Submillimetre compactness as a critical dimension to understand the main sequence of star-forming galaxies

Puglisi, Annagrazia; Daddi, Emanuele; Valentino, Francesco; Magdis, Georgios; Liu, Daizhong; Kokorev, Vasily; Circosta, Chiara; Elbaz, David; Bournaud, Frédéric; Gomez-Guijarro, Carlos; Jin, Shuowen; Madden, Suzanne; Sargent, Mark T.; Swinbank, Mark

Published in:
Monthly Notices of the Royal Astronomical Society

DOI:
10.1093/mnras/stab2914

Publication date:
2021

Document version
Publisher's PDF, also known as Version of record

Citation for published version (APA):
Submillimetre compactness as a critical dimension to understand the main sequence of star-forming galaxies

Annagrazia Puglisi,1,1* Emanuele Daddi,2 Francesco Valentino,3,4 Georgios Magdis,3,4,5,6 Daizhong Liu,7 Vasily Kokorev,3,4 Chiara Circosta,8 David Elbaz,2 Frederic Bournaud,2 Carlos Gomez-Guijarro,2 Shuowen Jin,9,10 Suzanne Madden,2 Mark T. Sargent11,12 and Mark Swinbank1

1 Centre for Extragalactic Astronomy, Department of Physics, Durham University, South Road, Durham DH1 3LE, UK
2 CEA, Irfa, Dap, Aim, Université Paris-Saclay, Université de Paris, CNRS, F-91191 Gif-sur-Yvette, France
3 Cosmic Dawn Center (Dawn), Copenhagen, Denmark
4 Niels Bohr Institute, University of Copenhagen, Jagtvej 128, DK-2200 Copenhagen, Denmark
5 DTU-Space, Technical University of Denmark, Elektrovej 327, DK-2800 Kgs. Lyngby, Denmark
6 Institute for Astronomy, Astrophysics, Space Applications and Remote Sensing, National Observatory of Athens, GR-15236 Athens, Greece
7 Max-Planck-Institut für Extraterrestrische Physik (MPE), Giessenbachstr. 1, D-85748 Garching, Germany
8 Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, UK
9 Instituto de Astrofísica de Canarias (IAC), E-38205 La Laguna, Tenerife, Spain
10 Universidad de La Laguna, Dpto. Astrofísica, E-38206 La Laguna, Tenerife, Spain
11 Astronomy Centre, Department of Physics and Astronomy, University of Sussex, Brighton BN1 9QH, UK
12 International Space Science Institute (ISSI), Hallerstrasse 6, CH-3012 Bern, Switzerland

Accepted 2021 October 4. Received 2021 October 4; in original form 2021 March 17

ABSTRACT
We study the interstellar medium (ISM) properties as a function of the molecular gas size for 77 infrared-selected galaxies at $z \sim 1.3$, having stellar masses $10^{9.4} \lesssim M_\star \lesssim 10^{12.0} M_\odot$ and star formation rates $12 \lesssim \text{SFR}_{\text{FIR}} \lesssim 1000 M_\odot \text{yr}^{-1}$. Molecular gas sizes are measured on ALMA images that combine CO(2-1), CO(5-4), and underlying continuum observations, and include CO(4-3), CO(7-6) $+\text{[CI]}(3P_2 - 3P_0)$, [CI]$\times(3P_1 - 3P_0)$ observations for a subset of the sample. The $\gtrsim 46$ per cent of our galaxies have a compact molecular gas reservoir, and lie below the optical discs mass–size relation. Compact galaxies on and above the main sequence have higher CO excitation and star formation efficiency than galaxies with extended molecular gas reservoirs, as traced by CO(5-4)/CO(2-1) and CO(2-1)/L_{\text{IR, SF}} ratios. Average CO $+\text{[CI]}$ spectral line energy distributions indicate higher excitation in compact relative to extended sources. Using CO(2-1) and dust masses as molecular gas mass tracers, and conversion factors tailored to their ISM conditions, we measure lower gas fractions in compact main-sequence galaxies compared to extended sources. We suggest that the submillimetre compactness, defined as the ratio between the molecular gas and the stellar size, is an unavoidable information to be used with the main sequence offset to describe the ISM properties of galaxies, at least above $M_\star \gtrsim 10^{10.6} M_\odot$, where our observations fully probe the main sequence scatter. Our results are consistent with mergers driving the gas in the nuclear regions, enhancing the CO excitation and star formation efficiency. Compact main-sequence galaxies are consistent with being an early post-starburst population following a merger-driven starburst episode, stressing the important role of mergers in the evolution of massive galaxies.

Key words: galaxies: evolution – galaxies: star formation – galaxies: ISM.

1 INTRODUCTION
The majority of star-forming galaxies (SFGs) are observed to follow a correlation in the stellar mass ($M_\star$) versus star formation rate (SFR) plane. In the mainstream scenario, the existence of this so-called Main Sequence (MS; Daddi et al. 2007; Elbaz et al. 2007; Noeske et al. 2007; Wuyts et al. 2011; Kashino et al. 2013; Rodighiero et al. 2014; Sargent et al. 2014; Speagle et al. 2014; Whitaker et al. 2014; Pannella et al. 2015; Renzini & Peng 2015; Schreiber et al. 2015, to mention a few) and its tight scatter (≃0.3 dex, e.g. Rodighiero et al. 2011; Sargent et al. 2012; Schreiber et al. 2015) are interpreted as evidence that star formation in most galaxies is a fairly ordered process. In particular, galaxies on the main sequence appear to be secularly evolving (Daddi et al. 2010a; Genzel et al. 2015) clumpy discs (Förster Schreiber et al. 2009) growing inside out (Nelson et al. 2012, 2016). Galaxies above the main sequence are believed to undergo a starburst mode of star formation associated with stochastic processes like major mergers (Kartaltepe et al. 2012; Hung et al. 2013; Silverman et al. 2015a, 2018a; Cibinel et al. 2019). These so-
called starbursts only represent a small percentage of the star-forming galaxy population and seem to have a minor impact on the cosmic star formation history (Rodighiero et al. 2011; Schreiber et al. 2015).

On the other hand, recent studies at far-infrared (FIR) submillimetre (sub-mm) wavelengths of galaxies at $z > 1$ are revealing a somewhat different picture. In particular, the properties of galaxies at long wavelengths seem to be poorly correlated with their main sequence position, in seeming contrast with the current interpretation of the main sequence. For example, the spatial extent of the molecular gas reservoir does not show correlations with the offset from the main sequence ($\Delta MS = \frac{SFR}{SFR_{MS}}$) or, almost equivalently, with the specific SFR ($sSFR$; Puglisi et al. 2019). The star formation efficiency (SFE) and molecular gas excitation are also somewhat weakly correlated with the main sequence offset (e.g. Tacconi, Genzel & Sternberg 2020, and references therein) and little difference is observed when comparing average CO spectral line energy distributions (SLEDs) of galaxies on and above the main sequence (Valentino et al. 2020). This echoes studies of the infrared spectral energy distribution (SED) showing that there is only a little difference between the far-infrared SED shape of main-sequence and starburst galaxies at $z \sim 2$ (Béthermin et al. 2015, see also Burnham et al. 2021).

In addition to an overall weak correlation between the interstellar medium (ISM) properties and the main sequence offset, it has been recently discovered that a large fraction of massive ($M_*>10^{10.5}$ $M_\odot$) star-forming galaxies within the main sequence have molecular gas reservoirs embedded in a more extended stellar structure (Tadaki et al. 2017a; Elbaz et al. 2018; Jiménez-Andrade et al. 2019, 2021; Puglisi et al. 2019; Franco et al. 2020; Tadaki et al. 2020; Gómez-Guijarro et al. 2021). These ‘submillimetre compact’ main-sequence galaxies have short depletion time-scales, as derived from dust-continuum measurements (Elbaz et al. 2018; Franco et al. 2020), and single-object studies show that these objects have highly excited ISM (Popping et al. 2017). These properties are consistent with the properties expected from merger-driven starbursts (Mihos & Hernquist 1999; Papadopoulos et al. 2012; Hodge et al. 2016). However, these sources display ‘main sequence’ levels of star-forming activity. Furthermore, ALMA observations reveal that some of these compact, star-forming cores are rotating and thus possibly discs (Tadaki et al. 2017a, b; Talia et al. 2018; Kaasinen et al. 2020). Understanding the formation mechanism of such objects is important since compact star-forming galaxies might represent a key phase in the passivation mechanisms of galaxies (Barro et al. 2013, 2014; Elbaz et al. 2018; Gómez-Guijarro et al. 2019; Puglisi et al. 2019).

Are galaxies within the main sequence uniquely associated with a secular mode of star-formation? Do we really observe two star-forming galaxy populations with different properties (extended discs in secular evolution and merger-driven starbursts) or do we rather observe a broad variety of properties in massive star-forming galaxies at $z \geq 1$? What are the mechanisms responsible for the formation of massive galaxies with a compact molecular gas reservoir within the main sequence?

In this paper, we aim to address the above questions by taking advantage of the unique coverage provided by our Atacama Large Millimeter Array (ALMA) survey presented in Valentino et al. (2020). This multicycle campaign has allowed us to obtain several carbon monoxide (CO) and neutral atomic carbon ([CII]) line detections enabling a characterization of the molecular gas excitation conditions in a statistical sample of far-infrared selected star-forming galaxies at $z \sim 1.3$. While CO scaling relations as a function of the main sequence position have been extensively discussed in Valentino et al. (2020), here we investigate the dependence of ISM properties on the spatial extent of the molecular gas reservoir. We also take advantage of the availability of different observables to estimate the gas content of our sample from multiple tracers (namely the CO(2-1) luminosity, $L_{\text{CO(2-1)}}$, and the dust mass, $M_{\text{dust}}$), building on the study of the ISM conditions to physically motivate the choice of the conversion factors required to convert the observables into gas masses.

The paper is organized as follows. In Section 2, we report information on the measurements that are essential for the analysis presented in this paper. These include details on the galaxy size measurements, galaxy integrated properties, and our sources classification criterion. In Section 3, we present the results on the ISM properties of the sample. We discuss these results in Section 4. Finally, we summarize the main findings of this paper in Section 5. Throughout this paper we use a Chabrier (2003) initial mass function and a standard ΛCDM cosmology ($H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_m = 0.3$, $\Omega_{\Lambda} = 0.7$).

2 THE SAMPLE

In this study, we use the molecular gas properties of a statistical sample of 123 galaxies at 1.1 $\leq z \leq 1.7$ in the COSMOS field (Scoville et al. 2007) selected in the far-infrared to have a total infrared luminosity $L_{IR} \gtrsim 10^{12} L_\odot$. These galaxies were observed with ALMA in Band 6 with an average circularized beam of $\sim 0.7$ arcsec to detect the CO(5-4) emission (Program-ID 2015.1.00260.S, PI: E. Daddi). For 75 of the 123 galaxies detected at high significance, we obtained follow-up observations of the CO(2-1) transition in ALMA Band 3 with an average circularized beam of $\sim 1.5$ arcsec (Program-ID 2016.1.00171.S, PI: E. Daddi). We also acquired CO(7-6) + [CII]$(^3P_2 − ^3P_1)$ observations from a follow-up program targeting 15 of the 123 galaxies in the main program (Program-ID 2019.1.01702.S, PI: F. Valentino). Finally, we complemented these observations with available CO(4-3) and [CII]$(^3P_1 − ^3P_0)$ observations from an independent campaign (Program-IDs 2016.1.00140.S and 2018.1.00635.S, PI: F. Valentino) for 15 out of 123 galaxies in the primary sample. For a detailed description of the selection criteria, the full ALMA data set, the data reduction, and measurements of emission line fluxes and flux upper limits we refer the reader to Valentino et al. (2020). Here we report details of the ALMA size measurements that are used in this paper (Section 2.1). We also provide details of the measurements of stellar masses, near-infrared sizes, star formation rates, dust masses, and the intensity of the radiation field for these galaxies (Section 2.2). Finally, we present the source classification criterion adopted in our analysis, which is based on the size of the molecular gas component relative to the position of the source with respect to the optical mass–size relation (Section 2.3).

2.1 ALMA size measurements

In this work, we measure sizes from ALMA images following the methodology detailed in Puglisi et al. (2019), Valentino et al. (2020), and references therein. As already discussed in these papers, the source sizes are measured by combining all the available ALMA observations in the $uv$ plane, allowing for an arbitrary re-normalization of the signal for all tracers. These include the CO(2-1), CO(4-3), CO(5-4), CO(7-6), [CI], and underlying continua. This is to maximize the accuracy of the size measurements and the sample statistics. To determine the best-fitting size and its $1\sigma$ uncertainty we compare the $uv$ distance versus amplitude distribution to circular Gaussian models. Our ALMA size measurement method is exemplified in fig. 1 of Puglisi et al. (2019). The best-fitting size of each galaxy
$R_{\text{eff}}$ is defined as half of the full-width half maximum (FWHM) of the best-fitting circular Gaussian model ($R_{\text{eff}} = \text{FWHM}/2$). This corresponds to the half-light radius for a 2D Gaussian profile. We quantify the probability $P_{\text{unres}}$ of each galaxy to be unresolved by comparing the best-fitting circular Gaussian $\chi^2$ to the $\chi^2$ for a point source. We consider a galaxy to be resolved when $P_{\text{unres}} \leq 0.1$ per cent with this threshold corresponding to 0.5 galaxies expected to be spuriously resolved in our total sample of 123 sources. Recent results have suggested that high-redshift galaxies have far-infrared profiles consistent with exponential discs (Hodge & da Cunha 2020, and references therein). However, most of these studies rely on higher resolution ALMA observations ($\lesssim 0.25$ arcsec, e.g. Hodge et al. 2016). A circular Gaussian profile provides a good fit to galaxies in our sample given the signal-to-noise ratio (SNR) and beam of our observations. In addition, fitting an exponential profile to our data gives consistent size measurements within the errors.

We measure sizes or size upper limits for 89 out of the 123 sources observed in our ALMA program. Of these, 39 galaxies are resolved with an average size/error ratio of $\sim 10$. We measure 1σ size upper limits for 50 galaxies in the sample. Finally, we cannot measure sizes for 34 sources in the sample. Of these, 18 objects lack a line detection and have uncertain emission line upper limits as a result of a poor optical redshift. In addition, 15 sources have uncertain upper limits on the CO(5-4) transition based on high-quality optical redshifts only and no continuum detection. Finally, we discard one object because of an unreliable size measurement ($P_{\text{unres}} = 29$ per cent) due to low SNR detections in CO and continuum. We include in Table 1 a summary of the ALMA size measurements statistics. We note that size measurements are updated with respect to those presented in Puglisi et al. (2019). This is because in this work we have included additional CO(7-6) + [CI]($^3P_1 - ^3P_0$) and CO(4-3) and [CI]($^3P_1 - ^3P_0$) observations in our average size estimate. This results in six additional sources with a measured size. Furthermore, this results in an average size/error that is $\sim 1.9$ times higher than in our previous analysis (average size/error of $\sim 5.3$ in Puglisi et al. 2019).

We extensively tested the reliability of our procedure to measure sizes using Monte Carlo simulations, as discussed in Puglisi et al. (2019) and Coogan et al. (2018). In brief, we created 1000 mock realizations of our data sets by perturbing the best-fitting models within the measured uncertainties in the $uv$-amplitudes plane, assuming a Gaussian distribution for the noise. We applied our procedure to measure sizes for each of these 1000 synthetic realizations. We performed this test on galaxies covering a broad range in sizes and flux SNR corresponding to that covered by our measurements ($R_{\text{eff}} = 0.1-1$ arcsec, flux SNR = 5-10). We find no residual systematic uncertainties in the recovered sizes within 5 per cent. For galaxies that are significantly larger than our average beam (i.e. above $\sim 1.2$ arcsec), we find a small systematic uncertainties with sizes from simulations that are $\sim 10$ per cent smaller than input sizes. This systems does not depend on the SNR of the source but it seems to be rather related to the intrinsic size of the object. However, we find that the largest discrepancy is $\sim 10$ per cent for the most extended sources, thus not affecting our results. We also find that the average size errorbars are consistent with the 1σ dispersion of simulated size measurement, validating the robustness of our 1σ size error bars. The robustness of the 1σ size errorbars is also indicated by the $\chi^2$ distribution and the corresponding probability of the individual fits in the $uv$ plane, which is consistent with pure noise.

### 2.1.1 Potential biases in our ALMA size estimates

Our method that combines several molecular gas tracers to measure sizes might be sensitive to different components of the ISM depending on the excitation properties of each individual galaxy. However, we verified that in all cases (but for three single notable exceptions) where multiple tracers were combined to a single size measurement, the tracers’ signal versus amplitude trends agree among themselves. This suggests that for our sample there are no significant size variations between low-to-high J CO tracers and underlying dust continuum within the beam of our observations ($\sim 0.7-1.5$ arcsec, see fig. 1 in Puglisi et al. 2019). Furthermore, we find a 1:1 correlation between the ALMA sizes and the sizes of CO(5-4) or CO(4-3) and/or the underlying dust continuum ($\lambda_{\text{rest}} \sim 520-650$ μm), suggesting that ALMA sizes are mostly driven by the high-J CO transitions and/or the underlying dust continuum. When comparing CO(5-4) or CO(4-3) and 1 mm dust continuum sizes, we find that the scatter among the two independent size measurements is consistent with pure noise, leaving no space for additional systematics. Finally, for both extended and compact galaxies with robust CO(5-4) sizes (i.e. sizes/error $\geq 3$), we have verified that the CO(5-4) size is consistent within the errors with the average size obtained as the weighted average of the individual size tracers and excluding the CO(5-4). These tests justify our method to combine different molecular gas size tracers and suggests that our ALMA sizes are representative of the typical extension of the molecular gas and cold dust in our galaxies. Future ALMA observations with higher spatial resolution in multiple tracers will help us to investigate the presence of size variations as a function of the molecular gas tracer for this sample.

Another potential source of bias in our size measurements might be associated to the presence of an active galactic nucleus (AGN), since we detect AGN features in the $\sim 40$ per cent of our sample (Valentino et al. 2020, see also Section 2.2). However, our ALMA sizes are mostly derived from the CO(5-4) emission and we do not expect a significant contribution of X-ray dominated regions (XDR) at these intermediate CO transitions (see also discussion in Section 3.1). Even when size measurements are driven by the continuum underlying the CO(5-4) emission, we expect the AGN to provide a marginal contribution to the size. In fact, our decomposition of the far-infrared SED (see below) shows a marginal AGN contribution at $\lambda_{\text{rest}} \sim 500$ μm. This is in line with other studies showing that there is currently

### Table 1. Summary of ALMA size measurements and CO emission lines statistics for our sample.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>$N_{\text{obj}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALMA size</td>
<td>39/123</td>
</tr>
<tr>
<td>ALMA size upper limit</td>
<td>50/123</td>
</tr>
<tr>
<td>ALMA size or size upper limit &amp; M$\star$ &amp; SFR$\text{^{d}}_{\text{phys}}$</td>
<td>82/123</td>
</tr>
<tr>
<td>ALMA size or size upper limit &amp; M$\star$ &amp; SFR$\text{^{d}}<em>{\text{phys}}$ &amp; $f</em>{\text{AGN}} &lt; 0.8$</td>
<td>77/123</td>
</tr>
<tr>
<td>This work with CO(5-4)$^b$</td>
<td>46/77</td>
</tr>
<tr>
<td>This work with CO(2-1)$^b$</td>
<td>33/77</td>
</tr>
<tr>
<td>This work with CO(5-4) and CO(2-1)$^d$</td>
<td>31/77</td>
</tr>
</tbody>
</table>

*Notes.* $a$We exclude 7 out of the 89 galaxies with a ALMA size or size upper limit from our analysis. Four of these galaxies have no far-infrared counterpart in the Jin et al. (2018) catalogue. Three objects are AGN and have no measurements of the stellar mass. $b$Galaxies with CO(5-4) detections or robust upper limits (corresponding to $\text{Flag}_{\text{CO(5-4)}} \geq 0.5$ in the Valentino et al. 2020 catalogue). $c$Galaxies with CO(2-1) detection or robust upper limits ($\text{Flag}_{\text{CO(2-1)}} \geq 0.5$). $d$Galaxies with CO(5-4) and CO(2-1) detection or robust upper limits.
The other physical properties that are relevant for this work are the total IR luminosity \( L_{\text{IR}} \) in the rest-frame wavelength range \( \lambda = 8-1000 \, \mu\text{m} \), the dust mass and the intensity of the radiation field (\( U \)). These quantities are obtained by modelling the IR photometry from the Jin et al. (2018) ‘super-deblended’ catalogue, and by adding the dust continuum emission observed with ALMA in the \( \lambda_{\text{obs}} \in [0.8 - 3.2] \, \text{mm} \) wavelength range. The FIR modelling is performed by using the customized \( \chi^2 \) minimization tool STARDUST (Kokorev, Magdis & Davidzon 2021) and it is based on the Magdis et al. (2012) approach which uses Draine & Li (2007) dust models and a mid-infrared AGN torus component from Mullaney et al. (2011). The total infrared luminosity obtained from the fitting thus consists of a component associated with star formation (\( L_{\text{IR, SF}} \)), and one component arising from the dusty torus (\( L_{\text{IR, AGN}} \)). This allows us to estimate the fraction of the total IR luminosity associated with the dusty torus as \( f_{\text{AGN}} = L_{\text{IR, AGN}}/L_{\text{IR}} \). Following Valentino et al. (2020), we consider that an AGN component is reliably detected when \( f_{\text{AGN}} + 1\sigma_{\text{f,AGN}} \geq 20 \) per cent, and we classify galaxies as AGN-dominated when \( f_{\text{AGN}} \geq 80 \) per cent. We finally measure star formation rates \( SFR_{\text{IR}} \) from \( L_{\text{IR, SF}} \) by using the Kennicutt (1998) calibration rescaled to a Chabrier (2003) IMF by a factor 1.7. For more details about the modelling procedure we refer the reader to Section 3.3 of Valentino et al. (2020) and references therein.

### 2.3 Submillimetre compactness

In Fig. 1, we show an updated version of the \( M_{\ast}-\text{Size} \) plane presented in Puglisi et al. (2019). In the left-hand panel we show the \( K_s \)-band sizes, roughly tracing the size of the stellar mass distribution at \( z \sim 1.25 \), corresponding to the average redshift of our sample. This plot shows that nearly all the galaxies in our sample are located within the scatter of the \( M_{\ast}-\text{Size} \) relation for \( z \sim 1.25 \) star-forming discs (or late type galaxies, LTG, van der Wel et al. 2014, blue line in Fig. 1). In the right-hand panel of Fig. 1 we show the ALMA sizes of our sources. From this plot we see that the ALMA measurements are skewed towards small size values.

We define the submillimetre compactness (or simply compactness) as:

\[
C_{\text{ALMA}} = \frac{R_{\text{eff, ALMA}}}{R_{\text{eff, LTG}}}.
\]

where \( R_{\text{eff, LTG}} \) is the size measured from the van der Wel et al. (2014) LTG relation at the stellar mass of the galaxy, and \( R_{\text{eff, ALMA}} \) is the ALMA size of the source. We then compute the \( \chi^2 \) of the compactness distribution for the 82 galaxies with an ALMA size (or size upper limit), a measurement of the stellar mass and of the star formation rate (see equation 1 in Puglisi et al. 2019). We find that 34 per cent of the source has an ALMA size\( \sim \)size upper limit consistent with the van der Wel et al. (2014) mass\( \sim \)size relation for star-forming discs. These galaxies have similar \( K_s \)-band and ALMA sizes. We identify these galaxies as ‘extended’. Instead, 46 per cent of the galaxies have molecular gas sizes that are \( \geq 1\sigma \) more compact relative to the stars as expected from the discs mass\( \sim \)size relation. These galaxies are highlighted in red in both panels of Fig. 1. Through the paper we will refer to these galaxies as ‘compacts’. These compact sources are on average 3.3 \times smaller in ALMA than in the \( K_s \)-band, on average, consistent with the measurements from our previous study (Puglisi et al. 2019). Finally, we find that 20 per cent of the sources have ALMA size upper limits within the LTG relation (black upper limits in the right-hand panel of Fig. 1). We dub those objects

---

1https://github.com/VasilyKokorev/stardust
as ‘ambiguous’ since we cannot place robust constraints on their compactness. We will consider these objects as a separate category through the paper.

While the stellar sizes of our sample are by and large consistent with those of typical $z \sim 1.25$ star-forming galaxies, the left-hand panel of Fig. 1 suggests that compact galaxies have slightly smaller $K_s$-band sizes than extended sources. To quantify this effect, we perform a two-sided Kolmogorov–Smirnov (KS) test after dividing the size inferred from the van der Wel et al. (2014) LTG relation at the stellar mass of the source by the $K_s$-band size. We consider only extended and compact galaxies for this test. We find median size ratios of $R_{\text{eff,LTG,Extended}} / R_{\text{eff,compact}} = 1.03 \pm 0.38$ and $R_{\text{eff,LTG,Compsacts}} / R_{\text{eff,compact}} = 1.23 \pm 0.43$ for extended and compact galaxies, respectively. That is, compact galaxies have slightly smaller $K_s$-band sizes than extended sources. However, we cannot reject the null hypothesis at the $\sim 10$ per cent level ($p$-value = 0.13). This suggests that the $K_s$-band size distribution of extended and compact sources in our sample is slightly but not substantially different, as expected if compact galaxies are caught in the process of building up a dense stellar core (see also discussion in Section 4.2). Recently, Popping et al. (2021) has challenged the idea that the dust-continuum size is more compact than the stellar half-mass radius in $z \geq 1$ galaxies, due to strong dust attenuation gradients affecting $H$-band observations. This however corresponds to an observed-frame wavelength of $\sim 0.5 \mu m$ at $z \sim 2$. Here we use $K_s$-band observations (corresponding to $\sim 1 \mu m$ at $z \sim 1.25$) to sample the stellar half-mass radius and we expect negligible dust attenuation effects in this case. In fact, the emission at these wavelengths has been observed to align with the CO and radio emission even for the most obscured starbursts (e.g. Tan et al. 2014).

We highlight with crosses in Fig. 1 the galaxies with a reliably detected AGN contribution to the far-infrared SED. Five of the sources with an ALMA size/size upper limit have $f_{\text{AGN}} \geq 80$ per cent (red crosses in Fig. 1). We exclude these galaxies from the analysis presented in Section 3 since we cannot derive robust SFR constraints and XDRs might contribute significantly to the CO line ratios. We find AGN signatures in $25 \pm 11$ per cent of the extended galaxies. Instead, we find AGN signatures in $45 \pm 13$ per cent of the compact sources. This suggests that compact galaxies have an enhanced AGN fraction. To test whether AGN are associated to more compact galaxies, we apply a log-rank test to the compactness distribution of galaxies with and without an AGN contribution to the far-infrared SED, accounting for the presence of compactness lower limits. We find a 94 per cent probability that the two distributions are different ($p$-value = 0.06). We note however that the AGN host population contains a larger fraction of compactness lower limits (62 per cent versus 47 per cent in galaxies without an AGN) and such lower limits are mostly associated with large compactness values. This implies that the AGN hosts’ compactness distribution is not well constrained at high compactness values and the log-rank test results might be unreliable. On the other hand, this may suggest that AGN are associated with more compact galaxies for which we have stringent size upper limits. We thus conclude that our observations provide marginal evidence that AGN are more likely associated to galaxies with a compact molecular gas reservoir, similarly to that suggested by previous analyses (Elbaz et al. 2018, Puglisi et al. 2019, Lamerti et al. 2021, see also Barro et al. 2014 for optically compact star-forming galaxies). This might indicate that the mechanisms responsible for fuelling the molecular gas to the nuclear regions is also efficient in feeding the AGN.

In Fig. 2, we report the star formation rate as a function of the stellar mass for our sample. We normalize the star formation rate of each galaxy to the SFR of the main sequence at the average redshift of the sample, to account for the redshift evolution of the main sequence normalization (Sargent et al. 2012). Similar to our previous analysis, we identify as ‘main sequence galaxies’ sources with $\Delta M < 3.5$, whereas we classify as ‘off main sequence’ or starbursts galaxies with $\Delta M > 3.5$. The colour code in Fig. 2 provides information on the sources’ compactness. In particular, blue circles represent galaxies with an ALMA size consistent with the LTG relation, red filled circles indicate compact sources and grey filled circles highlight ambiguous galaxies. Finally, small black dots display galaxies for which we cannot measure a size from our ALMA observations (see Table 1 and Section 2.1). We do not find a clear correlation between the compactness and the main sequence position, similarly to what reported in Puglisi et al. (2019). Galaxies with a compact molecular gas reservoir make up a significant fraction of the main sequence population above $\sim 5 \times 10^{10} M_\odot$, in line with previous studies (e.g. Tadaki et al. 2017b, 2020; Elbaz et al. 2018; Puglisi et al. 2019; Franco et al. 2020). Fig. 2 also shows that, while spanning a broad range of stellar masses and star formation rate in the main sequence plane, our observations consist of ‘upper main sequence’ galaxies, that is, strongly star-forming galaxies above the main sequence and galaxies that probe the high specific SFR stripe of the main sequence scatter. This is a result of the far-infrared selection, which corresponds to a horizontal cut in the main sequence plane (Rodighiero et al. 2014).

3 RESULTS

In the following, we study the molecular gas properties of our sample as a function of the compactness and the main sequence offset. For consistency with our previous studies, we quantify the main sequence offset using the main sequence parametrization from Sargent et al. (2014). We also checked that using the widely adopted
parametrisation from Schreiber et al. (2015), which accounts for the bending of the main sequence at high stellar masses, does not affect the main sequence position of our sample. For this analysis, unless stated otherwise, we consider only galaxies with reliable CO fluxes or robust CO flux upper limits. The latter corresponds to reliable upper limits on the line flux, given the presence of alternative sub-mm measurements from our public catalogue (Valentino et al. 2020). In Fig. 3, we show the average SLEDs for extended, ambiguous, and compact galaxies in our sample. Here we include as a reference the distribution of individual measurements in each subsample.

3.1 Gas excitation

3.1.1 The \(L'_{\text{CO}(5-4)}/L'_{\text{CO}(2-1)}\) ratio

In Fig. 3, we show the \(R_{52} = L'_{\text{CO}(5-4)}/L'_{\text{CO}(2-1)}\) ratio as a function of AMS (left-hand panel) and \(C_{\text{gas}}\) (right-hand panel). The \(R_{52}\) ratio is a proxy for the CO excitation and this plot allows us to infer the excitation properties of each galaxy as a function of their structural and star formation rate properties.

The left-hand panel of Fig. 3 shows that there is a substantial number of compact galaxies within a factor of \(\pm3.5\) around the main sequence. These compact galaxies within the main sequence have an enhanced \(R_{52}\) ratio with respect to their extended counterparts. The \(R_{52}\) ratio of compact main-sequence galaxies is instead similar to that of galaxies above the main sequence. This plot suggests that compact and ambiguous galaxies within the main sequence contribute to the scatter observed in the \(R_{52}-\text{AMS}\) relation reported in Valentino et al. (2020) and obtained using the same sample considered in this work (green dotted line in the left-hand panel of Fig. 3). To quantify the contribution of these galaxies to the scatter observed in the Valentino et al. (2020) relation, we fit \(R_{52}\) as a function of \(\Delta\text{AMS}\) excluding compact and ambiguous galaxies with \(\Delta\text{AMS} \leq \pm3.5\). We also exclude the strongest outlier above the main sequence, for consistency with our previous analysis. We apply a Bayesian regression analysis in the log–log space using the Python version of the LINMIX_ERR.FOR package (Kelly 2007). The results of this fit are listed in Table 2. The best-fitting line is shown as a dark red solid line in the left-hand panel of Fig. 3. The slope is \(\beta = 0.45 \pm 0.14\) and this is \(\sim 2\times\) steeper than the trend reported in Valentino et al. (2020) when considering all galaxies regardless of their compactness. The intrinsic scatter of the best-fitting relation (\(\sigma_{\text{int}} = 0.11 \pm 0.03\)) is slightly reduced. Finally, the correlation between \(R_{52}\) and \(\Delta\text{AMS}\) strengthens when excluding compact and ambiguous main sequence galaxies from the fit. This suggests that the presence of compact/ambiguous galaxies (i.e. non-typical discs) within the main sequence blurs the correlation between \(R_{52}\) and \(\Delta\text{AMS}\). We note however that the compactness classification criterion applied in this paper reduces the source statistics with respect to the analysis presented in Valentino et al. (2020).

The right-hand panel of Fig. 3 shows a trend of increasing \(R_{52}\) as a function of the compactness. To quantify the correlation between \(R_{52}\) and \(C_{\text{gas}}\) we apply a Bayesian regression analysis in the log–log space similarly to that described above. The results of this fit are listed in Table 2. We show the best-fitting line as a red line in the right-hand panel of Fig. 3. The slope of the \(C_{\text{gas}}-R_{52}\) trend is \(\beta = 0.40 \pm 0.30\) and this is similar to the slope of the \(\Delta\text{AMS}-R_{52}\) best-fitting relation discussed above. However, the slope of the \(C_{\text{gas}}-R_{52}\) relation is poorly constrained. The \(C_{\text{gas}}-R_{52}\) correlation index is smaller than the \(R_{52}-\Delta\text{AMS}\) correlation index obtained above. We note however that our data only allow us to probe a limited compactness range due to an average \(\sim 1\) arcsec beam. Observations with a smaller beam would be required to probe a wider range of sizes and explore in more details the relation between \(R_{52}\) and the compactness.

3.1.2 Average CO spectral line energy distributions as a function of the compactness

In the previous section, we used the \(R_{52}\) ratio to study the CO excitation as a function of the main sequence offset and the compactness in individual galaxies in our sample. However, CO(4-3) and CO(7-6) observations available for a subset of our sources (see Tables 3 and 4) allow us to construct average CO spectral line energy distributions (SLEDs) for the three classes of sources identified in Section 2.3. We thus construct average CO SLEDs for the galaxies that have CO(2-1) and CO(5-4) emission line detections/upper limits, and a measurement of the compactness. We split those sources into different subsamples according to their compactness and main sequence position and we compute average \(L'_{\text{CO}}\) luminosities using a survival analysis technique to account for the presence of upper limits (Kaplan & Meier 1958). We report in Tables 3 and 4 the average CO luminosities of each subsample along with the detection statistics for each transition. We then convert the average \(L'_{\text{CO}}\) luminosities into CO fluxes at \(z \sim 1.25\), corresponding to the average redshift of the sample. Errors on the average \(L'_{\text{CO}}\) luminosities and average \(I_{\text{CO}}\) fluxes quoted throughout this section correspond to the interquartile range from the distribution of individual measurements in each subsample.

In Fig. 4, we show the average SLEDs for extended, ambiguous, and compact galaxies in our sample. Here we include as a reference
the average CO SLEDs from Valentino et al. (2020), which uses the same sample considered in this work but classifies galaxies according to their main sequence offset. A comparison with other galaxy types and QSO SLEDs for this sample has already been performed in Valentino et al. (2020) and we refer the reader to this paper for more details in this regard (see in particular their Section 4.4). Here we find that the CO SLED of extended galaxies closely resembles that of main sequence galaxies at $z \sim 1.25$. The CO excitation ladder of compact galaxies is consistent with that of the strongest main-sequence outliers, defined as galaxies with $\Delta MS \geq 7$. Likewise, ambiguous galaxies have a CO SLED similar to that of strong starbursts, possibly suggesting that these sources are more compact than indicated by their loose ALMA size upper limits. We

Table 2. Scaling relations between CO properties, main sequence offset, and compactness.

<table>
<thead>
<tr>
<th>Relation</th>
<th>Slope $\beta$</th>
<th>Intercept $\alpha$</th>
<th>Intrinsic scatter $\sigma_{\text{Int}}$</th>
<th>Correlation $\rho$</th>
<th>$N_{\text{det}}, N_{\text{up}}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta MS, R_{\text{IS}}^2$</td>
<td>0.45 ± 0.14</td>
<td>−0.83 ± 0.09</td>
<td>0.11 ± 0.03</td>
<td>0.78</td>
<td>19.4</td>
</tr>
<tr>
<td>$\Delta MS, L_{\text{CO}(2-1)}/L_{\text{IR-SF}}^*$</td>
<td>−0.42 ± 0.14</td>
<td>−1.73 ± 0.09</td>
<td>0.22 ± 0.04</td>
<td>−0.58</td>
<td>21.4</td>
</tr>
</tbody>
</table>

Notes. In this table we quote the mean of the best-fitting parameters from LINMIX and corresponding 1σ error bars.

†Excluding compacts and ambiguous within the main sequence.

Table 3. Average CO and [CII] luminosities and detection statistics for galaxies classified as a function of their compactness. The $L_i$ luminosities are expressed in $10^{10}$ K km s$^{-1}$ pc$^2$. The average $I$ fluxes in Fig. 4 are expressed in Jy km s$^{-1}$ and are computed from $L_i$ luminosities by adopting $z = 1.25$.

Table 4. As in Table 3 but for extended and compact galaxies on and above the main sequence.

| Transition | Within LTG, on MS ($\Delta MS \leq 3.5$) 
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\text{det}}, N_{\text{up}}$</td>
<td>Mean</td>
</tr>
<tr>
<td>$L_{\text{CO}(2-1)}$</td>
<td>10.2</td>
</tr>
<tr>
<td>$L_{\text{CO}(4-3)}$</td>
<td>4.0</td>
</tr>
<tr>
<td>$L_{\text{CO}(5-4)}$</td>
<td>11.1</td>
</tr>
<tr>
<td>$L_{\text{CO}(7-6)}$</td>
<td>3.0</td>
</tr>
<tr>
<td>$L_{\text{CII}(1-0)}$</td>
<td>6.0</td>
</tr>
<tr>
<td>$L_{\text{CII}(2-1)}$</td>
<td>3.0</td>
</tr>
</tbody>
</table>

| Transition | Compacts, on MS ($\Delta MS \geq 3.5$) 
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\text{det}}, N_{\text{up}}$</td>
<td>Mean</td>
</tr>
<tr>
<td>$L_{\text{CO}(2-1)}$</td>
<td>6.1</td>
</tr>
<tr>
<td>$L_{\text{CO}(4-3)}$</td>
<td>7.0</td>
</tr>
<tr>
<td>$L_{\text{CO}(5-4)}$</td>
<td>1.0</td>
</tr>
<tr>
<td>$L_{\text{CII}(1-0)}$</td>
<td>2.0</td>
</tr>
<tr>
<td>$L_{\text{CII}(2-1)}$</td>
<td>2.0</td>
</tr>
</tbody>
</table>

| Transition | Compacts, off MS $\Delta MS > 3.5$ 
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\text{det}}, N_{\text{up}}$</td>
<td>Mean</td>
</tr>
<tr>
<td>$L_{\text{CO}(2-1)}$</td>
<td>5.0</td>
</tr>
<tr>
<td>$L_{\text{CO}(4-3)}$</td>
<td>5.0</td>
</tr>
<tr>
<td>$L_{\text{CO}(5-4)}$</td>
<td>2.0</td>
</tr>
<tr>
<td>$L_{\text{CII}(1-0)}$</td>
<td>1.0</td>
</tr>
<tr>
<td>$L_{\text{CII}(2-1)}$</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Figure 4. Average CO SLED of extended, ambiguous, and compact galaxies (blue, grey, and red circles, respectively). As a reference we show the average SLEDs of main sequence ($\Delta MS \leq 3.5$), off main sequence ($\Delta MS > 3.5$) and extreme off main sequence ($\Delta MS > 7$) galaxies from Valentino et al. (2020) with light blue, blue, and dark blue shaded squares, respectively. We normalize the data points to the mean CO(2-1) flux + 1$\sigma$ flux error of $z \sim 1.25$ main sequence galaxies.

Figure 5. Average CO SLED of extended (left-hand panel) and compact (right-hand panel) galaxies in our sample, split according to their main sequence position. In both panels, circles indicate the average SLED of galaxies with $\Delta MS < 3.5$ and stars show the SLED for galaxies with $\Delta MS \geq 3.5$.

in the Valentino et al. (2020) analysis (light blue squares in Fig. 5) is intermediate between that of extended galaxies on and above the main sequence presented here. This suggests that main sequence galaxies may not represent a homogeneous population, similarly to that reported in Section 3.1.1. On the other hand, compact galaxies on and above the main sequence have remarkably similar CO SLEDs and these are consistent with that of the most extreme main-sequence outliers (dark blue squares in Fig. 5).

Figs 4 and 5 suggest that the CO excitation increases as a function of the source compactness. This dependence seems more significant than the variations of the CO excitation with the main sequence offset. To quantify this effect, we compute line ratios as a function of the compactness and the main sequence position (see Table 5). The average $R_{52}$ and $R_{72}$ ratios of compact galaxies are $1.6 \times$ and $1.8 \times$ higher than that of extended galaxies. Instead, compact galaxies have $R_{52}$ and $R_{72}$ ratios that are $1.8 \times$ higher than in extended galaxies within the main sequence. Focusing on the off-main-sequence population, compact galaxies are $1.3 \times$ and $1.4 \times$ more excited in CO(5-4) and CO(7-6) than their few extended counterparts. The $R_{52}$ and $R_{72}$ excitation ratios of compact galaxies on and above the main sequence are consistent with each other within the error bars, suggesting homogeneous CO excitation properties. On the other hand, we find marginal evidence for variations in the excitation ratios of extended sources as a function of the MS position. In particular, extended galaxies above the main sequence have $R_{52}$ and $R_{72}$ ratios that are $1.3 \times$ and $1.4 \times$ higher than in extended main sequence galaxies. This might indicate an evolutionary trend with extended galaxies above the main sequence being merging pairs in an early stage of the interaction. Alternatively, this might suggest that extended off-main-sequence galaxies are gas-rich starbursting discs with enhanced star formation rates after anomalous gas accretion episodes (e.g. Scoville et al. 2016). Observations with increased spatial resolution will allow us to identify any unresolved merger pairs among extended galaxies above the main sequence.

These results suggest that main sequence sources do not represent a homogeneous population in terms of CO excitation properties, as also anticipated in Valentino et al. (2020). Thus, adopting a CO excitation correction that is based on the main sequence position of a galaxy would introduce significant uncertainties in, e.g. the derived CO fluxes due to the presence of a variety of CO SLEDs within the main sequence. This is indicated by the measured $R_{52}$ ratios within the main sequence ranging from a minimum value of...
Table 5. Average line luminosities ratios for galaxies at $z \sim 1.25$. The ratios and their 1σ uncertainties are computed analytically based on the mean $L$ luminosities in Tables 3 and 4.

<table>
<thead>
<tr>
<th>Transition</th>
<th>Within LTG</th>
<th>Within LTG, on MS</th>
<th>Within LTG, above MS</th>
<th>Compacts</th>
<th>Compacts, on MS</th>
<th>Compacts, above MS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{32}$</td>
<td>0.31 ± 0.06</td>
<td>0.30 ± 0.03</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$R_{52}$</td>
<td>0.25 ± 0.05</td>
<td>0.24 ± 0.06</td>
<td>0.31 ± 0.02</td>
<td>0.40 ± 0.08</td>
<td>0.43 ± 0.10</td>
<td>0.39 ± 0.09</td>
</tr>
<tr>
<td>$R_{72}$</td>
<td>0.09 ± 0.02</td>
<td>0.08 ± 0.03</td>
<td>0.11 ± 0.01</td>
<td>0.16 ± 0.04</td>
<td>0.14 ± 0.02</td>
<td>0.15 ± 0.04</td>
</tr>
</tbody>
</table>

$R_{32\text{, MS, min}} = 0.13 \pm 0.03$ to a maximum value of $R_{32\text{, MS, max}} \gtrsim 0.55$. The submillimetre compactness classification, on the other hand, allows us to select galaxies with more homogeneous CO excitation properties, in agreement with the results from individual CO line ratios discussed in the previous section. Given the relatively small number of sources, we caution the reader that the observed variations should be taken as indicative. Future studies with larger statistics will allow us to better investigate variations of the CO SLED with the main sequence offset and the galaxy compactness.

3.1.3 Large velocity gradient modelling

To better understand the CO excitation properties of our galaxies, we apply a Large Velocity Gradient (LVG) modelling to the observed average SLEDs of our sample. We report few details about the modelling below and we refer the reader to Daddi et al. (2015) and Liu et al. (2015b) for a detailed description of the approach. We create a grid of LVG models using the RADEX tool (van der Tak et al. 2007). As RADEX requires the input of the [CO/H$_2$] abundance ratio, turbulence Doppler line width ($\delta V$) and a H$_2$ column density ($N_{\text{H}_2}$) separately, we set them to [CO/H$_2$] = 5 x 10$^{-5}$, $\delta V$ = 50 km s$^{-1}$ and $N_{\text{H}_2}/n_{\text{H}_2}$ = 10 pc. This implies a velocity gradient of 5 km s$^{-1}$ pc$^{-1}$, consistent with or in the range of the findings for Galactic centre or relatively dense and warm clouds (Goldreich & Kwan 1974; Dahmen et al. 1998; Ao et al. 2013) and extragalactic molecular gas in local actively star-forming galaxies (Curran et al. 2001; Weiß et al. 2001; Weiß, Walter & Scoville 2005; Zhang et al. 2014) and high-redshift galaxies (Weiß et al. 2007; Dannerbauer et al. 2009).

We note that these quantities are degenerate. Given that the SLEDs of our galaxies are not fully sampled, leaving the velocity gradient or the abundance free to vary would result in much larger uncertainties. Due to the degeneracy of the models, a different set of assumed values of velocity gradients or abundances will lead to no difference in determining the $n_{\text{H}_2}$ and $T_{\text{kin}}$, but could systematically bias the optical depth, the filling factor and hence the total mass from LVG.

Thus in this work we only use the LVG fitting to infer $n_{\text{H}_2}$ and $T_{\text{kin}}$. To better constrain the fit, we include the [CI] transitions available for a subset of the sample (see Tables 3 and 4). This allows us to provide additional information to the total gas content, since this transition correlates with the total infrared luminosity similar to low-J CO transitions (Valentino et al. 2018). Here we assume that the neutral atomic carbon is co-spatial with CO and a fixed [C$^0$/H$_2$] = 3 x 10$^{-5}$ abundance (Weiß et al. 2003; Papadopoulos, Thi & Viti 2004).

We compute the grid at the median redshift of the sample spanning a density and temperature range of $n$(H$_2$) = 10$^2$ - 10$^6$ cm$^{-3}$ and $T_{\text{kin}}$ = 5 - 300 K, including the appropriate value of the temperature of the cosmic microwave background. We derive the best-fitting model, the best-fitting parameters and their 1σ uncertainties using a customized $\chi^2$ minimization algorithm, optimized for the exploration of highly multidimensional spaces (MICHI2; Liu et al. 2021).
from the two components modelling in Table 6. We find that all galaxies are characterized by low- and high-density gas components, with compacts having marginally higher gas densities than extended sources \( (n_{\text{H}_2,\text{low}} \sim 10^2 - 10^4 \text{ cm}^{-3} \) and \( n_{\text{H}_2,\text{high}} \sim 10^4 - 10^6 \) for the compacts, while \( n_{\text{H}_2,\text{low}} \sim 10^2 - 10^4 \text{ cm}^{-3} \) and \( n_{\text{H}_2,\text{high}} \sim 10^3 - 10^4 \) cm\(^{-3} \) for the extended sample, see also Table 6). We also find that \( \sim 40 \) per cent of the total molecular gas mass of extended galaxies is in the dense component. The best-fitting high-density component for the compacts is rising at high \( J \) (see top panel in Fig. 6 and right-hand panels in Fig. 7). While we caution that we have no constraints on the CO SLEDs of our sources beyond \( J = 7 \) and the LVG best-fitting models are highly extrapolated for higher-J CO observations, this might suggest that the average CO SLED of this population is dominated by an excited and dense gas component, similarly to what is observed in the starburst-dominated local ULIRG Arp220 (Rangwala et al. 2011) and high-redshift SMGs (Yang et al. 2017; Cañameras et al. 2018; Birkin et al. 2021). On the other hand, it is unlikely that the CO SLED of our sources up to \( J = 7 \) is dominated by the AGN component. In fact, even in local QSOs such as Mrk 231, the CO excitation up to \( J = 8 \) can be explained by heating from star formation (van der Werf et al. 2010). We also note that our best-fitting LVG models seem to suggest a somewhat different behaviour for compact galaxies on and above the main sequence (see top and bottom right-hand panels in Fig. 7). However, this is likely due to the lack of constraints on the [C\( \text{I} \)] transitions for the off main-sequence compact population, since the shape of the observed CO SLEDs is nearly identical for the two classes of galaxies up to \( J = 7 \) (see right-hand panel in Fig. 5). The best-fitting parameters derived from the LVG analysis of the two subsamples are consistent within the uncertainties (see Table 6).

### 3.2 Star formation efficiency and depletion time

In the left-hand panel of Fig. 8 we show the \( L'_{\text{CO}(2-1)} \) luminosity as a function of \( L_{\text{IR,SF}} \) for the galaxies with CO(2-1) detections or upper limits in our sample. This is the so-called integrated Schmidt–Kennicutt plane and it allows us to study the relation between the molecular gas content and star formation rate, hence the nature of star formation in our sample (e.g. Daddi et al. 2010b; Sargent et al. 2014). Extended galaxies are on average close to the region of discs (Sargent et al. 2014, solid black line in Fig. 8). Compacts seem to be globally shifted towards the dash-dotted line in Fig. 8, defining the ‘strong starbursts’ (AMS \( \gtrsim 15 \) in the \( L'_{\text{CO}(2-1)} \) versus \( L_{\text{IR,SF}} \) plane according to the Sargent et al. (2014) model (see also Solomon & Vanden Bout 2005; Greve et al. 2005). In the right-hand panel of Fig. 8 we show the \( L'_{\text{CO}(1-0)} \) luminosity as a function of \( L_{\text{IR,SF}} \). To construct this plot, we use CO(2-1) fluxes, where available. To increase the statistics, we also include galaxies without CO(2-1) observations by converting their CO(5-4) flux to CO(2-1) using a \( R_{52} \) that is appropriate for their average SLED shape (see Section 3.1 and Table 5). We do not include galaxies in the ambiguous sample because their average excitation properties are poorly constrained. We stress however that mid/high-J CO transitions sample the denser phase of the molecular gas associated with star formation (Daddi et al. 2015; Liu et al. 2015b; Valentino et al. 2020) and should not be used as a proxy of the molecular gas mass when no direct constraints on the excitation correction are available. We finally convert the CO(2-1) flux to \( L'_{\text{CO}(1-0)} \) using \( R_{21} = 0.85 \), which is the average CO(2-1)-to-CO(1-0) ratio measured in star-forming galaxies at high redshift (Bothwell et al. 2013; Daddi et al. 2015; Boogaard et al. 2020). Using an homogeneous excitation correction for the CO(2-1) luminosity is a conservative choice since galaxies with an enhanced \( R_{52} \) ratio are expected to also show an enhanced \( R_{21} \) ratio (see e.g. Daddi et al. 2015). Implementing a differential excitation correction would thus exacerbate the tension between the extended and compact populations. The right-hand panel of Fig. 8 shows that the offset between compacts and extended galaxies increases when accounting for differences in the excitation properties of the two populations.

To quantify the separation between extended and compact galaxies in the \( L_{\text{CO}(1-0)} \) versus \( L_{\text{IR,SF}} \) plane, we fit the two populations independently using a linear function in the log–log space and accounting for upper limits as described in Section 3.1. We then consider only the best-fitting solutions yielding a slope \( \beta = 0.81 \pm 0.03 \), i.e. consistent with the value reported in Sargent et al. (2014). We find that extended galaxies have a best-fitting normalization value of \( 0.46 \pm 0.22 \) which is consistent with the Sargent et al. (2014) value for discs. We find a best-fitting normalization of \( 0.18 \pm 0.22 \) for the compacts. This corresponds to a \( \approx 2 \times \) offset from Sargent et al. (2014) and the \( 1.7 \times \) offset reported by Silverman et al. (2015b) for \( z \approx 1.5 \) starbursts. We obtain consistent results when considering the \( L'_{\text{CO}(2-1)} \) luminosity in the left-hand panel of Fig. 8. To quantify the significance of the offset between compact and extended galaxies, we compare the distributions of the two subsamples after subtracting the \( \beta \times L_{\text{IR,SF}} \) trend from the observed \( L'_{\text{CO}(1-0)} \) luminosity. We perform a log-rank test accounting for the presence of upper limits as described in the previous section. We find a 99.4 per cent probability that the distributions of compact and extended galaxies in the right-hand panel of Fig. 8 are different (\( p\text{-value} = 0.0058 \)).

The low-J CO transitions are proxies for the molecular gas content (Carilli & Walter 2013; Tacconi et al. 2020) while \( L_{\text{IR,SF}} \) quantifies the star-forming activity of a galaxy. Therefore, Fig. 8 suggest that compact galaxies in our sample have an enhanced star formation efficiency (\( \text{SFE} = M_{\text{gas}}/\text{SFR} \)) and a lower depletion time (\( \tau_{\text{dep}} = \text{SFR}/M_{\text{gas}} = 1/\text{SFE} \)) than extended galaxies. It also appears that extended and compacts off-main sequence galaxies (open black squares in Fig. 8) preferentially occupy the region with high \( L_{\text{IR,SF}} \) and \( L'_{\text{CO}(2-1)} \) or \( L'_{\text{CO}(1-0)} \). This suggests that compact galaxies above the main sequence have enhanced star formation rates and gas masses, as expected from literature scaling relations (Magdis et al. 2021).

**Figure 7.** LVG fit to the observed CO + [C\( \text{I} \)] SLEDs for extended (left-hand panel) and compact (right-hand panel) galaxies in our sample, split according to their main sequence position as in Fig. 5. Colour code is analogous to Fig. 6.
Submillimetre compactness as a critical dimension to understand the main sequence

Table 6. Best-fitting parameters of a double-component LVG modelling of the average CO + [CI] SLEDs of z ∼ 1.25 galaxies presented in Figs 4 and 5, and Tables 3 and 4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Within LTG</th>
<th>Within LTG, on MS</th>
<th>Within LTG, above MS</th>
<th>Compacts</th>
<th>Compacts, on MS</th>
<th>Compacts, above MS</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(nH2,low/(cm−3))</td>
<td>2.2 ± 0.2</td>
<td>2.2 ± 0.3</td>
<td>3.3 ± 0.6</td>
<td>3.1 ± 0.8</td>
<td>3.3 ± 0.8</td>
<td>3.0 ± 0.8</td>
</tr>
<tr>
<td>log(nH2,high/(cm−3))</td>
<td>3.9 ± 0.1</td>
<td>3.9 ± 0.2</td>
<td>4.8 ± 1.2</td>
<td>6 ± 1.3</td>
<td>5.2 ± 1.3</td>
<td>4.1 ± 1.6</td>
</tr>
<tr>
<td>Tkin,low/[K]</td>
<td>45 ± 98</td>
<td>45 ± 113</td>
<td>45 ± 130</td>
<td>180 ± 135</td>
<td>80 ± 135</td>
<td>215 ± 138</td>
</tr>
<tr>
<td>Tkin,high/[K]</td>
<td>45 ± 10</td>
<td>45 ± 23</td>
<td>250 ± 130</td>
<td>135 ± 138</td>
<td>125 ± 138</td>
<td>105 ± 138</td>
</tr>
<tr>
<td>log(MH2,low/[M⊙])</td>
<td>10.3 ± 0.3</td>
<td>10.3 ± 0.3</td>
<td>10.1 ± 0.2</td>
<td>9.7 ± 0.4</td>
<td>9.7 ± 0.4</td>
<td>9.8 ± 0.6</td>
</tr>
<tr>
<td>log(MH2,high/[M⊙])</td>
<td>10.1 ± 0.2</td>
<td>10.1 ± 0.3</td>
<td>9.6 ± 1.1</td>
<td>11.1 ± 1.5</td>
<td>10.2 ± 1.5</td>
<td>9.6 ± 1.8</td>
</tr>
</tbody>
</table>

Notes: The average values and their uncertainties are the best-fitting estimates and their statistical 1σ errors, where we impose that nH2,low < nH2,high.

In Section 3.1, we plot the main sequence offset and compactness in our sample, we plot the log(L′CO(2−1)/L′IR,SF) ratio as a function of the main sequence position and compactness in Fig. 9. As already observed in Section 3.1 for the molecular gas excitation, we find that compact galaxies within the main sequence have a L′CO(2−1)/L′IR,SF ratio smaller than their extended counterparts, and similar to that measured in galaxies above the main sequence. Also in this case, compact main-sequence galaxies seem to contribute to the scatter observed in the L′CO(2−1)/L′IR,SF−ΔMS relation from Valentino et al. (2020).

To explore the relation between the star formation efficiency, main sequence offset and compactness in our sample, we plot the L′CO(2−1)/L′IR,SF ratio as a function of the main sequence position and compactness in Fig. 8. As already observed in Section 3.1 for the molecular gas excitation, we find that compact galaxies within the main sequence have a L′CO(2−1)/L′IR,SF ratio smaller than their extended counterparts, and similar to that measured in galaxies above the main sequence. Also in this case, compact main-sequence galaxies seem to contribute to the scatter observed in the L′CO(2−1)/L′IR,SF−ΔMS relation from Valentino et al. (2020).

Figure 8. L′CO(2−1) [K km s−1 pc2] (left-hand panel) and L′CO(1−0) [K km s−1 pc2] (right-hand panel) luminosity as a function of the infrared luminosity from star formation LIR,SF [L⊙]. The solid and dashed black lines represent the models for main sequence and starburst galaxies from Sargent et al. (2014). In the left-hand panel, we convert this equation to L′CO(2−1), using R21 = 0.85. In both panels, the blue and red solid lines are the best-fitting lines with a slope β = 0.81 obtained by fitting separately the extended and compact population, respectively. The shaded areas represent the 1σ confidence interval on the best-fitting normalization. The symbols colour-code is analogous to Fig. 3. In the right-hand panel we show with filled diamonds measurements of L′CO(2−1) extrapolated from the CO(5-4) observed flux. The error bars in this figure correspond to the 1σ uncertainty on the CO(2-1) or CO(5-4) flux measurements.

Figure 9. L′CO(2−1) [K km s−1 pc2] / LIR,SF [L⊙] as a function of the main sequence offset (left-hand panel) and the compactness (right-hand panel). The green dotted line and shaded area in the left-hand panel represent the best-fitting model and 1σ confidence interval from Valentino et al. (2020). The dark red solid line and shaded area in the left-hand panel indicate the L′CO(2−1) [K km s−1 pc2] / LIR,SF−ΔMS trend and 1σ confidence interval obtained after excluding compact and ambiguous main sequence galaxies from the fit. The dark red solid line and shaded area in the right-hand panel mark the best-fitting model and 1σ confidence interval of the L′CO(2−1) [K km s−1 pc2] / LIR,SF [L⊙]−C/nginx trend. The colour-code and symbols are analogous to Fig. 3. We measure the uncertainty on the L′CO(2−1)/LIR,SF ratio by propagating the 1σ errors on the CO(2-1) flux measurements and log(LIR,SF).

2012; Sargent et al. 2014; Scoville et al. 2014, 2016; Genzel et al. 2015; Silverman et al. 2015b, 2018a; Tacconi et al. 2017, 2020; Elbaz et al. 2018; Franco et al. 2020; Feldmann 2020). These galaxies also lie closer to the ‘strong starburst’ locus, which also implies a higher star formation efficiency. On the other hand, compact galaxies within the main sequence occupy the region with low LIR,SF and L′CO(2−1), or L′CO(1−0), along the red solid line in Fig. 8. This suggests that these sources have similar star formation efficiency (or τdep) to compact off-main sequence sources, but lower star formation rates and gas masses. Finally, we highlight that ambiguous objects have low CO(2-1) fluxes suggesting high star formation efficiency. Hence, these sources are likely adding to the compacts population (see also Section 3.1).

Therefore, they are subject to the uncertainties already described in Section 3.4. Relative comparisons between the two phases for each population still hold, under the assumption that dense and diffuse gas reservoirs share the same metallicity.
contribution of compact main-sequence galaxies to the scatter of the $L'_{\text{CO}(2-1)}/L_{\text{IR},\text{SF}} - \Delta MS$ trend by fitting the $L'_{\text{CO}(2-1)}/L_{\text{IR},\text{SF}}$ ratio as a function of the main sequence offset excluding the compact and ambiguous galaxies within the main sequence scatter and the strongest outlier. As for the previous case, excluding these sources from the fit improves the correlation between $L'_{\text{CO}(2-1)}/L_{\text{IR},\text{SF}}$ and $\Delta MS$ (see Table 2). We also find a slightly steeper slope and smaller intrinsic scatter than the trend reported in Valentino et al. (2020). The right-hand panel of Fig. 9 shows that the $L'_{\text{CO}(2-1)}/L_{\text{IR},\text{SF}}$ ratio decreases as a function of the compactness (red line in Fig. 9, see also Table 2). We note that the slope of this correlation ($\beta \approx -0.42$, red line in the left-hand panel of Fig. 9) is only a suggestion at this stage, however, due to the large uncertainties associated with this parameter.

3.3 Star formation rate surface density, intensity of the radiation field, and dust temperature

We measure the star formation rate surface density by dividing the far-infrared star formation rate by the area $\Sigma_{\text{SFR}} = \text{SFR}/(2\pi R^{2,\text{eff}})$, where $R^{2,\text{eff}}$ is the ALMA effective radius. In Fig. 10 we show the star formation rate surface density distribution for our sample. The median star formation rate surface density is $\Sigma_{\text{SFR}} = 242^{+222}_{-199}$ M$_{\odot}$ yr$^{-1}$ kpc$^{-2}$ for compact galaxies and $\Sigma_{\text{SFR}} = 5^{+3}_{-1}$ M$_{\odot}$ yr$^{-1}$ kpc$^{-2}$ for the extended population, with error bars indicating the interquartile range of the distribution. A log-rank test to the two distributions, accounting for the presence of upper limits on $\Sigma_{\text{SFR}}$ shows that the two distributions are significantly different (p-value < 0.001). This shows that compact galaxies have enhanced star formation rate surface density with respect to extended galaxies and similar to local (U)LIRGs (e.g. Liu, Gao & Greve 2015a) and galaxies above the main sequence at high redshift (Jiménez-Andrade et al. 2019). This is somewhat expected, since compact and extended galaxies have similar star formation rates (Fig. 2) while very different submillimetre sizes (Fig. 1). This fits the idea that the compactness allows us to identify starburst galaxies, as discussed in Section 4.

Figure 10. Star formation rate surface density distribution for compact (red) and extended (hatched blue) galaxies in our sample. The solid and dashed lines indicate the median and interquartile range of each distribution, respectively. Measurements and upper limits are highlighted with circles and triangles, respectively. The open black squares indicate galaxies above the main sequence, with $\Delta MS \geq 3.5$.

Figure 11. IR luminosity surface density as a function of the dust temperature for our sample. The colour-code is analogous to Fig. 3. The dashed line represents the Stefan-Boltzman law for optically thick dust clouds.

3.4 Gas content

The mass of the molecular gas reservoir is a crucial quantity to understand the future evolution of galaxies. However, measuring the amount of molecular gas in galaxies is notoriously a difficult task (see e.g. Birkin et al. 2021). This stems from the fact that measuring the molecular gas mass requires choosing an observable-to-gas conversion factor depending on the molecular gas tracer adopted. When inferring gas masses from the CO luminosity or the dust mass, for example, one needs to assume a CO-to-H$_2$ or a gas-to-dust ratio ($\alpha_{\text{CO}}$ and $\delta_{\text{GDR}}$, respectively). Both conversion factors have complex dependencies on e.g. the state of the ISM, the gas-phase metallicity,
or galaxy type (Leroy et al. 2011; Magdis et al. 2012; Narayanan et al. 2012; Bolatto, Wolfire & Leroy 2013; Genzel et al. 2015; Silverman et al. 2018b) that are not yet fully understood. For this reason, we first explore the molecular gas properties of our sample by considering observed $L_{\text{CO}(2-1)}$ luminosities as a proxy. We also use $M_{\text{dust}}$ as an independent tracer of the molecular gas reservoir.

### 3.4.1 Observed quantities

The analysis performed in the previous sections suggests that compacts are characterized by a highly excited ISM, enhanced star formation efficiency and shorter depletion time. Also, compact galaxies within the main sequence seem underluminous in CO(2-1), possibly suggesting that their gas content is reduced. However, the gas fraction is predicted to decrease with stellar mass (e.g. Magdis et al. 2012; Tacconi et al. 2017; Liu et al. 2019, Tacconi et al. 2020, and references therein). To understand whether the trends observed in the previous sections are driven by the stellar mass, we plot in Fig. 13 the $L_{\text{CO}(2-1)}$ luminosity as a function of the stellar mass for our sample. Despite the scatter, this plot confirms that compact main sequence galaxies are less luminous in $L_{\text{CO}(2-1)}$ than extended and compact off-main sequence galaxies with similar stellar mass, on average. To quantify this effect, we compute the average $L_{\text{CO}(2-1)}$ luminosity of compact and extended galaxies within the main sequence. For this computation, we consider only galaxies with $M_\star \geq 5 \times 10^{10} M_\odot$ to account for the fact that the gas fraction (hence $L_{\text{CO}(2-1)}$) decreases as a function of the stellar mass. This results in 7 compact and 13 extended galaxies within the main sequence with an average stellar mass of $M_\star = 10^{11} M_\odot$. We find $L_{\text{CO}(2-1),\text{compact}, MS} = 1.3 \pm 0.2 \times 10^{10} \text{[K km s}^{-1} \text{pc}^2]$ and $L_{\text{CO}(2-1),\text{extended}, MS} = 2.2 \pm 0.3 \times 10^{10} \text{[K km s}^{-1} \text{pc}^2]$. That is, compact main sequence galaxies are $\sim 1.7 \times$ less luminous in CO(2-1) at $\sim 5\sigma$ significance.

Finally, in Fig. 14 we show the ratio between the $L_{\text{CO}(2-1)}$ luminosity and stellar mass as a function of the main sequence offset and the compactness. In addition, we show the ratio between the dust and stellar mass as a function of the same quantities. The top left-hand panel of Fig. 14 shows that the $L_{\text{CO}(2-1)}/M_\star$ ratio increases as a function of the main sequence offset. Compact galaxies on the main sequence, on the other hand, seem to be underluminous in CO(2-1) with respect to extended sources. We find a similar trend for the $M_{\text{dust}}/M_\star$ ratio in the bottom left-hand panel of Fig. 14. In the right-hand panels of this figure we see that both the $L_{\text{CO}(2-1)}/M_\star$ and $M_{\text{dust}}/M_\star$ ratios decrease as a function of the compactness, albeit with a large scatter.

### 3.4.2 Derived quantities

Building on the results presented in the previous sections, we now convert CO luminosities and dust masses into gas masses by adopting conversion factors that are appropriate for the excitation conditions and star formation efficiency properties of each galaxy. Clearly our approach will still suffer from the classical uncertainties related to the choice of the observable-to-gas conversion factors. However, the detailed knowledge of the molecular gas properties of both individual galaxies in our sample (see Table 1) and the availability of average CO SLEDs for each of the galaxy sub-populations analysed in this work allow us to physically motivate the choice of $\alpha_{\text{CO}}/\beta_{\text{GDR}}$ for each galaxy class. A detailed analysis of the $\alpha_{\text{CO}}/\beta_{\text{GDR}}$ conversion factors in individual sources will be presented in future papers. We highlight here that a common approach in literature is to use the main sequence position of a source to define the preferred observable-to-gas conversion factor (e.g. Magnelli et al. 2012; Sargent et al. 2014; Accurso et al. 2017; Aravena et al. 2019; Cassata et al. 2020). While this might appear a reasonable assumption lacking additional constraints on the galaxy ISM conditions, we urge caution against this approach considering that galaxies within the main sequence display a wide range of compactnesses, CO excitations, and star formation efficiencies.

When inferring CO-based gas masses, we only consider galaxies with CO(2-1) observations because this low-J transition is directly tracing the total molecular gas reservoir. We convert $L_{\text{CO(2-1)}}$ to $L_{\text{CO(1-0)}}$ using $R_{21} = L_{\text{CO(2-1)}}/L_{\text{CO(1-0)}} = 0.85$. We then compute
the gas mass as $M_{\text{gas}} = \alpha_{\text{CO}} \times L'_{\text{CO}(2-1)}$. We adopt $\alpha_{\text{CO}} = 3.6 \, M_\odot/(K \, \text{km s}^{-1} \, \text{pc}^2)$ for extended galaxies since this value has been suggested to be appropriate for high-redshift, highly star-forming discs (e.g. Daddi et al. 2015). Ambiguous and compact galaxies are instead characterized by a compact CO emission, enhanced CO excitation, and high $L'_{\text{CO}(2-1)}/L_{\text{IR}}$ ratios, resembling the ISM conditions of starbursting objects in the local Universe for which $\alpha_{\text{CO}} = 0.8 \, M_\odot/(K \, \text{km s}^{-1} \, \text{pc}^2)$ (Solomon et al. 1987). Indeed, a starburst-like $\alpha_{\text{CO}}$ has been shown to be appropriate for compact galaxies at high redshift (Tadaki et al. 2017b).

When computing gas masses from the dust mass, we consider only galaxies with a reliable dust mass measurement ($M_{\text{dust}}/M_{\text{dust, err}} \geq 5$). This corresponds to 69 galaxies, significantly improving the statistics with respect to CO-based gas mass estimates. We compute the gas mass as $M_{\text{gas}} = \delta_{\text{GDR}} \times M_{\text{dust}}$, following Magdis et al. (2012). We adopt $\delta_{\text{GDR}} = 85$ for the extended population, corresponding to a metallicity-dependent gas-to-dust ratio at $Z = 12 + \log(O/H) = Z_\odot$ (Magdis et al. 2012). On the other hand, we use $\delta_{\text{GDR}} = 30$ for compact and ambiguous galaxies since we show that these have ‘starburst-like’ ISM conditions such as an enhanced CO excitation (as seen from both individual $R_2$ ratios and average CO SLEDs, see Section 3.1), enhanced efficiency (see Section 3.2), high SFR, and infrared surface density, dust temperatures, and intensity of the radiation field (see Section 3.3), and starbursts at high redshift are reported to have supersolar metallicities (Puglisi et al. 2017).

Figure 14. Top row: $L'_{\text{CO}(2-1)}/M_\star$ ratio as a function of the main sequence offset (left-hand panel) and the compactness (right-hand panel). Bottom row: $M_{\text{dust}}/M_\star$ ratio as a function of the main sequence offset (left-hand panel) and the compactness (right-hand panel). In the left-hand panels, large filled diamonds indicate median $L'_{\text{CO}(2-1)}/M_\star$ and $M_{\text{dust}}/M_\star$ ratios for extended and compact galaxies with $1 \leq \Delta MS \leq 3.5$ while large filled squares indicate median gas fractions for extended and compact sources with $\Delta MS \geq 3.5$ (dark blue and dark red symbols, respectively). The colour code and symbols for individual data points are analogous to Fig. 3. The error bars associated to the individual data points are obtained by propagating the $1\sigma$ uncertainty on the CO(2-1) flux measurements and a 0.2 dex uncertainty on the stellar mass.

Figure 15. Inverse, integrated Schmidt–Kennicutt relation between the star formation rate and the molecular gas mass. The coloured filled circles show gas masses derived from CO lines. The coloured filled diamonds indicate dust-based gas masses. We report both CO-based and dust-based gas masses when available. The blue and red solid (dotted) lines represent the best-fitting lines with a slope $\beta = 0.81$ obtained by fitting the CO- (dust-) based gas mass measurements for the extended and compact population, respectively. The shaded areas represent the $1\sigma$ confidence interval on the best-fitting normalizations.
In Fig. 15, we show the correlation between the molecular gas mass and the star formation rate for our sample, that, is, the integrated Schmidt–Kennicutt relation considering derived quantities rather than pure observables (as instead shown in Fig. 8). Here we measure gas masses from both CO lines and dust masses, when available. The inclusion of dust-based molecular gas masses allows us to study the relation between the gas mass and SFR with increased statistics with respect to gas mass measurements from the CO(2–1). We note that Fig. 15 would be equivalent to show the ‘resolved’ Schmidt–Kennicutt relation considering the molecular gas and star formation rate surface densities for our sample. This is because we find that molecular gas sizes (as sampled by the dust continuum) and SFR sizes (traced by the CO(5–4) emission) are nearly equivalent for our galaxies (see Section 2.1.1), and therefore both axes would be rescaled by the same quantity. This plot confirms the results of Fig. 8 that extended and compact galaxies have different star formation efficiency properties. These galaxies occupy distinct regions of the integrated Schmidt–Kennicutt plane and the offset increases when translating observables into physical quantities, accounting for the ISM properties of each source. In particular, if we consider CO-based molecular gas masses, we obtain that the normalization of the log(Mgas)−β×log(SFRgas) relation is 8.21 ± 0.08 for compact galaxies and 9.12 ± 0.07 for extended sources (red and blue solid lines in Fig. 15, respectively). The average offset of compact galaxies with respect to the locus for extended sources is 0.91 dex. This corresponds roughly to a factor of 8 enhancement in star formation efficiency, further stressing the starbursting nature of the ISM in these sources. When considering dust-based molecular gas masses, the offset of compact galaxies reduces to 0.5 dex or a factor of 3.2 star formation efficiency enhancement, on average (see red and blue dotted lines in Fig. 15). However, the fit seems to be driven by a small number of compact galaxies with a high gas mass in this case. These sources might have been misclassified as compact since they lie only ~1σ below the mass–size relation (see also Fig. 16). Alternatively, this might reflect the fact that the separation between compact and extended galaxies is not purely bimodal. However, we still observe that most compact galaxies are shifted towards the starburst locus in the Sargent et al. (2014) model.

We define the gas fraction as $\mu_{\text{gas}} = \frac{M_{\text{gas}}}{M_\star}$ and we show this quantity as a function of the main sequence offset and the compactness in Fig. 16. To quantify the difference in the gas fraction of compact and extended galaxies on and above the main sequence, we split our sample in two ΔMS bins and we compute the average gas fraction of compact and extended sources within each bin. When considering CO-based gas fractions, we find $\mu_{\text{gas, Compacts, MS}} = 0.12 \pm 0.05$ and $\mu_{\text{gas, Extended, MS}} = 1.04 \pm 0.34$ for compact and extended galaxies within the main sequence. If we consider dust-based gas masses, we find $\mu_{\text{gas, Compacts, MS}} = 0.12 \pm 0.09$ and $\mu_{\text{gas, Extended, MS}} = 0.66 \pm 0.34$. That is, compact galaxies have ~6–9 x reduced gas fractions with respect to extended sources, in agreement with previous results for small samples of one or two compact galaxies (Popping et al. 2017; Tadaki et al. 2017b; Brusa et al. 2018, for CO-based gas fractions). The median gas fractions are different at 1.2 – 2.7σ significance. On the other hand, galaxies above the main sequence (ΔMS ≥ 3.5) have similar gas fractions as we obtain $\mu_{\text{gas, Compacts, off-MS}} = 1.14 \pm 0.70$ and $\mu_{\text{gas, Extended, off-MS}} = 2.2 \pm 1.7$ when considering the CO(2–1) luminosity as a molecular gas tracer. This is confirmed when considering dust masses as proxies for the molecular gas ($\mu_{\text{gas, Compacts, off-MS}} = 0.97 \pm 0.80$ and $\mu_{\text{gas, Extended, off-MS}} = 0.93 \pm 0.52$). We also find a tentative trend of decreasing gas fraction as a function of the compactness when considering both CO(2–1) luminosities and dust masses. We explore the dependence of $\mu_{\text{gas}}$ on the compactness by applying a linear regression analysis to the bottom panel of Fig. 16, owing to the larger statistics available when considering the dust mass as a gas mass tracer. Indeed, we find a correlation between the gas fraction and the compactness. The results are reported in Table 2 and the best-fitting correlation is shown as a red solid line in the right-hand panel of Fig. 16.

A small number of compact galaxies within the main sequence show ‘main sequence like’ gas fractions when considering dust-based measurements. These sources lie only 1σ below the mass–size relation and are close to the compactness limit that we use to discriminate between compact and extended galaxies (red dash-dotted line in the right-hand panels of Fig. 16, e.g.). Hence, these could have been misclassified (due to e.g. noise in the ALMA size measurements) and might rather belong to the extended sample. Alternatively our data might suggest that galaxies do not follow a bimodal distribution, but are gradually distributed in the $\mu_{\text{gas}}$–Cgas plane similar to our previous findings (see Fig. 15 and e.g. fig. 3 in Valentino et al. 2020). This seems to be suggested by the right-hand panel of Fig. 16 where we see that $\mu_{\text{gas}}$ broadly decreases as a function of the compactness. However, we argue that Fig. 16 overall suggests a more complex dependence of the gas fraction on galaxy properties, possibly as a result of evolutionary trends. We will explore this aspect in Section 4.

We finally note that the difference in $\mu_{\text{gas}}$ between compact and extended galaxies on the main sequence depends on the choice of the $\delta_{\text{GDR}}$ or $\sigma_{\text{CO}}$ conversion factors. In particular, the tension between compact and extended galaxies within the main sequence is reduced to a factor of 2 when considering metallicity-dependent conversion factors (see Appendix A). However, the extensive analysis of the ISM conditions presented in the previous sections, the results obtained from the observables as well as the agreement between molecular gas tracers when using observables-to-gas conversion factors tailored to the compactness properties of each source (see Figs A2 and A3) corroborate our choice of the conversion factors for each galaxy subsample. Finally, the use of different conversion factors for compact and extended galaxies is also corroborated by the results presented in Fig. 8 and 15, suggesting that these sources have different star formation efficiency properties.

## 4 DISCUSSION

### 4.1 The role of compactness in galaxy evolution

While the ISM properties of galaxies at $z \sim 1.3$ are weakly correlated with the offset from the main sequence (Valentino et al. 2020), a diversity of ISM properties has been observed within the main sequence scatter itself (Elbaz et al. 2018; Puglisi et al. 2019). In agreement with these results, here we find that ~ 46 per cent$^4$ of galaxies in our sample have a compact molecular gas reservoir. These galaxies have excited CO line ratios, enhanced star formation efficiencies, and are spread on and above the main sequence scatter. The $R_{52}$ and $L_{\text{CO(2–1)}}/L_{\text{IR}}$ versus $\Delta$MS correlations (see Figs 3 and 9). These results suggest that galaxies within the main sequence scatter are not all largely unperturbed gas-rich discs. However, this does not seem to be simply due to a large scatter in the properties of star-forming galaxies. Our results suggest instead that the diversity...

---

$^4$This number would actually be even larger if one would consider galaxies with loose size upper limits as submillimetre compact sources, see also Section 3.1.
of gas excitation conditions and efficiency observed in MS galaxies is associated with the compactness of the molecular gas reservoir. Indeed, excluding submillimetre compact galaxies from the fits in Figs 3 and 9 improves the correlation between galaxy ISM properties and the main sequence offset. Furthermore, we find correlations between galaxy ISM properties and the submillimetre compactness (see Table 2). Distinguishing galaxies for their submillimetre compactness also allows us to select objects with significantly different CO SLEDs (see Figs 4 and 5). These results suggest that the compactness of the molecular gas reservoir traces the ISM state of a source. We thus suggest here that using a submillimetre compactness threshold: 

$$C_{\text{gas}} = \frac{R_{\text{eff,ALMA}}}{R_{\text{eff,ALMA}}} = \frac{A(z)}{(M_*/7 \times 10^{10} M_\odot)^{\alpha(z)}} \geq 2.2 \quad (2)$$

would allow us to better distinguish between high redshift, gas-rich discs and galaxies harbouring a highly excited, starbursting ISM. Here $A(z)$ and $\alpha(z)$ are the best-fitting coefficients for the mass–size relation of van der Wel et al. (2014). The value $C_{\text{gas}} = 2.2$ corresponds to $\sim 1\sigma$ below the optical mass–size relation of discs. We note that this proposed criterion is qualitatively similar to using the star formation rate surface density ($\Sigma_{\text{SFR}}$) to select starbursting sources (e.g. Jiménez-Andrade et al. 2019; Valentino et al. 2020, and references therein). In fact, compact galaxies have higher star formation rate surface density than extended sources (see e.g. Fig. 10). However, considering the submillimetre compactness as in equation (2) allows us to account for the stellar mass dependence of the star formation rate and size thus rescaling to the structural properties of each object.

In line with our results, various studies (e.g. Downes & Solomon 1998; Combes et al. 2013; Narayanan & Krumholz 2014; Bournaud et al. 2015) have highlighted the importance of the star formation rate surface density as a proxy for the star formation properties of galaxies since this parameter depends on the gas density, temperature, and optical depth (Narayanan & Krumholz 2014).

One caveat here is that, as a result of the far-infrared selection, our observations sample the upper stripe of the main sequence scatter at $M_* \sim 5 \times 10^{10} M_\odot$. Our observations fully probe the $1\sigma$ scatter of the main sequence ($\pm 0.3$ dex) only at $M_* \geq 10^{11} M_\odot$. Similar to previous ALMA studies at high redshift (e.g. Elbaz et al. 2018; Tadaki et al. 2020), our sample thus appears to be biased towards highly star-forming massive galaxies and it might be not trivial to extrapolate our results to lower stellar mass regimes. This is because the high-mass end of the main sequence is the locus where galaxies are expected to quench soon (Dekel & Birnboim 2006; Zolotov et al. 2015), and/or where the more numerous population of passive galaxies might be temporarily boosted by rejuvenation (Mancini et al. 2019). Deeper observations of mass-selected samples at $M_* \leq 10^{10} M_\odot$ will be required to understand if our results apply to the main sequence population at lower stellar mass.

4.2 On the nature of submillimetre compact galaxies within the MS

As discussed in the introduction, the discovery of a significant number of submillimetre compact galaxies within the scatter of
the main sequence conflicts with the idea that galaxies along this sequence are mostly secularly evolving discs. The lack of a clear trend between galaxy ISM properties and the main sequence offset seems to disfavour alternative scenarios according which galaxies oscillate around the main sequence as a result of compaction episodes (e.g. Tacchella et al. 2016). It has also been proposed that submillimetre compact main sequence galaxies are the result of major mergers with a moderate star formation rate enhancement because of the enhanced gas fractions (Jiménez-Andrade et al. 2019). While some of the submillimetre compact main sequence galaxies in our analysis might be consistent with being ‘failed burst’ mergers, this would fail to explain why most of these sources have a reduced gas fraction (see Fig. 16).

The investigation of the ISM conditions presented in this paper allows us to shed light on possible formation mechanisms of such objects. Submillimetre compact galaxies on and above the main sequence have remarkably similar excitation properties (see in particular Fig. 5 and Table 5) and star formation efficiency (see Section 3.2) and these properties are enhanced with respect to those of extended galaxies. This hints at a common origin between submillimetre compact galaxies on and above the main sequence, likely associated to a merger event. Mergers are in fact capable of inducing strong inflows to the nuclear regions reducing significantly the size of the molecular gas reservoir and enhancing the efficiency of star formation (Mihos & Hernquist 1996). Merger-driven starbursts are also predicted to have larger gas excitations than highly star-forming discs due to the prevalence of compressive tides enhancing the density of the gas (Bournaud et al. 2015). On the other hand, submillimetre compact galaxies within the main sequence with \( M_\bullet \geq 10^{10.3} M_\odot \) are underluminous in \( L_{\text{CO(2-1)}} \) for their stellar mass and SFR (see Figs 13 and 14) translating into reduced gas fractions (see Fig. 16). The reduced gas fractions of submillimetre compact galaxies within the main sequence would naturally result from efficient gas consumption during the preceding starburst phase. Furthermore, when fitting a linear function in the \( \log(M_{\text{gas}}) - \log(\Delta M) \) plane, we find that submillimetre compact galaxies have a larger intrinsic scatter than extended sources \( \sigma_{\text{int, Compacts}} = 0.49 \pm 0.08 \) versus \( \sigma_{\text{int, Extended}} = 0.22 \pm 0.09 \) in Fig. 16). This fits the idea that submillimetre compact galaxies within the main sequence are the relic of a previous starburst episode, as in this case we expect to detect sources in different stages of the post-starburst phase.

Therefore, the results of this paper support the scenario proposed in Puglisi et al. (2019) and suggest that submillimetre compact massive galaxies within the main sequence represent transient objects in an ‘early post-starburst phase’ following a merger-driven starburst episode (see also Elbaz et al. 2018 and Franco et al. 2020). We schematically summarize this proposed evolutionary trend in Fig. 17. Our results and proposed scenario are consistent with cosmological simulations suggesting that compact galaxies form by repeated major mergers of small progenitors (Chabanier et al. 2020). These simulations predict gas fractions of ~ 20 per cent in compact star-forming galaxies versus ~ 50 per cent for extended main-sequence galaxies at \( z \sim 2 \), as a result of gas consumption by star formation, as well as gas exhaustion in major mergers which can efficiently consume and/or expel gas and in good agreement with our results.

A visual inspection of the HST imaging available for this sample does not reveal any clear evidence of an enhanced merger fraction in the submillimetre compact population within the main sequence. However, this does not necessarily contradict the idea that submillimetre compact galaxies are associated to mergers. In fact, only F814W HST imaging is homogeneously available for the full sample, sampling the rest-frame UV emission at \( z \sim 1.3 \). Therefore, strong dust attenuation effects might hamper an accurate merger classification (e.g. Cibinel et al. 2019). Furthermore, in our proposed interpretation, submillimetre compact galaxies within the main sequence are ‘early post-starburst’ galaxies, hence likely observed at or somewhat past the coalescence phase. Therefore, we would not necessarily classify these sources as mergers. In fact, morphological classification criteria are able to identify mergers up to the coalescence phase, hence before disturbances or asymmetries in the imaging fade away (see discussion in Puglisi et al. 2019, and references therein). On the other hand, the \( L_{\text{CO(5-4)}}/L_{\text{IR, SF}} \) ratio might provide further indications on the evolutionary stage of our sources. Fig. 18 shows the distribution of the \( L_{\text{CO(5-4)}}/L_{\text{IR, SF}} \) logarithmic ratio for galaxies in our sample, distinguished for their submillimetre compactness. We measure \( \log(L_{\text{CO(5-4)}}/L_{\text{IR, SF}}) \) for \( \text{Kms}^{-1}\text{pc}^2 \) and we find a ~ 87 per cent probability that the two distributions are different (\( p=0.13 \)). That is, we find marginal evidence that submillimetre compact galaxies have lower \( L_{\text{CO(5-4)}}/L_{\text{IR, SF}} \) ratio compared to submillimetre extended sources. The CO(5-4) luminosity correlates linearly with the far-infrared luminosity from star formation and this has been interpreted as an evidence that \( L_{\text{CO(5-4)}} \) traces dense, star-forming molecular gas (Bayet et al. 2009; Greve et al. 2014; Daddi et al. 2015; Liu et al. 2015b; Cassata et al. 2020; Valentino et al. 2020). Therefore, this might suggest that the submillimetre compact population is caught in a declining phase of the starburst since the ‘instantaneous star formation rate’, as traced by the CO(5-4) luminosity (Daddi et al. 5217–5238 (2021).
The star formation rate averaged on a ∼100 Myr time-scale, as traced by the far-infrared luminosity (Kennicutt 1998). However, we measure a difference only at the ∼2σ level. Future studies with larger statistics will allow us to confirm this result.

We note that compact star-forming galaxies at high redshift have already been proposed as a key population for our understanding of massive, quenched galaxies formation (Barro et al. 2013, 2014, 2016; Nelson et al. 2014; van Dokkum et al. 2015). These so-called ‘blue nuggets’ are however selected for their compact size in the optical. This selection might be therefore biased to a later phase of the quenching process where the compact stellar core has been already formed. Conversely, our selection seems to identify an early phase of the compact core build-up (see Fig. 1). This might explain why extended SFGs and ‘blue nuggets’ present similar ISM conditions except than in the radio regime which samples the starburst activity on longer time-scales (0–400 Myr, see Gómez-Guijarro et al. 2019, and references therein). The radio properties of ‘blue nuggets’ suggest that these galaxies are old starbursts (Gómez-Guijarro et al. 2019) and might indicate the existence of an evolutionary link between submillimetre compacts, optically compact galaxies, and passive ellipticals.

Selecting compact galaxies by means of the molecular gas size likely allows us to identify galaxies in an early phase of the transition to a passive, bulge dominated galaxy retaining the imprints of their formation mechanism. Future studies of this population will provide a new perspective for our understanding of quenching processes and passive galaxies formation which typically display post-starburst conditions of a galaxy since large variations of the CO excitation are expected on to the central black hole (Springel & Hernquist 2005) affecting the future star formation processes in the remnant.

Finally, the existence of an evolutionary link between ‘submillimetre compact’ and quenched galaxies might also be suggested by the extremely compact size of high-redshift post-starburst galaxies that are likely to be formed through dissipative collapse of gas and rapid star formation prior to quenching (Almaini et al. 2017; Maltby et al. 2018), consistently with our findings.

5 SUMMARY

In this work, we presented a characterization of the molecular gas properties and molecular gas content of galaxies as a function of the compactness of the molecular gas reservoir by using ALMA observations of several CO and [C I] transitions for a sample of 82 far-infrared selected galaxies at z ∼ 1.3 in COSMOS. We measured the compactness of the molecular gas reservoir by comparing measurements of the molecular gas size to the optical M–Size relation for discs at z ∼ 1.25. We then investigated the relation between the molecular gas properties, the offset from the main sequence and the submillimetre compactness. We further measured molecular gas masses from multiple molecular gas tracers to gain insights on the origin of submillimetre compact galaxies within the main sequence. The main findings of this paper are as follows:

(i) The >46 per cent of galaxies in our sample have a molecular gas reservoir more compact than the stellar size of typical massive star-forming galaxies at z ∼ 1.3. The effective radius of the molecular gas reservoir in these sources is on average ≥3.3 × smaller than their K–band effective radius. The compactness of the molecular gas reservoir shows no significant correlation with the main sequence position.

(ii) Submillimetre compact galaxies have enhanced CO(5-4)/CO(2-1) and reduced CO(2-1)/LIR, SF luminosity ratios with respect to submillimetre extended galaxies, implying enhanced CO excitation and star formation efficiencies. A significant number of these sources are located within the scatter of the main sequence, blurring the correlation between these ratios and the main sequence offset reported in our previous analysis. Both the CO(5-4)/CO(2-1) and CO(2-1)/LIR, SF luminosity ratios correlate with the submillimetre compactness.

(iii) The average CO SLED of submillimetre compact galaxies up to CO(7-6) is consistent with that of the most extreme main sequence outliers at z ∼ 1.3. On the other hand, galaxies with an extended molecular gas reservoir have a less excited average CO SLED. This mirrors results from individual line ratios and suggest that the submillimetre compactness provides a good indicator for the CO excitation conditions of a galaxy. Furthermore, this stresses the fact that high-J CO transitions should not be used to derive total molecular gas masses without prior knowledge of the excitation conditions of a galaxy since large variations of the CO excitation are observed within the main sequence itself.

(iv) We find that submillimetre extended and compact galaxies occupy distinct regions in the integrated Schmidt–Kennicutt plane and, in particular, submillimetre compact galaxies have enhanced LIR, SF for a given CO(2-1) luminosity (Fig. 8) indicating enhanced star formation efficiencies similar to starbursts (Fig. 15). A higher star formation efficiency for the submillimetre compact population is also indicated by the intensity of the radiation field, inferred from the far-infrared spectral energy distribution.

(v) We find that submillimetre compact galaxies have higher SFR surface density than extended sources, similar to local starburst galaxies and high-redshift SMGs. We find indications that submillimetre extended galaxies have higher dust temperatures than extended sources, although the average dust temperatures are consistent within the error bars.
(vi) We find that submillimetre compact galaxies within the main sequence are underluminous in CO(2-1) with respect to submillimetre extended main sequence galaxies and off-main sequence sources with similar stellar mass (Fig. 13) and SFR (Fig. 14). Using both the CO(2-1) luminosity and the dust mass as molecular gas tracers, and using \( \epsilon_{\text{CO}} \) and \( \delta_{\text{DR}} \) conversion factors tailored to the ISM conditions of our sources, we find that the gas fraction mildly increases as a function of the main sequence offset, in qualitative agreement with published scaling relations. However, we find that submillimetre compact galaxies within the main sequence have reduced gas fractions on average, but with a large scatter. While the magnitude of this offset depends on the \( \epsilon_{\text{CO}} \) and \( \delta_{\text{DR}} \) prescriptions, these results suggest that submillimetre compact galaxies within the main sequence have lower gas fractions with respect to their extended main-sequence counterparts.

Overall, this study shows that the structural properties of galaxies at long wavelengths are a crucial ingredient for interpreting the main sequence of star-forming galaxies at \( z \sim 1 \). While currently limited to \( M_\star \geq 10^{11} M_\odot \) where our selection allows us to fully probe the main sequence scatter, our analysis suggests that the compactness of the molecular gas reservoir allows to identify sources with a highly excited, starbursting ISM. Similar to the star formation rate surface density, this parameter provides a good proxy for the ISM conditions of a galaxy while also allowing to rescale for its individual structural properties showing a dependence on the stellar mass. Future crucial steps for our understanding of star formation in distant galaxies will include to perform studies of the molecular gas properties in \( M_\star \)-selected samples of galaxies with lower stellar masses, to understand if these results also apply to the full main sequence population. Another critical aspect to understand the properties of star-forming galaxies on and above the main sequence at high-\( z \) will be to study large galaxy samples at high spatial resolution in the far-infrared/submillimetre regime, where spatially resolved studies have not yet reached the statistics of spatially resolved optical surveys (e.g. Förster Schreiber et al. 2009; Stott et al. 2016). This study has also allowed us to shed light on the origin of submillimetre compact galaxies within the main sequence which are now commonly detected in submillimetre/radio surveys at \( z \geq 1 \). We argue that their high CO excitation and star formation efficiency, and reduced gas fractions suggest that these sources are associated with an evolutionary phase of the merger. In particular, these properties are consistent with sources in an ‘early post-starburst’ phase in which the star formation rate has declined and the gas fraction has been reduced as a result of efficient gas consumption while the galaxy retains enhanced excitation and efficiency. Future studies of this population will be crucial for our understanding of passive galaxies formation.

**ACKNOWLEDGEMENTS**

We would like to thank the referee for their constructive report which significantly improved the content and clarity of the paper. AP acknowledges funding from Region Ile-de-France and Incoming CEA fellowship from the CEA-Enhanced Eurotalents program, co-funded by FP7 Marie-Skłodowska-Curie COFUND program (Grant Agreement 600382). AP also gratefully acknowledges financial support from STFC through grants ST/T000244/1 and ST/P000541/1. MTS acknowledges support from a Scientific Exchanges visitor fellowship (IZSEZO 202357) from the Swiss National Science Foundation. AP thanks Ivan Delvecchio and Ian Smail for helpful discussions.

**DATA AVAILABILITY**

The data used in this paper are available through the ALMA data archive.

**REFERENCES**

In this appendix, we explore the dependence of the results presented in Section 3.4 and Fig. 16 on the choice of the observable-to-gas conversion factor. The main goal of this test is to explore the difference in gas fraction between extended and compact galaxies on the main sequence, since this aspect is critical for our interpretation of the submillimetre compact population within the main sequence (see Section 4). To quantify the dependence of our results on the observable-to-gas conversion factors, we compute gas fractions for galaxies on the main sequence by adopting metallicity-dependent $\alpha_{CO}$ and $\delta_{GDR}$ conversion factors, following the prescription of Magdis et al. (2012). The metallicity of our galaxies is $Z = 8.78$ ± 0.27. In the bottom row of Fig. A1, we show the gas fractions from $L_{\text{CO}(2-1)}$ when adopting a metallicity-dependent $\alpha_{CO}$ for galaxies within the main sequence. We find median gas fractions of $M_{\text{gas}, \text{Compacts, MS}} = 0.46 \pm 0.20$ and $M_{\text{gas}, \text{Compacts, MS}} = 0.83 \pm 0.27$. In the bottom row of Fig. A1, we show the gas fractions from $M_{\text{gas}}$ by adopting a $\delta_{GDR}$ that varies as a function of the metallicity for galaxies within the main sequence. We find median gas fractions of $M_{\text{gas}, \text{Compacts, MS}} = 0.13 \pm 0.10$ and $M_{\text{gas}, \text{Compacts, MS}} = 0.26 \pm 0.16$. Finally, using a
Submillimetre compactness as a critical dimension to understand the main sequence

Figure A1. As Fig. 16 but using metallicity-dependent $\alpha_{\text{CO}}$ and $\delta_{\text{GDR}}$ conversion factors for galaxies on the main sequence. We consider constant conversion factors for galaxies above the main sequence, as in the main text.

metallicity-dependent $\delta_{\text{GDR}}$ provides a slope, normalization, and intrinsic scatter for the $C_{\text{gas}}-\mu_{\text{gas,dust}}$ relation that is consistent within the error bars to that reported in Table 2 while providing a lower correlation index ($\rho = -0.3$). This figure shows that compact main sequence galaxies show a reduced gas fraction, on average, although considering the metallicity dependence of the $\alpha_{\text{CO}}$ and $\delta_{\text{GDR}}$ reduces the difference with respect to the extended population. We summarize our results for the average gas fractions measured for galaxies on and above the main sequence with different tracers and observable-to-gas conversion factors prescriptions in Table A1.

Finally, in Fig. A2 we show the comparison between $M_{\text{gas}}$ computed from $L'_{\text{CO}(2-1)}$ and $M_{\text{dust}}$, using the $\alpha_{\text{CO}}$ and $\delta_{\text{GDR}}$ specified in the caption. This figure shows a very good agreement between CO-based and dust-based gas masses for the compact galaxies, supporting our choice of starburst-like conversion factors for this class of objects. At the same time, this figure suggests that the gas mass of a subset of extended galaxies might be overestimated (underestimated) by a factor of $\sim 0.25$ dex when adopting $\alpha_{\text{CO}} = 3.6$ ($\delta_{\text{GDR}} = 85$). This does not substantially affect our conclusions.

The small discrepancy in the gas masses measured from $L'_{\text{CO}(2-1)}$ and $M_{\text{dust}}$ might suggest variation in the $L'_{\text{CO}(2-1)}/M_{\text{dust}}$ ratio as a function of the main sequence offset and/or compactness. In Fig. A3, we thus investigate for the presence of variations in the $L'_{\text{CO}(2-1)}/M_{\text{dust}}$ ratio as a function of the main sequence offset and of the compactness for galaxies with detections or reliable $L'_{\text{CO}(2-1)}$ upper limits and robust measurements of $M_{\text{dust}}$. This plot shows that there is no clear dependence of the $L'_{\text{CO}(2-1)}/M_{\text{dust}}$ ratio on the main sequence offset nor the compactness. The median $L'_{\text{CO}(2-1)}/M_{\text{dust}}$ ratio for compact and extended galaxies on and above the main sequence is consistent within the error bars. This suggests that CO and dust give equivalent results in terms of the molecular gas mass of compact and extended galaxies, without introducing biases.
Table A1. Average gas fractions for extended and compact galaxies across the main sequence using different observable-to-gas conversion factors.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\mu_{\text{gas, Extended, MS}}$ (a)</th>
<th>$\mu_{\text{gas, Compacts, MS}}$ (a)</th>
<th>$\mu_{\text{gas, Extended, off-MS}}$ (b)</th>
<th>$\mu_{\text{gas, Compacts, off-MS}}$ (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bimodal $\alpha_{\text{CO}}$ - Compactness</td>
<td>1.04 ± 0.34</td>
<td>0.12 ± 0.05</td>
<td>2.2 ± 1.7</td>
<td>1.14 ± 0.7</td>
</tr>
<tr>
<td>$\alpha_{\text{CO}(Z)}$</td>
<td>0.83 ± 0.27</td>
<td>0.46 ± 0.20</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Bimodal $\delta_{\text{GDR}}$ - Compactness</td>
<td>0.66 ± 0.34</td>
<td>0.12 ± 0.09</td>
<td>0.93 ± 0.52</td>
<td>0.97 ± 0.80</td>
</tr>
<tr>
<td>$\delta_{\text{GRD}(Z)}$</td>
<td>0.73 ± 0.48</td>
<td>0.36 ± 0.25</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

Notes. (a) $1 \leq \Delta \text{MS} < 3.5$

(b) $\Delta \text{MS} \geq 3.5$

Figure A2. Comparison of the gas mass from $M_{\text{dust}}$ [$M_\odot$] and from $L'_{\text{CO}(2-1)}$ [K km s$^{-1}$ pc$^2$] for the galaxies with a robust dust mass estimate and a secure CO(2-1) detection. The solid line is the 1:1 relation.

Figure A3. The $L'_{\text{CO}(2-1)}/M_{\text{dust}}$ ratio as a function of the main sequence offset (left-hand panel) and of the compactness (right-hand panel) for galaxies with detections or reliable $L'_{\text{CO}(2-1)}$ upper limits and robust measurements of $M_{\text{dust}}$. In the left-hand panel, large filled diamonds and large filled squares show the median $L'_{\text{CO}(2-1)}/M_{\text{dust}}$ ratio for compact and extended galaxies on and above the main sequence, respectively. Large filled diamonds in the right-hand panel show the median $L'_{\text{CO}(2-1)}/M_{\text{dust}}$ ratio for extended and compact galaxies. The errors on the average measurements are the interquartile range of the $L'_{\text{CO}(2-1)}/M_{\text{dust}}$ distribution.

This paper has been typeset from a TeX/LaTeX file prepared by the author.