignment requests received,
which fulfill the following criteria:

- The Nonce inside can make the hash of this Assignment request fulfill the Intended diﬃculty.
- The HashPrevBlock is the hash of the Fub at Ti iterations before the current block height.

After selecting the Assignment requests, the following procedure is carried out:

1. Let InD be the Intended Difficulty indicated in a Assignment request. If \( bl(i + 1) > InD \geq bl(i) \) then place this
   Assignment request to list i. \( bl(S g) = +\infty \).
2. Rank the Assignment requests in every list \( i \) by ascending order of \( \text{abs}(I_n D - t i) \), where \( t i = \frac{b(t+1) + b(t)}{2} \). Specially, in this step, \( b(S_g) = b(S_g - 1) \).
3. Select \( K/S_g \) Assignment requests from the top of every list. If a list has less than \( K/S_g \) Assignment requests, then take all of them.
4. Rank the selected in descending order of their calculation power claim, and sum the front \( \frac{1}{4} \). If that is larger than half of the overall power of the selected Assignment requests, then delete the Assignment requests from the top until the front \( \frac{1}{4} \) of power claims are not more than half of the overall power of the selected Assignment requests.
5. Rank the remaining Assignment requests according to the alphabetical order of their hashes and place them into a list \( NAJ \). Create a new sequence \( RNAJ \),

\[
RNAJ_i = \text{Hash}(MGBH @ \text{Hash}(NAJ_i))
\]

Link \( NAJ_i \) with \( RNAJ_i \) and rank \( RNAJ \) by alphabetical order. After that, a new index of \( NAJ \) can be reached.
6. Let the “New participant section” in the assignment box assign Assignment requests to \( \text{min}(NC, K) \) chains. \( \text{NAJ}_i \bmod \text{min}(NC,K)e_j \) is assigned to chain \( j \) indicated in the assignment box, \( NC \) is the number of chains.
7. Write the assignment plan into the “New participant section” in the assignment box.

Determine the exact assignment Any chain accepts the Assignment requests which assigned to it if these Assignment requests are written in a “Re-assignment section”. If a Assignment request \( Alice \) claims she has been assigned to a chain \( Ben \) by a Pub \( Gary \) of \( C5 \) in “New participant section”, \( Ben \) verifies this information by the following steps:

1. Let \( LI \) be the number of subsections in the “New participant section” of the Assignment box of \( Gary \), and \( NC \) be the number of chains. Rank chains by the alphabetical order of their Chain ID.
2. The Assignment requests in subsection \( i, i \in [0, LI] \) of the Assignment box of \( Gary \) is assigned to chain \( \text{Hash}(Gary) \bmod NC \) in the ranked sequence.

If it is verified by the above procedure that \( Alice \) is assigned to \( Ben \), then \( Ben \) should accept \( Alice \).

3.4.5. Fuel-up block
Miners need to send a Entrance ticket to the chain which they have been assigned to. The Entrance tickets are embedded in the Fuel-up block, and Try Ranges are assigned afterward. Figure 12 is the structure of Entrance ticket. The Entrance ticket for any chain \( Ben \) is valid when:

1. The Merkle Branch and the hash of the Assignment request attached can form the Merkle root of the Assignment Box of the chain who has made the assignment.
2. The Assignment request is assigned to \( Ben \).
3. The Pab which made this assignment is the latest finally accepted Pab of that chain.
4. This Entrance ticket has not been used previously.
**Entrance ticket**

<table>
<thead>
<tr>
<th>Assignment</th>
<th>Chain ID</th>
<th>Merkle branch</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The Chain ID of the chain which made the assignment.</td>
<td>The Merkle Root of the Assignment Box.</td>
</tr>
<tr>
<td>Block header hash</td>
<td>The block header hash of the block which made the assignment.</td>
<td>multiple levels</td>
</tr>
</tbody>
</table>

**Figure 12: Entrance ticket**

**Power adjustment** A Fub of a chain Ben will record the valid Entrance ticket to the chain Ben. The miners of chain Ben acquire the Intended Difficulty of the Entrance tickets and add the nodes into the chain. However, after adding, if the restriction $2 \times \text{Threshold Chainpower} \geq \text{Chainpower}$ is not met, the Intended Difficulty of the new assigned nodes in $\text{group}(Sg - 1)$ is lowered to $bl(Sg - 1)$. If the restriction has still not been met, the chain will be merged with others. The adjusted Intended Difficulty is then used as the reference for assigning a Try Range.

4. Blockchain Sharding Security

While blockchain sharding is a simple and appealing concept, there are some substantial security challenges to be addressed. First, how do we distribute people to shards in a decentralised and unpredictable way? Second, how can people determine if a block in a shard is created by the people assigned to the shard? Third, without monitoring what happens in a shard, how can an outsider know if the majority in that shard supports a block or not? Fourth, to make collusion hard to happen, how large the population in a shard must be and how many shards we must have? In this section we discuss the security models and issues surrounding blockchain sharding.

4.1. $n/3$ Security Level Blockchain Sharding

4.1.1. Committee failure probability

This is about how likely node assignment to a shard may cause it to be compromised, or its failure probability. Given $n$ nodes, the probability of having no less than $X (X > m/2)$ adversary nodes in a shard when randomly picking a shard of size $m$ (the number of nodes inside the shard) can be calculated by the cumulative hypergeometric distribution function without replacement [16]. Let $X$ denote the random variable corresponding to the number of adversary nodes in a shard and $t$ be the number of adversary nodes in the system. The failure probability for one shard is

$$\Pr[X > \lfloor m/2 \rfloor] = \sum_{X=\lfloor m/2 \rfloor+1}^{m} \binom{t}{X} \binom{n-t}{m-X} \binom{n}{m}$$ (10)

where $n$ is the number of nodes and $t$ is the number of adversary nodes in the whole system, respectively. Figure 13 shows the maximum probability to fail with $n = 2000, t = n/3, t = n/2$ and $m = n/s$ where $s$ is the number of shards. As can be seen from the result, the system has a very high failure probability when there are $n/2$ adversary nodes. This is the main reason why most blockchain sharding approaches can only withstand up to $n/3$ nodes being adversary and only a few shards can exist.
4.1.2. Epoch failure probability

In practice, however, a blockchain is secure only when an adversary fails to take control of any shard in the system. Thus, we need to consider system security for a blockchain as a whole or epoch failure probability. Let \( f(x, y, z) \) denote the probability of the system remaining secure when there are \( x \) shards containing \( z \) adversary nodes in total and \( y \) of which are located in the last shard. Let \( O = n - t \) denote the number of honest nodes, and \( \alpha_x = (x - 1) \times m \) indicates the number of nodes located in the first \( x - 1 \) shards. For \( x > 0 \), \( 0 \leq z < \alpha_x / 2 \) and \( \max(0, z - (x - 1) \times \lfloor m/2 \rfloor) \leq y \leq \min(z, \lfloor m/2 \rfloor) \) (i.e. when the blockchain remains secure),

\[
f(x, y, z) = \sum_{j=0}^{\min(z, \lfloor m/2 \rfloor)} f(x - 1, j, z - y) \times \frac{\binom{x-1-j}{y} \times \binom{O-(\alpha_x-1-z-y)}{m-y}}{\binom{m}{m-y}}
\]

(11)

Specially, we have

\[ f(0, 0, 0) = 1 \]

and for other \( f(x, y, z) \)’s

\[ f(x, y, z) = 0 \]

Then, the probability that the system will fail is

\[
Pr_e(B) = 1 - \sum_{j=0}^{\min(m/2)} f(s, j, t)
\]

(12)

where \( B \) stands for the whole blockchain.

**Example 1.** We illustrate the calculation of \( Pr_e(B) \) with an example. Suppose that we have 12 nodes \( n = 12 \) of which 3 are adversary \( t = 3 \), and 3 shards \( s = 3 \) each having 4 nodes \( m = 4 \). We line up the shards and number them as follows, with \( O \) standing for honest and \( A \) adversary nodes. Assume that for the system to stay secured, we want no adversary having more than two nodes in a shard. Then:

- \( f(1, 0, 0) \) denotes the probability for the system to have zero adversary node in shard 1.

<table>
<thead>
<tr>
<th>Shard ID</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
</tbody>
</table>

\[ f(1, 0, 0) = f(0, 0, 0) \times \frac{\binom{3}{1} \times \binom{9}{2}}{\binom{12}{3}} = \frac{126}{495} = 0.255 \]

- \( f(1, 1, 1) \) denotes the probability for the system to have one adversary node in shard 1.

<table>
<thead>
<tr>
<th>Shard ID</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>O</td>
<td>O</td>
</tr>
</tbody>
</table>

\[ f(1, 1, 1) = f(0, 0, 0) \times \frac{\binom{3}{1} \times \binom{9}{3}}{\binom{12}{4}} = \frac{252}{495} = 0.509 \]
• $f(1, 2, 2)$ denotes the probability for the system to have two adversary nodes in shard 1.

$$
\begin{array}{c|ccc}
\text{Shard ID} & 1 & 2 & 3 \\
\hline
A & A & O & O \\
A & O & O & O \\
O & O & O & O \\
\end{array}
$$

$$
f(1, 2, 2) = f(0, 0, 0) \times \frac{\binom{1}{1} \times \binom{1}{1}}{\binom{3}{1}} = \frac{108}{495} \approx 0.218
$$

• $f(2, 0, 1)$ denotes the probability for the system to have zero adversary node in shard 2, while there is one adversary node in the first two shards.

$$
\begin{array}{c|ccc}
\text{Shard ID} & 1 & 2 & 3 \\
\hline
A & O & O & O \\
O & O & O & O \\
O & O & O & O \\
\end{array}
$$

$$
f(2, 0, 1) = f(1, 1, 1) \times \frac{\binom{0}{0} \times \binom{1}{1}}{\binom{3}{1}} = \frac{252}{495} \times \frac{15}{70} \approx 0.109
$$

• Accordingly, we have the following probabilities for other cases where the assignment of adversary nodes in the first two shards will leave the whole blockchain in a secure state:

$$
\begin{align*}
 f(2, 0, 2) &= f(1, 2, 2) \times \frac{\binom{1}{0} \times \binom{1}{1}}{\binom{3}{1}} = \frac{108}{495} \times \frac{35}{70} \approx 0.109 \\
 f(2, 1, 1) &= f(1, 0, 0) \times \frac{\binom{1}{1} \times \binom{1}{0}}{\binom{3}{1}} = \frac{126}{495} \times \frac{30}{70} \approx 0.109 \\
 f(2, 1, 2) &= f(1, 1, 1) \times \frac{\binom{1}{1} \times \binom{1}{1}}{\binom{3}{1}} = \frac{252}{495} \times \frac{40}{70} \approx 0.291 \\
 f(2, 1, 3) &= f(1, 2, 2) \times \frac{\binom{1}{0} \times \binom{1}{1}}{\binom{3}{1}} = \frac{108}{495} \times \frac{35}{70} \approx 0.109 \\
 f(2, 2, 2) &= f(1, 0, 0) \times \frac{\binom{1}{1} \times \binom{1}{1}}{\binom{3}{1}} = \frac{126}{495} \times \frac{30}{70} \approx 0.109 \\
 f(2, 2, 3) &= f(1, 1, 1) \times \frac{\binom{1}{1} \times \binom{1}{1}}{\binom{3}{1}} = \frac{252}{495} \times \frac{15}{70} \approx 0.109 \\
\end{align*}
$$

Note that $f(2, 0, 3)$ represents the case where the blockchain has been compromised already, hence not included in the above calculation.

• $f(3, 0, 3)$ denote the probability for the system to have zero adversaries in shard 3, while there are three adversary nodes in the first three shards.

$$
\begin{array}{c|ccc}
\text{Shard ID} & 1 & 2 & 3 \\
\hline
A & A & O & O \\
A & O & O & O \\
O & O & O & O \\
\end{array}
$$

$$
f(3, 0, 3) = (f(2, 1, 3) + f(2, 2, 3)) \times \frac{\binom{1}{0} \times \binom{1}{1}}{\binom{3}{1}} = 0.218
$$

• Similarly, we have the following probabilities for other cases where the assignment of adversary nodes in the first three shards will keep the whole blockchain secure, and these assignments have not been considered so far:

$$
\begin{align*}
 f(3, 1, 3) &= (f(2, 0, 2) + f(2, 1, 2) + f(2, 2, 2)) \times \frac{\binom{1}{0} \times \binom{1}{1}}{\binom{3}{1}} \approx 0.509 \\
 f(3, 2, 3) &= (f(2, 1, 1) + f(2, 0, 1)) \times \frac{\binom{1}{1} \times \binom{1}{1}}{\binom{3}{1}} \approx 0.218
\end{align*}
$$
From these calculations we obtain the probability of the whole system being secure as

\[ \sum_{i=1}^{m/2} f(s, i, t) = f(3, 0, 3) + f(3, 1, 3) + f(3, 2, 3) \approx 0.945 \]

and the probability of compromising the system is therefore

\[ \Pr_e(B) = 1 - \sum_{i=0}^{m/2} f(s, i, t) \approx 0.055 \]

4.2. \( n/2 \) Security level blockchain sharding

In this section, we introduce an improved sharding approach [18] that can withstand \( n/2 \) of nodes being adversary in an \( n \)-node system. With this approach, nodes are organised into \( m \) classes and \( s \) shards, and every shard must have one and only one node of each class. Table 1 shows one possible node assignment for a 5-shard and 5-class sharding, where \( A \) refers to an adversary node and \( O \) an honest one as before.

<table>
<thead>
<tr>
<th>Class</th>
<th>Shard 0</th>
<th>Shard 1</th>
<th>Shard 2</th>
<th>Shard 3</th>
<th>Shard 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>Class 2</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>A</td>
<td>O</td>
</tr>
<tr>
<td>Class 3</td>
<td>A</td>
<td>O</td>
<td>A</td>
<td>O</td>
<td>A</td>
</tr>
<tr>
<td>Class 4</td>
<td>O</td>
<td>A</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>Class 5</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
</tbody>
</table>

Note that as every shard must have one node from each class, the size of a shard is \( m \), the number of classes. The assignment of nodes into shards is carried out as follows. The system is separated into a working zone and a pending zone. The nodes inside the working zone are placed into the shards and can mine (verify transactions, propose, and approve blocks), while the nodes in the pending zone will wait to be assigned into the working zone. When new nodes join the system, they choose a class and are placed into the queue for that class in the pending zone. When the minimum number of nodes in every queue (class) has reached a pre-defined level, the first \( Q \) nodes of every queue are added to the working zone and all the nodes in the working zone are then reassigned to form shards.

A consensus on a statement is reached in a shard when at least a pre-defined \( T \) number of nodes in this shard agree on this statement, typically \( T > m/2 \). Thus, to manipulate a shard, an adversary must control \( T \) classes in a shard as each class can only have one member in a shard. Assume that the adversary puts all his or her nodes into the first \( T \) classes and has \( A_i \) nodes in class \( i \), \( i = 1, \ldots, T \), respectively. Then the probability for the adversary to control a shard is

\[ \Pr[X = T] = \prod_{i=1}^{T} \frac{A_i}{s} \]

where \( X \) is the number of nodes controlled by the adversary. To maximise this probability, it is sufficient to maximise \( \prod_{i=1}^{T} A_i \) as \( s \) is a constant. This is equivalent to splitting an integer \( t \) (the total number of adversary nodes) into a fixed number of factors \( A_1, A_2, \ldots, A_T \) so that their product \( \prod_{i=1}^{T} A_i \) is maximum, and is achieved by placing \( t \) nodes into \( T \) classes as equally as possible. We therefore have

\[ \Pr[X = T]_{\text{max}} \approx \left( \frac{t}{T \times s} \right)^T \]

Figure 14 shows the maximum failure chances with different numbers of shards \( s \), but a fixed total of \( n = s \times m = 2000 \) nodes and \( t = n/2 = 1000 \) adversary nodes in the system, and the security threshold set at \( T = 0.7 \times m \).

As can be seen from the result, when there are 10 shards and \( n/2 \) adversary nodes in the system, the failure chance is below \( 10^{-20} \), which significantly outperforms the previous sharding approaches that have a failure probability of \( 10^{-6} \) but only allow \( n/3 \) adversary nodes for the same setting (see Figure 13). On the other hand, if we accept failure chance at \( 10^{-6} \) with \( T = 0.7 \times m \), the \( n/2 \) approach can have 33 shards running in parallel, in contrast to 10 shards only with the \( n/3 \) solution.
4.2.1. Global halting

With the $n/2$ approach introduced above, an adversary will not be able to manipulate a consensus when he or she does not control at least $T$ nodes inside a shard but can still halt a consensus to be reached on a statement. Table 2 shows an example of system halting with $m = 5$ and $T = 4$,

Table 2: A system halting scenario

<table>
<thead>
<tr>
<th>Class</th>
<th>Shard 0</th>
<th>Shard 1</th>
<th>Shard 2</th>
<th>Shard 3</th>
<th>Shard 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>Class 2</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>Class 3</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>Class 4</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>Class 5</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
</tbody>
</table>

Here, the adversary does not have enough nodes to influence the consensus in a shard, but there are not enough honest nodes to deliver a consensus either. When this happens across all the shards, the system will stop to progress. That is, when an adversary takes control of $m - T + 1$ nodes in every shard, the system halts. Note that this halting problem cannot be resolved by adding more nodes, as the shard in charge of shard membership will also stop functioning and nodes re-assignment to shards will be needed.

A flexible $n/2$ security level blockchain sharding approach [19] has been proposed to solve this global halting problem. In this approach, instead of choosing a class when joining the system, every node chooses a colour code from the colour spectrum. The system maintains a number of base colours, and nodes are clustered into groups based on their closest base colour. In other words, we have replaced classes by base colours in this approach. While every class (base colour) will still have the same number of nodes and every shard still requires nodes from all the classes (base colours) as before, node assignment to a class will now depend on the colour code that the node has and the base colours that the system uses. This allows the membership of classes to be adjusted dynamically by changing the number of base colours used. When a global halting occurs, we can increase the number of base colours globally (which will decrease the number of shards as two are inversely proportional). Because the adversary is assumed to have no more resources than honest people in the system, in the worst case, the halting problem can be solved by reducing the number of shards to one. Once a system halting scenario is resolved, the system can then begin to split shards again to increase mining performance. The reader is referred to [19] for more details of this flexible sharding approach.

4.2.2. Epoch failure probability for $n/2$ security level blockchain sharding

We now analyse epoch failure probability for the $n/2$ security level sharding, which applies to both class-based and colour-based solutions described above. Let $f_1(x)$ denote the chance for an adversary to take control of any $x$ shards ($x > 0$) in the system. Then,

$$f_1(x) = \prod_{j=0}^{x-1} \prod_{i=1}^{T} \frac{\max(0, A_i - j)}{s - j}$$

$$\text{s} \in [2, 600], t = 1000 = n/2$$

Figure 14: The chance to fail with different $s$ when $n = 2000$ and $m = n/s$ where $s$ is the number of shards
As before, this chance is maximised if adversary nodes are placed equally across the $T$ classes. Let $f_2(x)$ denote the chance for the adversary to take the first $x$ shards while the honest take the rest $s - x$ shards.

$$f_2(x) = f_1(x) - \sum_{i=1}^{s-x} \binom{s-x}{i} f_2(x+i)$$ (16)

Specially, we have

$$f_2(s) = f_1(s)$$ (17)

So the chance for the system to fail is

$$Pr_e(B) = \sum_{x=1}^{s} \binom{s}{x} f_2(x)$$ (18)

**Example 2.** We illustrate the calculation of $Pr_e(B)$ with an example. Suppose that we have 16 nodes ($n = 16$), 4 shards, 8 adversary nodes ($t = n/2 = 8$) and $T = 3$. Then, $A_1 = 2$, $A_2 = 3$, $A_3 = 3$. If a shard is compromised, we denote it as $A$ otherwise we denote it as $O$.

- $f_1(4)$ denotes the chance of the adversary taking control of all 4 shards, and we have $f_2(4) = f_1(4) = 0.0$.

  \[
  \begin{array}{cccccc}
  \text{Shard ID} & 1 & 2 & 3 & 4 \\
  \hline
  \text{corresp. } f_1(4) & A & A & A & A \\
  \end{array}
  \]

- $f_1(3)$ denotes the chance of the adversary taking control of the first 3 shards, and we have $f_1(3) = 0.0$ and $f_2(3) = f_1(3) - f_2(4) = 0.0$.

  \[
  \begin{array}{cccccc}
  \text{Shard ID} & 1 & 2 & 3 & 4 \\
  \hline
  \text{corresp. } f_1(3) & A & A & A & A \\
  \text{corresp. } f_2(3) & A & A & O & A \\
  \end{array}
  \]

- $f_1(2)$ denotes the chance of the adversary taking control of the first 2 shards, and we have $f_1(2) \approx 0.0417$ and $f_2(2) = f_1(2) - f_2(4) \times 1 - f_2(3) \times 2 \approx 0.0417$.

  \[
  \begin{array}{cccccc}
  \text{Shard ID} & 1 & 2 & 3 & 4 \\
  \hline
  \text{corresp. } f_1(2) & A & A & A & A \\
  \text{corresp. } f_2(2) & A & A & O & A \\
  \end{array}
  \]

- $f_1(1)$ denotes the chance of the adversary taking control of the first shard, and we have $f_1(1) \approx 0.28125$ and $f_2(1) = f_1(1) - f_2(4) \times 1 - f_2(3) \times 3 - f_2(2) \approx 0.156$.

  \[
  \begin{array}{cccccc}
  \text{Shard ID} & 1 & 2 & 3 & 4 \\
  \hline
  \text{corresp. } f_1(1) & A & A & A & A \\
  \text{corresp. } f_1(1) & A & A & O & A \\
  \text{corresp. } f_1(1) & A & O & A & A \\
  \text{corresp. } f_1(1) & A & O & O & A \\
  \end{array}
  \]
Then, a delay time is shown in Figure 15. We simulated three calculation power scenarios MWPoW, we gave every connection a random delay time ranging from 1ms to 200ms. The distribution of connection 5.1. Experiment setup

connections and the network structure are dynamically adjusted to fit into the data flow to make data propagation fast. In this experiment, we maintain a 10−6 failure chance for every approach. We use a regulated layout of Distributed Ledger Network [25] as the communication protocol used for the essential P2P connections. The connections and the network structure are dynamically adjusted to fit into the data flow to make data propagation fast. 4.3. Multichain Epoch failure probability

Let \( f_3(x) \) denote the maximum chance for an adversary to take \( x \) shards. Select \( x \) shards (refer to them as \( Ss(k, 1), \ldots, Ss(k, x) \)), where \( k \) refers to the \( k \)-th selection from a total of \( \binom{s}{x} \) selection schemes. All other shards are listed as \( Ss(k, x + 1), \ldots, Ss(k, s) \). If there is at least one vote from class \( i \) on the latest block of any of the selected shards, then \( DG(i) = 1 \). We then use this \( DG(i) \) to calculate \( tt = \sum_{i=0}^{i=8} DG(i) \times bl(i) \) and calculate \( PSs \) for all selected shards \( Ss \). Let \( Tt(i, j) \) refer to the overall number of nodes in class \( i \) in the front \( j \) shards of the \( Ss \) list.

\[
PSs(k, j) = \prod_{i=0}^{i=Ss(k,j)} \frac{\binom{Sg(k,j)}{i} - DG(i) \times Tt(i, j - 1)}{\binom{Sg(k,j)}{i}}
\]

Let \( RPSs(k, x + 1 \ldots s) = 1 - PSs(k, x + 1 \ldots s) \) for all the uncollected shards.

\[
f_3(x) = \sum_{k=1}^{k=8} PSs(k, 1) \times PSs \times \cdots \times PSs(k, x) \times RPSs \times \cdots \times RPS (k, s)
\]

Then,

\[
Pr_e(B) = \sum_{x=1}^{x=21} f_3(x)
\]

\( Pr_e(B) \) can only be calculated in operation and its worst case is equal to that of \( n/2 \) Adversary Resistant Blockchain Shading (one node per class and all the votes on the blocks come from the same classes).

5. Experiment

In this section, we experimentally evaluate the performance of Multichain MWPoW and test its scalability. We compare its performance with RapidChain [16] and \( n/2 \) Byzantine node resistant blockchain sharding approaches [18, 19] regarding throughput and transaction confirmation time with different percentage of adversary power in the system. In this experiment, we maintain a 10−6 failure chance for every approach. We use a regulated layout of Distributed Ledger Network [25] as the communication protocol used for the essential P2P connections. The connections and the network structure are dynamically adjusted to fit into the data flow to make data propagation fast. 5.1. Experiment setup

We simulated 8000 nodes in a network with 10Mbytes/s bandwidth per node in our experiments. For Multichain MWPoW, we gave every connection a random delay time ranging from 1ms to 200ms. The distribution of connection delay time is shown in Figure 15. We simulated three calculation power scenarios \( A, B, \) and \( C \) for every node, which are shown in Figure 16. In our experiments, we set \( K \) to be 2000, meaning that blocks can contain up to 2000 transactions per block, and a Pab can contain up to 2000 Assignment requests in the New participant section of the assignment box. When blocks are broadcast inside a chain, the blocks will be encoded by Graphene [5] like the original MWPoW. If a block is requested by nodes outside the chain or is requested by a new participant when it is synchronising data, the block sent will be the one which is decoded using Graphene.

For other approaches, i.e., RapidChain and two existing \( n/2 \) blockchain sharding approaches, they are implemented exactly the same as the network setting described above for Multichain MWPoW, except that the protocol runs on every node is not Multichain MWPoW, but the respective protocol and every node is equal in voting.
We send $10^6$ transactions per iteration to the network by random nodes (equal allocation). The size of each transaction is fixed at 500 bytes. We record the number of chains (shards) in the system and the throughput (the number of transactions processed globally per iteration). We also record the transaction confirmation time and the frequency of data refreshing (the frequency of a node being re-assigned). For Multichain MWPoW, we increase the amount of malignant power from 0 to 50% of the overall power during an attack, the number of malignant nodes may be higher than half of the node population. For other approaches, we increase the number of malignant nodes from 0 to 50% of all the nodes during an attack. We set the block interval to be 10 seconds globally and the experiments were conducted over 1000 block intervals. $T_{i}$ is set to 20 block intervals and $S_g = 20$. For $n/2$ blockchain sharding and flexible $n/2$ blockchain sharding, $m$ is set to be 33 at the beginning, and $T = 0.7 \times m$. The flexible $n/2$ blockchain sharding approach may adjust this number during the experiment. The number of shards is set as 55 for RapidChain to maintain the security threshold. For RapidChain, 20% of the transactions are multiple input shard transactions: a transaction that must be confirmed by all the input shards to proceed. All the approaches used in our experiments maintain a $10^{-6}$ failure probability.

An adversary node in Multichain MWPoW will function like an honest node if it does not have enough companions in the chain. When there are enough adversary nodes inside a blockchain to halt the shards, the adversary nodes will function maliciously by attempting to create corrupted fork branches. The adversary node in RapidChain has a 50% chance to drop out in every ten iterations. The dropped node will apply to join RapidChain again immediately. The adversary nodes will start to create wrong blocks when they have gained control over a shard. The adversary nodes for the two $n/2$ blockchain sharding approaches will correctly function when they do not have enough nodes to halt the shards. They will halt a shard immediately when having enough number of adversary companions. We set a transaction in the first block as the initial transaction. The inputs of transactions are randomly selected from the transactions in previous blocks. In our experiments, Multichain MWPoW started with one chain. Nodes were added to the system following the rule of Multichain MWPoW as soon as possible. Nodes in other approaches were also added following the rules as quickly as possible. When increasing the adversary percentage, we randomly select the honest nodes in the system and turn them into adversary nodes to obtain the required percentage.

5.2. Experiment results

The experiment lasted 1000 block intervals. Figure 17 shows the changes of chains (shards) in the progress of block intervals. As can be seen from the results, for Multichain MWPoW, the number of chains and the processing capacity are dynamically adjusted to fit the data flow and to prevent adversary’s power from halting the chains. Power distribution scenario C is generally more stable than A and B, mostly because the power is more balanced.

From Figure 18, we can see that the $n/2$ sharding approach stopped functioning after an adversary took 33% of the nodes, although still produced correct results. We stopped RapidChain functioning after an adversary took 33% of the power because at that point the security of RapidChain was wholly broken. The flexible $n/2$ approach also uses $K$ to indicate pending transactions. We can see that transactions processed by the flexible $n/2$ approach is drastic, and this is different from the pattern in Figure 17. This indicates what number of processed transactions could be. We see this difference because when a shard halts in both $n/2$ and flexible $n/2$ approaches, the shard is frozen until
new memberships replace the old nodes in this shard. The transaction per second for the flexible \( n/2 \) and Multichain MWPoW largely depends on the percentage of nodes/power doing halting attack. As can be seen from Figure 17, the flexible \( n/2 \) and Multichain MWPoW approaches have a performance that is about 8 times better than RapidChain. When dealing with halting attacks, their performance decreased and approached to RapidChain’s performance when the adversary have close to 50% of the nodes/power. Note that the security of RapidChain has already broken after the adversary has gained 33% of the nodes/power. The halting problem also explains why the \( n/2 \) approach has a slight fluctuation in operation. When a global halting occurred, the transaction per second was reduced to zero, and the system took a few intervals to recover from halting.

![Figure 17: The number of chains/shards.](image1)

![Figure 18: The number of transactions processed per iteration.](image2)

In Multichain MWPoW, however, the system would not stop processing transactions: halting would only affect when the blocks would be finally confirmed. Figure 21 shows the times of data refreshing in our experiment. Recall from experiment setup, there is \( \frac{1}{20} \) chance for the adversary nodes in RapidChain to quit and rejoin a shard immediately. Since any time when some nodes are assigned to a shard, the same number of old nodes in this shard must be re-assigned to other shards, this design causes the majority of refreshing in RapidChain. As there is no limitation on how many times a node can join or leave the system, an attacker can make this attack on a large scale in practice. This attack could also work for both \( n/2 \) and flexible \( n/2 \). However, in our experiment, the adjustments employed by the \( n/2 \) approaches are used mainly to solve the halting problem. For Multichain MWPoW, the adjustments are primarily there for solving local halting as well as for adjusting to data flow (changes in the number of pending transactions).

Figure 19 shows the average number of transactions per iteration in our experiments. This came to 0 for the \( n/2 \) blockchain sharding approach and RapidChain after the adversary has taken 33% of the nodes. Because the Flexible \( n/2 \) and Multichain MWPoW can safely split to create more shards/chains, we see that their average performance is about 5 times better than RapidChain. Figure 20 shows the average pending time for nodes to accept a transaction finally.

6. Conclusion

We have presented the Multichain MWPoW approach to blockchain sharding in this paper. Our new solution achieves an efficient and robust decentralised autonomous organisation architecture. Multichain MWPoW is the first blockchain sharding approach that can withstand up to 50% of adversary power without assuming that honest people have to create as many nodes in the system as possible. Our experiments show that Multichain MWPoW largely outperforms Rapidchain, the \( n/2 \) blockchain sharding approach [18] as well as the flexible \( n/2 \) blockchain sharding approach [19] in terms of stability, throughput, and transaction confirmation time. We have proposed a secure random distribution mechanism which maintains a threshold distribution of power inside every chain. We categorise nodes into different classes dynamically and require at least one node per class per chain, the number of participants per chain (shard) is significantly reduced, allowing more chains to be be split. This brings a significant improvement in terms of scalability.
References

Appendix A. Intra-shard Protocol: Brief outline of MWPoW protocol

Appendix A.1. Calculation Power

The calculation power is defined as, in a fixed time window, how much PoW difficulty a machine can achieve. PoW difficulty is a measure of how difficult it is to generate a PoW:

\[
\text{Difficulty} = \frac{\text{difficulty}_\text{target}}{\text{current}_\text{target}}
\]  

(A.1)

where \(\text{difficulty}_\text{target}\) is a 256-bit constant and \(\text{current}_\text{target}\) is any 256-bit number. A machine can alter the hash of a given String by adding some random value to the end of the String and use the hash as the \(\text{current}_\text{target}\) to calculate the Difficulty. The machine’s calculation power implies the maximum Difficulty the machine guaranteed to achieve within a fixed time window.

Appendix A.2. Protocol outline

Multiple Winners Proof of Work protocol (MWPoW) [17] works with the following security assumptions.

- Just like in the Nakamoto blockchain, it assumes that the majority of participating calculation power is honest.
- The same as an asynchronous protocol, it assumes honest nodes would only vote for one block candidate at a block height while there can be multiple block candidates. The honest votes may arrive late but will arrive eventually.
- The block height can move on regardless if a consensus has been reached or not. Therefore, when MWPoW is implemented in a sharded system, the progress of block height can be synchronous for every shard, despite the progress for consensus is asynchronous among the shards.

The working procedure of MWPoW is divided into the following steps:

1. Power registration: When joining the system, nodes are required to declare an amount of calculation power (in PoW difficulty form) that they will put into the competition in every block interval, with an attached PoW to prove that.

2. Progress of block height: There are two PoW difficulties for a block of every block height: Entrance Difficulty and Acceptance Difficulty. The Entrance Difficulty is for proposing a block and the Acceptance difficulty is for the system to move on to the next block height. The two PoW difficulties are adjusted after every block interval. Every node is in charge of a different interval in \([0, 2^{256}]\) for the Nonce of the block. By adjusting the Nonce, the hash of block can be adjusted, then the PoW difficulty of the block can be adjusted. At the beginning of every block interval, nodes create their own blocks. When a node finds a Nonce in its Nonce interval for its block that makes the hash of the block fulfill the Entrance difficulty, it broadcasts the block and the Nonce. Every node then tries to find a Nonce of the Acceptance difficulty in their Nonce interval for the block that reaches the Entrance difficulty first. When such a Nonce is found and broadcast, the system moves to the next block height.
3. **Voting:** The node should vote four times at each block height by broadcasting its Nonces for a block. Each Nonce for the block should make the PoW difficulty of this block fulfill 25% of the node’s registered power. The Nonce is embedded in a data structure called *Share*. Note that a Nonce is only valid for one *Share*. Table A.3 shows the structure of a *Share*. Because every *Share* embeds the hash of the preceding *Share*, every node is linked to the preceding *Share* and therefore there is a **vote chain** for every voter. There are four vote chain heights within every block height, as shown in Table A.3.

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block candidate hash</td>
<td>The hash of the block candidate for which the voter votes.</td>
</tr>
<tr>
<td>Last vote hash</td>
<td>The hash of the last vote which the voter sent.</td>
</tr>
<tr>
<td>Vote Array</td>
<td>An array of the <em>Share</em> hashes at the vote chain height (i-1) from the nodes which together registered at least (\lfloor n/2 \rfloor + 1) of the calculation power.</td>
</tr>
<tr>
<td>Nonce</td>
<td>A 256-bit integer for adjusting the PoW difficulty of the block.</td>
</tr>
</tbody>
</table>

4. **Vote counting:** Instead of counting the number of *Shares* for a block in \(h\) block height, we calculate the accumulated PoW difficulty for this block as well as the whole branch of blocks stemming from it (from \(h\) block height to the latest block height \(H\)). Note that if the PoW difficulty is larger than 25% of the voter’s registered power, only 25% of the voter’s registered power will be counted. Let \(Rp\) be a block candidate of block height \(h\). Let \(W(i(Rp))\) be the number of PoW difficulties at block height \(i \geq h\) which voted for the descendent block candidates of \(Rp\) at block height \(i\) (if \(i = h\) then for \(Rp\) itself). The accumulation of PoW difficulties for \(Rp\) is

\[
P(Rq) = \sum_{i=h}^{H} W(i(Rq)) \tag{A.2}
\]

where \(H\) is the latest block height.

5. **Vote discounting:** According to the vote chain of a node, if this node has voted for multiple block candidates at \(i\) block height, the *Shares* in this vote chain for the block candidates from \(i\) to \(H\) block heights are discounted in Equation A.2. If a node has multiple vote chains (due to sending multiple *Shares* with the same *Last vote hash*), we only count the PoW difficulties of the longest vote chain. If the vote chains are of equal length, we only count the PoW difficulties before forking. According to a node’s vote chain, if this node has not voted four times in block height \(i\), its *Shares* for block height \(i + 1\) will not be counted.

6. **Consensus reaching:** A block *Alice* at block height \(h\) is accepted when the following two conditions are met at the same time.

(a) \[
P(Alice) > 0.5 \times \sum_{i=h}^{H} CP[i] \tag{A.3}
\]

where \(CP[i]\) is the overall registered power at block height \(i\), \(P(Alice)\) is the accumulation of PoW difficulties for *Alice* and its descendent block candidates, and \(H\) is the latest block height.

(b) \[
P(Alice) - P(Gary) - \sum_{i=h}^{H} P(U_i) > 1 \times \frac{1}{4} \sum_{i=h}^{H} CP[i] \tag{A.4}
\]

where *Gary* is the second most supported block candidate at block height \(h\). \(P(U_i) = CP[i] - VP_i\) and \(VP_i\) is the accumulation of PoW difficulties of the votes in block height \(i\). \(P(U_i)\) stands for the PoW difficulties of the nodes which have not yet voted in block height \(i\).
7. **Adversary eliminating:** A graph can be built over the Vote Array in a share sent from any node $q$. The vote chains of all the nodes can be derived from this graph, and these vote chains are the vote chains from the perspective of the sender node $q$ of this share. Suppose that we have two block candidate Alice and Gary at the $h$ block height, and they are both correct (acceptable). $P(Alice)$ and $P(Gary)$ from the perspective of node $q$ have been determined, and we refer to them as $P_q(Alice)$ and $P_q(Gary)$ respectively. Then node $q$ in the branch of Gary is an adversary if

$$P_q(Alice) - P_q(Gary) - \frac{\sum_{i=0}^{H} P_q(U_i)}{N} > \frac{CP[H]}{2}$$

(A.5)

The future votes from the node $q$ should not be counted in Equation A.2 after it has been considered as an adversary.

8. **Membership adjustment:** Every block records the Shares of blocks at the last block height. The membership of a node is cancelled from the branch of this block onward, provided that the block does not embed at least two of the node’s Shares.

Note that the sender of every information must use a digital signature.

**Appendix A.2.1. Security**

The Vote Array structure inside the share enables nodes to derive vote chains for all nodes from the perspective of voter (the sender of this share). In this way, MWPoW is able to determine if this node given a delayed vote for a wrong branch due to synchronisation delays or the node is actually an adversary. In another word, using the vote chains derived from the vote array, we are able to determine if a node has a reasonable ground to continuously support a minority branch. This feature enable MWPoW to tolerate up to $f = \lfloor (n - 1)/2 \rfloor$ adversary nodes in an $n$-node system if all the nodes are assumed to have the same calculation power. The reader is referred to [17] for more details of MWPoW.

**Appendix B. Multichain MWPoW Data analysis**

Miners are required to synchronise block headers of the announced blocks of all the chains and the blocks inside the chains they have been assigned to. Table B.4 shows the minimum size of a block header, Share, Entrance ticket, or Assignment request in Multichain MWPoW.

<table>
<thead>
<tr>
<th>Name</th>
<th>Size</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block header</td>
<td>124 + Sg×4 bytes</td>
<td>Three 256-bits hashes: the hash of the preceding block, MGBH, and the Markle root of the transactions.</td>
</tr>
<tr>
<td>Share</td>
<td>64.5 bytes</td>
<td>A 4-bits integer, the last four bits of the block hash, a 256-bits integer (Nonce), and a 256-bits signature.</td>
</tr>
<tr>
<td>Assignment request</td>
<td>132 bytes</td>
<td>A 256-bits hash (HashPrevBlock), a 32-bits integer (IntendedDifficulty), three 256-bits integers (Wallet address, IdentityKey and Nonce).</td>
</tr>
<tr>
<td>Entrance Ticket</td>
<td>12+32+log$_2(K)$×32 bytes</td>
<td>Three 32-bits integers (LL, IntendedDifficulty and Assignment Chain ID). A 256-bits hash (Block header hash), and log$_2(K)$ number of 256-bits hashes (Merkle branch).</td>
</tr>
</tbody>
</table>

The participants send shares during every iteration after joining in a chain. The majority of Assignment requests are only broadcast at block heights before a Pab in one iteration interval. The Entrance tickets are broadcast between a Pab is finally accepted, and before the next Fub comes out (also one mining interval). Thus, the minimum upload bandwidth required for a miner in the most data-intensive iteration is $Max(\text{SizeEntranceTicket}, 32 \times Sg \times \text{AssignmentRequest})$. The download bandwidth for a participant in the most data-intensive iteration in a system with NPC participants inside the chain is $NPC \times Max(\text{SizeEntranceTicket}, \text{SizeShare} \times 4, \text{SizeAssignmentRequest}) + \text{SizeTransactions} \times K$. Figure B.22 shows the download bandwidth requirement and transaction throughput globally with $n = 8000$ and different number of NC and $K$. NC is ranged from 1 to 400 ($\frac{n}{20}$, 20 participants per chain). $Sg = 20, NPC = \frac{n}{NC}$, while $K$ ranged from 2 to 1000.
Figure B.22: Data requirement and Throughput per iteration with different $K$ and $NC$ when $n = 8000$. 