Transport, destruction and growth of pebbles in the gas envelope of a protoplanet

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ABSTRACT

We analyse the size evolution of pebbles accreted into the gaseous envelope of a protoplanet growing in a protoplanetary disc, taking into account collisions driven by the relative sedimentation speed as well as the convective gas motion. Using a simple estimate of the convective gas speed based on the pebble accretion luminosity, we find that the speed of the convective gas is higher than the sedimentation speed for all particles smaller than 1 mm. This implies that both pebbles and pebble fragments are strongly affected by the convective gas motion and will be transported by large-scale convection cells both towards and away from the protoplanet’s surface. We present a simple scheme for evolving the characteristic size of the pebbles, taking into account the effects of erosion, mass transfer and fragmentation. Including the downwards motion of convective cells for the transport of pebbles with an initial radius of 1 millimeter, we find pebble sizes between 100 microns and 1 millimeter near the surface of the protoplanet. These sizes are generally amenable to accretion at the base of the convection flow. Small protoplanets far from the star (> 30 AU) nevertheless erode their pebbles to sizes below 10 microns; future hydrodynamical simulations will be needed to determine whether such small fragments can detach from the convection flow and become accreted by the protoplanet.

Keywords: planet-disk interactions, planets and satellites: formation, planets and satellites: gaseous planets

1. INTRODUCTION

The rapid accretion of millimeter-sized pebbles appears to be a necessary ingredient in forming the cores of cold gas giants and ice giants within the life-time of the gaseous protoplanetary disc (Johansen & Lambrechts 2017; Ormel 2017; Johansen et al. 2019; Johansen & Bitsch 2019). The formation of super-Earths in the inner regions of the protoplanetary disc may also be driven by accretion from the drifting pebble population (Lambrechts et al. 2019; Izidoro et al. 2019). The fate of the pebbles after entering the gaseous envelope of a protoplanet is nevertheless relatively poorly explored and poorly understood. A recent paper concluded that pebbles will be sandblasted to dust in the envelope of a protoplanet and transported back to the protoplanetary disc with the convective overshoot (Ali-Dib & Thompson 2019), resulting in a pebble accretion time-scale of at least 3 million years.

The goal of this paper is to perform an independent analysis of the evolution of the pebble sizes within the gas envelope. We were particularly interested in the role of convective gas flows for the dynamics of the pebbles and for collisions between them. Recent work on the hydrodynamics and radiative transfer of protoplanetary envelopes has been done using an adaptive mesh to resolve the gas flow down to the surface of the protoplanet (Popovas et al. 2018, 2019). These simulations demon-
strated that the convective motion of the gas on the one hand had a major influence on the dynamics of pebbles, while on the other hand – given the assumptions used – did not have a significant effect on the pebble accretion rates. Pebbles that are captured into the envelope are sometimes carried closer to the protoplanet with downwelling cold gas flows, while sometimes instead carried away in upwelling hot flows. The average accretion rates of pebbles onto the protoplanet were observed to be relatively unaffected by the convection. The smallest pebble sizes of 10 microns considered in Popovas et al. (2019) showed indications of a decreased accretion efficiency for some of the simulations, but this could simply indicate fluctuations in the rather low accretion efficiency of such small pebbles or that small dust cannot decouple from the convection flow close to the protoplanet surface. Importantly, the hydrodynamical studies by Popovas et al. (2018) and Popovas et al. (2019) ignored the evolution of the pebble size during the transport down to the protoplanet’s surface. We therefore focus in this paper on understanding the size evolution of the accreted pebbles, using 1-D models that either include or exclude the convective gas motion.

The paper is organized as follows. In Section 2 we introduce our gas envelope model and discuss the physics of pebble capture. In Section 3 we present results for the pebble-to-gas ratio and fragmentation-limited size of the pebbles in a model that ignores the convective motion of the gas. The following Section 4 includes the convective motion. In Section 5 we evolve the pebble size using a simple approach that takes into account erosion, fragmentation and mass transfer, both including and excluding the convective motion. We conclude on our results in Section 6. Appendix A contains a calculation of the fate of the pebbles for an earlier and later stage of the protoplanetary disc compared to the nominal case presented in the main paper. Appendix B contains an additional numerical experiment where we consider convective models with fixed fragment mass ratios relative to the gas.

2. ENVELOPE STRUCTURE AND PEBBLE CAPTURE

We adopt a 1-D model for the gas envelope of the protoplanet in hydrostatic and energy balance. We run models of the gas envelopes of accreting protoplanets using a constant gas accretion rate through the protoplanetary disc of \( \dot{M}_g = 3 \times 10^{-8} \, M_\odot \, \text{yr}^{-1} \) and a constant viscosity of \( \alpha = 0.01 \).

The protoplanetary accretion disc model is only used to set the column density, \( \Sigma_g \), of the disk in which the envelope is embedded, through the relation \( \dot{M}_g = 3\pi \alpha c_s H \Sigma_g \) (Pringle 1981), where \( c_s \) is the sound speed of the gas and \( H = c_s / \Omega \) is the local vertical scale-height (\( \Omega \) is the Keplerian frequency). We consider the mass accretion rate rather than an assumed column density profile in order to facilitate comparisons to the pebble accretion models of Johansen et al. (2019) who used an evolving alpha-disc. We use the cold temperature structure \( T = 121 \, K \, (r/\text{AU})^{-3/7} \) from Chiang & Youdin (2010). This yields then a gas column density of approximately \( 10^4 \, \text{kg m}^{-2} \) at 1 AU, similar to the minimum mass solar nebula, but with a shallow radial logarithmic slope of -15/14. This evolution stage of the protoplanetary disc is chosen as it constitutes the main growth phase of the cores of giant planets (Johansen et al. 2019). Planets could form at even earlier stages of the protoplanetary disc, with accretion rates in the range between \( 10^{-6} \, M_\odot \, \text{yr}^{-1} \) to \( 10^{-7} \, M_\odot \, \text{yr}^{-1} \) (Manara et al. 2018). We present the results of considering either an earlier or a later evolutionary stage of the protoplanetary disc in Appendix A.

We run simulations with planetary masses \( M = 0.1, 0.3, 1.0, 3.0 \, M_\oplus \) placed at distances \( a = 1.0, 3.0, 10.0 \) and 30.0 AU from the central star. The pebble accretion time-scale \( \tau \) is set to a fixed value of \( 10^6 \, \text{yr} \). We fix the incoming pebble size to 1 millimeter.

2.1. Protoplanet envelope

We calculate the structure of the gas envelope by integrating the gas density and temperature inwards from the Hill radius down to the planetary surface, setting the density and temperature at the outer boundary equal to the protoplanetary disc conditions at the relevant distance. The temperature gradient is set to be the minimum of the radiative and the convective gradient, using the opacity power-laws of Bell & Lin (1994) and an ideal gas equation of state with constant adiabatic index \( \gamma = 1.4 \). We use here an opacity corresponding to micron-sized dust with 1% mass relative to the gas and ignore for simplicity any increase in mean molecular weight and release of latent heat at the ice sublimation line (at a temperature of 170 K) and the silicate dust sublimation line (at temperatures of 2,000–3,000 K). We refer to Chambers (2017), Brouwers et al. (2018) and Brouwers & Ormel (2020) for the effect of the release of water vapour and silicate vapour on the structure of the envelope.

The resulting envelope structure is shown in Figure 1. Both the density and the temperature display wiggles in their profiles; this is due to opacity transitions in the Bell & Lin (1994) opacity. Increasing the protoplanet mass leads to an increase in both the temperature and the density of the envelope. The temperature never-
2.2. Pebble capture

Pebbles that pass the protoplanet with the Keplerian shear flow are captured with the help of gas drag when their terminal velocity is approximately equal to the Keplerian shear speed (Ormel & Klahr 2010; Lambrechts & Johansen 2012),

$$\tau_{t} \frac{GM}{b^2} \sim \Omega b.$$  

(1)

Here $\tau_{t}$ is the friction time of the pebbles and $b$ is the impact parameter along the radial axis from the central star. The maximum impact parameter for accretion is called the accretion radius $R_{\text{acc}}$. A more precise analysis (Morbidelli et al. 2015) yields the pebble accretion radius as a function of the Stokes number $\text{St} = \Omega \tau_{t}$ and Hill radius $R_{H} = \left(\frac{GM}{(3\Omega^2/2)}\right)^{1/3}$ as

$$R_{\text{acc}} = \left(\frac{\text{St}}{0.1}\right)^{1/3} R_{H}. \quad (2)$$

This scaling is nevertheless only valid under the assumption that the gas streamlines follow a pure Keplerian shear flow. In reality, the gravity of the planet will bend the streamlines, turning those streamlines with small impact parameter into horseshoe flows (Ormel 2013). The hydrodynamical simulations of Popovas et al. (2018) demonstrated that pebbles of all sizes penetrate the outer regions of the Hill sphere along bent gas streamlines. The smallest pebbles are then sorted away along the horseshoe streamlines and along more distant streamlines that pass relatively unperturbed through the Hill sphere. Larger pebbles detach from a wider interval of gas streamlines and sediment towards the protoplanet, encountering the gas envelope approximately at the distance of the Bondi radius, defined here as $R_{B} = GM/c_{s}^2$ where $c_{s}$ is the sound speed of the gas in the protoplanetary disc.

2.3. Recycling flows

Recycling flows are characterised by streamlines that penetrate into the Hill sphere and leave back to the protoplanetary disc again (Alibert 2017). Thus both horseshoe flows and the perturbed Keplerian shear can...
be considered recycling flows (Lambrechts & Lega 2017; Kurokawa & Tanigawa 2018). The recycling flows nevertheless penetrate only to the Bondi radius, unless the envelope is nearly adiabatic (Lambrechts & Lega 2017; Popovas et al. 2018). The reason why the Bondi radius marks the maximum penetration of the recycling flows is that the entropy can only be significantly reduced compared to the disc value interior of the Bondi radius (Rafikov 2006; Piso & Youdin 2014) – and buoyancy effects due to entropy gradients prevent the penetration of the recycling flows (Kurokawa & Tanigawa 2018). We therefore assume that any pebbles or pebble fragments that make it below the Bondi radius are protected from the recycling flows, unless they are pushed out of the Bondi radius again with rising convective gas plumes.

3. PEBBLE EVOLUTION WITHOUT CONVECTIVE GAS MOTION

We analyze in this section the characteristic pebble size while ignoring the convective gas motion. We use this approach to put into context the results including convection presented in the following section.

3.1. Sedimentation speed

Pebbles within the pebble capture radius sediment towards the protoplanet at the terminal speed

$$v_t = \frac{GM}{r^2} \tau_f. \quad (3)$$

Here $\tau_f$ is the friction time of the pebbles and $r$ is the distance from the protoplanet. The collision speed of pebbles is given by the differential sedimentation speed and the speed from the turbulent convection (which we ignore in this section). The shear speed does not contribute to the collision speed, since dust, pebbles and gas follow the same streamlines outside of the Bondi radius of the protoplanet. The terminal velocity of the pebbles is shown in Figure 2. We indicate also the critical speed for fragmentation in collisions between equal-sized pebbles (1 m/s). The general behaviour starting at the Hill radius is (a) an increase in the terminal velocity as the gravity increases, (b) for the low-mass protoplanets a change of slope where the free fall speed is lower than the terminal velocity, (c) a reduced sedimentation speed where the gas density increases closer to the protoplanet, and (d) for the high-mass protoplanets an increase in the terminal velocity starting at the transition between Epstein drag and Stokes drag. The highest sedimentation speeds are obtained for low-mass protoplanets far from the star, since these protoplanets have the lowest gas density in the envelope.

Bondi radius, yielding the expression

$$\dot{M} \equiv \frac{M}{\tau} = 4\pi r^2 \rho_p v_t = \text{constant}, \quad (4)$$

where $\dot{M}$ is the mass accretion rate, $M$ is the protoplanet mass, $\tau$ is the accretion time-scale and $\rho_p$ is the density of the particles. The pebble-to-gas mass ratio is therefore

$$\epsilon = \frac{\rho_p}{\rho_g} = \frac{\dot{M}}{4\pi r^2 v_t \rho_g} = \frac{\dot{M} c_s}{4\pi G M R \rho_p}. \quad (5)$$

Here the last step is valid for Epstein drag with friction time $\tau_f = \dot{R} \rho_p / (c_s \rho_g)$, where $\rho_p$ is the material density of the particles, and terminal velocity $v_t = \tau f GM/r^2$ (see Whipple 1972; Weidenschilling 1977; Johansen et al. 2014, for discussions of the different drag force regimes relevant for protoplanetary discs). The pebble-to-gas ratio therefore remains constant in the isothermal regions of the envelope and rises slowly proportional to the increase in sound speed in the deeper regions.

Outside of the Bondi radius we cannot assume spherical symmetry. We parameterize the degree of spherical
Figure 3. The mass density of the pebbles relative to the gas as a function of the distance from the protoplanet. We assumed here that the pebbles fall at their terminal velocity (or the free fall speed when that is slower than the terminal velocity); this gives rise to the increased pebble-to-gas ratio outside of the Bondi radius for the two lowest-mass protoplanets furthest from the star) and that the mass flux is independent of the distance from the protoplanet. The pebble-to-gas ratio is constructed to match an outer boundary value of 0.1. The sudden fall in the pebble-to-gas ratio marks the transition to spherical symmetry inside of the Bondi radius. This is followed by an increased pebble-to-gas ratio as the temperature increases, before the transition from Epstein to Stokes drag leads to a faster sedimentation speed and a decreased pebble-to-gas ratio near the surface of the protoplanet.

We show the calculated pebble-to-gas mass ratio for our models in Figure 3. The pebble-to-gas ratio stays below a few percent throughout most of the envelope. Thus our assumption in making Figure 1 that the opacity is given by micron-sized grains at 1% mass loading is not valid, since millimeter-sized particles yield a (geometric) opacity that is orders of magnitude lower than micron-sized particles. Ali-Dib & Thompson (2019) showed that small particles will pile up in the envelope until the envelope has high enough opacity to become convective. We therefore consider in the main paper only the two extremes where the envelope is either fully radiative or fully convective. We do not explore here further the feedback between opacity and temperature, since we will demonstrate in Section 4 that in the more realistic case where the large-scale convective motion of the gas is included, the speed of both pebbles, pebble fragments and dust is set mainly by the speed of the gas – and hence pebble fragments and dust cannot pile up in the envelope once the envelope becomes convective.

3.3. Mean collision distance

High relative speeds do not necessarily imply fragmenting collisions, since the pebbles must also have time to collide on the way towards the protoplanet. The mean collision distance can be calculated from the pebble number density \( n_p \) and cross section \( \sigma_p \), as

$$\ell_{\text{coll}} = \frac{1}{n_p \sigma_p} = \frac{(4/3)R \rho_\bullet}{\rho_p}.$$  

We assumed here spherical pebbles with a constant internal density \( \rho_\bullet \). We calculate the pebble density \( \rho_p \) from equation (4) and equation (6). The mean collision distance is shown in Figure 4, normalised by the distance to the protoplanet. The mean collision distance is generally longer than the distance to the protoplanet outside of the Bondi radius when the protoplanet is far from the star. These are also the regions of highest terminal velocity (compare to Figure 2). Protoplanets closer to the star are collisional also outside of the Bondi radius, but the sedimentation speeds are relative modest, in the 1–10 m/s range, at the higher gas densities closer to the star.

3.4. Erosion distance

We calculated the mean collision distance above based on pebble-pebble collisions. However, the envelope may contain a significant population of small grains as well. The maximum projectile size giving rise to erosion of a pebble, at the relative speed is \( v \), was measured in Schräpler et al. (2018) to be

$$R_{\text{eros}} = 2 \times 10^{-5} \text{ m } \left( \frac{v}{15 \text{ m s}^{-1}} \right)^{1.62}.$$  

Thus any impactor smaller than 20 \( \mu \text{m} \) will erode the target when the collision speed is 15 m/s and the limit rises to 0.4 mm at a collision speed of 100 m/s. The terminal velocity of the pebbles reaches high enough values...
Figure 4. The mean collision distance of the pebbles, relative to the distance to the protoplanet, as a function of distance from the protoplanet. The collision distance increases with distance from the star and is generally longer than the distance to the protoplanet in the regions of the Hill sphere and the Bondi sphere where the sedimentation speed is high.

The mean collision distance of the pebbles, relative to the distance to the protoplanet, as a function of distance from the protoplanet. The collision distance increases with distance from the star and is generally longer than the distance to the protoplanet in the regions of the Hill sphere and the Bondi sphere where the sedimentation speed is high.

high up in the envelope for collisions with smaller dust aggregates to be erosive. Large projectiles are less efficient than small projectiles at eroding dust aggregate pebbles. The mass loss observed in the experiments of Schräpler et al. (2018) was fitted as

\[
\frac{\Delta m_p}{m_{\text{proj}}} = \left( \frac{v}{15 \text{ m s}^{-1}} \right) \left( \frac{R_{\text{proj}}}{2 \times 10^{-5} \text{ m}} \right)^{-0.62}. \tag{9}
\]

This expression gives an erosion efficiency of 42 (200) for micron-sized grains impacting at 100 (500) m/s and 2.5 (12.3) for projectile grains of 100 microns in size. The erosion distance is given by

\[
\ell_{\text{eros}} = \frac{m_p / (f_{\text{proj}} m_{\text{proj}})}{\sigma_{\text{proj}}} = \frac{m_p}{f_{\text{proj}} \rho_{\text{proj}} \sigma_{\text{proj}}}, \tag{10}
\]

where \(m_p\) and \(\sigma_p\) are the mass and cross section of the pebble, \(m_{\text{proj}}\) is the mass of the projectile, \(\rho_{\text{proj}}\) and \(n_{\text{proj}}\) are the number density and mass density of the eroding grains and \(f_{\text{proj}} = \Delta m_p / m_{\text{proj}} - 1\) is the erosion efficiency. The erosion distance can now be written in terms of the pebble-pebble collision distance as

\[
\ell_{\text{eros}} = \frac{\ell_{\text{coll}}}{f_{\text{proj}} (\rho_{\text{proj}} / \rho_p)}, \tag{11}
\]

where \(\rho_p\) is the mass density of pebbles in the gas. Inserting the erosion efficiency from Schräpler et al. (2018) and adopting the size distribution \(dn_{\text{proj}}/dR \propto R^{-q}\) we arrive at

\[
\ell_{\text{eros}} / \ell_{\text{coll}} = \left( \frac{R_{\text{proj}}}{R_p} \right)^{q-4} \left( \frac{R_{\text{proj}}}{20 \mu\text{m}} \right)^{0.62} \left( \frac{15 \text{ m s}^{-1}}{v} \right). \tag{12}
\]

Here the exponent \(q - 4\) comes from multiplication by mass (scaling as \(R^3\)) and averaging over a logarithmic mass interval (by multiplying by an additional factor \(R\)) – the relevant integral is shown in equation (16). We see that for \(q = 3\), a size distribution with equal surface area in all particle sizes, the erosion distance by micron-sized grains is longer than the mean pebble-pebble collision distance for speeds slower than 2,300 m/s (which is well outside of the range of erosion experiments). For grains of 100 microns the limiting speed is 400 m/s. Adopting instead a size distribution power law of \(q = 3.5\), micron-sized grains erode as efficiently as pebble-pebble collisions at a speed of 70 m/s, while grains of 100 microns erode as well as pebbles at a speed of 130 m/s. We thus conclude that erosion by collisions with smaller grains can at most be as efficient as pebble-pebble collisions at destroying the pebbles and only at the highest sedimentation speeds experienced by the captured pebbles. Such high speeds occur mainly for our protoplanets growing at 30 AU, due to the low gas density there. Also, the mean pebble-pebble collision distance is typically 10 or more times the distance to the protoplanet where such high sedimentation speeds occur. We ignored in this analysis the possibility that the smaller dust aggregates could be very fluffy and would have a much more limited erosion capability (Seizinger et al. 2013).

This analysis implicitly assumed that the density of the projectiles (dust and pebble fragments) is similar to the density of the pebbles. This is a reasonable assumption in the initial capture process where pebbles of many sizes move along bent gas streamlines through the Hill radius. Inside of the Bondi radius, dust and fragments may nevertheless pile up to very high densities. We consider the effect of such pile ups on the erosion of pebbles in Section 5.

3.5. Fragmentation-limited pebble sizes (sedimentation)

Collisions between equal-sized pebbles are expected to be destructive when the collision speeds are higher than 1 m/s (Güttler et al. 2010). This is a well-established threshold for silicate dust aggregates and the same limit likely applies to icy pebbles at low temperatures as well (Musiolik & Wurm 2019). Obtaining the fragmentation-limited particle size from the sedimentation speed is nevertheless problematic, since equal-sized particles formally have zero relative speed and collisions with particles of 100 microns in size have an erosion threshold of 40 m/s (see equation 8). The threshold falls to 2–3 m/s for...
micron-sized projectiles (Schräpler et al. 2018). Taking the worst-case scenario where pebbles can have a maximum sedimentation speed of 1 m/s, we plot in Figure 5 the fragmentation-limited pebble size. The size falls steadily with depth, until reaching the regions of high gas density close to the protoplanet where the pebbles re-coagulate to sizes between 100 microns and 1 millimeter. The minimum pebble size is larger than 10 µm in the collisional regions of the envelope.

Full solutions to the coagulation equation presented in Schräpler et al. (2018) showed that particles in the protoplanetary disc grow to dm sizes where they move with approximately 10 m/s through the gas. The stalling of the growth at 10 m/s in Schräpler et al. (2018) seems to be the result of a balance between eroding collisions with small dust grains and sticking collisions with medium-sized dust aggregates. In Section 5 we also demonstrate that time-dependent solutions to the erosion and growth of the pebbles yield systematically larger pebble sizes than in Figure 5 (compare to the right-side plot of Figure 11).

4. PEBBLE EVOLUTION INCLUDING CONVECTIVE GAS MOTION

The luminosity of protoplanets accreting pebbles is in many cases large enough that the energy must be transported through the envelope by convection. When present, convective motions will have a large influence on the dynamics and collision speeds of the pebbles in the envelope. We acknowledge that for some of the cases considered here, under given assumptions the results indicate that there may not be a sufficient presence of small dust to provide the opacity necessary to trigger convection. Ali-Dib & Thompson (2019) derived a criterion for the dust-to-gas ratio needed to drive convection (their equation 33). In Appendix B we therefore present additional numerical experiments where we fix the density of pebble fragments at 1% and 10% of the gas density, to bracket the values found by Ali-Dib & Thompson (2019). While an increase in the fragment density leads to a reduction in the pebble size reached at the planetary surfaces, we find qualitatively similar results when calculating the fragment density from the local pebble density (as we do in this section) and when considering a fixed fragment fraction relative to the gas (as in Appendix B).

4.1. Convective speed

We use here a simple mixing length estimate of the luminosity transported by convection (Ali-Dib & Thompson 2019),

\[ L_c = 4\pi r^2 (0.36\alpha_{\text{mix}}) \rho_g v_c^3. \]

(13)

Here \( \alpha_{\text{mix}} \) is a mixing length coefficient that we take to be unity and \( v_c \) is the characteristic speed of the convection cells. Setting \( L_c \) equal to the luminosity of pebble accretion, \( L_c = GM(M/\tau)/R \), where \( \tau \) is assumed accretion time-scale and \( R \) is the radius of the solid protoplanet, we obtain an approximate value for \( v_c \). We fix in the main paper \( \tau = 10^6 \) yr and note that the choice of \( \tau \) affects the gas densities in the envelope and hence the collision speeds. In Appendix A we explore the effect of a lower value of \( \tau \).

We plot the convective speed estimates in Figure 6. The estimates generally lie between 100 m/s and 1,000 m/s, and are thus always higher than the local sedimentation speed. The dynamics of the pebbles is therefore dominated by transport with the convective gas rather than by sedimentation.

We point out here that, contrary to what is normally assumed when deriving mixing length estimates, the ratio of the local scale height to the radius is not small in the case of a protoplanet envelope. Hence the scale of the dominating convective motions can even be comparable to the system scale (cf. Popovas et al. (2018)), and should thus not be thought of as small-scale turbulent motions.

The capture of pebbles is ultimately determined from the streamlines that enter the Hill radius—smaller pebbles, whose paths deviate less from their streamlines,
must enter on streamlines that reach closer to the planet, to avoid being transported past the protoplanet (Ormel 2013). The streamline interval for accreted pebbles thus narrows for decreasing pebble size. When convection is present the pebbles are transported faster towards the protoplanet with downwelling cold gas, but also visit for a correspondingly shorter time, and as observed in the 3-D hydrodynamical simulations of Popovas et al. (2018) and Popovas et al. (2019) the accretion rates are therefore not strongly affected when constant particle size is assumed. How the value of the limiting fragment size that can detach at the base of the convection flow depends on the protoplanet’s mass and distance to the star is nevertheless relatively poorly understood, since the simulations of Popovas et al. (2019) focused on the 1-1.6 AU region, and did not include fragmentation, coagulation, and sublimation.

4.2. Pebble-to-gas ratio (convection)

The high speeds of the convective cells imply that pebbles are transported towards the protoplanet’s surface along downwelling cold flows with a low ambient pebble density. However, we can no longer assume that the transport is spherically symmetric within the Bondi radius, as the downwelling cold flows transport pebbles from the accretion radius all the way down to the surface in a single turn-over time-scale (Popovas et al. 2019). We therefore calculate the pebble-to-gas ratio inside of the Bondi radius in the convective case including the factor $f_{\text{sphere}}$ from equation (6), which maintains the degree of spherical symmetry of the flow defined at the Hill radius. The resulting pebble-to-gas ratio is shown in Figure 7. The increased speed of the gas flow is partially cancelled by the non-spherical-symmetry of the pebble component and hence the pebble-to-gas ratio appears quite similar to the non-convective case shown in Figure 3. The high gas speed nevertheless means that the pebbles have less time to collide on the way to the surface of the protoplanet.

4.3. Convective collision speeds

The convective gas motions also induce collisions between the particles. Inspired by calculations of the collision speed in generalized protoplanetary disc turbulence (Ormel & Cuzzi 2007), we take the collision speed $\Delta v_c$ to be

$$\Delta v_c = \sqrt{St_c v_c}.$$ 

(14)

Here $St_c = \omega_c \tau_f$ is the large-scale Stokes number of the pebbles and $\omega_c$ is the frequency of the large-scale turbulent convection cells. We use the simple expression $\omega_c = v_c/R_B$ for convection cells moving at speed $v_c$ over the Bondi radius $R_B$. The convective collision speeds between millimeter-sized pebbles are shown in Figure 8. The collision speed reaches several hundred m/s for protoplanets residing far from the star. But these regions of high convective collision speeds have long collision distances as we also saw for the collisions driven by the sedimentation (compare to Figure 4). The collisional regions of the envelope have convective collision speeds below 100 m/s.

4.4. Fragmentation-limited pebble sizes (convection)

The fragmentation-limited pebble size for convective collisions can be calculated from

$$St_c = \left(\frac{\nu_{\text{frag}}}{v_c}\right)^2.$$ 

(15)

Here we again take a conservative fragmentation threshold of $\nu_{\text{frag}} = 1$ m/s. The convective Stokes number can then be converted to a particle size when the gas density and temperature are known. The resulting particle sizes are shown in the upper panel of Figure 9. If convection is assumed to occur in the form of small scale turbulent motions it is much more efficient at fragmenting the pebbles than sedimentation, since the relative speed is only reduced as the square root of the particle size in the case of turbulence. Hence protoplanets at 10 AU and 30 AU have fragmentation-limited particle sizes smaller than the size of a monomer in the outer regions of the envelope. These regions are nevertheless optically thin to collisions. However, the fragmentation-limited pebble size increases further down in the envelope and reach millimeter sizes again close to the surface of the protoplanet.

The lower panel of Figure 9 shows the results of adopting the smallest of the particle sizes from the cases with sedimentation and convection acting separately. This is likely a worst case estimate, in that the large scale convection found in Popovas et al. (2019) also would act to reduce the fragmentation in downwards directed plumes, by shortening the transport time of particles down to the neighborhood of the surface.

5. SOLVING THE PEBBLE SIZE EVOLUTION

The analysis of the fragmentation-limited pebble sizes in the previous two sections ignored many aspects of pebble erosion and pebble growth as well as the timescale needed to reach the fragmentation-limited size. We therefore go a step further in analysing the size evolution of pebbles in the envelope by solving for the time-dependent pebble size, taking into account erosion, fragmentation and growth by mass transfer.

5.1. Size distribution and fragment density
Figure 6. The convection speed, from a mixing length expression based on the luminosity of the protoplanet, as a function of the distance from the protoplanet (left) and the terminal velocity as a function of the convection speed (right). The convective speed is generally larger than the sedimentation speed (the dotted line marks the equality between the convective speed and the terminal velocity). All particle sizes are thus strongly affected by transport with convection cells, as also seen in 3-D simulations in Popovas et al. (2018) and Popovas et al. (2019). Pebbles can therefore only accreted to the protoplanet with the downwelling cold gas, if the envelope is convective.

Figure 7. The pebble-to-gas ratio in the convective case. We assume here that the speed of the pebbles is given by the downwards speed of the convective cells, starting already at the pebble accretion radius due to convective overshoot. We furthermore assume that the mass flux is not spherically symmetric inside of the Bondi radius, since the pebbles are transported towards the surface of the protoplanet along downwelling cold flows. These two effects partially cancel each other so that the pebble-to-gas ratio is relatively similar to the non-convective case shown in Figure 3.

Figure 8. The convective collision speed between millimeter-sized pebbles as a function of the distance from the protoplanet. We mark the critical fragmentation speed for equal-sized particles (1 m/s). The collisional regions of the envelope generally have convective collision speeds slower than 100 m/s.

We follow the pebble size $R_p$ and assume that smaller particles are present as a continuous size distribution with the number density $dn/dR = KR^{-q}$ down to the smallest particle size $R_0 = 10^{-6}$ m (we fix $q = 3.5$ here).
Figure 9. Fragmentation-limited particle size assuming a critical fragmentation speed of 1 m/s and the relative speed from convection as a function of the distance from the protoplanet (top panel) and the fragmentation-limited particle sized resulting from adopting the smallest size resulting from turbulent convection and sedimentation acting separately (bottom panel).

The size distribution is normalised to give the total dust mass from equation (4), with

$$\int_{R_0}^{R_p} KR^{3-q} dR = \rho_p .$$

(16)

We make one of two assumptions about the sizes and total density of the particles: (i) that the fragment size distribution extends to the pebble size and that the total density of pebbles and fragments is given by the radial speed of the pebbles or (ii) that the fragment size distribution extends only to the fragmentation-limited size and that these fragments have the same flux as the pebbles within the Bondi radius, with their density given by the speed at the fragmentation-limited size. The first case is realistic if pebbles and fragments enter a collisional equilibrium whereby the net downwards motion is set by the largest objects. The second assumption is a worst-case scenario where the entire pebble flux is converted into fragments that achieve a high spatial density due to their slow sedimentation speed. We always assume assumption (i) to be the case outside of the Bondi radius, since the shear flows and recycling flows prevent the pile up of fragments in this region and additionally the pebble accretion process itself intrinsically picks up large particles out of the shear flow and transport them towards the protoplanet, leaving behind the smaller particles. We additionally limit the fragment density to be less than or equal to the gas density inside of the envelope, since any additional mass loading is dynamically unstable to the formation of rapidly sedimenting Rayleigh-Taylor-like dust blobs (Lambrechts et al. 2016; Capelo et al. 2019). We have done additional tests using the fragmentation-limited approach also outside of the Bondi radius as well as allowing pebble-to-gas ratios above unity and found only minor differences in the resulting pebble sizes.

5.2. Pebble size evolution model

We divide the particle size distribution into $N_{\text{spec}} = 100$ bins (or species), spaced logarithmically so that $dR \propto R$ when assigning mass to each bin using the binned equivalent of equation (16). The size of the particles in species $i$ is $R_i$, their mass is $m_i$ and their density is $\rho_i$.

We change the mass of the pebbles by summing over the equation

$$m_p = - \sum_i \pi (R_p + R_i)^2 \Delta v_i \rho_i m_i f_{\text{int},i} .$$

(17)

Here the relative speed $\Delta v_i$ is given by either the relative sedimentation speed (proportional to $R_p - R_i$) or the combination of relative sedimentation speed and the
convective collision speed. The latter is approximated as the collision speed of the pebbles, equation (14), as this represents well the collision speed with smaller particles as well. The interaction fraction $f_{\text{int},i}$ can be either positive (erosion or fragmentation) or negative (sticking or mass transfer). We follow Bukhari Syed et al. (2017) and distinguish between the situation where the projectile and the target have very dissimilar sizes ($R_{\text{p}}/R_{\text{t}} \geq 5.83$) and the situation where they have similar sizes ($R_{\text{p}}/R_{\text{t}} < 5.83$). The different collision outcomes are parameterised as

- Erosion ($R_{\text{p}}/R_{\text{t}} > 5.83$). The eroded mass fraction relative to the projectile is given in Schräpler et al. (2018), see also our equation (9). We subtract one from the mass fraction to get the relative mass loss.

- Mass transfer ($R_{\text{p}}/R_{\text{t}} > 5.83$). When the erosion coefficient of Schräpler et al. (2018) becomes less than one, we use the power law fit from Bukhari Syed et al. (2017) to calculate the mass transfer coefficient as a function of the relative velocity, capped at a maximum of 0.5 at high speeds (Wurm et al. 2005).

- Fragmentation ($R_{\text{p}}/R_{\text{t}} < 5.83$). When the projectile is larger than $1/5.83$ times the target size, we allow the projectile to fragment the target. We use the power law fit to collision experiments provided in Bukhari Syed et al. (2017) to calculate the erosion factor for fragmenting collisions. We include the probability $p_{\text{sur}} = 0.194R_{\text{p}}/R_{\text{t}} - 0.13$ that the target survives the collision (i.e., is not fragmented) and instead experiences mass transfer from the projectile. Mass transfer is not allowed when the kinetic energy in the collision is higher than the energy needed to reduce the target mass by a factor two. The final erosion factor is calculated as a weighted average of mass transfer and erosion, $f_{\text{int}} = -p_{\text{sur}} f_{\text{int}} + (1 - p_{\text{sur}}) f_{\text{frag}}$.

The interaction fraction $f_{\text{int}}$ as a function of the projectile size is shown for five different collision speeds in Figure 10.

5.3. **Pebble size evolution without convection**

In Figure 11 we show the pebble size as a function of the distance from the protoplanet. There is only little evolution in the pebble size outside of the Bondi radius. The pebbles inside of the Bondi radius reach a size near the fragmentation-limited expression near the surface of the protoplanet (compare to Figure 5) for both our assumptions about the fragment density – and in the case where the fragment mass flux equals the pebble mass flux the pebble sizes follow the fragmentation-limited size even in the regions of the envelope that we identified as optically thin to collisions. This is due to the high density of pebble fragments in those regions from our assumption that the fragment flux is equal to the pebble flux there.

5.4. **Pebble size evolution including convection**

In Figure 12 we include the effect of convection both for the speed of the pebbles as they move towards the protoplanet (by adding to the sedimentation speed the downwards motion of the cooling gas flows) and for the collision speed. The speed of the cool gas flows are so high (see Figure 6) that the pebbles do not have time to interact appreciably before they are accreted by the protoplanet, except for the lowest-mass proto-planets far from the star. The difference between our two fragment density prescriptions comes mainly here from the smaller size of the fragments in case (ii), since the convective gas speed makes the pebble and fragment density similar in both cases. In Appendix A we consider the evolution of pebbles at an earlier and a later evolution stage of the protoplanetary disc, respectively. The high gas densities in the protoplanetary disc early in the disc evolution reduces the collision speeds and leads to better pebble sur-
Figure 11. The pebble size in models without convection, resulting from two assumptions about the density of the pebble fragments: (i) that their density is given by the radial speed of the pebbles (left) and (ii) that their density is given by the radial speed of the largest fragments, assumed to be at the fragmentation-limited size and with the same mass flux as the pebbles (right). We mark the Bondi radius with black dots; outside of the Bondi radius we assume that the fragment density is equal to the pebble density since particles cannot pile up in this region. The pebbles undergo only minor erosion outside of the Bondi radius; this contrasts with the small size of the fragmentation-limited particles seen in Figure 5. Within the Bondi radius we observe that model (i) sees only limited size changes, while model (ii) approaches the fragmentation-limited sizes. The pebbles are nevertheless a factor 2–3 times larger than the fragmentation limit inside of the Bondi radius.

6. CONCLUSIONS

In this paper we have presented calculations of pebble erosion, fragmentation and regrowth inside of the envelope of an accreting protoplanet, covering a parameter range that includes both weakly and tightly coupled regimes (in terms of ratio of stopping time and collision time to transit time at the Bondi radius). Pebbles reach high relative speeds, up to of order 1,000 m/s, both by differential sedimentation and by acceleration in the convective gas. The speeds are highest for low-mass protoplanets (\(< 1 M_E\)) far from the star (\(a > 3 \text{ AU}\)), at low protoplanetary disk accretion rates, while the higher gas density obtained for higher accretion rates, and for high-mass protoplanets close to the star, reduces the relative speeds. However, the regions of the envelope where collision speeds are high are also typically optically thin to collisions.

We analysed the fragmentation-limited particle size, assuming a worst-case scenario where the threshold speed for fragmentation is just 1 m/s. This leads to very small pebble fragment sizes, down to micron sizes, in the regions of the envelope where the relative speed is high. The increased gas density near the surface of the protoplanet nevertheless allows regrowth to sizes up to a few hundred microns.

We also implemented a simple size evolution scheme that keeps track of the characteristic size of the pebbles as they move towards the surface of the protoplanet. The size evolution depends strongly on our assumptions on the mass density of the pebble fragments. Assuming that the fragment density follows the pebble density, we find a modest decrease in the pebble size from the original millimeter sizes to a few hundred microns near the surface of the protoplanet. Allowing instead the fragments to pile up in the envelope, the pebble size follows closely the fragmentation-limited size throughout the Bondi sphere. This case seems to be most similar to the model of Ali-Dib & Thompson (2019).

However, we have a significantly different approach to convection from Ali-Dib & Thompson (2019). Based on hydrodynamical simulations of accreting protoplanets (Popovas et al. 2019), we assume that the convective flow consists of large-scale flows that extend from the protoplanet’s surface all the way to the Bondi radius and that the convective overshoot is significant enough to reach the pebble accretion radius. Hence the pebbles and the fragments are transported down towards the protoplanet at speeds in the range 100-1,000 m/s, to
near-stagnation regions close to the planetary surface, from where they can hence sediment down to the planet more easily. This approach precludes the pile up of pebble fragments in the envelope and additionally lowers the residence time of the pebbles in the envelope, so that the erosion effect is lowered. We observe then that the pebbles are transported to the surface of the protoplanet at sizes between 100 microns and 1 millimeter. Low-mass protoplanets residing far from the star (> 30 AU) nevertheless still experience significant pebble erosion due to the low gas densities in their envelopes.

We have for simplicity ignored several physical effects in the model. We assumed the opacity to be a constant, irrespective of the pile up of pebble fragments in the envelope. The feedback from the opacity on the temperature structure was included by Ali-Dib & Thompson (2019), who concluded that the increased opacity leads to a large-scale expelling of dust grains from the envelope. Their approach to convection is nevertheless very different from ours, in that it was assumed that convection can be treated as a purely diffusive process. We believe that a more realistic view is to consider convection to be a large-scale circulation that transports pebbles and fragments down to the protoplanet’s surface, while upwelling hot flows work to remove remaining dust and fragments back to the protoplanetary disc.

We also ignored the effect of ice lines. Ice lines must be located interior of the Bondi radius, since the temperature of the envelope only starts to rise interior of the Bondi radius. Sublimation of ices (such as H$_2$O, CO$_2$ and CO) would lead to a restructuring of the dust aggregate pebbles, but we assume that the sublimation process is so slow (the sublimation time-scale is on the order of several days or even weeks) that loss of volatiles does not lead to a monomerisation of the aggregates. The silicate sublimation front at 2,000 K would under all circumstances destroy pebbles and dust close to the surface of massive protoplanets; however this sublimation is associated with a radiative zone, due to the strong decrease in opacity there (see Figure 1), which would separate the silicate vapour from the bulk envelope. In addition we ignored the latent heat from sublimation and deposition (Brouwers et al. 2018, 2019; Brouwers & Ormel 2020), which may also be important for the thermodynamics of the envelope.

From our study we conclude that (i) if the gas envelope is not convective, then the pebbles will fragment inside of the envelope and slowly settle towards the protoplanet’s surface while they regrow to macroscopic sizes, while (ii) if the gas envelope is convective, then the large-scale convection flows transport the pebbles quickly to the base of the convection flow – the pebbles then maintain their size relatively well for protoplanets growing within 10 AU, while low-mass protoplanets accreting late further out can have their pebbles reduced to 10 microns or less. More complete studies, including the resolved flow of the gas and a more advanced approach to the radiative energy transfer, fragmentation, coag-
ulation and sublimation equations will nevertheless be needed to fully understand the fate of pebbles accreted by a protoplanet and their detachment at the base of the convection flow.

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REFERENCES

Ormel, C. W. 2017, Astrophysics and Space Science Library, 197
Whipple, F. L. 1972, From Plasma to Planet, 211
APPENDIX

A. HIGHER AND LOWER GAS ACCRETION RATES

In this Appendix we demonstrate how the evolutionary stage of the protoplanetary disc affects the survival of the pebbles in the convective models. We consider first an earlier evolution stage where the gas accretion rate through the protoplanetary disc is $\dot{M}_* = 3 \times 10^{-7} \, M_\odot \, \text{yr}^{-1}$, three times larger than in the nominal model, with corresponding pebble accretion time-scale $\tau = 3 \times 10^5 \, \text{yr}$ three times smaller than in the nominal model. The resulting pebble sizes are shown in the top panel of Figure 13. The pebbles retain larger sizes on the way down to the protoplanet compared to the nominal model (Figure 12), due to the higher gas density and hence lower sedimentation and collision speeds.

We also consider here a variation of the high-accretion model where the opacity is constant and high enough that the envelope becomes fully convective. The middle panel of Figure 13 shows that this has only little effect on the pebble sizes, since the high-accretion rate case is already close to fully convective.

Our second parameter variation considers a later growth stage where the gas accretion rate through the protoplanetary disc is $\dot{M}_* = 1 \times 10^{-8} \, M_\odot \, \text{yr}^{-1}$, three times lower than in the nominal model. The lower gas densities lead to an increased destruction of the pebbles.

B. CONSTANT FRAGMENT DENSITY

The density and temperature profiles of the envelope were constructed in the main paper under the assumption of the opacity being provided by a constant dust-to-gas ratio of 0.01 and using the Bell & Lin (1994) opacity power laws. For the considered pebble accretion rates, the envelopes are close to fully convective (Figure 1). We nevertheless did not consider a fully self-consistent coupling between the pebbles, their fragments and the dust opacity. Ali-Dib & Thompson (2019) derived a criterion for the dust mass loading needed to make the envelope fully convective (their equation 33). Here we perform an additional numerical experiment where we fix the fragment density to 0.01 or 0.1 times the gas density. We present the results in Figure 14. Increasing the fragment density leads to a decrease in the characteristic pebble size, but the pebbles still maintain sizes above 100 $\mu$m within 1 AU and 5-50 $\mu$m further out.
\[ \dot{M}_* = 3 \times 10^{-7} M_\odot \text{yr}^{-1}, \tau = 3 \times 10^5 \text{yr} \]

\[ \dot{M}_* = 3 \times 10^{-7} M_\odot \text{yr}^{-1}, \tau = 3 \times 10^5 \text{yr}, \text{fully convective} \]

\[ \dot{M}_* = 1 \times 10^{-8} M_\odot \text{yr}^{-1}, \tau = 1 \times 10^6 \text{yr} \]

Figure 13. The pebble size in the models including convective collision speeds and large-scale transport for a gas accretion rate of \( \dot{M}_* = 3 \times 10^{-7} M_\odot \text{yr}^{-1} \) and a pebble accretion time-scale of \( \tau = 3 \times 10^5 \text{yr} \) (top), for a fully convective model with constant, high opacity (middle) and for a lower gas accretion rate \( \dot{M}_* = 10^{-8} M_\odot \text{yr}^{-1} \) and the nominal pebble accretion time-scale of \( \tau = 10^6 \text{yr} \) (bottom). The higher gas density leads to slower sedimentation and collision speeds and better pebble survival. The fully convective model gives a very similar result, although the slightly lower gas density leads to lower pebble sizes for the low-mass protoplanets far from the star. The model with lower gas accretion rate shown in the bottom panel has a much stronger effect on the pebble size than considering fully convective envelopes.
Figure 14. The pebble size in convective models when fixing the fragment density to either 0.01 (left plot) or 0.1 (right plot) times the gas density. The latter may represent the value needed for the envelope to be fully convective, according to Ali-Dib & Thompson (2019). The presence of 10% fragments in the gas leads to lower pebble sizes, with a characteristic size above 100 µm within 1 AU and falling down to 5-50 µm further from the star.