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Published in:
Optics Express

DOI:
10.1364/OE.27.018561

Publication date:
2019

Document version
Publisher's PDF, also known as Version of record

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Citation for published version (APA):
Sensitive optomechanical transduction of electric and magnetic signals to the optical domain

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Abstract: We report a radio-frequency-to-optical converter based on an electro-optomechanical transduction scheme where the electrical, optical, and mechanical interface was integrated on a chip and operated with a fiber-coupled optical setup. The device was designed for field tests in a magnetic resonance scanner where its small form-factor and simple operation is paramount. For the appurtenant magnetic resonance detection circuit at 32 MHz, we demonstrate transduction with an intrinsic magnetic field sensitivity of 8 fT/√Hz, noise figure 2.3 dB, noise temperature 210 K, voltage noise 99 pV/√Hz, and current noise 113 pA/√Hz, all in a 3 dB-bandwidth of 12 kHz. Such sensitivity and bandwidth make the transducer a valuable alternative to conventional electronic preamplifiers that additionally is directly compatible with fiber communication networks.

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1. Introduction

In the past decade, a promising research field has emerged from transduction of radio and microwave electromagnetic radiation onto light using nano- and micro-mechanical oscillators [1]. Such electro-mechano-optical transducers offer exciting opportunities. They are strong candidates for transduction at a single photon level when operated at cryogenic temperatures. When operated at room temperature, they have little added noise and are therefore useful for sensing. By up-converting electrical signals to a light carrier, these transducers can additionally be used to transmit radio-frequency (RF) and microwave radiation with optical fibers, thus replacing lossy wires and waveguides and extending the usual advantages of optical fibers to the RF and microwave domain.

The platform presented in this paper uses the electro-mechano-optical transduction scheme suggested in [2] and first implemented in [3]. Figure 1 depicts this scheme where a mechanical actuator, a membrane, serves both as a plate of a capacitor and as a mirror of an optical cavity. The RF signal that will be transduced is picked-up by an antenna (coil) connected to the membrane-capacitor, thus forming an electrical circuit tuned with an additional capacitor to resonate at the signal frequency. The circuit is also driven by an AC bias that induces charges on the membrane-capacitor along with the signal. The charges exert a force on the membrane proportional to the charges squared, so this force contains the beatnote between the signal and bias. With the right choice of frequencies, the beatnote will be at the mechanical resonance frequency and drive the membrane’s motion even though the electrical and mechanical frequencies are different. As the membrane vibrates it changes the capacitance and, in turn, both the charges on the capacitor and the electrical resonance frequency—the electrical and mechanical system are
parametrically coupled [4]. Furthermore, the vibrations affect the optical cavity by changing its length and, therefore, the light reflected from it, consequently transducing the RF signal into an optical modulation. This electro-opto-mechanical transduction scheme has recently been used to detect a magnetic resonance (MR) signal [5] but with a high level of noise. The phase-noise contribution from the AC bias was later reduced [6], but remained significant. Moreover, the detection bandwidth has been low compared to standard electronic amplifiers, ≤3 kHz in [3, 5, 6], but [7] has demonstrated that multiple mechanical resonances can increase the bandwidth.

Our work aims to transduce the signal generated in an MR scanner used for medical imaging. Specifically, we aim to detect the $^{13}$C isotope in a 3T magnetic field which corresponds to a Larmor frequency of 32 MHz. However, several sensing applications could benefit from the low-noise and low-voltage conversion to a light carrier as it grants access to the advantages offered by optical telecommunication technology, i.e. low-loss and immunity to electromagnetic fields and the associated noise. The latter is especially important in MR imaging [8]. For all real-world applications it will be crucial to have the transduction implemented in a compact, robust, and simple fashion. Therefore, we have worked towards a solution that is compact enough to fit into a standard circuit socket, that fixes all components robustly so both additional stabilization and alignment is unnecessary, and that requires only a simple optical setup that is easily transported and assembled outside a laboratory. Such miniaturization and ease-of-use is particularly relevant for MR detection arrays with more than one coil, which is the state-of-the-art technology commonly used in MR imaging systems.

In section 2, we present a device that integrates the membrane, capacitor, and optical cavity required for transduction on a chip coupled directly to fiber. Our transducer features a membrane-electrode distance around 500 nm; a key parameter realized with cleanroom processing techniques. This results in a large electro-mechanical coupling with just a couple of volts of driving AC, and we show it significantly improves the electro-mechanical interface over previous works on room-temperature transduction. In section 3, we use the device to transduce RF signals at 32 MHz onto light and show this transduction adds noise roughly equal to two thirds of the electrical Johnson noise. That noise-figure is much lower than [5–7] and, unlike them, we did not see a significant contribution from phase-noise. Finally, our detection bandwidth was nearly as large as in [7] but used only one mechanical resonance.

2. Transducer

To make the transducer, we followed the step-by-step fabrication procedure described in appendix A. Crucially, our membrane was comprised of Al and Al$_2$O$_3$ under high tensile stress and we targeted mechanical frequencies slightly above 1 MHz for the fundamental resonance, which we achieved with a nearly circular boundary at a diameter above 150 μm. As shown in Fig. 2(a), we had the membrane suspended over a second metal electrode to form a mechanically
compliant parallel-plate capacitor. In addition, the bottom capacitor plate had a hole through its center so light could pass through it, and all layers were deposited on top of a partially reflective dielectric mirror. That mirror formed an optical cavity together with the reflective—and moving—membrane, and the membrane-mirror distance was set by the thickness of a sacrificial layer beneath the membrane and tuned to give amplitude modulation for our operating wavelength 1064 nm. Specifically, our cavity had an optical path length around one wavelength. Note that our design caused the membrane to deform after its release which gave a small offset on cavity length that we estimate in section 2.3. To get light into the cavity, we aligned and permanently attached a gradient refractive-index lens to the pack of the membrane-capacitor substrate, and again aligned and permanently attached the lens to a single-mode optical fiber. We furthermore mounted and wirebonded the membrane-capacitor chip to a standard 8-pin circuit socket (see Fig. 2(b)) that enabled easy connection to a circuit. The socket included a switch that we used to short the capacitor to protect the membrane from electrostatic discharges.

In the subsections below, we first characterize the mechanical performance of the membrane with emphasis on the frequency and quality (Q) factor of the fundamental resonance, measured over a long time and for many samples. Then we introduce the basic theory behind the coupling between circuit and membrane and give two figures-of-merit that allow us to compare this work with other literature. Finally, we model and measure the integrated cavities, and account for an observed dynamical backaction effect from the interaction between light and the membrane. Additionally, we investigate what optimal transduction performance we expect for our integrated cavities and discuss how to account for the backaction in section 3.

### 2.1. Mechanical characterization

We measured the frequency and Q-factor of bare membranes to investigate the repeatability of our process. Those measurements were performed on thermally driven membranes in vacuum (pressure $\leq 1 \times 10^{-5}$ mbar) with a commercial vibrometer. We probed the mechanical motion at the membrane center and recorded the displacement power spectrum with a resolution bandwidth of 2 Hz—well below the mechanical linewidth—after averaging those spectra thirty times. The membrane gave a peak in the power spectrum and, from that, we obtained the mechanical resonance frequency and Q-factor using a peak-fit function built into the vibrometer. We investigated seven groups of over fabrication run—each group containing five or seven membranes with identical parameters, to a total of 45 samples—and found resonance frequencies between 1.0 MHz and 1.5 MHz for the fundamental mode, with a variation below 2 % for samples with identical geometry. The mechanical Q-factor showed values between 5 000 and 60 000, with the majority between 10 000 and 20 000. We believe this spread comes from a coupling between the membrane and the modes in the substrate that allow phonon tunneling [9], possibly combined...
with variation in film-quality due to micro-cracks along grain-boundaries, and that the Q-factor’s upper limit is set by material loss in Al [10, 11]. When repeating measurement immediately, the resonance frequencies had decreased slightly while the Q-factor fluctuated up to 50% for its highest values and by ca. 2000 for most others. Most likely because the vibrometer’s probe laser heats the membrane and substrate a little and the consequent thermal expansion causes the substrate and membrane modes to drift thus changing their coupling [12]. On every wafer there were some samples, less than ten, with mechanical resonances that were inconsistent or had Q-factors below 10000. The latter would usually degrade over time until no peak could be measured indicating severe defects. Many other samples showed a small decrease in Q-factor over time or from the continued fabrication process, but the final yield of membranes with Q ≥ 10000 was higher than 75%.

The mechanical frequencies drifted down slowly over time. Long-term observations revealed that, typically, the decrease started with 10% during the first two weeks, with only another 2% added after nearly a year suggesting it reached an equilibrium. We believe this comes from stress relaxation in the metal and therefore investigated the tensile stress in both the aluminum and alumina layers. The stress was extracted from the wafer curvature with and without those layers, measured with a profilometer that sweeps a stylus in contact with the surface over the entire wafer length. We always tried to scan along the same line in both measurements, but that was difficult in our setup. For alumina, we deposited the layer on both sides of a silicon wafer, but removed it from the backside, and measured the tensile stress to be 350 MPa. That agrees with [13] for our deposition parameters. In addition, we compared our measured and simulated frequencies for membranes of pure alumina and got good agreement assuming 400 MPa tensile stress in the simulation. For aluminum, the measured stress was approximately 400 MPa after annealing. When we compared our measured and simulated frequencies with the top electrode added, we found that the stress in Al must start around 350 MPa and fall to 200 MPa to agree with the frequency drift. This stress relaxation is similar to what [14] reported. The problem of frequency drift was not reported in [3, 5, 7, 11], probably because the high tensile stress of their LPCVD nitride membranes dominates the total stress of the compounded membrane thus making the effect small.

2.2. Electromechanical coupling

Two key figures of merit for transduction are the temperature $T_{\text{mech}}$ of the noise added by the mechanical intermediary, as well as the mechanical linewidth broadened by the electromechanical interaction $\Gamma_{\text{EM}}$. The latter determines the transduction bandwidth. It can be shown [3] that

$$T_{\text{mech}} = \frac{T_m \Omega_L}{C \Omega_m}, \quad \Gamma_{\text{EM}} = \Gamma_m (C + 1),$$

(1)

with $T_{\text{mech}}$ evaluated at the peak of the mechanical frequency response, i.e. at the most sensitive frequency. Therefore, all noise temperatures from now on will refer to the mechanical resonance frequency. In the expression, $T_m$ is the membrane temperature, $\Omega_L$ the electrical resonance frequency, $\Omega_m$ the mechanical resonance frequency, $\Gamma_m$ the intrinsic 3 dB mechanical linewidth, and $C$ is the electromechanical cooperativity [3]:

$$C = \frac{G_{\text{em}}}{m \Omega_m \Gamma_m \Omega_L \Gamma_L}.$$

(2)

Here, $m$ is the effective membrane mass [15], $L$ is the electrical inductance, and $\Gamma_L$ is the circuit resonance 3 dB linewidth. $G_{\text{em}}$ is the electromechanical coupling between the membrane and circuit resonance, induced by applying a bias across the capacitor $C$ for the resonance circuit. The bias $\bar{q}$ is the charge induced on $C$ for a DC bias or half the peak charge induced for an AC bias. The electromechanical coupling scales with the mechanical motion’s modulation of
Fig. 3. (a) Setup schematic showing the DC bias circuit and how the laser probed the non-fiber-coupled samples. (b) Optically measured spectrum of the thermal-driven mechanical motion. The resolution bandwidth was below 7 Hz and the spectra were averaged twenty to thirty times. The fit is a Lorentzian function plus an offset. (c) Mechanical resonance frequency versus DC bias with a parabola fitted to the data.

$C_{em} = \frac{\bar{q}^2}{C^2 C'_m}$ [16]. Here, $C'_m$ is a shorthand notation for the derivative of the membrane-capacitor $C_m$ with respect to the membrane displacement $x$ from its equilibrium, and we see that it alone characterizes how much the membrane-capacitor implementation affects the electromechanical coupling. In appendix B we derive how $C'_m$ may be estimated from measurements. That way we may compare different realizations of the membrane-capacitor. We obtain

$$C'_m \approx \frac{3.2 m \Omega_m \Delta \Omega_m}{V^2},$$

(3)

where we have introduced the effective electrical gap between the capacitor plates $d_{eff}$ and the mechanical frequency shift $\Delta \Omega_m$ for a given DC bias $V$. The gap is not just the distance between the bottom and top electrode because there is the dielectric alumina layer between. It is rather determined by the permittivity between the electrodes and alumina adds less to the gap than its thickness suggests because it has a higher permittivity than vacuum. Equation (3) also works for a square membrane because the proportionality constant is almost the same for those geometries. The result gets even smaller for the floating electrode geometry in [3, 5–7]. The reduction is a factor of four in the best case scenario.

Before fiber-coupling the samples, we measured the mechanical frequency shift versus DC bias as shown in Fig. 3(a) for membranes with effective mass $m = 3.9$ ng, frequency $\Omega_m = 1.16$ MHz and effective gap $d_{eff} = 560$ nm. Because our fabrication sets $d_{eff}$, we have an uncertainty on it from the layer deposition that should be below a few nanometer. However, we found a systematic offset, explained in section 2.3, so the uncertainty on $d_{eff}$ is instead closer to 10 nm, or about 2%. In contrast, the uncertainty is lower than 2% on the measured frequencies and comparable to 2% on the DC bias, but we set it higher for the physical mass because the exact Al and Al$_2$O$_3$ density is unknown and will depend on the deposition parameters. Altogether, we approximate the uncertainty on $C'_m$ to be 20%. All frequency measurements were in a vacuum of $<1 \times 10^{-5}$ mbar and we filtered the DC bias' electrical noise at the mechanical frequency using a low-pass filter formed between a 100 kΩ resistor and 220 nF capacitor connected in parallel to the transducer socket through a coaxial vacuum feed-through—see Fig. 3(a). Like above, we obtained the resonance frequencies from the thermally driven mechanical motion (see Fig. 3(b)), this time measured with a custom-built Michelson interferometer [17]. The frequency shift followed a parabola, as shown in Fig. 3(c), with the second-order coefficient of 1 kHz/V$^2$ and an offset of roughly 1 V. We did not expect this offset (see theory in appendix B, Eq. (8)) but attribute it to trapped charges on the membrane, like in [18] although we did not see any frequency hysteresis with bias. Such static charges will not mix with the electromechanical coupling induced by an AC bias, but they will for DC.

When combining the measured numbers, $C'_m$ for our device becomes $(50 \pm 10)$ nF/m. Even
with our assumed error, our device compares favorably with previous works for which we estimate 0.02 nF/m [3], 0.2 nF/m [7], and 4 nF/m [5] using Eq. (3). To make these estimates for [3, 5, 7], we read off a frequency shift for a given bias in their data and divided by four to adjust for floating electrode geometry. Furthermore, it is interesting to collect and evaluate the part of $C$ (Eq. (2)) that only depend on the membrane-capacitor using Eqs. (3) and (7). From that, we find

$$C \propto \left(\frac{C_{em}'}{m \Omega_m \Gamma_m}\right)^2 \approx \frac{10 m \Omega_m d_{eff}^2 \Delta \Omega_m^2}{\nu^4 \equiv \alpha}.$$ (4)

Our numbers then give $\alpha$ of $(130 \pm 25)$ fF/J which again is favorable compared to values $<1$ fF/J achieved in previous works [3, 5, 7]. We attribute this enhancement in performance primarily to the small gap between membrane and electrode in our implementation of the mechanical capacitor.

2.3. Optomechanical interaction

We investigated the reflection from the integrated cavity as a function of the cavity length and compared our measurements to a corresponding model. To scan the cavity, we applied a DC bias to the capacitor which gave the membrane a static displacement $\bar{x}$ given by

$$\bar{x}/d_{eff} \approx -1.6 \Delta \Omega_m/\Omega_m,$$ (5)

assuming $\bar{x} \ll d_{eff}$ (appendix B). Using Eq. 5 we extracted the displacement $\bar{x}$ directly from the measured frequency shift of the membrane. To model the cavity, we used the transducer’s unbiased gap and optical parameters for fabricated layers (see table 1 in appendix A), expressed as an optical propagation matrix, then multiplied them together and extracted the field reflection from the resulting matrix [19, ch. 6]. Making the membrane-mirror distance shorter by $\bar{x}$ will change both the cavity reflection and the optical modulation for given mechanical motion. The latter is proportional to the derivative of the cavity reflection versus displacement (the cavity slope), and we inferred it from the optical modulation due to thermal motion of the membrane assuming constant thermal driving power. To do that, we first needed to account for an optomechanical effect, that changes the linewidth of the membrane motion, as explained below.

We observed an increase (decrease) in the mechanical linewidth with optical power when the cavity resonance wavelength was below (above) the laser wavelength. When the linewidth narrowed there was a power threshold beyond which the membrane began to oscillate. We attribute these effects to dynamical backaction known from optomechanics [20] where a force out-of-phase with the displacement leads to work done by, or on, the mechanical resonator. Radiation pressure could be such a force, but there would not be a phase-lag between the intra-cavity photons and the membrane motion because the cavity linewidth is much larger than the mechanical resonance. Instead, we believe the dominant optical force comes from photothermal backaction [21] where light absorbed by the metal (about 5% at 1064 nm) makes the electrode expand thus changing the cavity length. The phase-lag then comes from a finite time constant of the thermal effects. Such optomechanical backaction changes the linewidth without adding noise to the thermal drive force [21]. From the fluctuation-dissipation theorem for the mechanical system, we know that the thermal drive power is proportional to the mechanical temperature and linewidth. So when the backaction rescales the linewidth and not the thermal drive power, that is equivalent to changing the mechanical mode’s temperature inversely proportional with the rescaled linewidth. Furthermore, the power spectral density for the thermal motion is proportional to the thermal drive power, but the rescaled linewidth cancels out when integrated over all frequencies. As a result, the variance of the thermally-driven motion will be proportional to the mechanical temperature rescaled by backaction, and the product between the measured variance and linewidth will be a constant. Therefore, we adjusted the measured optical modulation stemming from the membrane
motion accordingly. Finally, the linewidth also increased with the DC bias independently of the interaction with light. We accounted for this in the same way because it appeared to behave like dynamical backaction, but we are unsure of its origin.

The measured cavity reflection and its derivative agree qualitatively with the corresponding model with a small correction as shown in Fig. 4. To get this data, we used a setup that probes the cavity, shown in Fig. 3(a), with the fiber network in Fig. 2(c), and measured the reflection both from the cavity and right next to it, as indicated in Fig. 3(a), to calibrate out losses in the optical path. We obtained the mechanical frequency shift from fits to the power spectral density of the reflected light (see Fig. 3(b)) and converted it to \( \bar{x} \) with Eq. (5). The same fits gave the integral under the peak due to thermally driven mechanical motion, which is proportional to the variance of the membrane’s motion, and they gave the linewidth used to adjust for backaction as explained above. The outcome was used as a measure for the cavity derivative. To match the measurement and models, we had to correct the cavity length by adding 7 nm. This could come from two effects: either uncertainties in the deposition thickness and refractive index of the various layers—we typically have a few percents in the inferred parameters and \( \leq 1 \) nm in measured thicknesses. Or from the membrane deformation (appendix A, step 2). We verified through simulations that the deformation at the membrane’s center could fluctuate tens of nanometers out-of-plane at the membrane center, depending on the layers stress, Young’s modulus, and the simulation boundary condition.

We can extract the expected transduction performance, including both mechanical and optical noise, from the signal-to-noise ratio between the intrinsic mechanical peak \( \sigma_{x,x}^{pk} \) and the displacement noise floor from the optical detection \( \sigma_{x,x}^{light} \). In fact, there is an optimal cooperativity

![Fig. 4. Measured and modeled cavity reflection \( R \) (top) and cavity reflection derivative \( \Delta R \) (bottom) as a function of the membranes static deflection induced by a DC bias (inset). \( \Delta R \) was measured at several optical powers, and the errorbar is one standard deviation derived from the variance after the power has been normalized out. \( \Delta R \) is normalized such that both the data and adjusted model is one at zero bias.](image)
for transduction [3]:

\[
C_{\text{opt}} = \sqrt{\frac{S_{pk}^{xx}}{S_{\text{light}}^{xx}}} + 1 = \sqrt{\frac{4 k_B T_m}{m \Omega_m^2 \Gamma_m S_{\text{light}}^{xx}}} + 1
\]  

(6)

where \(k_B\) is Boltzmann’s constant. This optimum can be estimated directly from measured thermal-driven spectra by extracting the peak amplitude and the background offset, without calibrating the membrane motion in displacement units. We chose to operate our transducer close to \(C_{\text{opt}}\), and that was not necessarily where the cavity was most sensitive because the AC bias displaced the membrane. All samples had \(C_{\text{opt}} \gg 1\) so the +1 in Eqs. (1) and (6) can be neglected—we assume this limit from here on. At \(C_{\text{opt}}\) there is equal noise contributed from the light and mechanics at the membrane resonance frequency. That means the total noise contribution from the transducer optimally will be two times \(T_{\text{mech}}\) in Eq. (1) with \(C = C_{\text{opt}}\). For the fiber-coupled samples in general, we have measured peak-to-background ratios from 23 dB at 100 µW to 34 dB near 1 mW of optical power, which corresponds to \(C_{\text{opt}}\) from 14 to 50. We limited the optical power to 1 mW to avoid laser-induced degradation or damage of the membrane; already at 700 µW, the light shifted the mechanical frequency down by 6 kHz.

Optomechanical backaction can be included into the performance estimate by either using the mechanical linewidth broadened by backaction, and the corresponding effective mode temperature, or the intrinsic linewidth and temperature. These two approaches will lead to different cooperativities calculated from Eqs. (1) and (6). But the estimated transduction performance will be then same. This can be seen by inserting Eq. (6) into Eq. (1) and noting both \(T_{\text{mech}}\) and \(\Gamma_{\text{EM}}\) depend on the product of the mechanical linewidth and temperature. This is proportional to the thermal power which is constant even in the presence of backaction, as argued above. We chose to use the broadened linewidth to estimate the cooperativity in the following because that is what we measure during operation. However, we still needed to know the intrinsic linewidth, and consequently how much the backaction broadened the linewidth, in order to get the effective mode temperature. Therefore, we measured the intrinsic mechanical linewidth for the fiber-coupled sample we used for electro-mechano-optical transduction (section 3). It was found by fitting the membrane linewidth at different optical powers and extrapolating to zero power as shown in Fig. 5. The spectra fits were again like shown in the Fig. 3(b), measured in sets of five sequential spectra taken under identical settings. We used the weighted mean and unbiased variance from each set as data, and the fit confidence interval as a weight. The fit to the linewidths was a straight line and gave the offset \((124.5 \pm 0.6)\) Hz. Testing the fiber-coupled samples revealed that the product between the linewidth and thermally driven motional variance was not constant with optical power, as expected, but instead converged at higher power. The exact convergence was different between samples. We attribute this behaviour to poor alignment of the optical focus in the fiber-assembly, and have checked that deliberate misalignment of the
focus in the free-space setup (Fig. 2(c)) also gives a converging behaviour. But it is difficult
make a comparison due to the difference in convergence for different assemblies.

3. Signal detection
This section recount how we characterized the performance of the transducer for a circuit made
to detect MR signals at 32.19 MHz. First, we explain the design of the detection resonance (Fig.
6(a)) and how we injected a voltage signal into the pick-up coil. Then we describe an equivalent
model of the circuit (Fig. 6(b)) that fitted to measured scattering parameters (Fig. 6(c)), and how
that enabled us to extrapolate the intrinsic transduction noise coming from the optical readout
and mechanical noise (Fig. 7). Finally, the result is summarized in different units relevant for
evaluating the transducer performance.

3.1. Circuit design
On the circuit, the mechanically compliant capacitor $C_m$ was connected to a printed circuit board
(PCB) with the shortest wires possible and in parallel with a detection coil $L_d$ and a tuning capacitor
$C_p$, thus creating an electrical $LC$-resonance as shown in Fig. 6(a). The electromechanical
interaction between membrane and circuit required a drive bias at $\Omega_D = \Omega_L - \Omega_m$ because
$\Omega_L > \Omega_m$. For the actual transduction, this drive bias was filtered by an external bandpass
filter, before being connected to the PCB at port 2, to suppress sideband noise of the bias at $\Omega_L$.
Furthermore, the PCB included a bandpass filter at the frequency $\Omega_f$, formed by $L_f$ and $C_f$
and connected through the coupling capacitor $C_c$. This filter minimized how much port 2 loads the
detection resonance and it suppressed the sideband noise of the bias. As the detection resonance
and filter frequency get close they start to influence each other, so we targeted a minimum
difference of 1 MHz. Because $\Omega_f \approx \Omega_D$ that also constrains $\Omega_m$ to be larger than 1 MHz.

The PCB was placed inside a small vacuum chamber ($< 1 \times 10^{-3}$ mbar) terminated in a domed
glass tube. The detection coil was far away from any vacuum flange to prevent loading the coil
Q. The fiber went through a custom vacuum feedthrough [22], and the bias through a standard
vacuum feedthrough for coaxial cables. Everything was mounted inside a much larger aluminum
box to shield it from RF environment noise, and the setup was kept at room temperature. The
detection coil $L_d$ was near an excitation coil $L_e$ that probes the system through their mutual
inductance $M$ between the coils, with $L_e$ connected to port 1. Figure 6(b) shows an equivalent
model for the circuit between port 1 and 2. Here, all inductors includes a parasitic series resistance
and $L_e$ includes a parasitic parallel capacitance to account for its self-resonance. Note that
during transduction the membrane-capacitor corresponds to an equivalent circuit of a capacitor
connected in parallel with another series $LC$ resonator in [16], but without the drive bias it is a
capacitor.

3.2. Circuit model
To model the circuit, we expressed each element in Fig. 6(b) in a transmission (ABCD) matrix
formalism and multiplied the matrices together. A standard transformation converts the result
into scattering parameters [23, ch. 4.4] between port 1 and 2. We measured these with a network
analyzer. The model was fitted to all scattering parameters simultaneously with a Markov-chain
Monte Carlo minimization ([24] version 2.2.1 implemented in Python 3.6.3) of the linear
least-squares for all curves. In addition to the model in Fig. 6(b), we had to adjust for a phase
delay and absorption from cables, even though we calibrated the network analyzer as close to the
circuit as possible. However, both effects were small: the phaseshift was >11 mrad on each port,
and the amplitude scaling was $<0.25\%$ of $S_{11}$ and negligible for $S_{21}$ and $S_{22}$. Starting values of
the fit parameters came from the circuit component values, a separate fit to the excitation coil
without the detection circuit nearby, and a reasonable guess for the detection coil and tuning
 capacitors, but all parameters were left as free fit variables. The fitted numbers deviated from
the predetermined values by 1.5% to 12%, and the residuals were below 1% of the maximum value for the given curve—with most of them far below—indicating good agreement between fit and model. The fit also included a random vector that accounted for measurement noise, and that consisted of normal-distributed values where the width was a fit parameter. The fit gave the distribution-width of $3 \times 10^{-4}$ which is too small to solely account for the residuals. That means there are systematic offsets from imperfections in the model. Moreover, we estimated the filter-limited Q-factor of the detection circuit to be 91.4. That agrees with an independent measurement where we found 92 (119) with (without) the transducer connected from the 3 dB circuit linewidth measured with a double-loop probe, suggesting the membrane-capacitor chip has an electrical Q-factor of 405.

Using the circuit model, we calculated how a signal injected into the excitation coil translates into a voltage in series with the detection coil. On top of that, there also was an equivalent Johnson noise in series with the detection coil and we simulated it with the circuit model to be $117 \text{ pV}/\sqrt{\text{Hz}}$. Both the injected signal and Johnson noise drive the membrane motion and Fig. 7 (left panel) shows driven power spectral densities of mechanical motion, measured optically and averaged twenty times. The input signal was a white-noise drive around $\Omega_L$, injected into the excitation coil with different powers, and the drive bias at $\Omega_D$ was applied as in Fig. 6(a). We obtained the peak of the measured power spectral densities by fitting a Lorentzian (the mechanical motion) plus an offset (the light readout noise) to the spectra. Note that the measured frequency range is around $\Omega_m$ because the signal has been down-converted by the interaction with the bias. We extrapolated what the peak of the power spectral density would be without any electronic noise, both Johnson and the drive, by fitting the peak amplitudes as shown in Fig. 7 (right) and
taking the intersection with the y-axis. The remaining spectral noise is due to the membrane thermal motion and optical readout noise, and is the intrinsic noise of the transduction. These contributions can be separated in the fits because they have different frequency dependence. While Fig. 7 shows the extrapolation to the optical plus mechanical noise, we technically did the extrapolation without the optical noise (i.e. neglecting the fitted offset) and added it afterwards. As data points we took weighted means of the five sequential fits and used the fit confidence interval as weights. The errorbar were a weighted, unbiased estimate of one standard deviation obtained from the variance in the sets, the variance in simulated Johnson noise, and the variance in independently measured white-noise drive.

3.3. Results

For the sample we used to estimate the intrinsic noise, the mechanical linewidth broadened by backaction was (243 ± 4) Hz at the optical operating power before applying the drive bias. That means the mode temperature was (152 ± 3) K. From preliminary tests we estimated the optimal operating cooperativity and set the drive bias accordingly. With the bias on, the mechanical resonance broadened to (8260 ± 130) Hz as seen in Fig. 7. This new linewidth $\Gamma_{\text{mech}}^{\text{new}}$ relates to the cooperativity through $\Gamma_{\text{mech}}^{\text{new}} = (C + 1)\Gamma_m$ so it corresponds to $C = 33.0 ± 0.8$. For this cooperativity, the expected mechanical noise temperature $T_{\text{mech}}$ follows from Eq. (1) as $T_{\text{mech}} = (112 ± 3) K$ for $\Omega_m/2\pi = 1.31$ MHz and $\omega_{LC}/2\pi = 31.9$ MHz, while the extrapolated mechanical noise temperature was $(120 ± 6) K$. This is better than [5] by orders of magnitude, while [3] reported a lower mechanical noise temperature at the optimal cooperativity but only for a DC bias. We note that any noise at 32 MHz from the RF environment or the bias that gets through the filters will make the measured noise higher than the theory, as is the case here. But we expect these effect to be small because of the shielding and filtering. However, the expected and measured value agree nearly within one standard deviation, so we do believe that neither effect limits performance for our setup, in contrast to [5, 6].

On top of the mechanical noise contribution we extrapolated from the fit in Fig. 7, there is also the optical noise. We calculated it from the ratio between the background and the peak of the mechanical spectral density without a drive after the Lorentzian peak had been scaled down to the extrapolated mechanical noise. That gave $(90 ± 15) K$ optical noise temperature at the mechanical resonance frequency. Adding both optical and mechanical noise gave a total intrinsic transducer noise of $(210 ± 16) K$. That is equivalent to a noise-figure of $(2.33 ± 0.14) \text{ dB}$, with a voltage and current noise of $(98.7 ± 3.9) \text{ pV/}\sqrt{\text{Hz}}$ and $(113 ± 4) \text{ pA/}\sqrt{\text{Hz}}$ respectively. That noise-figure is about four times larger than a comparable state-of-the-art MR preamplifier (Watcom WMA32C, typical 0.7 dB noise-figure), but it can be expected to beat conventional electronic amplifier with further improvements [5]. In our case, improvements are needed on the optical cavity.

All the noises refer to the highest transduction sensitivity, i.e. the peak of the mechanical spectral density, as indicated in Fig. 7. The transducer also works in a bandwidth $BW$ around this peak. We define $BW$ as the frequency range over which the signal-to-noise ratio drops to half that at resonance. This bandwidth is different from the broadened linewidth $\Gamma_{\text{EM}}$ from Eq. (1) because the mechanical noise follows the mechanical frequency response and thus goes down together with the induced signal away from the peak mechanical response until it is limited by the optical background noise. The bandwidth at the optimal cooperativity is $\sqrt{2(C_{\text{opt}} + 1)\Gamma_m}$, but for our balancing of the mechanical and optical noise we find $BW = (12.3 ± 0.7) \text{ kHz}$. This is significantly larger than previously reported [3, 5, 6] for a single-mode device, and almost as high as a multi-mode transducer [7].

We can express the transducer noise as a magnetic field sensitivity using the relation $\Phi = LI$ between current $I$ in, and total flux $\Phi$ through, a coil. The total flux through the coil is the magnetic field component perpendicular to the coil surface $B_{\perp}$, integrated over the entire coil area $A$, i.e. $B_{\perp} = L_I I_0 / A$ where $I_0$ is the current noise. We determined the area to be $A = 5 \times 10^{-3} \text{ m}^2$. 

\[
\]
by parameterizing a picture of the coil and numerically integrating over it. Given that, we estimate the transducer has an intrinsic mechanical plus optical noise equivalent to a magnetic field sensitivity of $8 \text{fT}/\sqrt{\text{Hz}}$, beyond what the circuit adds. With the circuit’s Johnson noise, we find the magnetic field sensitivity $13 \text{fT}/\sqrt{\text{Hz}}$ for our room-temperature setup.

4. Summary and outlook

We report an integrated, fiber-coupled, RF-to-optical converter based on mechanical transduction. The device features an ultra-short optical cavity designed and fixed through cleanroom fabrication and using it for transduction requires only a simple optical setup—a laser, circulator, and detector, all coupled to fiber—and no alignment or active stabilization after assembly. Furthermore, it fits into a standard circuit socket and could maintain good performance over time despite our mounting and handling. When transducing at 32 MHz, the device had a sensitivity of $13 \text{fT}/\sqrt{\text{Hz}}$ and a noise figure of 2.3 dB. Combined with the intrinsic noise temperature of 210 K in a bandwidth of 12 kHz, those parameters make the transducer competitive with electronic amplifiers for weak signal detection in, for example, MRI. The reported sensitivity is an improvement of several orders-of-magnitude compared to [5–7]. Using multiple mechanical modes like [7], we could further enhance the bandwidth of our device beyond the limit set by the product $\Gamma_m C$.

The optical cavity was made for an optical wavelength of 1064 nm but can easily be modified to other wavelengths, including the telecom industry standards 1310 nm and 1550 nm, which would reduce the fiber attenuation from 1.5 dB/km to 0.2 dB/km. In fact, the optical cavity should only get better at those wavelengths because aluminum has a higher reflectivity at telecom wavelengths. Other straight-forward improvements—increasing the mirror’s reflectivity with different coatings, using a smaller membrane-electrode gap, and optimizing the cavity coupling—can make our platform better than the state-of-the-art electronic devices, while keeping its major advantage of up-converting RF signals to an optical carrier.

In its present form, our device needs to be placed in a vacuum setup. This is not a significant constraint for applications which require vacuum anyway, such as low temperature MR imaging.
Future work on vacuum packaging of the individual chips will be carried out to make the transducer a standalone solution. If successful, this becomes a viable alternative to standard electronic low-noise amplifiers that could start a new era in signal detection by using the transduction to harness the advantages of electrical, micromechanical, and optical technologies.

Appendix A: Fabrication

We fabricated the integrated membrane-capacitor chip with a fixed-length optical cavity in a class 10-100 cleanroom using the process flow shown step-by-step in Fig. 8. Each process step was carried out using standard deposition, lithography, and etch techniques as described below. We started from a commercial dielectric mirror coating—three alternating, quarter-wavelength TiO$_2$ and SiO$_2$ layers—on a 100 mm diameter, 500 µm thick fused silica wafer. The coating served as input/output mirror for the integrated cavity and had a reflectivity of approximately 79% at the target wavelength 1064 nm. On top of the mirror, we built the membrane-capacitor chips as follows:

**Step 1: Protective layer.** A ∼30 nm alumina (Al$_2$O$_3$) layer deposited on the mirror with atomic-layer-deposition (ALD) to protect it from subsequent etching. The thickness and refractive index was inferred from an ellipsometer measurement on a reference wafer (silicon) from the same deposition run.

**Step 2: Bottom electrode.** The first capacitor electrode was made with electron-beam evaporation of ∼50 nm aluminum, which was patterned with UV-lithography and lift-off. The thickness was measured by scanning an edge of the metal near the center with an atomic force microscope (AFM). The pattern defined both a pad for wirebonding close to the capacitor plate, and a 40 µm hole in the center of the plate to allow light through in the optical cavity. The center hole will deform the membrane because the surface topology is replicated in following layers. We wanted this deformation to be small compared to the membrane thickness, but also to limit ohmic losses in the capacitor plate, so we chose an electrode thickness about three times thinner than the full membrane as a compromise. After release we assumed that the tensile stress would flatten the membrane, however there could have been a small offset that changes the cavity length (see section 2.3).

**Step 3: Sacrificial layer.** A nitride (Si$_3$N$_4$) layer was deposited with plasma-enhanced chemical-vapor deposition (PECVD) using a timed, low-stress recipe. The thickness was measured by removing the nitride near the wafer center and scanning the edge with an AFM. To remove the nitride, we used the selective etch detailed in step 9 and a resist mask patterned with UV-lithography. The sacrificial layer defines the distance between mirror coating plus bottom electrode and the reflecting metal-membrane, and therefore both the optical cavity length and the gap between the capacitor plates—the shorter, the better. We determined the target thickness from a model of the cavity response (section 2.3) and typically ended between 550 nm and 610 nm. The lowest thickness we have succeeded with was 200 nm, but such a short distance is not compatible with with the dielectric mirror.

**Step 4: Alumina membrane.** Same as step 1. From the beginning, we aimed for a thickness of ∼50 nm, similar to [3], but the actual value was adjusted based on measurement of previous layers to fine-tune the cavity length. We chose an alumina membrane, unlike [3,5,7], because it enabled a simple fabrication flow. They used highly-stressed nitride membranes instead, made from low-pressure chemical vapor deposition (LPCVD).

**Step 5: Top electrode.** Same as step 2, but the thickness was ∼100 nm instead. Aluminum was chosen as the membrane metal because it is lightweight, has good conductivity, and has good reflectivity at our wavelength. The metal pattern was also different from step 2; the wirebond pad does not overlap with the bottom electrode and, instead of a hole in the middle of the capacitor plate, there were smaller holes (from 3 µm to 10 µm in diameter) arranged in circles around the center—see the microscope view in Fig. 8. These were used to underetch the membrane and
thus release it in step 9. The larger circle of holes sets the membrane diameter and thus also the fundamental mechanical resonance frequency. We aimed for diameters from 150 µm to 300 µm and frequencies \( \geq 1 \text{ MHz} \). The latter constraint came from the transduction circuit design—see section 3.1. This top electrode connects the membrane-metal directly to external circuitry in contrast to [3, 5, 7] where the membrane-metal was floating above a planar pair of electrodes. Furthermore, an optical cavity is formed between the first mirror coating and the center of the aluminum membrane.

**Step 6: Membrane perforation.** The alumina membrane layer was perforated to have the same array of holes as the top electrode with a chlorine-based dry-etch performed in an inductively-coupled plasma (ICP), and the etch was timed to stop in the sacrificial layer. The electrode was protected by a resist layer patterned with UV-lithography.

**Step 7: Alignment layer.** A layer was made on the backside of the wafer to enable alignment between a lens and the laser light to the optical cavity. The layer was made with e-beam evaporated aluminum, \(~100\text{ nm}\) thick, patterned with UV-lithography and lift-off.

**Step 8: Annealing.** As in [11], the wafer was annealed for one hour at 350°C to induce tensile stress in the membrane electrode.

**Step 9: Isotropic etch.** On diced out chips, the membrane was released by a timed, isotropic SF\(_6\)-based ICP dry-etch [25]. This etch removes the sacrificial nitride in concentric rings around the top holes until approximately circular membranes were released without notably attacking
the aluminum or alumina.

**Step 10: Assembly.** The backside of the membrane-capacitor chip was aligned and glued to a separate silicon chip with a hole through the center. The hole was later used to align the optics. This silicon chip was made as follows:

**Step A: Etch-stop layer.** An LPCVD nitride layer was deposited on both sides of a 100 mm diameter, (100), 350 μm thick silicon wafer. The layer was thick enough to protect the silicon in the following etch.

**Step B: Hole definition.** Circular holes, 1.8 mm diameter, were made in the nitride on both sides of the wafer. The nitride was protected by resist patterned with UV-lithography and dry-etched in an ICP with as little overetch as possible.

**Step C: Through-hole wet etch.** The holes through the wafer were etched anisotropically with KOH. The etch followed the silicon crystal planes and therefore made square hole even though the nitride was etched in circular ones. The sidewall of the hole started as crystal planes in {111}, but because the etch was from both sides the sidewall slowly straightened once the KOH had etched all the way through the wafer [26]. The etched was timed to have a sidewall near vertical but bending slightly in. After etching, the wafer was cleaved into 5 mm × 5 mm chips.

Figure 9(b) shows the complete chip mounted on an integrated circuit (IC) socket and wirebonded to the socket pins, while table 1 gives the layer parameters for the device used in section 2.3 as an example. The socket also has a switch connected in parallel to the capacitor such that it can be shorted. We found that shorting the membrane-capacitor or connecting circuitry to it prevented the membrane from suddenly collapsing onto the bottom electrode. We believe this collapse was caused by electrostatic charges built up between the floating membrane-capacitor pins. The chip and wirebonds was further protected by a 3D printed lid glued to the socket.

**Table 1. Thickness and Refractive Index of Layers in the Optical Path**

<table>
<thead>
<tr>
<th>Layer</th>
<th>Material</th>
<th>Refractive index</th>
<th>Layer thickness (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mirror (1, 3, and 5)</td>
<td>TiO₂</td>
<td>2.47</td>
<td>108</td>
</tr>
<tr>
<td>Mirror (2, 4, and 6)</td>
<td>SiO₂</td>
<td>1.45</td>
<td>183</td>
</tr>
<tr>
<td>Protective layer</td>
<td>Al₂O₃</td>
<td>1.65</td>
<td>30.1</td>
</tr>
<tr>
<td>Bottom electrode</td>
<td>Al</td>
<td>0 a</td>
<td>49.6</td>
</tr>
<tr>
<td>Spacer</td>
<td>Si₃N₄</td>
<td>1 a</td>
<td>605</td>
</tr>
<tr>
<td>Alumina membrane</td>
<td>Al₂O₃</td>
<td>1.65</td>
<td>50</td>
</tr>
<tr>
<td>Top electrode</td>
<td>Al</td>
<td>1.03 + 9.25i b</td>
<td>100</td>
</tr>
</tbody>
</table>

* a Value used for modelling because the layer was not in the optical path.
* b From [27].
We coupled laser light into the cavity, and the reflected light back into the fiber, as shown in Fig. 9. The light was focused onto the cavity by a gradient-refractive-index (GRIN) lens inserted into the hole in the silicon guiding chip. We fixed the lens to the fused silica chip with UV-curable adhesive, and used a glass tube that fit around the lens to align an optical fiber terminated in a glass ferrule (see Fig. 9(a)). We optimized the ferrule position by maximizing the light reflected back into the fiber thus ensuring the focus point was on the optical cavity. The fiber was single-mode at our operating wavelength, and the reflection measurement used an optical circulator to direct laser light to the chip while routing the reflected light to a detector (see Fig. 2(c)). Finally, we fixed all the glass components with UV-glue.

Appendix B: Electromechanical theory

Using the notation in section 2.2, we have [16]

\[ G_{em} = \tilde{q} \frac{\partial}{\partial x} \frac{1}{C(x)} = \frac{\tilde{q}}{C} \frac{\partial}{\partial x} C_m = \frac{\tilde{q}}{C^2} C'_m. \] (7)

To derive an expression for \( G_{em} \) in terms of simple and measurable quantities, we take the following two steps: first, we approximate the electrostatically-induced frequency shift of the membrane \( \Delta \Omega_m \) [16] as

\[ 2m \Omega_m \Delta \Omega_m = - \frac{q_{rms}^2}{2} \frac{\partial^2}{\partial x^2} \frac{1}{C(x)} \approx - \frac{V_{rms}^2}{2} C''_m. \] (8)

The last approximation assumes a large capacitor \( C_p \) is connected in parallel to \( C_m \) such that \( C = C_p + C_m \). We also assumed \( C_p \gg C_m \) and set \( V_{rms} = q_{rms}/C \). Second, we relate the first and second derivative of \( C_m \) to eliminate \( C''_m \) from Eq. (8). If \( C_m \) were a simple parallel-plate system, the capacitance would be inversely proportional to the plate distance \( d \), but we need to take into account both the membrane mode shape and the effective gap between the plates. The mode shape \( u(r) \) is approximately a zeroth-order Bessel-function \( J_0(j_{0,1} r/R) \) for the fundamental vibrational mode, where \( j_{0,1} \) is the first root of \( J_0(x) \), \( R \) is the membrane radius, and \( r \) is the radial coordinate. The membrane-capacitor distance is thus \( d(r) = d_{eff} + x u(r) \), and we get its capacitance by integrating over the membrane surface \( S \). A rigorous derivation, given in [28, ch. 2], leads to the relation

\[ \frac{\partial^k C_m}{\partial x^k} = (-1)^k k! \int_S dS \frac{\epsilon_0}{d(r)^k} u(r)^k. \] (9)

The approximation \( x \ll d_{eff} \) applies for thermal driven motion and our chip design, and leads to the result

\[ C'_m = - \frac{\epsilon_0 A 2 J_1(j_{0,1})}{d_{eff}} = \frac{-C''_m d_{eff}}{J_1(j_{0,1}) j_{0,1}}. \] (10)

Inserting Eq. (10) into Eq. (8) leaves

\[ C'_m = 4m \Omega_m d_{eff} \frac{\Delta \Omega_m}{V^2} \approx 3.2 m \Omega_m d_{eff} \frac{\Delta \Omega_m}{V^2} \] (11)

which relates \( C'_m \) to a measurable mechanical frequency shift.

We can use this result for \( C'_m \) to calculate how the applied bias statically displaces the membrane. The displacement comes from to a new force equilibrium between the tension in the membrane and electrostatic force on the capacitor [3]

\[ m \Omega_m^2 \ddot{x} = - \frac{\tilde{q}^2}{2} C' \approx - \frac{V^2}{2} C'_m. \] (12)

Using Eq. (3) to eliminate \( C'_m \), thus assuming \( \ddot{x} \ll d_{eff} \), yields

\[ \frac{\ddot{x}}{d_{eff}} = - \frac{2}{J_1(j_{0,1}) j_{0,1}} \frac{\Delta \Omega_m}{\Omega_m} \approx -1.6 \frac{\Delta \Omega_m}{\Omega_m}. \] (13)
Funding
European Union’s Horizon 2020 Research and Innovation programme (732894 HOT, 722923 OMT, 638765 Q-CEOM, 787520 QUANTUM-N, 768491 EMOT); Innovation Fund Denmark (5150-00004B Qubiz); Danish Council for Independent Research (4002-00060, 4005-00531); Danish National Research Foundation (DNRF124 HYPERMAG, DNRF139 Hy-Q).

Acknowledgments
We are grateful to E. Zeuthen, S. Schmid, R. Malureanu, J. Appel, J. H. Müller, and the technical staff at Danchip for fruitful discussions and collaborations.

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