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MeV-scale reheating temperature and cosmological production of light sterile neutrinos

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We investigate how sterile neutrinos with a range of masses influence cosmology in MeV-scale reheating temperature scenarios. By computing the production of sterile neutrinos through the combination of mixing and scattering in the early Universe, we find that light sterile neutrinos, with masses and mixings as inferred from short-baseline neutrino oscillation experiments, are consistent with big-bang nucleosynthesis (BBN) and cosmic microwave background (CMB) radiation for the reheating temperature of $\mathcal{O}(1)$ MeV if the parent particle responsible for reheating decays into electromagnetic components (radiative decay). In contrast, if the parent particle mainly decays into hadrons (hadronic decay), the bound from BBN becomes more stringent. In this case, the existence of the light sterile neutrinos can be cosmologically excluded, depending on the mass and the hadronic branching ratio of the parent particle.
I. INTRODUCTION

The anomaly in short-baseline (SBL) neutrino experiments is a long-standing problem in the neutrino sector. Since the LSND collaboration reported a 3.8-σ anomaly in their results in the 1990s [1], various experimental projects have been performed to investigate the origin. The Mini-BooNE collaboration found the similar anomaly in both the neutrino and anti-neutrino modes [2], and it remains after the update in the experiment [3]. In addition to the accelerator neutrino oscillation experiments, similar anomalies have been found in other types of experiments, e.g., reactor neutrino experiments such as Daya Bay [4] and Double Chooz [5], or Gallium experiments such as SAGE [6–8] and GALLEX [9–11].

The existence of the eV-scale sterile neutrino produced through the mixing with active neutrinos is a well-motivated scenario to explain the anomaly. This scenario has been tested in different kinds of experiments. In contrast to the appearance experiments, disappearance experiments such as MINOS/MINOS+ [12] and NOνA [13] reported the results disfavoring the existence of such a sterile neutrino. The IceCube collaboration also investigated a signature of the conversion from active to sterile neutrinos in the atmospheric neutrino spectrum and gave a strong constraint on the parameter space of the light sterile neutrino for the SBL neutrino anomaly [14]. The origin of the anomaly is still under debate, and future experimental programs such as the SBN experiment [15] and the JSNS$^2$ experiment [16] are expected to unveil the origin.

Cosmological observations are another important probe of sterile neutrinos. If such light sterile neutrinos exist and have an appreciable mixing with active neutrinos, they are abundantly produced in the early Universe and affect the big-bang nucleosynthesis (BBN) and the cosmic microwave background (CMB) radiation. In Refs. [17–20], it was shown that the light sterile neutrino inferred from the SBL anomaly is completely thermalized well-before the onset of BBN or the last scattering of CMB. This means the existence of the light sterile neutrino is strongly excluded from BBN and CMB. The tension could be alleviated by suppressing the thermalization of sterile neutrinos. Several scenarios have been proposed as the suppression mechanism: large chemical potentials of active neutrinos [18, 21–23], self-interaction or non-standard interaction of sterile neutrinos [24–25], or low reheating temperature of the Universe [30–35].

In this paper, we focus on the thermalization of sterile neutrinos in the Universe with an MeV-scale reheating temperature to solve the tension between the light sterile neutrino and cosmology. Since the sterile neutrino production through the weak interaction of active neutrinos effectively finishes when the cosmic temperature becomes $\sim \mathcal{O}(1)$ MeV, the MeV-scale reheating temperature
leads to the incomplete thermalization of sterile neutrinos, which offers the solution to the problem.

The reheating temperature of the Universe is much lower than that of the standard cosmology if there exists a long-lived massive particle and it causes reheating of the Universe. The existence of such particles is naturally expected in varieties of extensions of the standard model of particle physics. For example, curvaton, gravitino, flaton, modulus, or dilaton are well-motivated candidates for this particle. If such a long-lived particle dominates the energy density in the early epoch, the Universe experiences the early matter-dominated era before the ordinary radiation-dominated epoch, which modifies the initial condition of the standard cosmology.

This solution to the tension has been proposed in Ref. [30], and Refs. [32–34] later revisited the same scenario. In order to probe this scenario, it is necessary to simultaneously solve reheating of the Universe and the thermalization of neutrinos to accurately compute the abundance of sterile neutrinos. This is because most of the active and sterile neutrinos are produced during reheating, and matter effects on the thermalization of sterile neutrinos, cannot be neglected. However, such a computation is technically difficult, and Refs. [30, 32–34] assumed a simplified picture where sterile neutrinos are produced via vacuum oscillations after the completion of reheating. Also, the reheating temperature is fixed to be 5 MeV by hand in the studies. If sterile neutrinos are completely absent from the thermal bath, the lower bound on the reheating temperature is known to be almost 5 MeV [36, 37], but this is not true if sterile neutrinos exist, and it contributes to the energy density of the Universe. Therefore, we should not fix the reheating temperature to the typical value in advance.

Ref. [34] later updated the sterile neutrino production in the MeV-scale reheating scenario by calculating the semi-classical Boltzmann equation with effective collision terms, which include the matter effects, and provided a more detailed analysis of the thermalization of sterile neutrinos. It is however necessary to calculate the original quantum kinetic equation (QKE) instead of the semi-classical Boltzmann equation, to correctly follow the sterile neutrino thermalization unless the off-diagonal components of the collision term for neutrinos (i.e. the collisional damping term) dominate those of the neutrino Hamiltonian [38, 39].

The purpose of this study is to revisit the sterile neutrino thermalization in the cosmological model with an MeV-scale reheating temperature and refine the cosmological constraint on sterile neutrinos obtained in the previous studies [30, 32, 33] by performing a detailed computation of QKE and BBN. The main focus of this study is the eV-scale sterile neutrinos, motivated by the SBL neutrino anomaly.

The structure of this paper is as follows. In Sec. II we introduce our formulation for calculating
the production of active and sterile neutrinos during reheating. In Secs. III and IV we show our numerical results of the neutrino thermalization and BBN, respectively. In Sec. V we summarize the constraint on sterile neutrinos obtained from cosmological observations and ground-based experiments. Sec. VI is devoted to the conclusion.
II. STERILE NEUTRINO PRODUCTION DURING REHEATING

In this section, we explain the dynamics of cosmological models with late-time entropy production, which results in the MeV-scale reheating temperature. Also, we introduce key equations for calculating the production of sterile neutrinos during reheating.

We assume that a long-lived massive particle $\phi$ is responsible for reheating. In this case, the decay of $\phi$ induces the late-time entropy production and the subsequent dramatic particle production of the standard-model particles. Photons and charged leptons are rapidly thermalized through the electromagnetic interaction during reheating, while active neutrinos are slowly produced through the weak interaction. If the reheating temperature of the Universe is lower than the QCD scale $\sim 100$ MeV and the radiation-dominated epoch therefore realizes after the hadronization, active neutrinos are solely produced in the annihilation process of charged leptons $l + \bar{l} \rightarrow \nu_\alpha + \bar{\nu}_\alpha$ ($\alpha = e, \mu, \tau$), where $l$ and $\bar{l}$ denote the charged leptons and corresponding anti-particles, respectively. Sterile neutrinos are generated from active neutrinos through the flavor mixing as reheating proceeds. Therefore, we need to consider both neutrino collisions and neutrino oscillations in the thermalization calculations of active and sterile neutrinos. Ref. [40] provided an analytical expression to estimate the sterile neutrino abundance produced in the non-resonant active-sterile mixing. The production rate of sterile neutrinos has a sharp peak at temperature $T_{\text{max}}$:

$$T_{\text{max}} \sim 13 \text{ MeV} \left( \frac{m_s}{1 \text{ eV}} \right)^{1/3}, \quad (2.1)$$

where $m_s$ is the mass of the sterile neutrino. Hence, the abundance of sterile neutrinos is strongly suppressed compared to those obtained in the standard cosmology case if the reheating temperature is lower than $T_{\text{max}}$.

The states of active and sterile neutrinos are expressed in terms of a one-body-irreducible density matrix, which is expressed in an $N_f \times N_f$ Hermitian matrix, where $N_f$ is the number of neutrino flavors to mix. In this study, we adopt the so-called 1+1 mixing scheme in which one sterile neutrino species mixes with one active species. This approximation is reasonable when the mixing of the sterile neutrino with one active species dominates the mixing with the other active species. For the current calculation, we assume sterile neutrinos to mix with electron neutrinos, and we assume that $\mu$ neutrinos ($\nu_\mu$) and $\tau$ neutrinos ($\nu_\tau$) decouple from the neutrino oscillations. Under these assumptions, the states of spectator neutrinos, namely $\nu_\mu$ and $\nu_\tau$, are degenerate, and

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1 Even if a primordial component of sterile neutrinos exists before reheating, such a component is completely diluted by the entropy production associated with reheating. Also, we do not consider any other exotic interactions among the standard-model particles and sterile neutrinos. Therefore, sterile neutrinos are produced only through the active-sterile neutrino oscillation.
it is unnecessary to separately calculate dynamical equations for each. This is because the cosmic temperature is always below $O(1)$ MeV after reheating for $T_{RH} \sim O(1)$ MeV, and muons and $\tau$ leptons, which are heavier than the cosmic temperature, do not exist in the thermal bath of the Universe. In the following, quantities of the spectator neutrinos are multiplied by a factor of two for summing up contributions from $\nu_\mu$ and $\nu_\tau$.

Since we assume the 1+1 mixing, the density matrix of neutrinos with energy $E$ can be expressed as a $2 \times 2$ matrix:

$$\varrho_p(t) \equiv \varrho(E, t) = \begin{pmatrix} \varrho_{aa} & \varrho_{as} \\ \varrho_{as}^* & \varrho_{ss} \end{pmatrix}. \quad (2.2)$$

The energy of active and sterile neutrinos $E$ is replaced with their absolute momentum $p$, i.e. $E \rightarrow p \equiv |p|$, where $p$ is the three-momenta of neutrinos. This is because masses of the active neutrinos are known to be sub-eV scale [41] and safely neglected in a thermal bath of $T \sim O(1)$ MeV. In addition, we restrict ourselves to the mass range of sterile neutrinos below 10 keV so that they are always relativistic before their production effectively finishes at around a temperature of the neutrino decoupling $T \sim T_{\text{dec}}$. In Eq. (2.2), the diagonal elements of the density matrix correspond to the distribution functions of active and sterile neutrinos, i.e. $\varrho_{aa} = f_a$ and $\varrho_{ss} = f_s$, while the off-diagonal elements correspond to a quantum coherence between them.

The time evolution of the density matrix is governed by the momentum-dependent quantum kinetic equation (QKE) in the following [43, 44]:

$$\frac{d\varrho_p(t)}{dt} = \left( \frac{\partial}{\partial t} - H_p \frac{\partial}{\partial p} \right) \varrho_p(t) = -i [\mathcal{H}_\nu, \varrho_p(t)] + C[\varrho_p(t), t], \quad (2.3)$$

where $C[\varrho_p(t), t]$ is the collision term for the active-mixed neutrinos, $H$ is the Hubble parameter, and $\mathcal{H}_\nu$ in the commutator is the neutrino Hamiltonian.

In this study, we neglect the neutrino chemical potentials. Then, it is unnecessary to follow the time evolution of anti-neutrinos separately from corresponding neutrinos, and the neutrino Hamiltonian on the right-hand side of Eq. (2.3) is reduced to

$$\mathcal{H}_\nu = \frac{M^2}{2p} - \frac{8}{3} \sqrt{2} G_F p \left[ \frac{E_{CC}}{m_W^2} + \frac{E_{NC}}{m_Z^2} \right], \quad (2.4)$$

where $G_F$ is the Fermi coupling constant, and $m_W$ ($m_Z$) is the mass of $W$ ($Z$) boson. On the right-hand side of Eq. (2.4), the first term corresponds to the vacuum oscillation of neutrinos. The

---

2 This limitation is mandatory because non-relativistic neutrinos do not oscillate into another flavor [42], and we cannot rely on the QKE for neutrinos (Eq. (2.3)) in such cases.

3 This is a reasonable assumption since its effect on the neutrino oscillation can be safely neglected for the chemical potentials of $O(10^{-10})$, as is naturally attained in the standard mechanism of baryogenesis associated with the sphaleron process. For those interested in the effect of the neutrino chemical potentials on the sterile neutrino thermalization, see e.g. Refs. [18, 22, 23, 45, 46].
mass matrix $M$ in the flavor basis is related to that in the mass basis $\mathcal{M}$ as $M^2 = U\mathcal{M}^2 U^\dagger$ where $U$ is the flavor-mixing matrix. The mass matrix $\mathcal{M}$ for the 1+1 mixing has the explicit form of

$$\mathcal{M}^2 = \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}, \quad U = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix},$$

(2.5)

where $m_1$ and $m_2 (> m_1)$ are the mass eigenvalues for active and sterile neutrinos, whereas $\theta$ is the active-sterile mixing angle in a vacuum. Throughout this paper, we consider the normal mass ordering for sterile neutrinos $m_2 > m_1$, which is favored in cosmological observations \[47\].

The flavor-mixing matrix $U$ uniquely determines the relation between the mass and the flavor eigenstates as

$$|\nu_a\rangle = \cos \theta |\nu_1\rangle - \sin \theta |\nu_2\rangle,$$

(2.6)

$$|\nu_b\rangle = \sin \theta |\nu_1\rangle + \cos \theta |\nu_2\rangle,$$

(2.7)

where $|\nu_a\rangle$ and $|\nu_b\rangle$ are flavor eigenstates of active and sterile neutrinos, while $|\nu_1\rangle$ and $|\nu_2\rangle$ are the mass eigenstates of lighter and heavier states, respectively. The second and third terms in Eq. (2.4) correspond to the matter effects induced by the coherent scatterings of the active-mixed neutrinos with electrons $\nu_a + e^\pm \rightarrow \nu_a + e^\pm$. The matter effect modifies the relation between the mass and the flavor eigenstates. Particularly, the second (third) term arises from the charged- (neutral-) current interaction of $\nu_a$ ($= \nu_e$) with electrons, where $E_{CC} \equiv \text{diag}(\rho_e, 0)$ and $E_{NC} \equiv \text{diag}(\rho_{\nu_a}, 0)$ with $\rho_e$ and $\rho_{\nu_a}$ the energy densities of electrons and the active-mixed neutrinos, respectively.

The collision term of the QKE, Eq. (2.3), is written as

$$C[\rho_p(t), t] = \begin{pmatrix} R_{\nu_a} & -D \rho_{\nu_a} \\ -D \rho_{\nu_a}^* & 0 \end{pmatrix},$$

(2.8)

where $R_{\nu_a}$ is the production rate of the active-mixed neutrinos, and $D$ is the collisional-damping factor, which gives the decoherence between states of $\nu_a$ and $\nu_b$. We take into account the production of active neutrinos from the electron-pair annihilation, the neutrino-electron scattering, and the neutrino self-interaction. These processes are summarized in Table I of Ref. \[48\]. For each reaction process, we analytically reduce the dimension of momentum integrals from nine to two, without imposing any simplifying assumptions in the same way as in Ref. \[48\].

\[4\] In a thermal bath of $T \sim \mathcal{O}(1)$ MeV, abundances of muons and $\tau$ leptons are much smaller than that of electrons due to the Boltzmann suppression. Therefore, we do not consider contributions from muons or $\tau$ leptons to the scatterings.
For numerical implementations, we expand the density matrix with Pauli matrices $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ and convert the QKE into a set of scalar equations:

$$\rho_p(t) = \begin{pmatrix} \rho_{aa} & \rho_{as} \\ \rho_{as}^* & \rho_{ss} \end{pmatrix} = \frac{1}{2} \left[ P_0 \sigma_0 + \mathbf{P} \cdot \mathbf{\sigma} \right].$$ (2.9)

where $P_0$ and $\mathbf{P} = (P_x, P_y, P_z)$ are expansion coefficients referred to as the polarization vectors and $\sigma_0 = 1$ is the identity matrix. Since the diagonal components of the density matrix correspond to the distribution functions for the active-mixed and sterile neutrinos, we have

$$f_{\nu_a} = \frac{1}{2} (P_0 + P_z), \quad f_{\nu_s} = \frac{1}{2} (P_0 - P_z).$$ (2.10)

The QKE (Eq. (2.3)) is rewritten with polarization vectors as

$$\dot{\mathbf{P}} = \vec{\mathbf{H}} \times \mathbf{P} - D (P_x \mathbf{x} + P_y \mathbf{y}) + \dot{P}_0 \mathbf{z},$$ (2.11)

$$\dot{P}_0 = R_{\nu_a},$$ (2.12)

where $\vec{\mathbf{H}} = (\mathbf{H}_x, \mathbf{H}_y, \mathbf{H}_z)$ is the neutrino Hamiltonian. We define $P_{\nu_a} \equiv P_0 + P_z$ and $P_{\nu_s} \equiv P_0 - P_z$ and rewrite the above equation into

$$\dot{P}_{\nu_a} = \mathbf{H}_x P_y + R_{\nu_a},$$ (2.13)

$$\dot{P}_{\nu_s} = - \mathbf{H}_x P_y,$$ (2.14)

$$\dot{P}_x = - \mathbf{H}_z P_y - D P_x,$$ (2.15)

$$\dot{P}_y = \mathbf{H}_z P_x - \frac{1}{2} \mathbf{H}_x (P_{\nu_a} - P_{\nu_s}) - D P_y.$$ (2.16)

Given the squared-mass difference between the mass eigenstates $\delta m^2 \equiv m_2^2 - m_1^2$ and the mixing angle in a vacuum $\theta$, each component of the neutrino Hamiltonian is explicitly expressed as

$$\mathbf{H}_x = \frac{\delta m^2}{2p} \sin 2\theta,$$ (2.17)

$$\mathbf{H}_y = 0,$$ (2.18)

$$\mathbf{H}_z = - \frac{\delta m^2}{2p} \cos 2\theta + \mathbf{H}_{\text{mat}}.$$ (2.19)

The matter effect appears as the potential term $\mathbf{H}_{\text{mat}}$, which can be written as

$$\mathbf{H}_{\text{mat}} = - \frac{8\sqrt{2}}{3} G_F p \left[ \frac{\rho_e}{m_W^2} + \frac{\rho_{\nu_a}}{m_{2/2}} \right],$$

$$= - \frac{4\sqrt{2}}{3\pi^2} G_F p \left[ \frac{g_e}{m_W^2} \int_0^\infty dp' p'^2 \frac{E_e}{\exp(E_e/T_\gamma) + 1} + \frac{g_{\nu_a}}{m_{2/2}} \int_0^\infty dp' p'^4 f_{\nu_a} \right].$$ (2.20)

In the above expression, $T_\gamma$ is the photon temperature, and $E_e = \sqrt{p^2 + m_e^2}$ is the energy of electrons. Also, $g_e = 4$ is the statistical degree of freedom of electrons and $g_{\nu_a} = 2$ is that for
each flavor of neutrinos. The first and second terms in the bracket correspond to the charged- and neutral-current interactions of $\nu_e$ with electrons, respectively.

The active-spectator neutrinos $\nu_{sp}$ are irrelevant to the neutrino oscillation. Therefore, their time evolution can be described by the momentum-dependent classical Boltzmann equation:

$$
\frac{df_{\nu_{sp}}(t)}{dt} = \left( \frac{\partial}{\partial t} - H p \frac{\partial}{\partial p} \right) f_{\nu_{sp}}(t) = C[f_{\nu_{sp}}(t), t],
$$

where $f_{\nu_{sp}}$ is the distribution function of the active-spectator neutrino, and $C[f_{\nu_{sp}}(t), t]$ is the collision term for $\nu_{sp}$, whose expression is given by the same equation as the production rate for $\nu_a$, but with $f_{\nu_{sp}}$.

In order to calculate the thermalization of active and sterile neutrinos in the expanding Universe, it is necessary to solve the Friedman equation,

$$
H \equiv \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G \rho}{3}},
$$

(2.22)

to give the time evolution of the scale factor $a(t)$. The total energy density $\rho$ is written as

$$
\rho = \rho_\gamma + \rho_e + \rho_\nu + \rho_\phi
= \frac{\pi^2}{15} T_\gamma^4 + \frac{g_e}{2\pi^2} \int_0^\infty dp \frac{p^2}{\exp(E_e/T_\gamma) + 1}
+ \frac{g_\nu}{2\pi^2} \int_0^\infty dp \frac{p^3}{\exp(E_e/T_\gamma) + 1} f_{\nu_a} + 2 f_{\nu_{sp}} + f_{\nu_s} + \rho_\phi.
$$

(2.23)

In the above expression, $\rho_\gamma$, $\rho_e$, $\rho_\nu$, and $\rho_\phi$ are the energy densities of photons, electrons, neutrinos, and the parent particle, respectively. All flavors of neutrinos contribute to the total energy density of neutrinos, i.e. $\rho_\nu = \rho_{\nu_a} + \rho_{\nu_{sp}} + \rho_{\nu_s}$.

The evolution of $\rho_\phi$ can be obtained by solving the integrated Boltzmann equation for $\phi$:

$$
\frac{d\rho_\phi}{dt} = -\Gamma_\phi \rho_\phi - 3H \rho_\phi,
$$

(2.24)

where $\Gamma_\phi$ is the decay rate of $\phi$, and the lifetime of $\phi$ is given by its inverse, i.e. $\tau_\phi = \Gamma_\phi^{-1}$. This equation can be integrated analytically for the non-relativistic particle $\phi$, and we obtain

$$
\frac{\rho_\phi}{s} = \frac{\rho_{\phi,0}}{s_0} e^{-\Gamma_\phi t},
$$

(2.25)

where $\rho_{\phi,0}$ and $s_0$ are the energy density of $\phi$ and the total entropy density at the initial time $t_0$, respectively. In Eq. (2.25), we have assumed $\rho_{\phi,0}$ dominates the energy densities of other background particles, i.e. $\rho_{\phi,0} >> (\rho_\gamma + \rho_e + \rho_\nu)_{t=t_0}$. 

The energy and entropy injected from the decay of the parent particle $\phi$ during reheating are taken into account by solving the energy conservation equation:

$$\frac{d\rho}{dt} = -3H(\rho + P).$$

(2.26)

The total pressure $P$ can be expressed as

$$P = P_\gamma + P_e + P_{\nu},$$

$$= \frac{\pi^2}{45}T_\gamma^4 + \frac{g_\gamma}{6\pi^2} \int_0^\infty dp \frac{p^4}{E_e} \frac{1}{\exp(E_e/T_\gamma) + 1}$$

$$+ \frac{g_{\nu}}{6\pi^2} \int_0^\infty dp \ p^3 (f_{\nu_{\alpha}} + 2f_{\nu_{\text{mp}}} + f_{\nu_{\text{s}}}).$$

(2.27)

Since all electromagnetic particles are instantaneously thermalized during reheating, they have a common temperature $T_\gamma$. Hence, we can rewrite Eq. (2.26) into the differential equation for the time evolution of $T_\gamma$:

$$\frac{dT_\gamma}{dt} = -\Gamma_\phi \rho_\phi + 4H(\rho_\gamma + \rho_e + \rho_{\nu}) + 3H(\rho_{\nu_e} + P_e) + \frac{d\rho_{\nu}}{dt},$$

(2.28)

where $\Gamma_\phi$ and $T_{\text{RH}}$ are uniquely related through the relation:

$$\Gamma_\phi = 3H(T_{\text{RH}}).$$

(2.29)

The energy density is dominated by radiation after reheating, and hence the Hubble expansion rate can be written as

$$H = \sqrt{\frac{g^* \pi^2}{90} \frac{T_{\text{RH}}^2}{m_{\text{pl}}}},$$

(2.30)

where $m_{\text{pl}} \sim 2.4 \times 10^{18}$ GeV is the reduced Planck mass, and $g^* = 10.75$ is the canonical value of the relativistic degrees of freedom at the cosmic temperature of $O(1)$ MeV. Substituting Eq. (2.30) into Eq. (2.29) yields the one-to-one correspondence between the reheating temperature and the lifetime of the parent particle $\phi$:

$$T_{\text{RH}} \sim 0.7 \text{ MeV} \left( \frac{\Gamma_\phi}{\text{sec}^{-1}} \right)^{1/2}.$$  

(2.31)

This gives a reasonable estimate of the cosmic temperature at which the radiation-dominated epoch attains.

The results of the thermalization of active and sterile neutrinos are obtained by simultaneously solving Eqs. (2.13)–(2.16), (2.21), (2.22), (2.25), (2.28). For this purpose, we use the LASAGNA code [18, 46], which is an efficient ordinary differential equation solver optimized for the sterile neutrino production in the early Universe, with suitable modifications. Also, we utilize the SuperLU MT package [49, 50] to make use of the multicore CPU servers for numerical performance.
III. NUMERICAL RESULT: STERILE NEUTRINO THERMALIZATION

The abundance of neutrinos is often described in terms of the effective number of neutrino species $N_{\text{eff}}$. In the case of $\nu_e - \nu_{s}$ mixing, it is defined as

$$N_{\text{eff}} = N_{\text{eff}, \nu_e} + N_{\text{eff}, \nu_s} + 2 N_{\text{eff}, \nu_{sp}} = \rho_{\nu_e}/\rho_{\nu_{\text{std}}} + \rho_{\nu_s}/\rho_{\nu_{\text{std}}} + 2 \rho_{\nu_{sp}}/\rho_{\nu_{\text{std}}},$$  \hspace{1cm} (3.1)

where $N_{\text{eff}, \nu_\alpha}$ ($\alpha = e, s, \text{sp}$) is the contribution of each neutrino species to the total $N_{\text{eff}}$, and $\rho_{\nu_{\text{std}}}$ is the energy density of one species of neutrinos in the standard big-bang model. The factor of two in front of $N_{\text{eff}, \nu_{sp}}$ accounts for the contribution from $\nu_\mu$ and $\nu_\tau$. By definition, $N_{\text{eff}, \nu_e} = 1$ corresponds to the full thermalization of $\nu_\alpha$, i.e. the energy spectrum of $\nu_\alpha$ can be expressed as the thermal (Fermi-Dirac) distribution.

Figure 1 shows the relation between the reheating temperature $T_{\text{RH}}$ and the effective number of the neutrino species $N_{\text{eff}}$. The mass difference $\delta m^2$ and the mixing angle $\theta$ between the active and sterile neutrinos are fixed to $(\delta m^2, \sin^2 2\theta) = (1.29 \text{ eV}^2, 0.035)$, which is the best-fit value obtained from data analysis of $\nu_e$ disappearance experiments in Ref. [51]. In the figure, both $N_{\text{eff}}$ and $N_{\text{eff}, \nu_\alpha}$ increase with $T_{\text{RH}}$ since the active neutrino production via $e^- + e^+ \rightarrow \nu_\alpha + \bar{\nu}_\alpha$ is more efficient at a higher temperature, and neutrinos have more time to be produced during reheating. The abundance of sterile neutrinos decreases for $T_{\text{RH}} < T_{\text{max}}$ ($\sim 13 \text{ MeV for } m_s \sim 1 \text{ eV}$) and vanishes at $T_{\text{RH}} < 1 \text{ MeV}$ due to the decoupling of active neutrinos from the thermal plasma. An inequality $N_{\text{eff}, \nu_e} > N_{\text{eff}, \nu_{sp}}$ always holds because the neutrino interaction with the background electrons is stronger for $\nu_e$ than $\nu_\mu$ or $\nu_\tau$. The figure also reveals that the neutrino self-interaction enhances the production efficiency of the sterile neutrino. This is because the neutrino self-interaction increases the collisional-damping rate $D$ and increases the effective production rate of sterile neutrinos, which approximately scales as $D \sin^2 2\theta M$, where $\theta M$ is the mixing angle in a medium [52]. The effect of the neutrino self-interaction was neglected in Ref. [34], or approximately considered in Refs. [30, 32, 33].

Figure 2 shows the time evolution of $N_{\text{eff}, \nu_\alpha}$ for typical values of $T_{\text{RH}}$. The competition between the dilution of neutrinos due to the entropy production induced by the decay of $\phi$ and the production of neutrinos determines the behavior of $N_{\text{eff}, \nu_\alpha}$. Since neutrinos are only weakly produced in the thermal bath of photons and electrons, the former is dominant for $T_\gamma > T_{\text{RH}}$, and the effect takes its maximum at $T_\gamma \sim T_{\text{RH}}$, when the cosmic time is comparable to the lifetime of

$^5$ Here we normalize the contribution of $\nu_\alpha$ ($\alpha = e, \mu, \tau$) to the effective number of neutrino species $N_{\text{eff}, \nu_\alpha}$ in units of the energy density of $\nu_e$ in the standard big-bang model, $\rho_{\nu_e, \text{std}}$. Another choice for the normalization is to take the standard energy density of $\nu_\mu$ or $\nu_\tau$, but the difference between $\rho_{\nu_e}/\rho_{\nu_{\text{std}}}$ and $\rho_{\nu_e}/\rho_{\nu_{\mu, \text{std}}}$ (or $\rho_{\nu_e}/\rho_{\nu_{\tau, \text{std}}}$) should be quite small (< 1%), and hence negligible. The difference in $N_{\text{eff}}$ due to its definition is therefore irrelevant to our final results.
$N_{\text{eff}} = N_{\text{eff}, \nu_{\alpha}} + N_{\text{eff}, \nu_{s}} + 2N_{\text{eff}, \nu_{sp}}$ as a function of the reheating temperature $T_{RH}$, for the case of $\nu_{e} - \nu_{s}$ mixing. The narrow (bold) line corresponds to the case without (with) the neutrino self-interaction. The black line is for $N_{\text{eff}}$, whereas the red, green, and blue lines are for $N_{\text{eff}, \nu_{\alpha}}$, $N_{\text{eff}, \nu_{sp}}$, and $N_{\text{eff}, \nu_{s}}$, respectively.

This leads the local minimum of $N_{\text{eff}, \nu_{\alpha}}$ in Fig. 2. For $T_{\gamma} < T_{RH}$, neutrino production becomes dominant compared to the dilution effect, which is negligible after the decay of $\phi$. The value of $N_{\text{eff}, \nu_{\alpha}}$ increases for $T_{\gamma} < T_{RH}$ until neutrinos decouple from other particles at around $T_{\gamma} \sim 1$ MeV. This explains the behavior of the time evolution of $N_{\text{eff}}$. Also, Fig. 2 shows that eV-scale sterile neutrinos start to be produced through neutrino oscillation at around $T_{\gamma} \sim 13$ MeV ($= T_{\text{max}}$), as discussed in the previous section (see Eq. (2.1)).

In Fig. 3, we plot the energy spectrum of each flavor of neutrinos for the typical values of $T_{RH}$. We evaluate the spectra at $T_{\gamma} = 10^{-2}$ MeV, which is much later than the electron-pair annihilation and the neutrino decoupling. It can be seen from Fig. 3 that a peak position of the neutrino energy spectra is shifted to lower than $p/T_{\gamma} \sim 3.15$, which corresponds to the thermal value for fermions. This is because the photon temperature $T_{\gamma}$ increases by a factor of $(11/4)^{1/3} \sim 1.4$ compared to those of neutrinos after the annihilation of electrons.
FIG. 2. Temperature evolution of $N_{\text{eff, } \nu_{\alpha}} (\alpha = a, s, \text{sp})$, for the case of $\nu_e$-$\nu_s$ mixing. The left panel is for $N_{\text{eff, } \nu_a}$, the middle panel is for $N_{\text{eff, } \nu_{\text{sp}}}$, and the right panel is for $N_{\text{eff, } \nu_s}$. In each panel, the red long-dashed line is for $T_{\text{RH}} = 1 \text{ MeV}$, the blue middle-dashed line is for $T_{\text{RH}} = 2 \text{ MeV}$, the green short-dashed line is for $T_{\text{RH}} = 5 \text{ MeV}$, and the black solid line is for $T_{\text{RH}} = 10 \text{ MeV}$.

The averaged energy of neutrinos is expressed by the distortion parameter $R_{\text{dist, } \nu_{\alpha}}$, defined by

$$R_{\text{dist, } \nu_{\alpha}} = \frac{1}{3.15 T_{\nu_{\alpha}, \text{eff}}} \frac{\rho_{\nu_{\alpha}}}{n_{\nu_{\alpha}}} \tag{3.2}$$

where $T_{\nu_{\alpha}, \text{eff}} (= \sqrt[3]{4\pi^2 n_{\nu_{\alpha}} / 3\zeta(3)})$ is the effective temperature for each flavor of neutrinos. The thermal spectrum corresponds to $R_{\text{dist, } \nu_{\alpha}} = 1$ by definition. In Fig. 4, we show the dependence of the distortion parameter $R_{\text{dist}}$ on the reheating temperature. The figure reveals that $R_{\text{dist}}$ increases as the reheating temperature decreases, and particularly it goes to unity for active neutrinos at $T_{\text{RH}} > 10 \text{ MeV}$. The production mechanism of active neutrinos is responsible for this feature. Active neutrinos are produced only from the electron-pair annihilation, and each neutrino in the final state has energy larger than the electron mass $m_e \sim 0.5 \text{ MeV}$. Therefore, the value of $R_{\text{dist, } \nu_{\alpha}}$ becomes larger than unity due to the large contribution from neutrinos produced when the electron is still relativistic. The scattering rate of the process $e^\pm + \nu_{\alpha} \rightarrow e^\pm + \nu_{\alpha} (\alpha = a, s, \text{sp})$, is not sufficient to fully equilibrate the neutrino spectrum since it is of the order of $\mathcal{O}(G_F^2)$, which is the same as that of the neutrino-pair production. As seen in Fig. 4, the distortion parameter for the spectator neutrino is always larger than that of the active-mixed neutrino since we consider the mixing between $\nu_e$ and $\nu_s$. The scattering between the background electron and the electron neutrino is more frequent than those of the other active neutrino species. Also, the energy spectrum of sterile
neutrinos is heavily distorted compared to active neutrinos since electrons do not interact with sterile neutrinos. We note that the value of $R_{\text{dist}, \nu_s} \sim 1.6$ for $T_{\text{RH}} = 5$ MeV is $O(10)\%$ larger than that obtained in Ref. [32], $R_{\text{dist}, \nu_s} \sim 1.3$. This is possibly due to assumptions which they adopted to simplify the thermalization calculation of sterile neutrinos, namely the sterile neutrino abundance is negligible compared to the thermal abundance $f_{\nu_s} \ll f_{\text{FD}} = 1/\exp[p/T_\gamma + 1]$, where $T_\nu = (4/11)^{1/3} T_\gamma \sim T_\gamma/1.4$.

Figure 5 shows the mass dependence of $N_{\text{eff}}$. As can be seen from Fig. 5, the mass dependence is negligible for $T_{\text{RH}} \sim O(1)$ MeV, which justifies the assumption in Ref. [34]. The reason is that the matter effect is almost negligible at a low temperature of $O(1)$ MeV, and the effective production rate of the sterile neutrino is therefore given by $\sim D \sin^2 \theta$ for both cases of $m_\nu = 1$ eV and 1 keV. The damping rate $D$ is associated only with active neutrinos, and it has no sensitivity to the sterile neutrino property.
FIG. 4. Dependence of the distortion parameter for each flavor of neutrinos $R_{\text{dist}, \nu_{\alpha}} \ (\alpha = a, s, \text{sp})$ on the reheating temperature $T_{\text{RH}}$, for the case of $\nu_e - \nu_s$ mixing. The red short-dashed line is for $\nu_a$, the green middle-dashed line is for $\nu_{\text{sp}}$, and the blue long-dashed line is for $\nu_s$. $R_{\text{dist}} = 1$ corresponds to the thermal Fermi-Dirac spectrum.
FIG. 5. Mass dependence of the effective number of neutrino species for all flavors of neutrinos (left) and sterile neutrinos (right) for each value of the active-sterile mixing $\sin^2 2\theta$, for the case of $\nu_e-\nu_s$ mixing. The solid lines are for $m_s = 1$ eV, while dashed lines are for $m_s = 1$ keV. The red, blue, and black lines correspond to $\sin^2 2\theta = 10^{-3}, 10^{-2}$, and $10^{-1}$, respectively. There are no apparent mass dependences for the cases of $\sin^2 2\theta = 10^{-1}$. 
IV. NUMERICAL RESULT: BIG BANG NUCLEOSYNTHESIS

The thermalization of neutrinos is closely associated with BBN. In this section, we introduce the formalism for the calculation of BBN after a brief introduction to the role of neutrinos in BBN. For the detail of the theoretical framework, we refer the reader to Ref. [37]. The numerical results of the BBN calculation in cosmological models with MeV-scale reheating temperature is presented in the latter part of this section.

A. Neutrino thermalization and neutron-to-proton ratio

Neutrinos affect the light-element abundances synthesized in BBN since they are involved in the exchange reactions between protons and neutrons:

\[ n \leftrightarrow p + e^- + \bar{\nu}_e, \]  
\[ e^+ + n \leftrightarrow p + \nu_e, \]  
\[ \nu_e + n \leftrightarrow p + e^- , \]

which set the neutron-to-proton ratio \((n/p)\) before the nucleosynthesis. The neutron-to-proton ratio is one of the most important parameters in BBN, which determines the final abundances of light elements. In particular, the mass fraction of \(^4\)He, which is denoted as \(Y_p\), is written in a simple analytical form of \(Y_p \sim 1/(1 + (n/p)_{bbn}^{-1})\). The neutron-to-proton ratio \((n/p)_{bbn}\) is the value just before the deuterium bottleneck opens and the synthesis of light elements becomes effective, which corresponds to the cosmic time of \(t_{bbn} \sim 200\) sec (or the temperature of \(T_{bbn} \sim 80\) keV).

The freeze-out value of the ratio \((n/p)_{f}\) is related to \((n/p)_{bbn}\) as

\[(n/p)_{bbn} = (n/p)_f e^{-\tau_{bbn}/\tau_n},\]

where \(\tau_n = 880.2 \pm 1.0\) sec (68\% C.L.) is the neutron lifetime [41]. The freeze-out value is \((n/p)_f \sim 1/6\) in the standard big-bang model, where all active neutrinos are fully thermalized well-before BBN, and hence \((n/p)_{bbn} \sim 1/7\) and \(Y_p \sim 0.25\) [41].

As discussed in Secs. [III] and [III], neutrinos are not completely thermalized in cosmological models with MeV-scale reheating temperature. The incomplete thermalization of neutrinos changes both the freeze-out of the processes (4.1a)–(4.1c) and the Hubble expansion rate. Therefore, we expect a different \((n/p)_f\) value from that attained in the standard big-bang model in this case. Refs. [53, 54] provided a comprehensive discussion of the mechanism. The abundances of the other light elements
such as D, $^3$He, $^6$Li, and $^7$Li are very sensitive to the production abundance of $^4$He, which is the second most abundant element in the Universe. Consequently, light-element abundances in the low-reheating scenario are different from those of the standard big-bang model. Also, among the neutrino species only $\nu_e$ is relevant to the processes (4.1a)–(4.1c). For that reason, light-element abundances are highly sensitive to the $\nu_e$ spectrum, and any physics changing the neutrino flavors in the early Universe such as the neutrino self-interaction and neutrino oscillation plays an important role in the synthesis of light elements.

B. Observational abundances

The abundances of deuterium and helium in the current Universe are measured with $O(1)$% accuracy. The baryon-to-proton ratio $\eta_B$, which is the only free parameter of the standard theory of BBN, is also determined with high precision using these measurements [41]. In this study, we adopt the primordial mass fraction of $^4$He reported in Ref. [55]:

$$Y_p = 0.2449 \pm 0.0040 \ (68\% \ C.L.) ,$$

which was obtained from the observation of the recombination line of metal-poor stars in the extra-galactic HII regions or blue compact galaxies. For deuterium, we adopt [57]

$$D/H = (2.545 \pm 0.025) \times 10^{-5} \ (68\% \ C.L.) ,$$

which was determined using the absorption spectra in high-redshift metal-poor quasar absorption systems.

C. Numerical calculation

We numerically solve the code of the reaction network for light elements based on the Kawano code [59] with the updated nuclear reaction rates reported in Refs. [60–64] (see Ref. [65] for more details). The contribution of $\phi$ is accounted for in the Friedman equation, Eq. (2.22), and the energy conservation equation, Eq. (2.26). Also, we pre-evaluate the energy densities of active and sterile neutrinos together with weak reaction rates of the processes (4.1a)–(4.1c) with the LASAGNA code.

6 A slightly large value of the $^4$He abundance was reported in Ref. [56], $Y_p = 0.2551 \pm 0.0022 \ (68\% \ C.L.)$. Since the authors of Ref. [56] reanalyzed the same dataset used in Ref. [55], it should be reasonable to adopt the value reported in Ref. [55].

7 The authors of Ref. [58] reported $D/H = (2.527 \pm 0.030) \times 10^{-5} \ (68\% \ C.L.)$, which is similar to (4.4). Since the difference of the mean values between this value and (4.4) falls within a 1-$\sigma$ error, our result does not change even if we adopted this value to be the primordial $D/H$. 
and interpolate the data in the BBN code. In the standard BBN, the baryon-to-photon ratio \( \eta_B \) is the only free parameter of the theory. Currently, the value of \( \eta_B \) is precisely determined from the observation of CMB in the Planck collaboration \(^47\). In this study, we adopt the Planck bound on \( \eta_B \) for the base \( \Lambda \)CDM model extended by two additional parameters, namely the effective number of neutrino species, \( N_{\text{eff}} \), and the effective mass of sterile neutrinos, \( m_{\text{eff}} \):

\[
\eta_B = (6.14 \pm 0.04) \times 10^{-10} \quad (68\% \text{ C.L.}),
\]

as a prior for the calculation of BBN.

The light-element abundance is sensitive to the decay mode of the parent particle \( \phi \). \(^37, 54, 66\) In this study, we assume that the parent particle \( \phi \) decays into both radiation (\(i.e.\) photons and charged leptons) and hadrons (\(i.e.\) quarks and gluons) as in Refs. \(^37, 54\). For the case of the direct decay of the parent particle into active neutrino pairs \( \phi \rightarrow \nu_\alpha + \bar{\nu}_\alpha (\alpha = e, \mu, \tau) \), see Ref. \(^66\).

The effect of the particle injection from the parent particle \( \phi \) is more significant in the case where the hadronic branching ratio \( B_h \neq 0 \). In the case where the parent particle decays exclusively into radiation, the thermal bath of photons and electrons is instantaneously produced by cascade reactions through electromagnetic interaction, and active neutrinos are gradually produced from the thermal bath through the weak interaction. For \( B_h \neq 0 \), mesons, baryons, and their anti-particles are copiously produced from quarks and gluons after the hadronization, and they induce additional exchange reactions between neutrons and protons. Consequently, it affects the freeze-out value of the neutron-to-proton ratio \((n/p)_f\) and leads to different outcomes of BBN \(^37, 54, 67\). In our calculation, we take into account the hadronic effect induced by charged pions \((\pi^\pm)\) and nucleons \((n, \bar{n}, p, \bar{p})\) injected from the decay of \( \phi \) and neglect the effect of other hadronic particles for conservative treatment. We use the thermal reaction rates for the hadronic processes interchanging neutrons and protons. This treatment is justified since most of the injected hadrons are instantaneously stopped by inverse Compton-like scattering or Coulomb interaction with background particles (\(i.e.\) mainly photons and electrons) and reach equilibrium \(^68, 69\). Also, we evaluate the number of hadrons produced in the hadronic decay using the Pythia 8.2 code \(^70, 71\), assuming the decays of \( \phi \) into the \( u\bar{u} \) pair as a specific process for the quark production.

Figure \(4\) shows the abundances of helium \((Y_p)\) and deuterium \((\text{D/H})\) as a function of the reheating temperature \( T_{\text{RH}} \) for the 100% radiative decay cases. The mixing parameters of sterile neutrinos are fixed to the best-fit values in Ref. \(^51\), as in the previous section. Also, the baryon-
with mixing
no mixing

FIG. 6. Mass fraction of $^4$He, $Y_p$, and the deuterium-to-hydrogen ratio, D/H, as functions of the reheating temperature $T_{RH}$ for the cases corresponding to the 100% radiative decay of the parent particle $\phi$, with the assumption of $\nu_e-\nu_s$ mixing. The value of the baryon-to-photon ratio is fixed to $\eta_B = 6.14 \times 10^{-10}$ in the figure. The red solid line is for sterile neutrinos with the best-fit mixing parameters $(\delta m^2, \sin^2 2\theta) = (1.29 \text{ eV}^2, 0.035)$ reported in Ref. [51], and the case without sterile neutrinos is plotted by the black dotted line for reference. The 2-σ observational bounds on $Y_p$ (Ref. [55]) and D/H (Ref. [57]) are also shown by the gray-shaded regions.

The baryon-to-photon ratio $\eta_B$ is set to be the median value of Eq. (4.5). As seen in Fig. 6, the light-element abundance increases with the reheating temperature. This is because sterile neutrinos are more abundantly produced for a large $T_{RH}$ (see Fig. 1), and therefore the expansion rate $H$ increases. This leads to the early decoupling of the exchange reactions between protons and neutrons (4.1a)–(4.1c). Consequently, the freeze-out temperature $T_c$ increases and more neutrons remain unburnt, which increases $(n/p)_f$ and light-element abundances, i.e. $Y_p$ and D/H. In Fig. 6, a similar dependence of $Y_p$ and D/H on the reheating temperature can be seen even for the case without sterile neutrinos. For small $T_{RH}$ an incomplete thermalization of active neutrinos decreases the expansion rate, and $Y_p$ and D/H decrease due to the effect. The increase of $Y_p$ for small $T_{RH}$ is caused by a decrease in the weak rates responsible for the interconversion between protons and neutrons, $\Gamma_{np}$. This accelerates the decoupling of the processes (4.1a)–(4.1c) and plays a role in increasing the freeze-out value of the neutron-to-proton ratio $(n/p)_f$ and $Y_p$. The increase and decrease of $Y_p$ due to small $T_{RH}$ are competing, but the former dominates the latter for large $T_{RH}$ and the opposite is true for small $T_{RH}$ (see Refs. [37, 54] for more detail).
Figure 7 shows the results of the hadronic decay cases. It can be seen that light-element abundances increase for $T_{RH} \lesssim 10$ MeV due to the hadronic decay effect compared to those for the 100% radiative decay cases. This is because the injection of the high-energy hadrons induces additional exchange reactions between protons and neutrons, $p + N \leftrightarrow n + N'$, where $N$ and $N'$ are mesons or baryons, to equilibrate the number densities of protons and neutrons [37, 54, 67, 68]. This is not true for $T_{RH} \gtrsim 10$ MeV because the hadronic decay occurs much before the decoupling of the processes (4.1a)–(4.1c), and the neutron-to-proton ratio is subsequently equilibrated by the weak processes again, which erases the hadronic decay effect.
V. COSMOLOGICAL CONSTRAINT ON STERILE NEUTRINOS

In this section, we summarize the cosmological constraint on sterile neutrinos, especially focusing on the eV-scale sterile neutrinos motivated by the SBL anomaly. Refs. [30, 32, 33] have shown that cosmological observations place the most stringent bound on the existence of eV-scale sterile neutrinos. Here we summarize the latest results of cosmological observations of light elements and the CMB radiation as well as ground-based neutrino experiments.

A. Constraints from BBN

Sterile neutrinos affect the synthesis of light elements as discussed in Sec. IV. The BBN bound on sterile neutrinos can be obtained by requiring the agreement between theoretical predictions and observed abundances of light elements by performing a $\chi^2$ analysis. In this study, a $\chi^2$ function is defined as follows:

$$
\chi^2 \equiv \chi^2_{D/H} + \chi^2_{Y_p} = \frac{\{(D/H)_{\text{th}} - (D/H)_{\text{obs}}\}^2}{\sigma^2_{D,\text{th}} + \sigma^2_{D,\text{obs}}} + \frac{\{Y_{p,\text{th}} - Y_{p,\text{obs}}\}^2}{\sigma^2_{Y_p,\text{th}} + \sigma^2_{Y_p,\text{obs}}},
$$

where $\chi^2_{D/H}$ and $\chi^2_{Y_p}$ are $\chi^2$ functions for each of D/H and $Y_p$. We use the suffix “obs” and “th” to denote the “observational” and “theoretical”, respectively. Also, $\sigma$ is the uncertainty of the light-element abundance. The theoretical prediction of light-element abundances is defined at each point on the three-dimensional grid of $(m_s, \sin^2 2\theta, T_{RH})$. The theoretical error on each grid point is estimated by propagating the experimental errors in the nuclear reaction rates, the free neutron lifetime, and the hadronic reaction rates in the Monte-Carlo calculation of BBN. For the thermal cross-sections of the hadronic reactions, we use the results in Ref. [67] and assume a 30% error in each cross-section. We define the region of 95% confidence level as a parameter space satisfying

$$
\chi^2(m_s, \sin^2 2\theta, T_{RH}) < 5.991. \quad (5.2)
$$

B. Constraints from CMB

Thermalized sterile neutrinos contribute to $N_{\text{eff}}$ and affect the recombination. Searching the signature in the CMB spectra, the Planck collaboration derived an upper limit on the effective mass of sterile neutrinos $m_{s,\text{eff}}$ [47]:

$$
m_{s,\text{eff}} < 0.65 \text{ eV}. \quad (5.3)
$$
As we assume an absence of the neutrino chemical potential, sterile neutrinos are non-resonantly produced through its mixing with active neutrinos by the Dodelson-Widrow mechanism. In this case, the physical mass \( m_s \) is related to the effective mass as \( m_{s}^{\text{eff}} = m_s N_{\text{eff}, \nu_s} \), where \( N_{\text{eff}, \nu_s} \) is the thermalization degree of sterile neutrinos.

Figure 8–10 summarize the constraints on sterile neutrinos in the parameter space of \( (m_s, \sin^2 2\theta) \). Figure 8 corresponds to the case assuming the standard big-bang model, while Figs. 9 and 10 correspond to the low-reheating scenario assuming the 100% radiative and hadronic decays of the parent particle \( \phi \). In each figure, we also show the current and future sensitivities of the ground-based experiments. The region denoted by R is already excluded from the reactor experiments \( \langle R \rangle \) \cite{72–74}. In the future, KATRIN (KA) \cite{75} and PTOLEMY (P for 10 mg-yr and P2 for 100 g-yr exposures) \cite{76} will prove much smaller mixing angles. In these figures, we also show the 95% C.L. preferred regions of the sterile neutrino reported in Refs. \cite{51} and \cite{77}. Such sterile neutrinos that explain the SBL anomaly are excluded in the standard big-bang model from cosmological observations (Fig. 8), but the low reheating temperature scenario changes the picture. If all the energy of \( \phi \) goes to radiation after the decay, such sterile neutrinos are still compatible with cosmological observations (Fig. 9), which is consistent with the previous studies with simplified treatments \cite{30, 32–34}. In such a case, the production of sterile neutrinos is strongly suppressed, relaxing the BBN and CMB bounds. In contrast, if some part of the energy of \( \phi \) goes to hadrons, injected hadrons induce additional interconversion between neutrons and protons, and the BBN bound gets more severe (Fig. 10), as we discussed in Sec. \cite{IV}. We note that in Fig. 10 we intentionally choose the mass and the hadronic branching ratio of \( \phi \) to make the effect of the hadronic decay clear. For a heavier \( \phi, m_\phi > 10 \text{ GeV} \), or a smaller hadronic branching ratio, \( B_h < 1 \), the effect of the hadronic decay should be smaller (See Refs. \cite{37, 54, 67} for further discussions).

In light of our results, the existence of the light sterile neutrino explaining the SBL anomaly is still compatible with the observations of BBN and CMB, although it depends on the reheating temperature \( T_{\text{RH}} \), the mass of the parent particle \( m_\phi \), and the hadronic branching ratio \( B_h \) of the decay. In the future, such light sterile neutrinos could be detected by direct detection experiments of the cosmic neutrino background such as the PTOLEMY project \cite{76}. Also, energy spectra of active neutrinos are sensitive to the reheating temperature as well as the mass and the mixing angle of sterile neutrinos. A direct detection of the cosmic neutrino background should bring us reliable

---

\footnote{We refrain from plotting the result in the region with \( m_s > 10^{-1} \text{ keV} \) in the case of the standard big-bang model. This is because the peak production of sterile neutrinos with such a large mass occurs at or above 50 MeV (see Eq. \cite{24}) and therefore collisions between neutrinos and muons or light mesons, which are not considered in our computation, are non-negligible in the mass region.}
information on the values of these parameters, associated with the theory beyond the standard model of particle physics.
FIG. 8. Constraints on sterile neutrinos in the parameter space of $(m_s, \sin^2 2\theta)$ assuming the standard big-bang model, for the case of $\nu_e-\nu_s$ mixing. The 95% C.L. bound based on the BBN calculation is shown by the blue-hatched region while that on the CMB by the red region. We fix the reheating temperature $T_{RH} = 5$ MeV to plot the CMB bound. Other excluded regions come from Daya Bay [72], Bugey-3 [73], and PROSPECT [74] (R, gray region), the KATRIN neutrino mass experiment (KA, solid-black line) [75]. The narrow vertically-long region colored in red corresponds to the best-fit region in the $\nu_e$ disappearance only (left, Ref. [51]) and the global (right, Ref. [77]) data analysis. The sensitivities with the future cosmic neutrino background experiment [74] are also shown (P and P2, green region).
FIG. 9. The same as Fig. 8 but for the low reheating temperature case assuming 100% radiative decay of the parent particle $\phi$. 

Low reheating temperature:

100% radiative decay, $\phi \rightarrow \gamma + \ldots$ or $l^{\pm} + \ldots$
FIG. 10. The same as Fig. 9 but for the case corresponding to the $100\%$ hadronic decay of the parent particle $\phi$ with $m_\phi = 10$ GeV. Compared to Fig. 9, the BBN bound is more severe and the preferred regions in Refs. [51] and [78] are excluded.
VI. CONCLUSIONS

We have investigated the cosmological production of light sterile neutrinos with masses and mixings consistent with those needed to explain the anomaly in short-baseline neutrino experiments, assuming a low reheating temperature of the Universe $T_{RH} \sim \mathcal{O}(1)$ MeV. Considering the sterile neutrino production through the combination of scatterings and non-resonant oscillations, we have numerically solved its evolution and found that the existence of such sterile neutrinos becomes consistent with Big Bang nucleosynthesis if the parent particle responsible for reheating decays exclusively into electromagnetically interacting radiation. In contrast, if the parent particle mainly decays into hadrons, the BBN bound gets tighter and the preferred regions for explaining the anomaly are excluded for a wide range of the mass and the hadronic branching ratio of the parent particle.
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