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Hansen, Steen H.

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Accelerated expansion induced by dark matter with two charges

Steen H. Hansen
Dark Cosmology Centre, Niels Bohr Institute, University of Copenhagen, Jagtvej 155, DK-2100 Copenhagen, Denmark

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ABSTRACT
The accelerated expansion of the Universe has been established through observations of supernovae, the growth of structure, and the cosmic microwave background. The most popular explanation is Einstein’s cosmological constant, or dynamic variations thereof. A recent paper demonstrated that if dark matter particles are endowed with a repulsive force proportional to the internal velocity dispersion of galaxies, then the corresponding acceleration of the Universe may follow that of a cosmological constant fairly closely. However, no such long-range force is known to exist. A concrete example of such a force is derived here, by equipping the dark matter particles with two new dark charges. This result lends support to the possibility that the current acceleration of the Universe may be explained without the need for a cosmological constant.

Key words: acceleration of particles – gravitation – dark energy – dark matter.

1 INTRODUCTION
The acceleration of the Universe was first observed in SN1a data (Riess et al. 1998; Perlmutter et al. 1999) and has since been confirmed by a range of independent observations including growth of the large scale structure and the cosmic microwave background (Hicken et al. 2009; Percival et al. 2010; Blake et al. 2011a,b; Komatsu et al. 2011; Larson et al. 2011). Recent observational data is sufficiently abundant and precise, that one may start seeing indications that possibly not even dynamic versions of the cosmological constant provide a self-consistent explanation of all the data (Dainotti et al. 2021; Krolewski et al. 2021; Teng, Lee & Ng 2021). It is therefore relevant to investigate alternatives to (dynamics versions of) the cosmological constant. A large number of such models have been discussed over the years, see e.g. references in (Motta et al. 2021). Most of these proposals conform with the idea that (Peebles & Ratra 2003) ‘...this cosmic repulsion is a gravitational effect of the negative gravitational mass density, not a new force law.’ A recent paper, however, challenged this view, by demonstrating that if a new inverse square-distance force exists amongst the dark matter particles, then the resulting acceleration of the Universe is consistent with the acceleration that is induced by a cosmological constant (Loeve et al. 2021). In that paper, the temporal evolution came about by letting the force depend on the internal velocity dispersion of the dark matter haloes: since structure formation proceeds bottom-up, then the later structures will be more massive and have higher velocity dispersions (Loeve et al. 2021). The main problem is, however, that no such long-range force is known to exist in nature.

This paper shows that such a force indeed may exist. Using established tools from spacetime algebra (STA) it is shown, that if one equips the dark matter particles with two new dark-charges, where the two forces are attractive and repulsive, respectively (likely because the carriers of the forces have even and odd spins) then the non-cancelled part of the forces has exactly the form needed to generate an accelerated universe without a cosmological constant.

2 ANGULAR MOMENTUM
To introduce the practical tool needed, namely STA, let us start by addressing how angular momentum can be made Lorentz invariant.

Even though both time and space separately are changed under a Lorentz transformation, then the time-space vector, (ct, r), is a proper 4-dimensional vector and hence has invariant length under a change of reference. Similarly, the energy-momentum vector, (ε/c, p), is a proper 4-vector.

This is starkly contrasted by the 3-dimensional angular momentum, \( \vec{L} = \vec{r} \times \vec{p} \), where \( \vec{r} \) and \( \vec{p} \) are the 3-dimensional distance and momentum. This object cannot be combined with anything to make a proper 4-dimensional vector. Similarly, the dynamic mass moment, \( \vec{N} = ct\vec{p} - \varepsilon\vec{x}/c \), cannot be combined with anything to make a proper 4-vector (Landau & Lifshitz 1975). Instead, there exists a rank-2 antisymmetric tensor, \( M_{\mu\nu} \), which is the combination of \( \vec{L} \) and \( \vec{N} \), which has nice transformation properties under a Lorentz transformation. The properties of \( M_{\mu\nu} \) are made most simple and explicit in the language of STA (Hestenes 1966, 2003), where the relevant object is written as

\[
\mathbf{M} = r \wedge p,
\]

where \( r \) and \( p \) are proper 4-dimensional vectors. The wedge-operator, \( \wedge \), is the natural 4-dimensional antisymmetric generalization of the 3-dimensional cross-product, \( \times \). A brief teaser to STA calculations will be given now, and then the technical details will be discussed correctly in the next section. The basis of STA calculations is to acknowledge that all observables exist in 4-dimensional Minkowski space. In STA there is one unique derivative, \( \nabla \), which contains both time- and space-derivatives, and hence the only natural equation to
write for $M$ is
\[ \nabla M = j_{\mu}, \]
where $j_{\mu}$ is some source. Forces appear in STA by contracting $M$ with a proper 4-dimensional vector, $w$, to get $M \cdot w$, which for instance gives Newton’s gravitational law. The only thing missing is, that one needs observations to determine the constant in front of the force, which in this case is the gravitational constant, $G$.

### 3 Spacetime Algebra

After this brief teaser, the details of the algebra can now be introduced correctly. STA starts with Minkowski space, $M_{1,3}$, using the metric signature $(+, −, −, −)$, and a chosen basis $\{\gamma_{\mu}\}_{\mu=0}^{3}$ of $M_{1,3}$.

The four orthonormal vectors constitute the basis for 1-blades. The six antisymmetric products $\gamma_{\mu\nu} \equiv \gamma_{\mu} \gamma_{\nu}$, are called the 2-blades. The product is in general given by the sum of the dot and wedge products: $ab = (a \cdot b + a \wedge b)$ (Doran & Lasenby 2007; Hestenes 2015). The four 3-blades, $\gamma_{\mu\nu\lambda}$, are given by $\gamma_{\mu\nu\lambda} = \gamma_{\mu} \gamma_{\nu} \gamma_{\lambda}$. Finally one reaches the highest grade, the pseudoscalar $I \equiv \gamma_{0} \gamma_{1} \gamma_{2} \gamma_{3}$, which represents the unit 4-volume in any basis, with the property that $F = -1$.

The generalized angular momentum mentioned above is called a bi-vector. The bi-vectors are oriented plane segments, and examples also include the electromagnetic field $F$ (Doran & Lasenby 2007; Hestenes 2015). Vector-arrows are used above spatial 3-vectors like $\vec{E}$ or $\vec{p}$, no-vector-arrows are used for proper 4-vectors like $r$ and $p$, and boldface is used for bi-vectors like $F$ and $M$.

One of the reasons for the success of STA is that the derivative $\nabla F = \nabla \cdot F + \nabla \wedge F$ naturally contains both space and time derivations. When choosing a time-direction, $\gamma_{0}$, one can decompose the derivative along a direction parallel to and perpendicular to $\gamma_{0}$, $\nabla = (\delta_{0} - \vec{V}) \gamma_{0}$, where $\vec{V}$ is the frame-dependent relative 3-vector derivative. This choice of frame also allows one to connect a 4-vector $w$ with its para-vector, $w_{0} + \vec{w}$, via $\gamma_{0}$ (Doran & Lasenby 2007), namely $w = (w_{0} + \vec{w}) \gamma_{0}$. Furthermore, the right-multiplication by the timelike vector $\gamma_{0}$ isolates the relative quantities of that frame (Dressel, Bloikh & Nori 2015), e.g. $r \gamma_{0} = (ct + \vec{r})$.

### 4 Electromagnetism in STA

The case of electromagnetism in STA is well described in the literature (Dressel et al. 2015; Hestenes 2015). The starting point may be taken with the bi-vector (Hansen, in preparation)
\[ F = \frac{q}{m} r \wedge p. \]

When comparing to equation (1), one notes that the only difference is the exchange of mass by charge.

The simplest non-trivial equation is in this case given by
\[ \nabla F = j_{\mu}, \]
where the current, $j_{\mu}$, only contains an electric part in the absence of magnetic monopoles. By making the identification
\[ F = \vec{E} + \vec{B} I, \]
it is straightforward to derive the 4 Maxwells equations (Dressel et al. 2015; Hestenes 2015), which include
\[ \vec{V} \cdot \vec{E} = \frac{\rho_{e}}{\varepsilon_{0}}, \]
\[ \partial_{0} \vec{B} + \vec{V} \times \vec{E} = 0. \]

Whereas $F$ is a proper geometric object of electromagnetism, then the separation into $\vec{E}$ and $\vec{B}$ fields requires specification of a frame by the choice of $\gamma_{0}$. At this point, the connection with the tensor $F_{\mu\nu}$ can be made explicit by noticing (Dressel et al. 2015)
\[ F = (F \cdot \gamma_{0}) \gamma_{0} + (F \wedge \gamma_{0}) \gamma_{0} = \vec{E} + \vec{B} I, \]
\[ = E_{1} \gamma_{10} + E_{2} \gamma_{20} + E_{3} \gamma_{30} \]
\[ + B_{1} \gamma_{12} + B_{2} \gamma_{21} + B_{3} \gamma_{31}, \]
where $E_{i}$ and $B_{i}$ are the components of the corresponding 3-vectors, and $\gamma_{\mu\nu}$ are the 2-blades as described in Section 3.

It is important to emphasize, that equation (4) is not merely a matter of compact notation: it is instead the only logical extension beyond the most trivial equation in STA, $\vec{V} F = 0$ (which leads to the four Maxwell's equations without sources). The only thing remaining is to connect the bi-vector field to observables: the radial dependence is found from Gauss' law, equation (6), and the units of $\vec{E}$ and $\vec{B}$ are established through measurements.

### 5 Appearance of Forces

In STA, forces are derived by contracting the bi-vectors with a proper velocity vector, $v$, e.g. the Lorentz force is derived directly from $F \cdot w$ (Dressel et al. 2015)
\[ (F \cdot w) \frac{d\tau}{dt} \gamma_{0} = q \vec{E} \cdot \vec{v} + q \left( \vec{E} + \vec{v} \times \vec{B} \right), \]
where the first term on the r.h.s. is the rate of work, $d\vec{E} dt (ct)$, and the last parenthesis on the r.h.s. is the classical Lorentz force, where $\vec{w} = \vec{v} \gamma_{0}$. One notes that the $\gamma$ here is the relativistic Lorentz factor, and should not be confused with the bases for Minkowski space, $\gamma_{\mu}$.

Similarly the gravitational force appears from the contraction of $M$ and a proper 4-velocity
\[ (M \cdot w) m \frac{d\tau}{dt} \gamma_{0} = -m \vec{n} \cdot \frac{\vec{v}}{c} + m \left( \vec{n} - \frac{\vec{v}}{c} \times \vec{L} \right). \]
The first term on the r.h.s. is similar to a rate of work, and the first parenthesis in the second term leads to Newton’s gravitational force (Hansen 2021).

When one considers gravitational forces, then there is an extra detail, which arises if the structure under consideration contains a dynamical term proportional to the velocity dispersion, $\sigma^{2}$. This could, for instance arise in a galaxy cluster where the galaxies and dark matter particles are orbiting in the local gravitational potential. In this case the potential is minus 2 times the kinetic energy according to the virial theorem (Binney & Tremaine 2008), $2T + U = 0$, and hence one can write the energy as
\[ \varepsilon = mc^{2} - \frac{1}{2} m \sigma^{2}. \]
The correction in equation (12) above is the first order correction. It is similar to the first order relativistic correction to the kinetic energy of a gas, where the difference between the relativistic and restenergies typically reads
\[ mc^{2} \left( 1 - \frac{1}{\sqrt{1 - \frac{v^{2}}{c^{2}}} - 1} \right) = \frac{3}{2} kT. \]

The fact that it is exactly this correction to the energy which is relevant, comes from the dynamic mass moment, $N$, as shown in Hansen (2021). It is the expectation that the exact same expression
can be derived from general relativity (GR) in a weak field limit, as the natural correction to the energy momentum tensor when one includes a structure with non-zero internal dispersion.

From this extra term there will be a correction to the normal Newtonian force, which is proportional to the velocity dispersion squared (Hansen 2021). This correction can be of the order $10^{-5}$ for a galaxy cluster of mass $M_{\text{cluster}} = 10^{15} M_\odot$, and of the order $10^{-9}$ for a dwarf galaxy of mass $M_{\text{dwarf}} = 10^{9} M_\odot$.

6 A NEW REPELLENT FORCE FOR DARK MATTER PARTICLES

Having seen how the Lorentz force of electromagnetism and Newton's gravitational law appear naturally from the bi-vectors $\mathbf{F}$ and $\mathbf{M}$, in equations (3, 1), it is straightforward to generalize to new forces. Let us imagine that the dark matter particles are equipped with a new charge, $q_d$. The index, $r$, refers to repulsive, and one imagines that a quantum field theoretical description of the force-carrier is that of a spin-1 particle, such that equal charges repel each other. At this point, let us consider an asymmetric creation of dark matter, such that all dark matter particles carry the same charge.

A new bi-vector is defined, $\mathbf{D}_r = \hat{D}_r + \hat{C}_r \mathbf{I}$, (compare with the case of electromagnetism in equations 5, 3) where the $\hat{D}$ refers to the dark matter particle

$$\mathbf{D}_r = \frac{q_r}{m} \mathbf{r} \wedge \mathbf{p}. \quad \text{(14)}$$

From the dynamical equation

$$\mathbf{V} \mathbf{D}_r = \dot{j}_d, \quad \text{(15)}$$

where $\dot{j}_d$ is a source term, one gets 4 equations, of which one is $\mathbf{V} \cdot \mathbf{D}_r = \rho_d j_d$, where $\rho_d$ is the number density of the dark matter particle. $\sigma_d$ is similar to the vacuum permittivity, $\varepsilon_0$, of electromagnetism. Integrating this equation (just like Gauss' law) one gets that the field $\hat{D}_r$ is inverse square-distance. The corresponding forces are found from $\mathbf{D}_r \cdot w$, of which the main term gives a force

$$\frac{dp}{d\tau} \gamma_0 = q_r (\mathbf{D}_r \cdot w) \approx q_r \gamma \hat{D}_r. \quad \text{(16)}$$

The magnitude of the force must be determined from observations, in the same way that one measures $G$ for the gravitational force, and $\varepsilon_0$ for the Coulomb force.

There is one very important detail, namely the sign of the minor correction in equation (12). Since the structure was already created through gravity, the correction will come with a negative sign (as in equation 12) because the particles have repulsive forces: the particles try to push each other apart, and are therefore in a slightly higher energy-state than they would prefer.

7 A NEW ATTRACTIVE FORCE FOR THE DARK MATTER PARTICLES

The dark matter particle is now also equipped with a second charge, $q_a$. The index $a$ indicates that this is an attractive force, and hence one imagines it is generated by an even-spin force carrier. Everything repeats itself from Section 6, except for two details: that one is using $q_a$ and $\varepsilon_a$, and that the minor correction from equation (12) will come with the opposite sign: the particles are attracted to each other, and are hence in a lower energy state by being close together.

This sign difference will be very important in the next section. The force again turns out to be inverse square-distance, however, it will be attractive. The magnitude of the force, expressed through $\varepsilon_a$, to make the connection with $\varepsilon_0$ of electromagnetism explicit, must be determined from measurements.

8 TWO COMBINED FORCES

In the discussion above, there is nothing indicating the magnitude of the forces: both were inverse distance-square forces, and their magnitude could be virtually anything.

If each dark matter particle happens to be equipped with both charges, and for some reason the magnitude of the two forces happen to be identical, $q_a^2/\varepsilon_a = q_d^2/\varepsilon_d$, then the main force terms will cancel, where the main terms correspond to the first terms on the r.h.s. of equation (12). This is because the one force was (constructed to be) attractive and the other repulsive, but with equal strengths. However, the minor corrections (the last term in equation 12) have opposite signs, and the combined force will therefore be non-zero. The resulting combined force is repulsive and of the form

$$\frac{q_d^2}{\varepsilon_d} \left(1 - \frac{1}{\varepsilon_d} \frac{1}{c^2} \right) \hat{p} = \frac{q_a^2}{\varepsilon_a} \left(1 + \frac{1}{\varepsilon_a} \frac{1}{c^2} \right) \hat{p} = \frac{q_d^2 q_a^2}{\varepsilon_d \varepsilon_a c^2 r^2}. \quad \text{(17)}$$

where the dispersion has been normalized to the speed of light.

This form is exactly of the shape needed for the suggestion of Loeve et al. (2021)

$$\frac{\sigma^2}{c^2} Gm_1 m_2 \frac{\hat{p}}{r^2}. \quad \text{(18)}$$

By comparing the two expressions above, one sees that the combination of the dark matter charge and their strength, must be equal to the gravitational force times a constant $\kappa$, which according to Loeve et al. (2021) should be of the order $\kappa \sim 10^6 – 10^9$.

9 OBSERVATIONAL CONSTRAINTS

A standard comparison of the strength of forces gives that the gravitational attraction between 2 protons is about 36 orders of magnitude smaller than the electromagnetic repulsion. One notices that a force only 6–8 orders of magnitude stronger than gravity is needed, which means 28–30 orders of magnitude weaker than electromagnetism. Since the above calculations were imagined in an asymmetric dark matter model (all dark matter particles are created with the same charge), then that can be imagined as a charge which is 14–15 orders of magnitude smaller than the electric charge.

Since all main forces cancel between the attractive and repulsive forces, this implies that the evolution of the universe will be entirely unaffected by the charges themselves. The only difference is that in the late universe, when the dispersions in galaxies start being significant, there will be an overall repulsive force, which may accelerate the expansion of the universe.

Dark charges and the corresponding dark radiation have been discussed for many years (Ackerman et al. 2009). Impressively many models had been considered, some with massive dark photons (which is not relevant for the model considered here), some models with massless dark (or hidden) photons, with symmetric and asymmetric dark matter, millicharged dark matter, and naturally a very large number of models where there is some kind of interaction between the dark and visible sectors. Most of the constraints are highly model dependent and cannot be reviewed here, instead the
reader is referred to the references in the reviews (Battaglieri et al. 2017; Fabbrichesi, Gabrielli & Lanfranchi 2020).

The calculations above are entirely classical, in the sense that STA only allows one to derive Maxwell’s equations and the Lorentz force. In order to perform a quantum field theoretical derivation of the forces, one would have to define and calculate the relevant Feynman diagrams. It is the expectation that in the exact same way as QED generalized classical electromagnetism, a similar quantum field theoretical generalization would allow one to derive the dark forces described above. In this connection, it will be particularly interesting how one at the quantum level will make the two different forces (mediated by different spin particles) cancel. Naturally this point is beyond the scope of the classical derivation presented above.

Any quantum field contributes zero-point energy, which might add a contribution to the cosmological constant that is many orders of magnitude larger than observationally acceptable (Weinberg 1989). This goes very much counter to the suggestion of this paper, namely to avoid the need for a cosmological constant. The new dark charges proposed above are only likely to worsen this situation, and the present paper does in no way attempt to solve this issue.

One significant concern mentioned in Loeve et al. (2021) is the stability of structures like galaxies and dwarf galaxies: if the force would be a standard ‘electromagnetic-like’ force, then there would be large forces internally in a dwarf galaxy, which would rip the structure apart. In the present model all these large internal forces are exactly cancelled, and hence that concern is completely avoided. In order to test if this model is indeed able to explain the observed accelerating universe, it will be necessary to perform a numerical simulation, where the acceleration is accurately calculated in each time-step, from the actual velocity dispersions. Such a calculation is very likely to result in a temporal evolution, which is not an exact match to the evolution of a (dynamic versions of a) cosmological constant. The resulting observables can then be compared with accurate data from supernovae, growth of structures, and the cosmic microwave background, and in this way the models can be compared, tested and potentially observationally rejected.

10 CONCLUSION

Using the framework of STA it is shown that when equipping the dark matter particles with two different charges (attractive and repulsive, respectively) with identical strengths, then the non-cancelled part of the long-range forces is proportional to velocity dispersion squared. This is an example of a force which was recently suggested to be able to accelerate the expansion of the universe without the need for a cosmological constant. The magnitude of the strength is about 30 orders of magnitude smaller than that of electromagnetism, and hence perfectly allowed observationally.

DATA AVAILABILITY

No new data were generated or analysed in support of this research.

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