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Logit, CES, and rational inattention

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HIGHLIGHTS

- We microfound the CES utility function using the rational inattention model.
- We obtain a new interpretation for the parameters of the CES utility function.
- We provide new predictions about the price elasticities of expected demand.

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ABSTRACT

We study the fundamental links between two popular approaches to consumer choice: the multinomial logit model of individual discrete choice and the CES utility function, which describes a diversified choice of a representative consumer. We base our analysis on the rational inattention (RI) model and show that the demand system of RI agents, each of which chooses a single option, coincides with the demand system of a fictitious representative agent with CES utility function. Thus, the diversified choice of the representative agent may be explained by the heterogeneity in signals received by the RI agents. We obtain a new interpretation for the elasticity of substitution and the weighting coefficients of the CES utility function. Specifically, we provide a correspondence between parameters of the CES utility function, prior knowledge and marginal cost of information.

1. Introduction

People choose different products for different reasons, and, perhaps, the two most important are variation in preferences and in information. Correspondingly, there are models of individual choice based either on heterogeneous idiosyncratic preferences or on variation in information received by agents. Both types of models have become workhorses in microeconomics, decision making and related topics. However, for the analysis of the behavior of a set of consumers, rather than a single consumer, an "as if" model of a fictitious representative consumer with an aggregate utility function is commonly used, often having the shape of a constant elasticity of substitution (CES). The existing microfoundation of the CES utility function is based exclusively on preference heterogeneity, and thus any change in its parameters is interpreted as a change in the idiosyncratic preferences of underlying agents, while possible informational reasons are ignored.

In this paper we broaden the approach to the microfoundation of the CES utility function and show that this functional form might be obtained by aggregation of choices of rationally inattentive (RI) consumers who make a discrete choice with costly information acquisition. Our approach explains the origins of both the weighting coefficients (which have previously been interpreted as a consumer's preferences for separate goods) and of the elasticity of substitution of the CES utility function.

The existing literature that provides the microfoundation of the CES utility function and relates it to the multinomial logit model is based on a random utility model (Anderson et al., 1987, 1988). Hence, the elasticity of substitution of the aggregate utility is determined by an exogenous parameter of a specific (extreme value Gumbel) cumulative distribution function of taste dispersion. Since this parameter reflects idiosyncratic differences in preferences, it is difficult to forecast its changes under economic shocks.

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This paper, in contrast, uses rational inattention (RI) (Sims, 1998, 2003) as a microfoundation, and reveals the link between the parameters of the RI model and the elasticity of substitution and weighting coefficients of the CES utility function.

We model a situation in which a consumer is facing a discrete choice problem: she possesses some income and spends it to purchase only one kind of divisible good. We assume that, despite the goods having certain prices, the consumer may not observe them perfectly due to her inattention to prices. Limitations in consumers' attention to prices are confirmed empirically (e.g. Dickson and Sawyer (1990) and Rosa-Díaz (2004)).

We model the price vector as a random draw from a known to the consumer distribution. The RI consumer receives some information about its realization, and the information structure (any joint distribution of signal and state) is chosen by her. As is usually assumed in RI models, the information is costly, and the cost is proportional to the expected entropy-based reduction in uncertainty between the prior and the posterior beliefs.

We assume that the consumer has logarithmic utility and the marginal cost of information λ. We show that, conditional on price realization, the expected demand structure is the same as that generated by the CES utility function, that belongs to an aggregate representative consumer who possesses perfect information, and for which the elasticity of substitution is $\sigma = 1/\lambda + 1$. We show that the weighting coefficients of the CES utility function are defined by the prior belief of the RI consumers and the marginal cost of information. It is important to note that the resulting CES utility function is the same for a fixed distribution of prices; that is, if the price vector changes due to a new draw from the same distribution, the consumer is still solving the same problem and the demand corresponds to the same CES utility function of a representative consumer.

Importantly, our model leads to new implications. For instance, in our model the weighting coefficients of the resulting aggregate CES utility function are endogenous. That means, for example, that if the distribution from which the prices are coming changes,[1] then the parameters of the information acquisition and decision problem of the RI agent change; thus, the representative CES utility changes and the magnitude of consumers' reactions would be different from the magnitude implied by the CES function with exogenous coefficients. The other implication of our model concerns the reaction of demand to change in the marginal cost of information, λ. We show that the weighting coefficients of the CES utility depend on it; that is, the marginal cost of information enters the model on a more complex level than the parameter of taste heterogeneity, μ, in the model of Anderson et al. (1987, 1988).

Our paper is related to several strands of literature. The microfoundation of the utility function of the representative consumer is still an open question (see Kirman (1992), Sheu (2014) and Tito (2016)). It is especially important to microfound the CES shape because the CES function is used in many models of macroeconomics, international trade, economic geography and industrial organization (see, e.g., Atkin et al. (2018), Mrázová and Neary (2014) and Sheu (2014)). It is notable that one of the reasons for the critique of the welfare analysis based on models with CES utility (see Kirman (1992) and Tito (2016)) is that the relation between the fictitious representative consumer and the real consumer is not clear, and the welfare of the representative consumer seems not to be informative.

The model of RI, first introduced by Sims (1998, 2003), was applied to consumer behavior by Caplin and Dean (2015), Martin (2017), Matějka and McKay (2012), Matějka (2015) and Tutino (2013). Matějka and McKay (2015) proposed a foundation for the multinomial logit model based on RI, and we use their model in this paper.

[1] For example, because of the introduction of a new trade barrier on a foreign good.

2. The model

There are N types of goods that are perfect substitutes for the individual consumer. The consumer is endowed with budget y, which she spends entirely on one type of good.[2] The consumer would like to purchase the cheapest type of good to have as large a quantity of it as possible; however, at the moment of choosing the good she does not observe prices perfectly.[3] The true payoffs related to the chosen good are revealed only after the choice is made. One can think of the following interpretation of the model: different goods are sold by different vendors, who are located in different places. The consumer learns the price when she arrives at the location of the vendor and it is too late to change the vendor.

Following Anderson et al. (1987),[4] we assume that the utility of consumption of good i by the individual is

$$v_i = \ln q_i, \quad i = 1, \ldots, N,$$

where q_i is the quantity. If the individual chooses good i to purchase, then, obviously, the consumed quantity is

$$q_i^* = \frac{y}{p_i}, \tag{1}$$

where p_i is the price, and the indirect utility is

$$V(y, p_i) = \ln\left(\frac{y}{p_i}\right). \tag{2}$$

2.1. Choice of good

As we noted before, the consumer does not know the realization of prices. More precisely, the price vector $\mathbf{p} = (p_1, \ldots, p_n)$ is random, which makes $v_i = V(y, p_i) = \ln\left(\frac{y}{p_i}\right)$, $(i = 1, \ldots, N)$ random variables. That is, being inattentive to prices is equivalent to being inattentive to indirect utilities.

Following Matějka and McKay (2015), the consumer chooses from N products characterized by an unknown realization of utility values $\mathbf{v} = (v_1, \ldots, v_N)$ from probability distribution $G(\mathbf{v}) \in \Delta(\mathbb{R}^N)$, where $\Delta(\mathbb{R}^N)$ is the set of all probability distributions on $\mathbb{R}^N$. The belief about $\mathbf{v}$, i.e. $G(\mathbf{v})$, is given exogenously by the agent's prior knowledge of prices.

The agent is able, in principle, to obtain precise information about the realization of the random price vector $\mathbf{p} = (p_1, \ldots, p_N)$ (and, correspondingly, about the realization of the random vector of utilities $\mathbf{v} = (v_1, \ldots, v_N)$). However, for the agent the information about the realization is costly. She constructs her information-action strategy in advance by solving the problem of maximization of the expected utility less the information cost.

The information-action strategy includes the choice of information (signal) about the realization and the choice of action (selected product) conditional on the signal. The second choice is standard: the agent simply chooses the product which provides

[2] We could also assume that there are N types of goods which are perfect substitutes and one more good with a known price $p = 1$ (a numeraire good). The consumer has an opportunity to spend part of her budget on the numeraire good. Following Anderson et al. (1988), we could assume that the utility of consumption good i and good 0 is $v_i = \ln q_i + \alpha \ln q_0$. The resulting utility function of the representative consumer would have a shape of CES utility function multiplied by q_0^α. This function is similar to the utility function which was considered in the original (Dixit and Stiglitz, 1977) paper.

[3] Alternatively, we could assume uncertain quantities instead of prices.

[4] Anderson et al. (1987) build their analysis on a random utility model (McFadden, 1974). Specifically, they assume that the choice is made in order to maximize the stochastic utility, which consists of the sum of indirect utility and a stochastic component, $U_i = V(y, p_i) + \mu\varepsilon_i, \quad i = 1, \ldots, n$, where $\mu > 0$ and ε_i is a random variable with zero mean and unit variance.

the highest expected value. The first choice is the hallmark of rational inattention.

It is assumed that, to reduce uncertainty, the consumer has to pay a cost $\lambda\kappa$, where $\lambda > 0$ is the marginal cost of information, and $\kappa > 0$ is the amount of information processed. The latter is the expected entropy[5] reduction between the agent's prior and posterior beliefs about $\mathbf{v}$.[6]

According to Lemma 1 from Matějka and McKay (2015) the state-contingent choice behavior of the rationally inattentive consumer can be found as the solution to a simpler maximization problem that does not make reference to signals or posterior beliefs. That is, each information-action strategy may be characterized by a vector function $(\mathcal{P}_1(\mathbf{v}), \ldots, \mathcal{P}_N(\mathbf{v}))$, where $\mathcal{P}_i(\mathbf{v})$ is a conditional probability that product i will be chosen under the realization $\mathbf{v}$. The probabilities reflect the agent's choice under incomplete information, when she receives a noisy signal but does not know the realization of $\mathbf{v}$ precisely.

Formally, the consumer's problem is described in the following way.

Consumer's problem. The consumer's problem is to find an information processing strategy maximizing expected utility less the information cost:

$$\max_{(\mathcal{P}_1(\mathbf{v}),\mathcal{P}_2(\mathbf{v}),\ldots,\mathcal{P}_N(\mathbf{v}))} \left\{ \sum_{i=1}^{N} \int_{\mathbf{v}} v_i \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}) - \lambda\kappa(\mathcal{P}, G) \right\}, \tag{3}$$

subject to

$$\forall i: \quad \mathcal{P}_i(\mathbf{v}) \geq 0 \quad \forall \mathbf{v} \in \mathbb{R}^N,$$

$$\sum_{i=1}^{N} \mathcal{P}_i(\mathbf{v}) = 1 \quad \forall \mathbf{v} \in \mathbb{R}^N,$$

where

$$\kappa(\mathcal{P}, G) = -\sum_{i=1}^{N} \mathcal{P}_i^0 \ln \mathcal{P}_i^0 + \int_{\mathbf{v}} \left(\sum_{i=1}^{N} \mathcal{P}_i(\mathbf{v}) \ln \mathcal{P}_i(\mathbf{v}) \right) G(d\mathbf{v}),$$

$\mathcal{P}_i(\mathbf{v})$ is the probability of choosing good i conditional on the realized vector $\mathbf{v}$ and $\mathcal{P}_i^0$ is the unconditional probability that the good i will be chosen,

$$\mathcal{P}_i^0 = \int_{\mathbf{v}} \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}), \quad i = 1, \ldots, N.$$

Probabilities $\mathcal{P}_i^0$ depend on the marginal cost of information λ, on prior belief $G(\mathbf{v})$ and do not depend on the realization of $\mathbf{v}$.

It is shown by Matějka and McKay (2015) that the conditional on the realized vector $\mathbf{v}$ choice probabilities, $\mathcal{P}_i(\mathbf{v})$, follow the modified logit formula:

$$\mathcal{P}_i(\mathbf{v}) = \frac{\mathcal{P}_i^0 e^{\frac{v_i}{\lambda}}}{\sum_{j=1}^{N} \mathcal{P}_j^0 e^{\frac{v_j}{\lambda}}}, \quad i = 1, \ldots, N. \tag{4}$$

By plugging (2) into (4) we obtain the probability of choosing product i as a function of price vector and prior beliefs:

$$\mathcal{P}_i(\mathbf{v}(\mathbf{p})) = \frac{\mathcal{P}_i^0 p_i^{-\frac{1}{\lambda}}}{\sum_{j=1}^{N} \mathcal{P}_j^0 p_j^{-\frac{1}{\lambda}}}, \quad i = 1, \ldots, N. \tag{5}$$

The conditional expected demand for good i is $D_i = \mathcal{P}_i(\mathbf{v}(\mathbf{p}))q_i^*$, q_i^* is the amount of good i which is purchased by the consumer if the good i was chosen (see Eq. (1)). Eqs. (1) and (5) imply the following.

Lemma 1. *The expectation of conditional demand for good i, $D_i = \mathcal{P}_i(\mathbf{v}(\mathbf{p}))q_i^*$, is*

$$D_i = \frac{\mathcal{P}_i^0 p_i^{-\frac{1}{\lambda}-1}}{\sum_{j=1}^{N} \mathcal{P}_j^0 p_j^{-\frac{1}{\lambda}}} y, \quad i = 1, \ldots, N. \tag{6}$$

It is important to note that the solution to the consumer's problem may result in $\mathcal{P}_i^0 = 0$ for some $i \in 1, 2, \ldots, N$; which means that the consumer never chooses the good i, an example of such situation is provided in Appendix C.

2.2. The link between rational inattention and the CES utility function

In the following proposition we show that the demand of the aggregate of RI agents is the same as if there was a fictitious representative consumer maximizing the CES utility function under full information.[7] The aggregation here can be understood in the following way: there is a continuum of rationally inattentive agents who are solving the same problem and who are receiving independent signals about the realization of prices.[8]

Proposition 1 (*The CES Demand Structure of Rationally Inattentive Agents*)**.** *The demand structure (6) representing the rational inattention model of discrete choice with logarithmic preferences is generated by the CES utility function*

$$U = \left(\sum_{j=1}^{N} \beta_j q_j^{\rho} \right)^{\frac{1}{\rho}},$$

which is maximized by the representative consumer subject to the budget constraint

$$\sum_{j=1}^{N} p_j q_j \leq y,$$

where the elasticity of substitution is

$$\sigma = \frac{1}{1-\rho} = \frac{1}{\lambda} + 1, \tag{7}$$

and the weighting coefficients are

$$\beta_i = \gamma \left(\mathcal{P}_i^0\right)^{1-\rho} = \gamma \left(\mathcal{P}_i^0\right)^{\frac{\lambda}{1+\lambda}}, \quad i = 1, \ldots, N, \tag{8}$$

where γ is any positive coefficient.

Proof. see Appendix A.

Thus, the goods seem as if they are not perfect substitutes for the representative consumer, despite being perfect substitutes for each of the underlying RI consumers.

From (7) we see that the elasticity of substitution σ is higher than 1 and depends negatively on the marginal cost of information λ. If the cost of information λ increases, then the behavior

[5] The entropy of a continuous random variable X with probability density function $f(x)$ with respect to a probability measure m is $H(X) = -\int f(x) \log f(x) m(dx)$.

[6] The case in which the cost of information is not linear in the expected entropy reduction, but is determined by a convex, continuously differentiable and nonnegative function of the expected entropy reduction $f(\kappa)$, for which $f'(0) > 0$, is considered in Appendix B. In this case, the result is similar, but the role of λ in the CES will be taken by the marginal cost of information at the optimum, $f'(\kappa^*)$, thus, it will depend on the prior belief.

[7] The discussion on whether the opposite holds, that is, whether for any CES demand system there is a corresponding RI problem, can be found in Appendix C.

[8] The independence assumption is used in many leading papers in the literature on RI, e.g., Maćkowiak and Wiederholt (2009) or Hellwig and Veldkamp (2009). It is intuitive when it comes to private information processing, but it is true that if all agents received information from the same source, e.g., CNN, then it might be violated.

of the representative (aggregate) consumer is the same as if the elasticity of substitution went down. The reason is that the individual consumer inspects prices less, and consequently she is more likely to make errors, and thus react less to changes in prices.

The weighting coefficients β_i depend positively on the corresponding unconditional probabilities $\mathcal{P}_i^0$.

2.3. The effect of change in price distribution on demand structure

Let us consider a situation in which an RI consumer is choosing between two types of good to purchase. Let us assume that the belief about the price distribution and the marginal cost of information are such that $\mathcal{P}_1^0 > 0$ and $\mathcal{P}_2^0 > 0$. Let us also assume that the government has introduced an extra tax on good one. This means that the distribution, from which the prices (p_1, p_2) are drawn, has changed.

If the consumer is unaware of the extra tax on good one, her expected demand structure would look as if she is maximizing the same CES utility function as before the tax introduction. The reaction to change in price would be the same as the reaction of the representative consumer who is maximizing CES utility function.

However, if the consumer is aware of the change in the price distribution, changes in beliefs lead to a new expected demand structure (different CES function with new weighting coefficients). In such a case, the reaction to the change in price would be different.

Proposition 2 (*Distinction between Price Elasticity of Rationally Inattentive Agents and Price Elasticity for the CES Utility Function*). *Let $N = 2$ and initially (before the change in price distribution) the feasible unconditional probability $0 < \mathcal{P}_1^0 < 1$ is unique, and the change in price distribution is such that in all states of the world price p_1 increases. Then the price elasticity of the expected demand of the RI agent, who is aware of the change in the price distribution, is lower than the price elasticity for the CES utility function.*

Proof. see Appendix D.

3. Conclusion

It is often assumed that changes in the aggregate consumer's demand are due to changes in the idiosyncratic preferences of individual consumers. We propose an alternative story: the demand shifts for particular goods might sometimes be better explained by a change in information about goods.

According to our model, the demand structure changes due to shifts in information costs and the structure of prior knowledge of consumers, not in the idiosyncratic preferences. In many markets there was a reduction in the costs of information (due to the appearance of websites with information on products, such as google.com/shopping, special search engines to compare the prices of airline tickets, hotels, restaurants, etc.). All this directly affects the information costs and the consumer's prior beliefs. The information coming from different countries or regions and making their products salient might also change a consumer's priors. Accordingly, we can anticipate changes in the structure of the CES utility function and the aggregate consumer behavior. Thus, our model extends the understanding of why changes in demand, which are usually interpreted as a change in preferences, often occur after certain events (shocks) in the economy, such as crises, opening of new markets, and changes in the advertising policy of certain firms.

We show that the demand system generated by the CES utility function is equivalent to a model of rational inattention to discrete choice. That is, we endogenize (microfound) the CES utility function with the RI model. We show that the elasticity of substitution and weighting coefficients of the CES function are determined by the parameters of the RI model, namely marginal cost of information and prior beliefs. Such a link helps us to connect the intensively developing RI theory with neoclassical economic models.

Also, our results may help to find estimates for the cost of information. In the literature there are estimations of elasticities of substitution for the CES function (e.g. Bergstrand et al. (2013), Coloma (2009) and Redding and Weinstein (2016)). Based on such estimations and using formula (7), which connects elasticity of substitution, σ, and marginal cost of information, λ, it is now possible to obtain the estimates for the parameter of cost of information.

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Appendix A. Proofs, discussion and example

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