Pebble dynamics and accretion on to rocky planets – II. Radiative models

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ABSTRACT

We investigate the effects of radiative energy transfer on a series of nested-grid, high-resolution hydrodynamic simulations of gas and particle dynamics in the vicinity of an Earth-mass planetary embryo. We include heating due to the accretion of solids and the subsequent convective motions. Using a constant embryo surface temperature, we show that radiative energy transport results in a tendency to reduce the entropy in the primordial atmosphere, but this tendency is alleviated by an increase in the strength of convective energy transport, triggered by a correspondingly increased superadiabatic temperature gradient. As a consequence, the amplitude of the convective motions increase by roughly an order of magnitude in the vicinity of the embryo. In the cases investigated here, where the optical depth towards the disc surface is larger than unity, the reduction of the temperature in the outer parts of the Hill sphere relative to cases without radiative energy transport is only \( \sim 100 \) K, while the mass density increase is of the order of a factor of two in the inner parts of the Hill sphere. Our results demonstrate that, unless unrealistically low dust opacities are assumed, radiative cooling in the context of primordial rocky planet atmospheres can only become important after the disc surface density has dropped significantly below minimum-mass-solar-nebula values.

Key words: hydrodynamics (HD) – radiative transfer – planets and satellites: formation – protoplanetary discs.

1 INTRODUCTION

Planetary embryos of sufficient mass embedded in optically thick protoplanetary accretion discs are necessarily accompanied by extended primordial atmospheres in near hydrostatic equilibrium, merging smoothly with the disc background at distances of the order of the Hill radius, \( R_H = a \sqrt{M_p / M_*} \), where \( a \) is the semimajor axis of the embryo’s orbit, and \( M_p \) and \( M_* \) are the masses of the embryo and the central star, respectively (Perri & Cameron 1974; Bodenheimer & Pollack 1986; Pollack et al. 1996; D’Angelo & Bodenheimer 2013; Alibert 2017).

The existence of dust and solid particles in the protoplanetary disc leads to accretion of solids on to such embryos, resulting in growth of the embryo, accretion heating at the embryo surface, and frictional heating of the surrounding primordial atmosphere (e.g. Brouwers, Vazan & Ormel 2018). As long as the embryo atmosphere and surrounding disc remain optically thick, convective motions are the most effective means of transporting heat through the atmosphere, with efficient convective transport leading to a near-adiabatic stratification of the atmosphere (Stevenson 1982; Wuchterl 1993). Near-adiabaticity implies that the inner parts of the atmosphere are relatively hot, which when combined with the reduction of the embryo’s gravity as the square of the distance, results in extended, relatively low-mass atmospheres (e.g. Venturini et al. 2015).

If, on the other hand, the optical depth of the primordial atmosphere is not very large, radiative energy transport leads to cooling, resulting in possibly significant changes to the stratification of the atmosphere. Specifically, radiative cooling would lead to smaller pressures and density scale heights, thus increasing the mass of the atmosphere. Significant cooling would eventually lead to a situation where hydrostatic equilibrium is no longer sustainable, resulting in gravitational collapse and the formation of gas giants (e.g. Mizuno, Nakazawa & Hayashi 1978; Bodenheimer & Pollack 1986; Hori & Ikoma 2011). Opposing this effect is the replenishment of gas inside the Hill sphere with disc material (Ormel, Shi & Kuiper 2015; Cimerman, Kuiper & Ormel 2017).

In cases where catastrophic collapse of the atmosphere does not occur, the ultimate formation outcome depends crucially on the balance between loss of atmospheric gas caused by the reduction

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Table 1. Simulation parameters. $\kappa$ is the total opacity (absorption plus scattering), $\varepsilon$ is the fraction of absorption, and $M$ is the solid accretion rate adopted for the embryo heating term.

<table>
<thead>
<tr>
<th>Run</th>
<th>$\kappa$ [cm$^2$ g$^{-1}$]</th>
<th>$\varepsilon$</th>
<th>$M$ [M$_\oplus$ yr$^{-1}$]</th>
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<tr>
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<td>m095-conv-2e-6</td>
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<tr>
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<td>0.2</td>
<td>$2 \times 10^{-5}$</td>
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<td>m095-conv-2e-5-rt</td>
<td>1.0</td>
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of the density and pressure of the surrounding disc over time, and the opposing tendency caused by the increased radiative cooling – occurring for essentially the same reason. As shown by Ginzburg, Schlichting & Sari (2016), this balance may, in the end, dictate if the final outcome is a gas-rich planet or a rocky planet with a thin remnant atmosphere.

Properly accounting for the radiative transfer (RT) of energy and the corresponding effects on the near-hydrostatic equilibrium of primordial atmospheres is thus crucial for realistic and accurate modelling of early planet formation. Such efforts are additionally complicated by effects of scattering, which are known to be important at the wavelengths and temperatures of relevance (e.g. Pinte et al. 2009).

In this Letter, we present first-of-a-kind results, using ray-tracing RT with scattering to demonstrate its effects on primordial atmospheres, and the resulting effects on particle dynamics and pebble accretion on to rocky planets.

2 METHODS

As in Popovas et al. (2018, hereafter PNRO18), this study is carried out using the DISPATCH framework (Nordlund et al. 2018), employing a three-dimensional Cartesian (shearing box) domain with a set of static, nested patches. We continue to use an ideal gas equation of state (EOS) with adiabatic index $\gamma = 1.4$ and molecular weight $\mu = 2$, and embedment in a disc with a nominal surface mass density of 170 g cm$^{-2}$, corresponding to 1/10 of the minimum-mass-solar-nebula (MMSN). The grid set-up is the same as in PNRO18 for an $M_p = 0.95M_{\oplus}$ planet at 1 au distance from the central star. We conduct a series of radiative-convective simulations, for which the basic RT and accretion heating parameters are summarized in Table 1.

To investigate the effects of radiative cooling while maintaining a nearly adiabatic (i.e. convective) atmosphere, we deliberately choose a combination of disc surface density and opacity such that the disc and embryo atmosphere are slightly optically thick. To this end, we adopt a total opacity (with 80 percent scattering and 20 percent absorption) of 0.1 cm$^2$ g$^{-1}$ and a disc surface density of 170 g cm$^{-2}$. We thus obtain a midplane optical depth in the unperturbed disc of $1/3 \times 0.1 \times 1700/10 = 8.5$. The additional optical depth of the initial, adiabatic atmosphere is $\approx 6.0$, and the optical properties are such that we expect to begin to see the effects of radiative cooling.

The optical properties of dust and gas in hot, primordial atmospheres are quite uncertain, mainly due to uncertainties related to dust coagulation and thermal processing. Nevertheless, if we consider the opacities from Semenov et al. (2003) at densities and temperatures relevant to this work (e.g. Fig. 1), then the vast majority of our domain has opacities of the order of 0.1–1 cm$^2$ g$^{-1}$.

Figure 1. Temperature and density profiles, averaged over radial shells and time (see PNRO18), when neither RT nor accretion heating is considered (m095, black curves), when accretion heating ($M = 2 \times 10^{-6} M_{\oplus}$ yr$^{-1}$) and driven convection but no RT is considered (m095-conv-2e-6, red), when accretion heating, convection, and RT (with $\kappa = 0.1$ cm$^2$ g$^{-1}$) are considered (m095-conv-2e-6-rt, blue), when the same accretion heating and radiative cooling are considered, but $\kappa = 1.0$ cm$^2$ g$^{-1}$ (m095-conv-2e-6-rt-k1, magenta), and when the accretion heating is increased to rates consistent with pebble-sized particle accretion ($M = 2 \times 10^{-5} M_{\oplus}$ yr$^{-1}$) and RT (with $\kappa = 0.1$ cm$^2$ g$^{-1}$) is considered (m095-conv-2e-5-rt, green).

2.1 Initial and boundary conditions

For initial conditions we take a fiducial, fully relaxed MMSN/10 run with adiabatic stratification (m095t10 in PNRO18). The external and spherical boundary conditions for density $\rho$, entropy per unit mass, and mass flux are also the same as in PNRO18. However, as we also consider radiative energy transport in this work, appropriate boundary conditions must be considered. The disc is optically thick in the radial and azimuthal directions and is initially in radiative equilibrium in those directions, i.e. no heat exchange occurs radially or azimuthally. However, protoplanetary discs do cool radiatively in the vertical direction. Therefore, we adopt the following external boundary conditions for the RT $Q = I - S = 0$ at external horizontal (radial and azimuthal) boundaries; $P_{\text{incoming}} = 0 \Rightarrow Q_{\text{incoming}} = -S$ at external vertical boundaries, where $I$ is the radiation intensity in a specific direction, $S$ is the local source function, and $Q$ is the heating/cooling rate.

2.1.1 Heating from the embryo

Accretion heating due to solids is included via a source term in the entropy equation (PNRO18). $Q_{\text{acc}} = \rho L_{\text{acc}}$, where $Q_{\text{acc}} = GM\dot{M}p_g/4\pi r^4$, $r$ is the distance from the embryo, $G$ is the gravitational constant, and $\dot{M}$ is the adopted accretion rate (Table 1).
As the envelope surrounding the embryo is optically thick for most of the disc evolution, the embryo and its atmosphere cannot radiate heat away effectively when solids are accreted. The atmosphere is thus expected to remain nearly adiabatic through most of the build-up of the embryo mass, and the embryo must be correspondingly hot, especially towards the end of the build-up period. Detailed predictions of the temperature and its history require realistic equations of state for both the primordial atmosphere and the planet, in combination with modelling of the embryo thermal evolution over disc evolution time-scales. This is a formidable task by itself, where sequences of simulations such as the ones reported here may be used to provide the required estimates of instantaneous cooling rates.

In any case, since the heat capacity of a planet is large, the planet is expected to remain hot for a considerable time, even after the envelope becomes optically thin (Ginzburg et al. 2016). For the relatively brief periods of time covered by the current simulations, it is thus appropriate to consider the surface temperature of the embryo to be fixed, and we therefore adopt the surface temperature predicted by the adiabatic atmosphere model as a fixed lower boundary condition in all models.

2.2 Radiative energy transport

We use a hybrid-characteristics ray-tracing scheme (Nordlund et al. 2018, Appendix A, available online only) with 26 ray directions (forward and reverse directions along three axes, six face diagonals, and four space diagonals) and a single frequency bin (constant opacity $\kappa$). Since scattering is an important effect at the temperatures and wavelengths of relevance here, we split the opacity into an absorption part, $\kappa_A$, and a scattering part, $(1 - \varepsilon)\kappa$. At the levels of scattering typically assumed – from 50 per cent to 80 per cent – and with the relatively slow time evolution dictated by convective motions, scattering can be handled with what is essentially ‘A iteration’, i.e. iteratively feeding the mean intensity from previous time steps into the source function of the next time step (Hubeny 2003). Here, the time slices stored in DISPATCH may be used to predict the mean intensity in the next time step, thus efficiently reducing the time lag otherwise resulting from the use of the previous mean intensity.

In order to avoid that the intense radiation from the embryo surface results in narrow, parallel beams of strong radiation in the 26 angular directions used in the ray-based radiation solver, we replace the solution from the ray-based solver with a diffusion approximation in the region near the embryo. In the optically thick limit, the integral solution of the RT equation along any direction may be approximated by

$$Q = \frac{1}{2}(I^+ + I^-) - S \approx \frac{d^2 S}{d\tau^2}, \quad (1)$$

where $I^+$ and $I^-$ are the specific radiation intensities in the forward and reverse directions, and $\tau$ is the optical depth along the ray direction. A term proportional to the first derivative of the source function is omitted, since it cancels out when averaging forward and reverse solutions along a given ray direction. Given the availability of the source function and opacity values on the Cartesian mesh, it is trivial to evaluate this expression in the three axis directions and estimate the full space angle, integrated heat exchange rate per unit mass:

$$q \approx \kappa \frac{4\pi}{3} \left( \frac{d^2 S}{d\tau_x^2} + \frac{d^2 S}{d\tau_y^2} + \frac{d^2 S}{d\tau_z^2} \right), \quad (2)$$

The heat exchange rate for the ray-based radiation solver, after averaging over the 26 forward and reverse ray directions, is meanwhile

$$q \approx \kappa \frac{4\pi}{26} \sum_{j=1}^{13} (Q_j^+ + Q_j^-). \quad (3)$$

We apply the ray-based solver everywhere, but we weigh the resultant heat exchange rate by

$$w = e^{-(r/r_0)}^\delta, \quad (4)$$

and the diffusion-based value of $q$ from equation (2) by $(1 - w)$, where $r_0$ is 10 times the radius of the embryo.

2.3 Particles

We use $\sim$12 million macro-particles, each representing a swarm of real particles with a given size and mass. The initial spatial distribution of macro-particles is proportional to the local gas density, with particle sizes ranging from 10 $\mu$m to 1 cm, and a constant number of macro-particles per logarithmic size bin. Rather than making assumptions about the settling and actual size distribution, we instead analyse subpopulations of our initial distribution. Specifically, to measure accretion rate as a function of particle size, we tag and follow only the particles that initially reside within one Hill radius of the midplane. See PNRO18 for more details about the particle distribution, their motion, particle size distribution within macro-particles, and selection.

3 RESULTS AND DISCUSSION

3.1 Gas dynamics

Fig. 1 shows the thermal and density structures of the envelopes for the five cases considered.

Convective motions effectively transport the excess heating from accretion of solids, even when RT is not considered (or $\kappa$ is sufficiently high), and therefore the differences in thermal structure relative to the purely convective case are small. When radiative energy transport is used, radiative cooling effects become important in the outer parts of the atmosphere and the temperature decreases. However, cooling effects do not penetrate all the way to the embryo, and the thermal structure near the embryo remains close to the adiabatic one. As the envelope adjusts its near-hydrostatic equilibrium to the lower temperature, it becomes on average denser. Since this is an effect that accumulates with depth – pressure scale height is proportional to the local temperature – the effect on the density is larger than on the temperature. Indeed, in the radiatively cooled models, the total mass of the atmosphere is increased by about 4 per cent and 77 per cent for $M = 2 \times 10^{-5}$ and $2 \times 10^{-6}$ M$_\odot$ yr$^{-1}$, respectively (with $\kappa = 0.1$ cm$^2$ g$^{-1}$), relative to adiabatic and purely convective models.

Fig. 2 (middle) shows an example of the convective velocity flow for a simulation where radiative energy transfer is included (m095-conv-2e-6-rt). The fluid motions are similar to the case with pure convection (left-hand panel of Fig. 2), but the velocity amplitudes are larger – especially at small radii – when radiative energy transfer is included. The reason for the increased velocity amplitudes is illustrated in the right-hand panel of Fig. 2. The radiative cooling tends to lower the entropy in the neighbourhood of the embryo relative to runs without RT, while accretion heating simultaneously keeps adding entropy, and the temperature of the embryo surface is held fixed. The net effect is a tendency to increase the
temperature gradient, thus making it more superadiabatic than in the case with only accretion heating, resulting in stronger driving of the convection. Near the embryo, the increase of the convective energy transport is largely able to compensate for the increased radiative cooling. Further out, where the convection motions are weaker and the vertical optical depth is smaller, the effects of radiative cooling become more visible.

The increase of convective velocity amplitudes is illustrated in Fig. 3 against a background of particle drift speeds. The figure shows that velocity amplitudes near the embryo for simulations that include RT are increased by nearly an order of magnitude over the simulation that does not, becoming comparable to the vertical drift speed of 2–3 mm particles. Conversely, however, increasing the accretion heating further does not significantly increase the dispersion. Further out, where the horseshoe and shear flow patterns start to contribute to the velocity dispersion, the effects of radiative cooling on the velocity dispersion diminish.

3.2 Solid accretion rates

In our previous paper (PNRO18), we determined the accretion rates of solids and showed that they scale linearly with the particle size. In Fig. 4, we affirm that the previously determined accretion rates are robust. Indeed, the accretion rates in simulation runs with convective motions and radiative cooling do not differ significantly from runs without; deviations are mainly due to increased noise as the convective motions of the gas stir the solids. Moreover, we find that stronger convective motions due to a larger accretion heating (e.g. that could result from changing the particle size distribution) do not significantly affect the accretion rates. Although the amplitude of the convective motions increase with accretion heating, they remain confined to a region $\lesssim 30R_p$ (Fig. 3). Whether a particle will be accreted or not is, meanwhile, determined at larger radii (cf. fig. 19 of PNRO18), and thus the strength of the convection does not significantly affect the accretion rate of particles.

4 CONCLUSIONS AND OUTLOOK

In these first-of-a-kind, three-dimensional, radiative–convective models of hot and extended primordial atmospheres of Earth-mass embryos, with realistic – albeit schematic – scattering included, we find that radiative cooling leads to a local increase of the shell-averaged mass density of the order of a factor of two, but only a slight decrease in the temperature. For the conditions modelled here, cooling effects do not penetrate to the bottom of the atmosphere where convection is the dominant mechanism of energy transport. In response to the tendency from cooling to increase the radial temperature gradient, however, the amplitude of convective
motions increase by nearly an order of magnitude near the embryo surface relative to runs without RT.

Even though radiative cooling modifies the atmosphere structure, the accretion heating and resulting convective energy transport still dominate near the planetary embryo, where the atmosphere temperature remains ‘anchored’ to the surface temperature of the embryo. This is encouraging, since it implies that planetary embryos embedded in protoplanetary discs can retain hot atmospheres by adjusting their nearly adiabatic stratifications only slightly, throughout much of the evolution of the disc. For cooling to become important at earlier times, the opacities would have to be much lower than assumed here ($0.1–1$ g cm$^{-2}$).

When the disc and primordial atmosphere finally become genuinely optically thin, the atmospheres that remain around low-mass planetary embryos are therefore relatively light, with their future fate depending on the relative balance between the slow cooling of the still hot embryo, and the secular loss of the remaining atmosphere. As argued by Ginzburg et al. (2016), it is the outcome of this balance that ultimately determines if a planet ends up as a gas-rich planet with a significant H + He atmosphere, or becomes a rocky planet with an atmosphere consisting of heavier gas molecules, possibly even dominated by out-gassing.

The current modelling should be seen as a pilot effort, exploring effects and probing what is currently possible. As briefly discussed in Appendix B (available online only), the current results were obtained with a moderate computational effort. Obvious factors that could use improvement are the EOS, the constant and grey opacity, and the spatial resolution, which could stand to be increased in order to better resolve the convective motions. Improving all these factors is certainly possible, and will be the subject of future work.

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**SUPPORTING INFORMATION**

Supplementary data are available at *MNRASL* online.

appendix.pdf

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