Extracting the Energy-Dependent Neutrino-Nucleon Cross Section above 10 TeV Using IceCube Showers

Bustamante, Mauricio; Connolly, Amy

Published in:
Physical Review Letters

DOI:
10.1103/PhysRevLett.122.041101

Publication date:
2019

Document version
Publisher's PDF, also known as Version of record

Citation for published version (APA):
Extracting the Energy-Dependent Neutrino-Nucleon Cross Section above 10 TeV Using IceCube Showers

Mauricio Bustamante1,2,3,* and Amy Connolly2,3,†

1Niels Bohr International Academy & Discovery Centre, Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen, Denmark
2Center for Cosmology and AstroParticle Physics (CCAPP), The Ohio State University, Columbus, Ohio 43210, USA
3Department of Physics, The Ohio State University, Columbus, Ohio 43210, USA

(Received 11 January 2018; revised manuscript received 17 November 2018; published 28 January 2019)

Neutrinos are key to probing the deep structure of matter and the high-energy Universe. Yet, until recently, their interactions had only been measured at laboratory energies up to about 350 GeV. An opportunity to measure their interactions at higher energies opened up with the detection of high-energy neutrinos in IceCube, partially of astrophysical origin. Scattering off matter inside Earth affects the distribution of their arrival directions—from this, we extract the neutrino-nucleon cross section at energies from 18 TeV to 2 PeV, in four energy bins, in spite of uncertainties in the neutrino flux. Using six years of public IceCube High-Energy Starting Events, we explicitly show for the first time that the energy dependence of the cross section above 18 TeV agrees with the predicted softer-than-linear dependence, and reaffirm the absence of new physics that would make the cross section rise sharply, up to a center-of-mass energy $\sqrt{s} \approx 1$ TeV.

DOI: 10.1103/PhysRevLett.122.041101

Introduction.---Neutrino interactions, though feeble, are important for particle physics and astrophysics. They provide precise tests of the standard model [1–3], probes of new physics [4–6], and windows to otherwise veiled regions of the Universe. Yet, at neutrino energies above 350 GeV there had been no measurement of their interactions. This changed recently when the IceCube Collaboration found that the neutrino-nucleon cross section from 6.3 to 980 TeV agrees with predictions [7].

Because there is no artificial neutrino beam at a TeV and above, IceCube used atmospheric and astrophysical neutrinos, the latter discovered by them up to a few PeV [8–16]. References [4,6,17–20] showed that, because IceCube neutrinos interact significantly with matter inside Earth, their distribution in energy and arrival direction carries information about neutrino-nucleon cross sections, which, like IceCube [7], we extract.

However, Ref. [7] extracted the cross section in a single, wide energy bin, so its energy dependence in that range remains untested. A significant deviation from the predicted softer-than-linear dependence could signal the presence of new physics, so we extract the cross section in intervals from 18 TeV to 2 PeV. While Ref. [7] used only events born outside of IceCube we use instead only events born inside of it, which leads to a better handle on the neutrino energy.

Figure 1 shows that the cross section that we extract is compatible with the standard prediction. There is no indication of the sharp rise, at least below 1 PeV, predicted by some models of new physics [6,21–29].

Neutrino-nucleon cross section.---Above a few GeV, neutrino-nucleon interactions are typically deep inelastic scattering. 

FIG. 1. Charged-current inclusive neutrino-nucleon cross section measurements [30–47]. The new results from this work, based on 6 years of IceCube HESE showers [48–50], are an average across cross sections for $\nu$ and $\bar{\nu}$, assuming equal astrophysical fluxes of each. In the highest-energy bin, we only set a lower limit ($1\sigma$ shown). The thick dashed curve is a standard prediction of deep inelastic scattering (DIS), averaged between $\nu$ and $\bar{\nu}$. Horizontal thin dashed lines are global averages from Ref. [51], which do not include the new results.
scattering (DIS), where the neutrino scatters off one of the constituent partons of the nucleon—either a quark or a gluon. In both the charged-current (CC, $\nu \rightarrow N \rightarrow l^+ + X$) and neutral-current (NC, $\nu + N \rightarrow \nu + X$) forms of this interaction, the nucleon $N$ is broken up into partons that hadronize into a final state $X$. The final-state hadrons carry a fraction $y$—the inelasticity—of the initial neutrino energy, while the final-state lepton carries the remaining fraction $(1 - y)$.

Calculation of the cross section $\sigma_{\nu N}$ requires knowing the parton distribution functions (PDFs) in the nucleon. PDFs depend on two kinematic variables: $Q^2 \equiv -q^2$, the four-momentum transferred to the mediating $W$ or $Z$ boson, and the Bjorken scaling $x$, the fraction of nucleon momentum carried by the interacting parton [52].

To compute cross sections at neutrino energies $E_\nu$ between TeV and PeV, we need PDFs evaluated at $x \gtrsim m_W/E_\nu \sim 10^{-4}$. Because these are known—at low $x$, from $e p$ collisions in HERA [53,54]—the uncertainty in the predicted TeV-PeV cross sections is small. References [4,55-65] have performed such calculations, some of which are shown in Fig. 3. Below $\sim 10$ TeV, they yield $\sigma_{\nu N} \propto E_\nu$, revelatory of hard scattering off partons, and in agreement with data. Above $\sim 10$ TeV, where $Q^2 \sim m_W^2$, they yield a softer-than-linear energy dependence, which has only been glimpsed in the available data up to 350 GeV [1-3].

Detecting high-energy neutrinos.—IceCube is the largest optical-Cherenkov neutrino detector. It consists of strings of photomultipliers buried deep in the clear Antarctic ice, instrumenting a volume of about 1 km$^3$.

Above TeV, CC interactions of $\nu_\mu$ and $\nu_\tau$ with nucleons in the ice, and NC interactions of all flavors, create localized particle showers, with roughly spherical Cherenkov-light profiles centered on the interaction vertex. CC interactions of $\nu_\mu$ additionally create muons that make elongated tracks of Cherenkov light, several kilometers long and easily identifiable. (Other, flavor-specific signatures require energies higher than in our analysis [66-74].)

From the amount of collected light in a detected event, and its spatial and temporal profiles, IceCube infers its energy and arrival direction. But it cannot distinguish neutrinos from antineutrinos, or NC from CC showers, since they make similar light signals.

Using contained showers only.—Because cross sections vary with neutrino energy, we use exclusively a class of IceCube events where the incoming neutrino energy can be inferred using as few assumptions as possible. These are “starting events,” where the neutrino interaction was contained in the detector. Of these, we use only showers, not tracks, due not to a fundamental limitation, but to the IceCube data that is publicly available. Our approach differs from that of Ref. [7], which used only through-going muons, born in neutrino interactions outside the detector, for which estimation of the neutrino energy requires making important assumptions.

We use the publicly available 6-year sample of IceCube High-Energy Starting Events (HESE) [8,48-50], consisting of 58 contained showers with deposited energies $E_{\text{dep}}$ from 18 TeV to 2 PeV. Below a few tens of TeV, about half of the showers are due to atmospheric neutrinos and half to astrophysical neutrinos [50]; above, showers from astrophysical neutrinos dominate [75,76].

Figure 2 shows the HESE showers distributed in $E_{\text{dep}}$ and zenith angle $\theta_z$. Representative uncertainties are 10% in $E_{\text{dep}}$ and 15° in $\theta_z$ [77], which we adopt to describe the detector resolution. Showers are scarce above 200 TeV because the neutrino flux falls steeply with $E_\nu$.

In CC showers, the full neutrino energy is deposited in the ice, i.e., $E_{\text{dep}} \approx E_\nu$, because both the outgoing electron or tau and the final-state hadrons shower. In NC showers, only a fraction $y$ of the neutrino energy is deposited in the ice, i.e., $E_{\text{dep}} = yE_\nu$, because only final-state hadrons shower. Standard calculations yield an average $\langle y \rangle = 0.35$ at 10 TeV and 0.25 at 1 PeV [55]. Because of this low value and because the neutrino fluxes fall steeply with $E_\nu$, NC showers are nominally subdominant at any value of $E_{\text{dep}}$.

In starting tracks, the shower made by final-state hadrons is contained by the detector, but the muon track typically exits

![FIG. 2. Neutrino-induced showers from the IceCube 6-year HESE 8,48-50 sample. Neutrinos arrive from above (cos $\theta_z$ > 0); from below, through Earth (cos $\theta_z$ < 0); and horizontally (cos $\theta_z$ = 0). They travel a distance $D$ inside Earth (of radius $R_\oplus = 6371$ km) to IceCube, buried at a depth of 1.5 km. The background shading represents the fraction of isotropic neutrino flux that survives after being attenuated by $\nu N$ interactions inside Earth, calculated using cross sections predicted in Ref. 59.)](image-url)
it. An assumption-free reconstruction of $E_\nu$ requires knowing separately the energy of the hadronic shower $E_{sh}$ and the muon energy loss rate $dE_\mu/dX$, which is proportional to the muon energy $E_\mu$ [77]. Yet, while these quantities are known internally to the IceCube Collaboration, public data only provide, for each starting track, the total deposited energy, $E_{sh} + |dE_\mu/dX|\Delta X$, where $\Delta X$ is the track length in the detector. Without additional information, in order to deduce $E_\nu$, we would need to assume values of $\gamma$ and $\Delta X$ for each event [78]. Hence, in keeping to our tenet of using few assumptions to deduce $E_\nu$, we do not include starting tracks in our analysis. This choice also reduces the chance of erroneously using a track created by an atmospheric muon, not a neutrino.

To use through-going muons in extracting the cross section, IceCube [7] inferred the most likely parent neutrino energy from the measured muon energy [77] by assuming the inelasticity distribution $d\sigma_{\nu N}/dy$ from Ref. [59]. By using only contained showers, we forgo the need to assume an inelasticity distribution and remain more sensitive to potential new physics that could modify it.

Sensitivity to the cross section.—Neutrino-nucleon interactions make Earth opaque to neutrinos above 10 TeV, so neutrino fluxes are attenuated upon reaching IceCube. More neutrinos reach it from above—after crossing a few kilometers of ice—than from below—after crossing up to the diameter of Earth.

A flux of incoming neutrinos with energy $E_\nu$ and zenith angle $\theta_z$ is attenuated by a factor $e^{-\tau_{\nu\nu}(E_\nu, \theta_z)} = \exp[-D(\theta_z)/L_{\nu N}(E_\nu, \theta_z)]$, where $\tau_{\nu\nu}$ is the opacity to $\nu N$ interactions, $D$ is the distance from the point of entry into Earth to IceCube, and $L_{\nu N} = m_N/[(\sigma_{\nu N}^{CC}(E_\nu) + \sigma_{\nu N}^{NC}(E_\nu))\langle\rho_{\nu\nu}(\theta_z)\rangle]$ is the neutrino interaction length. Here, $\sigma_{\nu N}^{CC}$ and $\sigma_{\nu N}^{NC}$ are, respectively, the CC and NC cross sections, $m_N$ is the average nucleon mass in isoscalar matter, and $\langle\rho_{\nu\nu}(\theta_z)\rangle$ is the average matter density along this direction, calculated using the density profile from the preliminary reference Earth model [55,79]. Details are in the Supplemental Material [80], which includes Refs. [81–94].

Attenuation grows with the cross sections—which grow with $E_\nu$—and with $D$; both effects are evident in the background shading in Fig. 2.

Within an energy interval, the number of events induced by a neutrino flux $\Phi_\nu$ is $N_{sh} \propto \Phi_\nu e^{-\tau_{\nu\nu}\sigma_{\nu N}}$. Down-going showers (cos $\theta_z > 0$)—unaffected by attenuation—fix the product $\Phi_\nu \sigma_{\nu N}$, while up-going showers (cos $\theta_z < 0$)—affected by attenuation—break the degeneracy between $\Phi_\nu$ and $\sigma_{\nu N}$ via $e^{-\tau_{\nu\nu}}$, providing sensitivity to the cross sections.

Extracting cross sections.—We propagate atmospheric and astrophysical neutrinos through Earth and produce test samples of contained showers in IceCube, taking into account its energy and angular resolution; see the Supplemental Material [80]. To extract the cross sections, we compare the distributions in $E_{\text{dep}}$ and cos $\theta_\nu$ of the test showers—generated with varying values of the cross sections—to the distribution observed by IceCube.

To probe the energy dependence of the cross sections, we bin showers in $E_{\text{dep}}$ and extract the cross section from data in each bin independently of the others. Except for global assumptions on detector resolution and the choice of atmospheric neutrino spectrum (see below), parameters extracted in different bins are uncorrelated.

The first three bins contain comparable numbers of showers: 18–50 TeV (17 showers), 50–100 TeV (18 showers), and 100–400 TeV (20 showers). The final bin, 400–2004 TeV, contains only 3 downgoing showers, between 1–2 PeV. Because of their short travel distances ($D \lesssim 10 \text{ km}$) and negligible expected attenuation, in this bin we only set a lower limit on the cross section. This stresses the need for upgoing HESE events above 400 TeV.

For astrophysical neutrinos, we use the most recent calculation of the $\nu_e$, $\bar{\nu}_e$, $\nu_\mu$, and $\bar{\nu}_\mu$ fluxes from pion and kaon decays from Ref. [95]. Their zenith-angle distribution at the South Pole, though anisotropic, is symmetric around $\cos \theta = 0$, so it does not introduce spurious directional asymmetries. We do not include a contribution from prompt atmospheric neutrinos [96–109], since searches have failed to find evidence of them [8–16]. We include the self-veto [110,111] used by the HESE analysis to reduce the atmospheric contribution.

For astrophysical neutrinos, we assume, independently in each energy bin, an isotropic power-law energy spectrum $\Phi_\nu \propto E_\nu^{-\gamma}$ for all flavors of neutrinos and antineutrinos, in agreement with theoretical expectations [112] and IceCube findings [113]. The value of $\gamma$ is obtained from a fit to data in each bin. This makes our results robust against variations with energy of the spectral shape of astrophysical neutrinos, unlike Ref. [7], which assumed a single power law spanning the range 6.3–980 TeV. We assume flavor equipartition, as expected from standard mixing [68,78,114–118] and in agreement with data [14,119]. Because IceCube cannot distinguish neutrinos from antineutrinos, we can only extract a combination of their cross sections, each weighted by its corresponding flux. We assume the likely case [120,121] of equal fluxes, coming, e.g., from proton-proton interactions [122].

Assumptions.—Because data are scant, to reduce the number of free parameters to fit, we make three reasonable assumptions inspired on standard high-energy predictions. With more data, they could be tested.

First, the rate of CC showers dominates over the rate of NC showers at any value of $E_{\text{dep}}$, based on the arguments above. For simplicity, we adopt a constant $\langle \gamma \rangle = 0.25$ for NC showers. This assumption allows us to express the extracted cross section as a function of $E_\nu \approx E_{\text{dep}}$.

Second, CC cross sections dominate over NC cross sections. We assume $\sigma_{\nu N}^{\text{NC}} = \sigma_{\nu N}^{\text{CC}}/3$ and $\sigma_{\nu N}^{\text{EN}} = \sigma_{\nu N}^{\text{CC}}/3$, following, e.g., Ref. [4]. This assumption allows us to fit only for CC cross sections.

Third, the ratio of $\bar{\nu} N$ to $\nu N$ cross sections is fixed in each bin. Hence, when fitting, $\sigma_{\nu N}^{\text{CC}} = (\sigma_{\nu N}^{\text{CC}}/\sigma_{\nu N}^{\text{CC}})\sigma_{\nu N}^{\text{CC}}$, $\sigma_{\nu N}^{\text{EN}} = (\sigma_{\nu N}^{\text{EN}}/\sigma_{\nu N}^{\text{EN}})\sigma_{\nu N}^{\text{EN}}$.
TABLE I. Neutrino-nucleon charged-current inclusive cross sections, averaged between neutrinos ($\sigma_{\nu N}^{CC}$) and antineutrinos ($\sigma_{\bar{\nu} N}^{CC}$), extracted from 6 years of IceCube HESE showers. To obtain these results, we fixed $\sigma_{\nu N}^{CC} = (\sigma_{\nu N}^{CC} / \sigma_{\bar{\nu} N}^{CC}) \sigma_{\bar{\nu} N}^{CC}$—where $\langle \sigma_{\nu N}^{CC} / \sigma_{\bar{\nu} N}^{CC} \rangle$ is the average ratio of $\bar{\nu}$ to $\nu$ cross sections calculated using the standard prediction from Ref. [59]—and $\sigma_{\bar{\nu} N}^{CC} = \sigma_{\nu N}^{CC} / 3$, $\sigma_{\nu N}^{NC} = \sigma_{\bar{\nu} N}^{CC} / 3$. Uncertainties are 1σ, statistical plus systematic, added in quadrature.

<table>
<thead>
<tr>
<th>$E_{\nu}$ [TeV]</th>
<th>$&lt;E_{\nu}&gt;$ [TeV]</th>
<th>$\langle \sigma_{\nu N}^{CC} / \sigma_{\bar{\nu} N}^{CC} \rangle$</th>
<th>$\log_{10}[\sqrt[3]{\frac{1}{2} (\sigma_{\nu N}^{CC} + \sigma_{\bar{\nu} N}^{CC})}]$ / cm$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>18–50</td>
<td>32</td>
<td>0.752</td>
<td>−34.35 ± 0.53</td>
</tr>
<tr>
<td>50–100</td>
<td>75</td>
<td>0.825</td>
<td>−33.80 ± 0.67</td>
</tr>
<tr>
<td>100–400</td>
<td>250</td>
<td>0.888</td>
<td>−33.84 ± 0.67</td>
</tr>
<tr>
<td>400–2004</td>
<td>1202</td>
<td>0.957</td>
<td>&gt; −33.21 (1σ)</td>
</tr>
</tbody>
</table>

where $\langle \sigma_{\nu N}^{CC} / \sigma_{\bar{\nu} N}^{CC} \rangle$ is the average ratio in that bin predicted by Ref. [59] (see Table I). This assumption allows us to fit only for $\nu N$ cross sections.

Thus, within each energy bin, we independently vary only the $\nu N$ CC cross section $\sigma_{\nu N}^{CC}$ and three nuisance parameters—the number of showers due to atmospheric neutrinos $N_{s_{sh}}^{\nu}$, the number of showers due to astrophysical neutrinos $N_{s_{sh}}^{\nu_{ast}}$, and the astrophysical spectral index $\gamma$. To avoid introducing bias, we assume flat priors for all of them. For each choice of values, we compare our test shower spectrum to the HESE shower spectrum via a likelihood. To find the best-fit values of the parameters, we maximize the likelihood. The Supplemental Material describes the statistical analysis in detail [80].

Results.—Table I shows the extracted cross section, marginalized over the nuisance parameters. Because $\sigma_{\nu N}$ and $\sigma_{\bar{\nu} N}$ are not independent in the fit, we present their average there and in Figs. 2 and 3.

Figure 3 shows that, in each bin, results agree within 1σ with widely used standard predictions. The IceCube Collaboration has adopted the cross section from Cooper-Sarkar et al. [59]. We include other calculations for comparison [4,55,56,60,62]. All predictions are consistent with our measurements within errors.

Our results are consistent with the IceCube analysis [7], which found a cross section compatible with Ref. [59]. Their smaller uncertainty is due to using $\sim10^4$ through-going muons. However, by grouping all events in a single energy bin, their analysis did not probe the energy dependence of the cross section. Like that analysis, our results are also consistent with standard cross-section predictions, but in narrower energy intervals.

Because the number of showers in each bin is small, statistical fluctuations weaken the interplay of down-going versus up-going showers described above. To isolate the dominant statistical uncertainty, we minimized again the likelihood, this time keeping the nuisance parameters fixed at their best-fit values (see Table II in the Supplemental Material [80]). The resulting uncertainty, attributed to statistics only, is 0.51, 0.63, and 0.62 in the first three bins, where we have a measurement. The systematic uncertainty, obtained by subtracting these values in quadrature from the total uncertainties in Table I is 0.14, 0.23, and 0.25 in each bin, slightly higher than in Ref. [7], due to a less detailed modeling of the detector. While Ref. [7] found comparable statistical and systematic uncertainties, we are presently dominated by statistics, since it uses an event sample that is smaller by a factor of $\sim200$.

Nevertheless, our results disfavor new-physics models where the cross section rises sharply below 1 PeV [6,22–29]. Figure 3 shows as example a model of TeV-scale gravity with large extra dimensions [21]. While this model was disfavored by the LHC [123,124], we provide independent confirmation via a different channel. More stringent tests of new-physics models, beyond the scope of this Letter, should also consider the effect of modifications to the inelasticity distribution.

Limitations and improvements.—IceCube is sparsely instrumented and designed to detect the enormous light imprints made by high-energy neutrinos. Except for high-energy muons, it cannot track individual particles or reconstruct $Q^2$ and $x$, unlike densely instrumented detectors. Hence, we can only extract the cross section as a

FIG. 3. Neutrino-nucleon charged-current cross section, averaged for neutrinos and antineutrinos, from different predictions (lines) [4,56,59,60,62], compared to measurements from this work (stars). The low-energy global average [51] has the linear dependence on $E_{\nu}$ appropriate below $\sim10$ TeV. The model of large extra dimensions, included for illustration, is from Ref. [21] (quantum-gravity scale of 1 TeV and all partial waves summed), corrected here to match modern standard predictions of the cross section below 1 PeV.
function of energy, integrated over other kinematic variables. While we cannot extract individual PDFs, we can test their combination in the cross section.

Further, IceCube cannot distinguish if a particular shower was made in a CC or an NC interaction, and by a neutrino or an antineutrino. The differences are too subtle to unequivocally identify them in individual showers [125], but it might be possible to extract them statistically from a large enough data sample [126].

Lastly, we assumed that the astrophysical neutrino flux is isotropic [14,127,128]. Nevertheless, there are hints of a Galactic contribution [14,127,129,130], with data allowing < 14% of the all-sky flux to come from the Galactic Plane [128]. If a Galactic flux is discovered, future cross-section analyses will need to acknowledge its anisotropy to avoid incorrectly attributing the distribution of arrival directions solely to in-Earth attenuation.

Summary and outlook.—We have extracted the energy dependence of the neutrino-nucleon cross section at energies beyond those available in man-made neutrino beams, making use of the high-energy reach of IceCube. Our results are compatible with predictions based on nucleon structure extracted from scattering experiments at lower energies and disfavor extreme deviations that could stem from new physics in the TeV–PeV range.

It would be straightforward to repeat the present analysis using a larger HESE shower sample. The proposed upgrade IceCube-Gen2 [131] could have an event rate 5–7 times higher, thus reducing the impact of random fluctuations. These showers could be combined with showers from the upcoming KM3NeT detector [132]; their improved angular resolution of \( \sim 2^\circ \) above 50 TeV would allow for better estimates of in-Earth attenuation. Starting tracks can also be considered, as long as one does not rely on predictions of the inelasticity distribution to reconstruct the parent neutrino energy.

An interesting possibility is to measure the inelasticity distribution [133]. This can be done using starting tracks where the hadronic shower energy \( E_{\text{sh}} \) and the outgoing muon energy \( E_{\mu} \) are known individually, in order to reconstruct the inelasticity \( y = (1 + E_{\mu}/E_{\text{sh}})^{-1} \) [134,135].

At the EeV scale, differences between cross-section predictions increase. Measuring \( \sigma_{xN} \) at these energies would probe \( x \sim m_\nu/E_\nu \lesssim 10^{-6} \), beyond the reach of laboratory scattering experiments. This would prove instrumental in testing not only new physics, but also predictions of the potentially nonlinear behavior of PDFs at low \( x \), such as from BFKL theory [136–139] and color-glass condensates [140]; see, e.g., Refs. [63,141,142]. However, because the predicted neutrino flux at these energies, while uncertain, is smaller than at PeV, precision measurements of the cross section will likely be limited by statistics; see Ref. [143] for details. Nevertheless, large-volume neutrino detectors like ARA [144–146], ARIANNA [147,148], GRAND [149], and POEMMA [150] might differentiate between predictions provided the event rate is high enough.

M.B. is supported in part by NSF Grants No. PHY-1404311 and No. PHY-1714479, and by the Danmarks Grundforskningsfond Grant No. 1041811001. A.C. is supported by NSF CAREER Grant No. 1255557. This work used resources provided by the Ohio Supercomputer Center. We thank Patrick Allison, John Beacom, Francis Halzen, Spencer Klein, Shirley Li, and Subir Sarkar for useful feedback and discussion.

*mbustamante@nbi.ku.dk; ORCID: 0000-0001-6923-0865
\(^c\)connolly@phys.osu.edu; ORCID: 0000-0003-0499-5448


[60] M. M. Block, L. Durand, and P. Ha, Connection of the virtual $\gamma^*p$ cross section of ep deep inelastic scattering to real $\gamma p$ scattering, and the implications for $\nu N$ and $ep$ total cross sections, Phys. Rev. D 89, 094027 (2014).


