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New Developments in Flavor Evolution of a Dense Neutrino Gas

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Abstract
Neutrino–neutrino refraction dominates the flavor evolution in core-collapse supernovae, neutron star mergers, and the early Universe. Ordinary neutrino flavor conversions develop on timescales determined by the vacuum oscillation frequency. However, when the neutrino density is large enough, collective flavor conversions may arise because of pairwise neutrino scattering. Pairwise conversions are deemed fast because they are expected to occur on timescales that depend on the neutrino–neutrino interaction energy (i.e., on the neutrino number density) and are regulated by the angular distributions of electron neutrinos and antineutrinos. The enigmatic phenomenon of fast pairwise conversions has been overlooked for a long time. However, because of the fast conversion rate, pairwise conversions could occur in the proximity of the neutrino decoupling region with yet-to-be-understood implications for the hydrodynamics of astrophysical sources and the synthesis of the heavy elements. We review the physics of this fascinating phenomenon and its implications for neutrino-dense sources.
1. INTRODUCTION

Neutrinos are among the most abundant particles in our Universe and play a pivotal role in a variety of astrophysical environments that range from the sun to the most energetic transients (1). Neutrinos interact weakly and have the unique property of changing their flavor while propagating. In neutrino-dense environments, such as core-collapse supernovae, neutron star mergers, and the early Universe, the flavor evolution is vastly affected by the interactions of neutrinos among themselves (2–4) as well as ordinary neutrino interactions with matter. Under certain circumstances, neutrino–matter interactions can lead to the resonant conversion of (anti)neutrinos—the so-called Mikheyev–Smirnov–Wolfenstein (MSW) resonance (5–7).

It is especially relevant to grasp the physics of neutrino–neutrino interactions because it may provide insights into astrophysics, fundamental neutrino properties, and nonstandard scenarios that involve neutrinos. The forward coherent scattering of neutrinos among themselves (8, 9), which is misleadingly called neutrino self-interaction, crucially differs from neutrino interactions with matter. Neutrino–neutrino interactions lead to nonlinear evolution with positive feedback of the neutrino field onto itself (10, 11). As a consequence, the flavor evolution develops a collective nature that manifests itself through the coupling of all momentum modes.

The physics of neutrino–neutrino interactions has mostly been explored in the context of core-collapse supernovae (3, 12–15) that originate from the death of massive stars. Neutrino–neutrino interactions lead to so-called slow conversions (2, 3), which occur on a timescale that is determined by the vacuum oscillation frequency, $\omega = \Delta m^2/2E \simeq 6.3 \text{ km}^{-1}/E (\text{MeV})$, where $\Delta m^2$ is the largest squared mass difference of neutrinos and $E \simeq O(15) \text{ MeV}$ is the typical neutrino energy. The neutrino–neutrino interaction strength, $\mu = \sqrt{2} G_F (n_\nu + n_{\bar{\nu}})$, is determined by the Fermi constant ($G_F$) and the neutrino and antineutrino number density ($n_\nu$ and $n_{\bar{\nu}}$, respectively) for all six flavors; $\mu$ scales with the distance from the neutrino emission surface, but it is $O(10^5) \text{ km}^{-1}$ in the proximity of the neutrino decoupling region. In the widely investigated spherically symmetric supernova model, slow neutrino–neutrino interactions are expected to be relevant away from the
Slow flavor conversion: flavor conversion that develops on a timescale determined mainly by the neutrino vacuum frequency.

Spatial region where (anti)neutrinos decouple, as shown schematically in Figure 1, and they may lead to characteristic signatures in the (anti)neutrino energy distributions, such as the so-called spectral splits (i.e., certain energy modes swap their flavor content according to the neutrino mass ordering) (16, 17). For a long time, spectral splits were considered a characteristic imprint of neutrino–neutrino interactions.

This neat picture has become more complex in the past decade. In fact, given the challenges related to the numerical implementation of the neutrino feedback onto itself, many symmetries were initially imposed to solve the equations of motion. It was soon realized that these symmetries could be broken spontaneously, affect the flavor stability conditions, and trigger flavor conversions at higher densities (18–22). In addition, it became clear that flavor conversions are affected not only by the energy distribution but also by the angular distribution of (anti)neutrinos (3, 4, 16, 17).

The rough understanding of the phenomenology of neutrino–neutrino interactions has been shaken by the realization that, when the neutrino density is large enough, neutral current interactions of the type \( \nu_e(p) + \nu_x(q') \leftrightarrow \nu_x(p) + \nu_e(q') \) and \( \bar{\nu}_x(p) + \bar{\nu}_e(q') \leftrightarrow \bar{\nu}_e(p) + \bar{\nu}_x(q') \), where \( \nu_x = \nu_\mu \) or \( \nu_\tau \) and \( p \) and \( q' \) are the neutrino momenta, are not negligible (23, 24). Pairwise interactions can, strictly speaking, occur even if the vacuum term is zero (different from slow flavor conversions);
they preserve the net neutrino flavor, but they can modify the subsequent charged current interactions. The characteristic scale of pairwise conversions is the neutrino–neutrino interaction strength, $\mu$. Flavor conversions triggered by pairwise neutrino scattering are deemed fast flavor conversions because they develop on timescales that are orders of magnitude smaller than those of slow neutrino–neutrino interactions. Only recently has it been appreciated that, if fast pairwise conversions are at play, they could drastically affect the flavor content in the proximity of the neutrino decoupling region (see Figure 1) with significant possible implications regarding the source dynamics and nucleosynthesis (4, 23–27).

Despite intense theoretical work, our understanding of the role of neutrino–neutrino interactions in dense media, and especially of fast conversions, is still very approximate. A numerical solution entails solving a seven-dimensional transport problem (involving time, three spatial coordinates, and three momentum coordinates) with characteristic quantities that span several orders of magnitude. As such, a realistic, self-consistent study is out of reach given the computational methods and resources currently available. Rather, one of the main goals is to predict whether maximal or negligible flavor mixing could be achieved, in order to grasp the implications of neutrino–neutrino interactions in astrophysics and cosmology.

In this review, we focus on simple examples to outline the phenomenology of fast pairwise conversions, which are the most recent and less explored phenomenon characteristic of neutrino propagation in dense media. In Section 2, the neutrino equations of motion are introduced together with the linear stability analysis and the conditions that might lead to the growth of fast flavor instabilities. Section 3 focuses on the phenomenology of fast pairwise conversions and outlines their dependence on the neutrino vacuum frequency, electron lepton number (ELN), and neutrino advection and collisions. The possible occurrence of fast pairwise conversions in core-collapse supernovae, compact binary mergers, and the early Universe is discussed in Sections 4, 5, and 6, respectively. At the end of the review, we provide a summary and outlook.

2. NEUTRINO FLAVOR CONVERSIONS

In this section, we introduce the equations of motion for neutrinos in the mean-field approximation, although many-body descriptions can also be adopted (28–36). The onset of flavor conversions can be examined through a linear stability analysis applied to the linearized equations of motion. To grasp the development of fast pairwise conversions, we introduce the linear stability analysis technique, discuss the conditions under which flavor conversions develop, and classify the types of flavor instability.

2.1. Equations of Motion

The neutrino evolution in dense media is described in terms of the neutrino flavor field represented by the Wigner-transformed density matrix in the flavor space $\rho(t, \vec{x}, \vec{p})$—or $\tilde{\rho}(t, \vec{x}, \vec{p})$ for antineutrinos—expressed as a function of time ($t$), location ($\vec{x}$), and momentum ($\vec{p}$). The density matrix in the flavor basis, $\rho(t, \vec{x}, \vec{p})$, has elements that are expectation values of bilinear creation and annihilation operators $\langle a_i^\dagger a_j \rangle$, where $i$ and $j$ are flavor indices. The diagonal elements of the density matrix are the occupation numbers, and the off-diagonal elements describe the flavor correlations. The seven-dimensional phase space is not tractable numerically, and it is usually reduced through symmetry assumptions. For example, one often looks for stationary solutions depending only on the radial coordinate, energy, and zenith angle, thus reducing the problem to three dimensions.

For simplicity, in what follows, we rely on a two-flavor approximation in the weak-interaction basis: $(\nu_e, \nu_x)$, where $x$ stands for $\mu$, $\tau$, or a linear combination of the two. The following equations
of motion describe the flavor evolution of ultrarelativistic neutrinos (37, 38):

\[
\begin{align*}
\frac{d}{dt} + \vec{v} \cdot \vec{\nabla}_x + \vec{F} \cdot \vec{\nabla}_{\vec{p}} \rho(\vec{x}, \vec{p}, t) &= -i[H(\vec{x}, \vec{p}, t), \rho(\vec{x}, \vec{p}, t)] + \mathcal{C} \left( \rho(\vec{x}, \vec{p}, t), \tilde{\rho}(\vec{x}, \vec{p}, t) \right), \\
\frac{d}{dt} + \vec{v} \cdot \vec{\nabla}_x + \vec{F} \cdot \vec{\nabla}_{\vec{p}} \tilde{\rho}(\vec{x}, \vec{p}, t) &= -i[H(\vec{x}, \vec{p}, t), \tilde{\rho}(\vec{x}, \vec{p}, t)] + \tilde{\mathcal{C}} \left( \rho(\vec{x}, \vec{p}, t), \tilde{\rho}(\vec{x}, \vec{p}, t) \right),
\end{align*}
\]

where the advective term, \(\vec{v} \cdot \vec{\nabla}_x\), depends on the velocity of the neutrino or antineutrino field \(\vec{v}\) and is nonzero in the presence of spatial variations. The term proportional to the force \(\vec{F}\) is mathematically similar to the advective term; however, it depends on the gradient in the momentum space (39). This term takes into account the change in energy, direction, or both during propagation; for instance, it could result from an external force, such as the gravitational force that can bend the neutrino trajectory. On the right-hand side, \([\ldots, \ldots]\) denotes the commutator of two matrices. The term \(\mathcal{C}\) (or \(\tilde{\mathcal{C}}\)) schematically represents the collision term, which takes into account the nonforward scattering of neutrinos with the medium or with other neutrinos.

The Hamiltonian that describes the temporal evolution of the density matrix is made of three terms (the vacuum term, the matter term, and the neutrino–neutrino term):

\[
H(\vec{x}, \vec{p}, t) = H_{\text{vac}} + H_{\text{mat}} + H_{\nu\nu} \quad \text{and} \quad \dot{H}(\vec{x}, \vec{p}, t) = -H_{\text{vac}} + H_{\text{mat}} + H_{\nu\nu}.
\]

The various terms of the Hamiltonian have the following form:

\[
H_{\text{vac}} = \frac{\omega}{2} \begin{pmatrix}
-\cos 2\theta_V & \sin 2\theta_V \\
\sin 2\theta_V & \cos 2\theta_V
\end{pmatrix},
\]

\[
H_{\text{mat}} = \begin{pmatrix}
\sqrt{2} G_F n_e & 0 \\
0 & 0
\end{pmatrix},
\]

\[
H_{\nu\nu} = \mu \int d\vec{p}' [\rho(\vec{p}') - \tilde{\rho}(\vec{p}')] (1 - \vec{v} \cdot \vec{v}').
\]

The vacuum frequency is \(\omega = \Delta m^2/2E\) (where \(E\) is the neutrino energy and \(\Delta m^2 > 0\) for normal mass ordering and \(\Delta m^2 < 0\) for inverted ordering), \(\theta_V\) is the vacuum mixing angle, \(n_e\) is the effective number density of electrons in the medium, and \(\mu = \sqrt{2} G_F (n_e + n_{\nu})\) is the strength of the \(\nu\nu\) potential, which is proportional to the neutrino and antineutrino number density (\(n_\nu\) and \(n_{\bar{\nu}}\), respectively) for all flavors. Note that we have chosen a basis such that \(H_{\text{vac}}\) and \(H_{\text{mat}}\) have the same sign for neutrinos and the opposite sign for antineutrinos (16). Moreover, we have dropped the terms proportional to the identity matrix in the Hamiltonian because they contribute only to overall phases; these include the neutral current interactions in the matter term as well as a term proportional to the trace in the \(\nu\nu\) interaction Hamiltonian. However, these terms may become nondiagonal—for example, when the Hamiltonian is expanded to include physics beyond the Standard Model (see, e.g., 40, 41)—but such scenarios are not considered here. We also have excluded nonlocal terms from the Hamiltonian where the lepton asymmetry is large, which are not significant in compact astrophysical objects but become important in the early Universe (42–44).

One of the most significant developments in the field of neutrino flavor conversion physics was the observation by Mikheyev, Smirnov, and Wolfenstein that, under certain conditions, the diagonal components of \(H_{\text{vac}}\) and \(H_{\text{mat}}\) can cancel each other, a process that leads to the MSW resonant conversion of (anti)neutrinos (5–7). Similarly, the nonlinear \(\nu\nu\) term in the Hamiltonian can be responsible for nonnegligible flavor conversions because neutrino self-interactions may
trigger an exponential growth of the off-diagonal terms of the Hamiltonian in time. However, a characteristic feature of $\nu\nu$ interactions is that they depend not only on the energy distributions (like MSW oscillations) but also on the angular distributions of neutrinos and antineutrinos.

### 2.2. Linear Stability Analysis

The phenomenology of neutrino–neutrino interactions may seem counterintuitive because of the nonlinear nature of the problem. Reference 45 proposed extending the linear stability analysis technique, which has been widely adopted to understand whether a system has unstable solutions in many areas of physics, to the linearized equations of motion of neutrinos. The linear stability analysis allows us to predict the conditions under which $\nu\nu$ interactions lead to the growth of flavor instabilities (i.e., possibly to flavor mixing).

For the sake of simplicity, let us consider a homogeneous gas with no collisions and no external force. In addition, let us assume that neutrinos are monoenergetic and that the momentum depends on two angles: the polar angle $\theta$ and the azimuthal angle $\phi$. For each given tuple $(\theta, \phi)$, the density matrix encoding the flavor information of (anti)neutrinos traveling in that direction at a given time can be defined as follows:

$$
\rho(x, \theta, \phi, t) = \begin{pmatrix}
\rho_{ee}(x, \theta, \phi, t) & \epsilon(x, \theta, \phi, t) \\
\epsilon^*(x, \theta, \phi, t) & \rho_{\nu\nu}(x, \theta, \phi, t)
\end{pmatrix},
$$

$$
\tilde{\rho}(x, \theta, \phi, t) = \begin{pmatrix}
\tilde{\rho}_{ee}(x, \theta, \phi, t) & \tilde{\epsilon}(x, \theta, \phi, t) \\
\tilde{\epsilon}^*(x, \theta, \phi, t) & \tilde{\rho}_{\nu\nu}(x, \theta, \phi, t)
\end{pmatrix},
$$

where $|\epsilon(x, \theta, \phi, t)| \ll |\rho_{ee}(x, \theta, \phi, t) - \rho_{\nu\nu}(x, \theta, \phi, t)|$ and $|\tilde{\epsilon}(x, \theta, \phi, t)| \ll |\tilde{\rho}_{ee}(x, \theta, \phi, t) - \tilde{\rho}_{\nu\nu}(x, \theta, \phi, t)|$.

The equations of motion (Equations 1 and 2) can be expanded in series in $\epsilon$ (and $\tilde{\epsilon}$), and it can easily be seen that the evolution of $\Delta(x, \theta, \phi, t)$ goes like $O(\epsilon^2)$. By focusing on the leading order (and therefore neglecting $\Delta$), the equation of motion for $\epsilon(x, \theta, \phi, t)$ is

$$
\left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{V}\right) \epsilon(x, \theta, \phi, t) = -t \int d\cos \theta' d\phi' U(x, \theta, \phi, \theta', \phi', t) \epsilon(x, \theta', \phi', t).
$$

A similar equation holds for $\tilde{\epsilon}$. The evolution of the equation above suggests that the off-diagonal terms of the density matrix may rapidly become comparable to the diagonal terms. When this happens, the evolution of the off-diagonal terms is no longer exponential; this is called the nonlinear regime in the literature, and we have to rely on numerical techniques to gain any insight.

If the matrix $U$ has a complex eigenvalue, an exponential growth of the corresponding eigenvector will occur, which in turn contributes to the off-diagonal component of the Hamiltonian triggering the onset of neutrino flavor conversions. The exponential growth of a particular eigenvector is called flavor instability (45, 46). Although the existence of flavor instability is a promising indicator of flavor conversions, significant flavor conversions are not guaranteed if a flavor instability exists.

If we assume that $\epsilon$ and $U$ do not depend on $\phi$, the linearized equations of motion for neutrinos and antineutrinos become

$$
t \left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{V}\right) \epsilon(x, \theta, t) = (H_{ee} - H_{\nu\nu}) \epsilon(x, \theta, t)
$$

$$
+ (\rho_{\nu\nu} - \rho_{ee}) \mu \int d\cos \theta' \left[\epsilon(x, \theta', t) - \tilde{\epsilon}(x, \theta', t)\right] \left(1 - \vec{v} \cdot \vec{V}\right),
$$
The following equation can be computed by solving the following equation (4):

\[ \frac{d}{dt} \left( \frac{1}{\Omega_1} \right) \epsilon(\bar{x}, \theta, t) = (\bar{H}_{ee} - \bar{H}_{xx}) \epsilon(\bar{x}, \theta, t) + (\bar{\rho}_{xx} - \bar{\rho}_{ee}) \mu \int d \cos \theta' \left[ \epsilon(\bar{x}, \theta', t) - \epsilon(\bar{x}, \theta', t) \right] (1 - \bar{\nu} \cdot \bar{v}), \tag{11} \]

where the terms \( \rho_y \) and \( H_y \) denote the elements of the 2 \times 2 density matrix and the Hamiltonian, respectively. The evolution of neutrinos and antineutrinos is coupled when the \( \nu \bar{\nu} \) term dominates; hence, the flavor instabilities of (anti)neutrinos grow at the same rate (45):

\[ \epsilon(\bar{x}, \theta, t) = Q_\theta e^{-i(\Omega - \bar{k} \cdot \bar{x})} \quad \text{and} \quad \bar{\epsilon}(\bar{x}, \theta, t) = \bar{Q}_\theta e^{-i(\Omega - \bar{k} \cdot \bar{x})}. \tag{12} \]

When \( \Omega \) and \( \bar{k} \) have nonzero imaginary solutions, the flavor instability grows exponentially. In the homogeneous case \((\bar{k} = 0)\), the so-called temporal instabilities occur for \( \text{Im}(\bar{k}) \neq 0 \) and grow in time exponentially. If \( \Omega = 0 \), the so-called spatial instabilities occur for \( \text{Im}(\bar{k}) \neq 0 \); in this case, the flavor instabilities grow in space (4).

In the homogeneous case \((\bar{k} = 0)\), substituting Equation 12 in Equations 10 and 11, we obtain the following:

\[ [\Omega - (\bar{H}_{ee} - \bar{H}_{xx})] Q_\theta e^{-i\Omega t} = (\bar{\rho}_{xx} - \bar{\rho}_{ee}) \mu \int d \cos \theta' (Q_{\theta'} - \bar{Q}_{\theta'}) e^{-i\Omega t} (1 - \bar{\nu} \cdot \bar{v}), \tag{13} \]

\[ [\Omega - (\bar{H}_{ee} - \bar{H}_{xx})] \bar{Q}_\theta e^{-i\Omega t} = (\bar{\rho}_{xx} - \bar{\rho}_{ee}) \mu \int d \cos \theta' (\bar{Q}_{\theta'} - \bar{Q}_{\theta'}) e^{-i\Omega t} (1 - \bar{\nu} \cdot \bar{v}). \tag{14} \]

These equations allow us to express \( Q_\theta \) and \( \bar{Q}_\theta \) in the following parametric form:

\[ Q_\theta = (\bar{\rho}_{xx} - \bar{\rho}_{ee}) - \frac{(a - b \cos \theta)}{\Omega - (\bar{H}_{ee} - \bar{H}_{xx})} \quad \text{and} \quad \bar{Q}_\theta = (\bar{\rho}_{xx} - \bar{\rho}_{ee}) - \frac{(a - b \cos \theta)}{\Omega - (\bar{H}_{ee} - \bar{H}_{xx})}, \tag{15} \]

which depends on the parameters \( a \) and \( b \) as well as \( \Omega \). From here, the eigenvalue \( \Omega \) can be computed by solving the following equation (4):

\[ \begin{vmatrix} I[1] - 1 & -I[\cos \theta] \\ I[\cos \theta] & -I[\cos^2 \theta] - 1 \end{vmatrix} = 0, \tag{16} \]

where

\[ I[f(\theta)] = \int d \cos \theta f(\theta) \left[ \frac{(\bar{\rho}_{ee} - \bar{\rho}_{xx})}{\Omega - (\bar{H}_{ee} - \bar{H}_{xx})} - \frac{(\bar{\rho}_{ee} - \bar{\rho}_{xx})}{\Omega - (\bar{H}_{ee} - \bar{H}_{xx})} \right]. \tag{17} \]

Equation 16 can be solved analytically with respect to \( \Omega \) for special classes of angular distributions of (anti)neutrinos (see, e.g., 22, 26). Similarly, one can follow the same procedure and look for \( \text{Im}(\bar{k}) \neq 0 \) when \( \Omega = 0 \) or, more generally, for imaginary solutions of \( (\Omega, \bar{k}) \).

To investigate the growth of flavor instabilities, we consider a perfectly homogeneous neutrino gas, ignore the advective term \( \bar{v} \cdot \bar{V}_r \), and consider the following angular distributions (Case 1):

\[ \rho_{ee}(t = 0) = 0.5, \quad \bar{\rho}_{ee}(t = 0) = \begin{cases} 1 & \theta \in [0, \pi/3] \\ 0.25 & \theta \in [\pi/3, \pi] \end{cases} \quad \text{and} \quad \rho_{xx}(t = 0) = \bar{\rho}_{xx}(t = 0) = 0. \tag{18} \]
such that \( \int \cos \theta \rho_{\alpha \beta} (t=0) = 1 \). The blue line in Figure 2a shows \( \text{Im}(\Omega) \) as a function of \( \mu \); one can see that the flavor instability exists irrespective of the value of the self-interaction potential and grows linearly with \( \mu \). A similar trend is found for \( \text{Im}(\vec{k}) \) (see the purple line in Figure 2a).

Assuming \( \vec{k} = 0, \mu = 10^3 \text{ km}^{-1}, \Delta m^2 = 2.5 \times 10^{-3} \text{ eV}^2, \theta_V = 10^{-6}, \) and \( E = 10^4 \text{ MeV}, \) Figure 2b shows the temporal evolution of the angle-integrated off-diagonal term of the density matrix based on the predictions of the linear stability analysis and the numerical solution of the equations of motion (Equations 1 and 2).\(^1\) The linear stability analysis perfectly agrees with the numerical solution in the linear regime, but it cannot give any insight into the nonlinear evolution of the density matrix. The linear stability analysis aims to predict whether the (anti)neutrino ensemble may develop flavor instabilities for given initial conditions. However, the linear stability analysis cannot predict the final flavor outcome because this outcome is strongly affected by the nonlinear regime of neutrino–neutrino interactions.

2.3. Fast Pairwise Conversions

While slow \( \nu \nu \) conversions become relevant relatively far from the source (see Figure 1), fast pairwise conversions are expected to occur at high densities in the proximity of the decoupling region (23–25). Being determined by pairwise scattering of (anti)neutrinos, they could occur even for \( \Delta m^2 = 0 \); hence, in the absence of external perturbations and for \( \omega \to 0 \), fast pairwise conversions are such that the net electron, mu, or tau flavor number carried by neutrinos is separately conserved in addition to the total lepton number.

The linear stability analysis predicts that fast pairwise conversions of neutrinos should arise, for an arbitrarily large self-interaction potential, in a neutrino gas where the number density of

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\(^1\)To speed up the numerical computations, the value of \( \mu \) adopted in our examples is smaller than the typical neutrino self-interaction strength in the decoupling region \([O(10^5)] \text{ km}^{-1}\). A larger \( \mu \) would lead to the development of flavor conversions on scales smaller than what is shown here, but it would not affect the overall flavor phenomenology.
LIMITATIONS OF THE LINEAR STABILITY ANALYSIS

The linear stability analysis, and therefore the dispersion relation in the flavor space, predicts the existence of flavor instabilities on the basis of the initial setup of the neutrino ensemble at a given location \( \vec{x} \) and time \( t \); it is a local analysis technique. The existence of flavor instabilities is a necessary, but not sufficient, condition for significant flavor conversions. The flavor conversion physics is strongly affected by the nonlinear regime of flavor conversions that is not captured by the linearized equations of motion.

\( \tilde{\nu}_e \) is the same as that of \( \nu_e \) along some direction, but not along other directions. That is, there is at least one angle at which a crossing between the angular distributions of \( \nu_e \) and \( \tilde{\nu}_e \) occurs (ELN crossing, temporal instability), or there is a nonnegligible backward flux of neutrinos (spatial instability) (4, 27). Because the main parameters determining the flavor evolution are \( \omega \) and \( \mu \), with \( \mu \gg \omega \), we can deduce that the fast flavor instability has a growth rate proportional to \( \mu \) (4, 26, 27). Therefore, it is expected to lead to fast neutrino conversions. The linear stability analysis can be performed in the context of fast pairwise conversions through the formalism described in Section 2.2; however, in the \( \omega \rightarrow 0 \) limit, a formally elegant dispersion relation in the flavor space can be obtained (27).

The favorable conditions that trigger fast flavor instabilities [i.e., an ELN crossing or a nonnegligible backward flux of (anti)neutrinos] are formally connected (47). To show this, let us assume an axially symmetric and stationary configuration with the axis of symmetry along the radial direction and ignore the collision term. Equations 1 and 2 then become

\[
\frac{\partial}{\partial r} \rho(\vec{x}, \vec{p}) = -\frac{i}{\cos \theta} [H(\vec{x}, \vec{p}), \rho(\vec{x}, \vec{p})] \quad \text{and} \quad \frac{\partial}{\partial r} \tilde{\rho}(\vec{x}, \vec{p}) = -\frac{i}{\cos \theta} [\bar{H}(\vec{x}, \vec{p}), \tilde{\rho}(\vec{x}, \vec{p})],
\]

where \( \vec{x} \) is defined by \((r, \cos \theta)\) in polar coordinates, and the term \( \cos \theta \) in the denominator is due to the inner product of the neutrino velocity and the spatial gradient. If \( \cos \theta \) is absorbed in the Hamiltonian, the condition that is required for fast neutrino conversions to occur is a crossing between \( \rho_{ee}/\cos \theta \) and \( \tilde{\rho}_{ee}/\cos \theta \). Because \( \cos \theta \) can have negative values for a flux going radially inward, spatial fast flavor instabilities may exist when a backward neutrino flux is present, even in the absence of ELN crossings. The purple line in Figure 2a shows the growth rate for the spatial instability (see Section 2.2). Because of the additional \( \cos \theta \) factor in the denominator, the growth rate is substantially modified with respect to the temporal instability (blue line in Figure 2a). In this sense, it is appropriate to consider effective ELN crossings in \((\rho_{ee} - \tilde{\rho}_{ee})/\cos \theta \) (47).

Because the evolution of fast neutrino conversions depends on the shape of the angular distributions, quantifying the entity of the flavor instability is not straightforward. However, a heuristic parameter, the so-called ELN parameter, has been proposed (48):

\[
\zeta = \frac{I_1 I_2}{(I_1 + I_2)^2},
\]

with

\[
I_1 = \int_0^\pi \Theta[\rho_{ee}(\theta) - \tilde{\rho}_{ee}(\theta)]d\cos \theta \quad \text{and} \quad I_2 = \int_0^\pi \Theta[\tilde{\rho}_{ee}(\theta) - \rho_{ee}(\theta)]d\cos \theta,
\]

ELN crossing: crossing between the angular distributions of electron neutrinos and antineutrinos.

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where $\Theta$ is the Heaviside function. It can easily be seen that $\xi$ vanishes for no ELN crossings; however, the growth rate may not be proportional to $\xi$. In the steady-state configuration, an effective ELN parameter is obtained by replacing $\rho \rightarrow \rho / \cos \theta$.

We stress that although the presence of effective ELN crossings typically gives rise to fast flavor instabilities, their existence is a necessary, but not sufficient, condition for the occurrence of fast neutrino conversions. In particular, while each ELN crossing triggers an axially symmetry-breaking instability, if more than one crossing occurs, it is not known whether axially symmetric instabilities exist (49, 50). A simple example, which can be verified analytically, is that obtained for the following initial conditions: $\rho_{ee}(\theta) = \text{constant}$ and $\bar{\rho}_{ee}(\theta) \propto \sin \theta$ for $\theta \in [0, \pi]$. This configuration can clearly give rise to two ELN crossings for the right normalization of $\bar{\rho}_{ee}$, but it does not lead to fast flavor conversions.

The dispersion relation approach (27) allows us to classify fast flavor instabilities according to two categories (51). The first type, an absolute instability, is one that causes exponential growth of the off-diagonal elements at the point of the initial perturbation. The second type, a convective instability, is one in which the off-diagonal elements decay at the point of the initial perturbation and the instability moves away faster than it spreads (49, 51–53). The existence of ELN crossings is expected to lead to an absolute instability (51). Note, however, that the classification of an instability as convective or absolute is predicted on the basis of the initial angular distributions; it is an approximation because neutrino advection is not negligible in a realistic framework. The impact of neutrino advection on the classification of the nature of the instability is largely unexplored (49, 54) (see the sidebar title Limitations of the Linear Stability Analysis).

The dispersion relation approach (27) has been extended to a three-flavor framework in References 55 and 56, leading to an interesting phenomenology in the limit where the $\nu_\mu$ and $\nu_\tau$ fluxes are not identical. The dispersion relation has also been applied to scenarios that involve nonstandard neutrino interactions (57).

3. PHENOmenology OF FAST PAIRWISE CONVERSIONS

In this section, we discuss the dependence of fast pairwise conversions on neutrino energy and the eventual presence of ELN crossings in the angular distributions. We then explore the interplay between fast pairwise conversions, neutrino advection, and collisions. To explore the phenomenology of fast pairwise conversions, we rely on the simple system introduced in Equation 18 (Case 1); in addition, we adopt $\mu = 10^3 \text{ km}^{-1}$, $\Delta m^2 = +2.5 \times 10^{-3} \text{ eV}^2$ (normal ordering), and $\theta_V = 10^{-6}$ to take into account the effective mixing suppression due to matter effects (58), unless otherwise specified.

3.1. Dependence on Neutrino Energy

Fast pairwise conversions have traditionally been explored by neglecting the vacuum term in the Hamiltonian (see, e.g., 26) under the assumption that the vacuum frequency is negligible with respect to the $\nu \nu$ interaction strength. Reference 59 has extended the dispersion relation approach of Reference 27 to the most general case when slow and fast conversions can both occur. By focusing on the linear regime, the dispersion relation can exhibit fast modes for $\omega \neq 0$, and the growth rate of the instability can be $O(\mu)$ (fast instability) or $O(\sqrt{\omega \mu})$ (slow instability).

Intriguingly, while the dependence of the linear growth rate on $\omega$ is not dramatic for neutrino energies typical of neutrino-dense environments, and therefore the approximation adopted in References 26 and 27 is good enough, the onset of flavor conversions is affected by $\omega$ (60–62). This is shown in Figure 3a, where the angle-integrated off-diagonal term of the density matrix is plotted.
as a function of time for Case 1 and three different neutrino energies in normal ordering ($E = 1$, 10, and $10^4$ MeV, the last of which mimics the case of $\omega \rightarrow 0$ often adopted in the literature). Figure 3b shows the angle-averaged transition probability,

$$\langle P_{ex}\rangle(t) = 1 - \frac{\int \rho_{ee}(\theta,t)d\cos\theta - \int \rho_{xx}(\theta,t=0)d\cos\theta}{\int \rho_{ee}(\theta,t=0)d\cos\theta - \int \rho_{xx}(\theta,t=0)d\cos\theta},$$

where $\langle P_{ex}\rangle$ describes the average amount of flavor conversion even though the transition probability may be larger or smaller along any specific angular direction. For $E = 10$ and $10^4$ MeV, one can see a clear periodic trend in $\langle P_{ex}\rangle$ (60–62). In addition, an earlier onset of flavor conversions and an increase in the oscillation frequency occur as $\omega$ increases (60, 62). This is due to the fact that the system is driven by two characteristic frequencies, $\omega$ and $\mu$, and as $\omega$ increases, the onset of the nonlinear regime occurs earlier. Hence, the pendulum analogy, introduced to explain $\nu\nu$ interactions in the case of slow conversions (63), does not hold for fast pairwise conversions unless $\omega \rightarrow 0$ (60–62). For $E = 1$ MeV, the oscillation frequency is higher and the bipolar behavior is partially lost because of the overlap of two different frequencies dominating the precession (62). For Case 1 with $E = 1$ MeV, Figure 4 displays the angular distributions of $\nu_e$ and $\bar{\nu}_e$ before (Figure 4a) and after (Figure 4b) fast pairwise conversion. One can clearly see that the flavor instability arises in the proximity of the ELN angular crossing and spreads through the neighboring angular bins.

As opposed to slow neutrino self-interactions, a peculiar feature of fast pairwise conversions is that they are not strongly affected by the energy distribution of (anti)neutrinos, and the typical average vacuum frequency accurately reflects the behavior of the system (62). However, as shown by the comparison between normal and inverted mass ordering for $E = 1$ MeV in Figure 3a, the growth rate is steeper in inverted ordering, and the onset of flavor conversions is reached earlier for the setup we consider, in analogy to the slow conversion case (16, 63).
3.2. Dependence on Electron Lepton Number Crossings

The occurrence of ELN crossings is deemed to be one of the crucial factors triggering fast pairwise conversions (27). The dependence of flavor conversions on ELN crossings has been explored in References 53 and 64 in the linear regime. As the ELN distribution changes continuously, a theory of the critical points of the dispersion relation relies on the development and evolution of the branches of this dispersion relation to classify the instability type (53). However, for flavor conversions to grow, the ELN crossings have to be self-sustained in time (48). For example, neutrino advection can smear the ELN crossings and thereby hinder the development of fast pairwise conversions.

To explore how the flavor conversion physics is affected by the ELN crossings in the nonlinear regime, we consider two other configurations (Cases 2 and 3) in addition to Case 1 (introduced in Equation 18). Cases 2 and 3 are defined by keeping $\rho_{ee}(t = 0)$ the same as in Case 1, $\rho_{xx}(t = 0) = \bar{\rho}_{xx}(t = 0) = 0$, and

$$
\bar{\rho}_{ee}, \text{ Case 2} (t = 0) = \begin{cases} 
2 \rho_{ee}(t = 0) & \theta \in [0, \pi/4] \\
0.5 \rho_{ee}(t = 0) & \theta \in [\pi/4, \pi]
\end{cases},
$$

$$
\bar{\rho}_{ee}, \text{ Case 3} (t = 0) = \begin{cases} 
1.5 \rho_{ee}(t = 0) & \theta \in [0, \pi/3] \\
0.5 \rho_{ee}(t = 0) & \theta \in [\pi/3, \pi]
\end{cases}.
$$

Cases 1, 2, and 3 are shown in Figure 4a; note that compared with Case 1, the top-hat distribution of $\bar{\nu}_e$ is different for Case 2 in terms of width and for Case 3 in terms of height. Hence, the three cases display different ELN crossings, as indicated by the instability parameter in the plot legend (see Equation 20).

Figure 5 shows the temporal evolution of the angle-averaged off-diagonal term of the density matrix (Figure 5a) and survival probability (Figure 5b) for Cases 1, 2, and 3. One can see that, for larger $\zeta$, the onset of flavor conversions occurs earlier. Note, however, that the transition probability has the largest oscillation amplitude for the smallest $\zeta$ parameter (Case 3), and it is not a monotonic function of $\zeta$ (48). The behavior of Case 3 highlights the limitations intrinsic to the

---

**Figure 4**

(a) Initial angular distributions for Cases 1, 2, and 3 (see Section 3.2). The instability parameter $\zeta$ (Equation 20) is reported in the legend. (b) Angular distributions of $\nu_e$ and $\bar{\nu}_e$ for $E = 1$ MeV at $t = 5 \times 10^{-6}$ s (solid lines) and $t = 0$ (dashed lines). Flavor conversions develop in the proximity of the electron lepton number crossing and spread in the neighboring angular bins.
Case 1
Case 2
Case 3

Figure 5
Dependence of fast pairwise conversions on electron lepton number crossings. (a) Temporal evolution of the angle-integrated off-diagonal term of the density matrix for Cases 1, 2, and 3 (see Section 3.2). The onset of the nonlinear regime occurs earlier for larger ζ. (b) Temporal evolution of the angle-averaged transition probability. The oscillation amplitude does not grow monotonically with ζ.

predictive power of ζ; in fact, to fully predict the flavor conversion outcome, one should take into account the slope in addition to the width of the ELN crossing.

3.3. Dependence on Neutrino Advection and Collisions

The angular distributions of (anti)neutrinos play a major role in determining the development of fast neutrino conversions, but the origin of the angular distributions is a largely unexplored topic in the context of flavor conversions (for examples of approximate implementations, see 65–67; see also the sidebar titled Numerical Artifacts in the Evolution of Neutrino Flavor). The collision term in the equations of motion is used to incorporate absorption, emission, and momentum-changing scatterings of neutrinos. The spatial distribution of these terms in conjunction with the advective term (\(\vec{v} \cdot \nabla_x\)) determines the angular distribution of neutrinos at any given location. A flavor-dependent collision term has the effect of reducing the coherence between the flavor eigenstates, whereas a flavor-independent collision term will transport all angular modes.

In the deep interior of compact objects, the angular distributions of all neutrino flavors are isotropic because of the high collisional rate. As the matter density decreases, neutrinos approach the free-streaming regime, and their angular distribution becomes forward peaked (68, 69). The intermediate region (i.e., before decoupling is complete) is of significance to fast conversions because flavor evolution, advection, and collisions may be at play simultaneously (61, 65, 69). In the

**NUMERICAL ARTIFACTS IN THE EVOLUTION OF NEUTRINO FLAVOR**

It is well known that multiangle calculations of \(\nu\nu\) interactions may be plagued by spurious instabilities (72, 73); this problem could also arise in the case of fast pairwise conversions. In addition, it has been shown that fast conversions may enhance a cascade of flavor field power from large angular scales to small scales, hastening relaxation (74, 75). The momentum-space cascade can then induce numerical errors that may propagate back to larger scales, with possible major consequences on the isotropic moment. This problem has not yet been studied in the presence of collisions, and therefore it is not known whether collisions could partially cure the back-reaction triggered by the instabilities that develop on small scales.
earlier literature on $\nu\bar{\nu}$ interactions, neutrinos were assumed to decouple at a well-defined radius (16). This assumption is reasonable in the case of slow $\nu\nu$ interactions because neutrino flavor conversions occur away from the decoupling region (see, e.g., Figure 1). For fast conversions, however, this assumption needs renewed scrutiny because the distinction between the region of neutrino trapping (isotropic angular distributions) and free streaming (forward-peaked angular distributions) is gradual, and fast flavor conversions may occur in between.

The $\bar{\nu}_e$ cross section is typically smaller than the $\nu_e$ one; hence, $\bar{\nu}_e$s start decoupling earlier than $\nu_e$s. This can result in ELN crossings, as discussed in Reference 66. Reference 66, however, did not consider the occurrence of ELN crossings (and therefore flavor conversions) that eventually are caused by large-scale asymmetries (70) or turbulent matter density fluctuations (71). The occurrence of ELN crossings or lack thereof depends on the ratio of electron neutrino and antineutrino number densities (66). A large difference between $n_{\nu_e}$ and $n_{\bar{\nu}_e}$ implies that no ELN crossing can occur. However, if ELN crossings exist, flavor conversions may dynamically modify the angular distributions and, in turn, the ELN crossings. Importantly, the ELN crossings are also dynamically affected by collisions (66).

The effect of collisions on fast neutrino conversions is largely unexplored despite its likely relevance in triggering flavor conversions (65) and in determining the final flavor outcome (67). For simplicity, we assume a flavor-independent collision term, which is number conserving and energy independent, and assume that the collision term for neutrinos is twice that of antineutrinos and is equal to the inverse of the neutrino mean free path for all angular bins ($C = \bar{C}/2 = 1 \text{ km}^{-1}$) for Case 1 with $E = 10$ MeV. As shown in Figure 6, the collision term can significantly enhance the flavor conversion probability because of its dynamical effect on the angular distributions (67). The results shown in Figure 6 ignore the advective term in the equations of motion. Advection could mix different angular modes as a function of time. Although created within a simplified framework, Figure 6 suggests a possible nonnegligible interplay of fast flavor conversions with collisions and advection, which may imply a need to tackle flavor evolution as a time-dependent boundary value problem.
In this section, we first provide a brief overview of core-collapse supernova physics and then describe the conditions favorable to the development of fast pairwise conversions in core-collapse supernovae, as well as their possible implications.

4.1. Core-Collapse Supernovae

Core-collapse supernovae represent the final life stage for stars with masses of at least $8 \, M_\odot$ (1, 12–15). A supernova is effectively a blackbody source of (anti)neutrinos of all flavors; when the stellar core collapses, neutrinos carry 99% of the gravitational binding energy ($E_b \approx 3 \times 10^{53}$ erg). Our current understanding of stellar core collapse is based on the delayed neutrino-driven explosion mechanism (76). A massive star has an onion-like structure with an iron core and lighter elements in the outer shells. The iron core undergoes a homologous collapse until it reaches a nuclear density of $\mathcal{O}(3 \times 10^{14})$ g cm$^{-3}$, at which point the sudden halt of the collapse results in a shock wave (77). The shock wave propagates radially outward, but it stalls as it loses energy by photo-dissociating iron group nuclei. Neutrinos revive the shock by depositing energy, and a successful explosion takes place.

The typical neutrino signal lasts $\mathcal{O}(10)$ s, and the nonelectron neutrinos and antineutrinos have very similar energy distributions because of the typical energies involved, although some muons could be present (78). At first, the prompt neutronization burst of $\nu_s$ occurs when the shock wave breaks through the iron shell. The accretion phase follows, during which $\nu_e$ and $\bar{\nu}_e$ are emitted with similar energy luminosities and different average energies. The main production and interaction channel goes through beta reactions that involve the electron flavors, whereas the nonelectron flavors are produced in pairs in the inner layers and have lower energy luminosities. Soon after the explosion, the newly formed neutron star cools and deleptonizes, and the neutrino fluxes of all flavors become similar to each other. For detailed overviews, we refer the reader to, for instance, References 1, 3, 12, and 15.

Hydrodynamical simulations of core collapse have approached the 3D frontier (79, 80). Given the challenges involved in the modeling of neutrino flavor conversion physics, the neutrino transport equations in hydrodynamical simulations do not include flavor conversions (81). Instead, flavor conversions are investigated in a postprocessing phase. This approach is justified because, according to the classic picture, MSW conversions and $\nu\bar{\nu}$ interactions in the slow regime should occur beyond the shock radius in a spherically symmetric supernova model, as shown in Figure 1. As such, flavor conversions would be relevant only for detection purposes and for the nucleosynthesis in the neutrino-driven wind (3). However, this simplified picture neglects the occurrence of large-scale asymmetries (79, 82) and effective ELN crossings; fast flavor conversions could then potentially occur in the proximity of the decoupling region during the supernova accretion phase and possibly affect the supernova physics itself (25, 27).

4.2. Fast Pairwise Conversions in Supernovae

Because of the potential implications of fast pairwise conversions for supernova physics, various groups have looked for the occurrence of ELN crossings in hydrodynamical simulations of core-collapse supernovae. The first attempt, which was carried out in Reference 69, looked for ELN crossings in a set of spherically symmetric (one-dimensional) hydrodynamical simulations; no evidence of ELN crossings was found. Multidimensional simulations are naturally more prone to develop large-scale asymmetries in the ELN emission—for instance, in the presence
of lepton-emission self-sustained asymmetry (LESA) (70). However, with some exceptions (e.g., Reference 83), most of the available multidimensional simulations (e.g., 70, 83–92) track only the “moments”—that is, the energy-dependent angular integrals of the neutrino phase space distributions—because it is computationally too demanding to store and self-consistently compute the fully angle-dependent distributions as functions of time. Therefore, to diagnose the possible presence of ELN crossings (which, however, does not automatically imply the existence of flavor instabilities), alternative methods have been proposed (93, 94). For example, a Fourier mode of the flavor instability (the “zero mode,” which has a growth rate depending on the ELN angular moments up to the second order) was identified in Reference 93. The growth rate of this mode approximates that of flavor conversions (93). Along the same lines, a method based on higher angular moments was proposed recently (94).

As discussed in Reference 66, ELN crossings are expected to occur when the asymmetry parameter $\gamma = n_{\nu_\mu}/n_{\nu_e} \simeq 1$. References 95 and 96 found ELN crossings deep in the proto–neutron star region for a number of isolated points where $\gamma \simeq 1$. They linked the occurrence of ELN crossings with locations where the $\nu_e$ chemical potential $\mu_{\nu_e} \simeq 0$ and the electron fraction is relatively low. In addition, it has been suggested (97) that the appearance of light nuclei (mostly $\alpha$ particles) may support the development of ELN crossings by enhancing the chemical potential difference between protons and neutrons.

A similar analysis has been carried out in Reference 98, which searched for ELN crossings by adopting the method proposed in Reference 93. Favorable conditions for the development of fast flavor instabilities were found deep in the convective layer of the proto–neutron star; the decline of the electron fraction and the increases in density and temperature drive the electron–neutrino chemical potential to negative values, and hence an excess of $\bar{\nu}_e$ over $\nu_e$ occurs. The findings of Reference 98 confirm the overall conclusions of References 97 and 99. However, Reference 98 points out that the spatial locations of the ELN crossings develop in time within the boundary layers of large-scale and long-lasting volumes where the $\bar{\nu}_e$ density exceeds the $\nu_e$ density.

The ELN crossings may also occur in the postshock flows (100); in the proximity of the proto–neutron star, $\bar{\nu}_e$ dominates over $\nu_e$ in the forward direction, whereas the converse is true at larger radii where $\nu_e$ dominates over $\bar{\nu}_e$ in the forward direction because of the residual coherent neutrino-nucleus scattering (101). The small ELN crossings generated at large radii seem to lead to flavor instabilities according to the stability analysis. These instabilities would not have major consequences on the supernova dynamics but may still affect the nucleosynthesis in the neutrino-driven ejecta.

The consequences of the occurrence of fast pairwise conversions on the supernova neutrino-driven wind nucleosynthesis have been explored under the extreme assumption that fast pairwise conversions may lead to flavor equilibration (102). Flavor equilibration may significantly favor proton-rich conditions with an enhancement of the total mass loss by a factor of $O(1.5)$. This would have important implications for abundances in metal-poor stars and Galactic chemical evolution.

It is worth noting that if fast pairwise conversions occurred in the proto–neutron star layer and its surroundings, they would affect the neutrino spectra formation and the supernova dynamical evolution. Moreover, neutrino advection could smear the ELN crossings (48) (unless they are self-sustained) and therefore hinder the development of fast pairwise conversions. This section should thus be considered a discussion of potentially interesting effects of pairwise conversions on the supernova physics; only a fully self-consistent solution of the neutrino transport could shed light on the role of neutrino conversions in the stellar core and in supernova nucleosynthesis.
5. FAST PAIRWISE CONVERSIONS IN NEUTRON STAR Mergers

In this section, after a brief overview of the neutron star merger physics, the role of fast pairwise conversions is outlined together with the implications for the nucleosynthesis of the heavy elements.

5.1. Compact Binary Merger Remnants

A compact binary merger originates from the coalescence of two neutron stars or a neutron star and a black hole. The central remnant could be a massive neutron star or a black hole. Because of the merging, an accretion disk forms surrounding the central remnant. Neutrinos are copiously produced during the coalescence, and the neutrino energy luminosity may reach up to $10^{54}$ erg s$^{-1}$ within $\mathcal{O}(100)$ ms (103, 104).

Because of the low local merger rate, the probability of detecting thermal neutrinos from neutron star mergers is negligible (105). However, neutrinos may play an important indirect role. The neutron richness of the ejecta may be affected by neutrino absorption on matter dynamically ejected during the merger, and the r-process nucleosynthesis and the related kilonova light curve may be affected by the neutrino field (106–110). In addition, neutrinos play an important role in the merger cooling, and neutrino pair annihilation may aid the short $\gamma$-ray burst harbored by the merger (107, 111–121). Multimessenger observations (122–124) of the gravitational wave event GW170817 have confirmed the theory according to which compact binary mergers are among the main sites where elements heavier than iron form through the r-process; however, many unknowns remain.

Once the accretion disk forms, up to 20% of the initial disk mass can be ejected. The dynamical ejecta are the earliest matter outflows (114, 115, 125); they originate from the outermost layers, which are unbounded by means of tidal torques. For the first few hundred milliseconds, a neutrino-driven wind is emitted from the hot inner disk (112, 126). Within a few seconds, viscously driven ejecta are then emitted (126–129). The neutrino-driven wind can be relevant in the surroundings of the polar region, where it dominates the ejecta in a cone centered around the polar axis with a half-opening angle of 10–40° (130). This picture is schematically represented in Figure 7, and it holds independently of the nature of the central compact object, although a more massive neutrino-driven wind is expected in the case of a hypermassive neutron star remnant.

The evolution of the accretion disk can be divided into three stages based on the neutrino emission properties (126). At first, the environment is dense enough to be optically thick to neutrinos; neutrinos are trapped and advected in the flow, and cooling is inefficient. This phase is followed by a period of neutrino-dominated accretion, when the mass of the torus decreases and the density drops; neutrinos radiate most of the gravitational energy converted to internal energy. As mass, temperature, and density decrease, the neutrino production rate decreases until neutrino cooling eventually becomes inefficient. When the remnant compact object consists of a massive neutron star instead of a black hole, the neutrino energy luminosity reaches a plateau rather than decreasing in time (112, 131, 132).

The computational requirements for running three-dimensional, general-relativistic magnetohydrodynamical simulations of compact binary mergers with detailed neutrino transport are not yet available. Hence, the exploration of the role of neutrinos in merger remnants is extremely preliminary. A generic feature of all simulations is the protonization of the merger remnant, which leads to an excess of $\bar{\nu}_e$ over $\nu_e$. Since the neutrino density in compact binary mergers is comparable to that of core-collapse supernovae, $\nu\nu$ interactions should not be negligible. Because of the disk protonization, a cancelation between the $\nu\nu$ interaction strength and the matter potential
may occur in the neutrino equations of motion. This phenomenon is known as matter–neutrino resonance and may affect the final flavor configuration (133–138).

5.2. Fast Pairwise Conversions in Compact Binary Mergers

In neutron star mergers, as in supernovae, the $\bar{\nu}_e$ decoupling occurs at radii smaller than the $\nu_e$ radii. However, the overall flux of $\bar{\nu}_e$ is larger than that of $\nu_e$. As a consequence of the disk protonization and of the toroidal geometry, ELN crossings occur anywhere above the disk remnant (139). By relying on the linear stability analysis, one finds that fast flavor instabilities should occur everywhere above the merger remnant (130, 139). In particular, while temporal flavor instabilities are expected in an extended region above the remnant because of the ubiquitous appearance of ELN crossings, spatial instabilities take place in smaller spatial regions (overlapping with those of temporal instabilities) but with a larger growth rate. In the case of a black hole remnant, the region where flavor instabilities occur tends to shrink after the first $O(10)$ ms (130). For a massive neutron star remnant, the region where the instabilities occur remains stable for a longer time interval (132).

The occurrence of fast flavor instabilities (130, 139), however, does not imply that flavor equilibration can be achieved. For example, by relying on a simplified toy model, Reference 140 explored the nonlinear regime of fast pairwise conversions above the merger remnant disk and found that negligible mixing is achieved ($<1\%$) despite the large growth rate of the fast flavor instability. This result should not be considered a firm conclusion because the collisional term in the equations of
motion was neglected in Reference 140, and this could enhance the flavor conversion probability (67); however, it suggests that we are far from having a clear picture of fast pairwise conversions in compact binary mergers.

If we assume that flavor equilibration is achieved as a result of fast pairwise conversion above the merger remnant disk, it could have consequences on the r-process nucleosynthesis. Reference 130 explored the impact of flavor equilibration on a black hole binary merger remnant and found that the fraction of lanthanides produced in the neutrino-driven wind could be boosted by a factor of $O(10^3)$ with possible implications for the kilonova emission. However, in the case of a massive neutron star compact merger remnant, Reference 132 found that flavor equipartition may induce variations in the polar ejecta by enhancing the iron peak abundances and reducing the first peak abundances, but the fraction of emitted lanthanides is negligibly affected. The different outcomes in the two configurations are mostly due to the different dynamical evolution of the two remnant models, with the trajectories exposed for a longer time to the neutrino wind in the black hole merger remnant case, despite the flavor conversion outcomes being very similar.

6. FAST PAIRWISE CONVERSIONS IN THE EARLY UNIVERSE

The early Universe was rich in neutrinos (1, 141, 142). The relic background of neutrinos from the early Universe has not been detected yet, but we have indirect evidence from Big Bang nucleosynthesis, cosmic microwave background anisotropies, and the formation of cosmic structures. It is important to grasp the neutrino flavor conversion physics in the early Universe as this could have major implications regarding cosmological observables such as the effective number of radiation species ($N_{\rm eff}$), the eventual existence of dark radiation, and, indirectly, the Hubble parameter ($H_0$) and the sum of the neutrino masses.

In the absence of physics beyond the Standard Model, the lepton asymmetry is expected to be on the same order as the baryon asymmetry, making $\nu\nu$ interactions negligible. However, it is possible that electron–positron annihilation favors an excess of $\nu_e$ and $\bar{\nu}_e$ over the other flavors, and this excess might affect neutrino mixing (143). According to the standard picture, the excess of $\nu_e$s and $\bar{\nu}_e$s is small enough to have a negligible effect on the flavor mixing; this finding has also been confirmed by relaxing the assumption of small lepton asymmetry and assuming homogeneity and isotropy (42, 43, 144, 145). However, it is worth noting that current limits on the lepton asymmetry assume that we know how to calculate neutrino flavor conversions in the early Universe and how flavor conversions, in turn, affect the primordial abundances. In the context of slow neutrino self-interactions, by adopting the linear stability analysis, it has been found that anisotropic and inhomogeneous modes can lead to the development of instabilities despite the existence of approximately isotropic and homogeneous initial conditions (22). Thus, synchronized flavor conversions (37, 43, 146, 147) may be part of a much bigger picture (22, 44), and doubts are cast on the lepton asymmetry limits (148).

Numerical work on inhomogeneous and anisotropic neutrino flavor modes in the nonlinear regime in the presence of a self-consistent treatment of collisions has not been attempted yet. The vacuum frequency scales as the inverse of the temperature, whereas the collision term depends on the fifth power of temperature (149); this leads to an enormous range of scales over which numerical simulations have to be performed, making the problem technically challenging to solve. However, the presence of a transient localized flavor instability along with collisions and advection could trigger fast flavor conversions. The presence of inhomogeneous modes in the context of fast flavor conversions could affect the entropy evolution in the early Universe in yet unknown ways. If fast pairwise conversions induce large mixing, an analysis of this sort may bring new insights regarding flavor conversion physics in the early Universe and related cosmological observables.
SUMMARY POINTS

1. Neutrino flavor conversions may play a crucial role in dense objects such as core-collapse supernovae, compact binary mergers, and the early Universe. In such environments, neutrino–neutrino interactions induce nonlinear effects in the flavor evolution, and these effects make the flavor evolution challenging and counterintuitive to grasp. A recent development in the field concerns the possible existence of fast flavor conversions stemming from the pairwise scattering of neutrinos.

2. Existing analytical approaches employed to gauge the existence of instabilities in the flavor space rely on the linear stability analysis. The numerical solution of the equations of motion is challenged mainly by the presence of characteristic frequencies that differ from each other by many orders of magnitude.

3. Fast pairwise conversions have received much attention in the past few years because they may occur in the neutrino decoupling region and possibly affect the source physics in addition to the observable neutrino signal. The growth of the flavor instability is driven mainly by the (anti)neutrino number density; hence, the development of flavor conversions could occur on very small timescales. A peculiar aspect regarding fast pairwise conversions is that they do not strongly depend on the neutrino energy distribution, as in the case of classical flavor conversion, but instead on the angular distribution of the scattering neutrinos.

4. Fast pairwise conversions are expected to develop when crossings occur in the effective electron lepton number angular distribution of (anti)neutrinos. Moreover, the nonlinear regime of fast conversions is strongly dependent on the neutrino vacuum frequency (when this frequency is not vanishingly small), on the strength of the electron neutrino lepton number crossings, and, eventually, on collisions.

5. Favorable conditions for the occurrence of fast pairwise conversions exist in core-collapse supernovae (with potential implications for the explosion mechanism) as well as in compact binary mergers; however, preliminary work shows that the possible implications for the nucleosynthesis of the heavy elements in compact mergers (and the kilonova light curve) appear to depend on the nature of the compact object at the center of the remnant disk—that is, whether the latter is a hypermassive neutron star or a black hole. The physics of fast pairwise conversions in the early Universe has not yet been explored despite the fact that local inhomogeneities may lead to favorable conditions for the occurrence of pairwise conversions.

FUTURE ISSUES

1. Recent work highlights intrinsic limitations of the predictive power of the linear stability analysis. A deeper understanding of fast pairwise conversions, jointly with progress in numerical modeling, is necessary to finally grasp whether this phenomenon leads to large flavor mixing, as initially postulated. The validity of the mean-field approximation should also be scrutinized in light of our renewed understanding of neutrino self-interactions.
2. In the context of slow neutrino conversions, it has been shown that the flavor phenomenology is strongly affected when symmetry assumptions are relaxed. Similar findings might hold for pairwise conversions and can further affect current conjectures.

3. It remains to be understood whether an interplay between fast and slow conversions or matter effects can occur and how this would affect the flavor outcome. The physics picture discussed here may also be drastically altered by nonstandard physics.

4. If fast conversions occur in dense environments, leading to large flavor mixing, then their coupling to the source physics may be not negligible; this issue remains to be explored.

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Errata

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