Ward identities for the standard model effective field theory

Corbett, Tyler; Helset, Andreas; Trott, Michael

Published in:
Physical Review D

DOI:
10.1103/PhysRevD.101.013005

Publication date:
2020

Document version
Publisher's PDF, also known as Version of record

Document license:
CC BY

Citation for published version (APA):
I. INTRODUCTION

The standard model (SM) is an incomplete description of observed phenomena in nature. However, explicit evidence of new long-distance propagating states is lacking. Consequently, the SM is usefully thought of as an effective field theory (EFT) for measurements and data analysis, with characteristic energies proximate to the electroweak scale ($\Lambda_\text{EW}$) scales based on assuming that physics beyond the SM is present at scales $\Lambda$ above $\Lambda_\text{EW}$ at the LHC or lower energies.

The standard model effective field theory (SMEFT) is defined as

$$\mathcal{L}_\text{SMEFT} = \mathcal{L}_\text{SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \ldots,$$

$$\mathcal{L}^{(d)} = \sum_i \frac{C^{(d)}_i}{\Lambda^{d-4}} Q^{(d)}_i$$ for $d > 4$. (1)

The higher-dimensional operators $Q^{(d)}_i$ are labeled with a mass dimension $d$ superscript, and multiply unknown, dimensionless Wilson coefficients $C^{(d)}_i$. The sum over $i$, after nonredundant operators are removed with field redefinitions of the SM fields, runs over the operators in a particular operator basis. In this paper we use the Warsaw basis [1]. However, the main results are formulated in a basis independent manner and constrain relationships between Lagrangian parameters due to the linear realization of $\text{SU}(2)_L \times U(1)_Y$ in the SMEFT.

The SMEFT is a powerful practical tool, but it is also a well-defined field theory. Many formal field-theory issues also have a new representation in the SMEFT. This can lead to interesting subtleties, particularly when developing SMEFT analyses beyond leading order. When calculating beyond leading order in the loop ($\bar{h}$) expansion, renormalization is required. The counterterms for the SMEFT at dimension five [2,3], and six [4–7] are known and preserve the $\text{SU}(3) \times \text{SU}(2) \times U(1)$ symmetry of the SM. Such unbroken (but nonmanifest in some cases) symmetries are also represented in the naive Ward-Takahashi identities [8,9] when the background-field method (BFM) [10–15] is used to gauge fix the theory. In Ref. [16] it was shown how to gauge fix the SMEFT in the BFM in $R_\xi$ gauges, and we use this gauge-fixing procedure in this work.

The BFM gauge-fixing procedure in the SMEFT relies on a geometric description of the field connections, and real representations for the $\text{SU}(2)_L \times U(1)_Y$ generators. Using this formulation of the SMEFT allows a simple Ward-Takahashi identity to be derived, which constrains the...
\( n \)-point vertex functions. The purpose of this paper is to report this result and derivation.\(^1\)

II. PATH INTEGRAL FORMULATION

The BFM generating functional of the SMEFT is given by

\[
Z(\hat{F}, J) = \int Df \det \left[ \Delta G^\alpha_{\beta} \right] e^{i(S[F+\hat{F}]+\mathcal{L}\text{GF}+\text{source terms})}.
\]

The integration over \( d^4x \) is implicit in \( \mathcal{L}_{\text{GF}} \). The generating functional is integrated over the quantum-field configurations via \( Df \), with \( F \) field coordinates describing all long-distance propagating states. \( J \) stands for the dependence on the sources that only couple to the quantum fields. The background fields also effectively act as sources of the quantum fields. \( S \) is the action, initially classical, and augmented with a renormalization prescription to define loop corrections.

The scalar Higgs doublet is decomposed into field coordinates \( \phi_{1,2,3,4} \), defined with the normalization

\[
H = \frac{1}{\sqrt{2}} \left[ \phi_2 + i \phi_1 \right] \left[ \phi_4 - i \phi_3 \right].
\]

The scalar kinetic term is defined with a field space metric introduced as

\[
\mathcal{L}_{\text{scalar,kin}} = \frac{1}{2} h_{IJ}(\phi)(D^\mu \phi)^I(D^\mu \phi)^J,
\]

where \( (D^\mu \phi)^I = (\partial^\mu \phi_I - \frac{i}{2} \mathcal{W}^{A \mu}_{\beta I} \phi^\beta) \), with real generators (\( \hat{\gamma} \)) and structure constants (\( \hat{\varepsilon}_{\beta c} \)) defined in the appendix. The corresponding kinetic term for the SU(2) \(_L \times U(1)_Y \) spin-one fields is

\[
\mathcal{L}_{\text{gauge,kin}} = -\frac{1}{4} g_{AB}(\phi) \mathcal{W}^A_{\mu \nu} \mathcal{W}^B_{\mu \nu},
\]

where \( A, B, C, \ldots \) run over \{1,2,3,4\}, (as do \( I, J \)) and \( \mathcal{W}^A_{\mu \nu} = B_{\mu \nu} \). Extending this definition to include the gluons is straightforward.

A quantum-field gauge transformation involving these fields is indicated with a \( \Delta \), with an infinitesimal quantum gauge parameter \( \Delta \alpha^A \). Explicitly, the transformations are

\[
\Delta \mathcal{W}^A_{\mu \nu} = -\varepsilon_{BC} \Delta \alpha^B (\mathcal{Y}^B_{\mu \nu} + \mathcal{W}^B_{\mu \nu}) - \partial^\mu (\Delta \alpha^A),
\]

\[
\Delta \phi^I = -\Delta \alpha^A \hat{\gamma}^A_{IJ} \left( \phi^J + \hat{\phi}^J \right).
\]

The BFM gauge-fixing term of the quantum fields \( \mathcal{W}^A \) is [16]

\[
\mathcal{L}_{\text{GF}} = -\frac{g_{AB}}{2} \frac{G^A G^B}{S_{138}}
\]

\[
G^A = \partial_I \mathcal{W}^{A \mu}_{\nu} - \mathcal{C}_D \mathcal{V}^{A \mu}_{\nu} \mathcal{W}^{D \mu}_{\nu} + \frac{\hat{g}}{2} \mathcal{V}^{A \mu}_{\nu} \mathcal{W}^{B \mu}_{\nu} + \frac{\hat{g}}{2} \mathcal{W}^{A \mu}_{\nu} \mathcal{W}^{B \mu}_{\nu} + \frac{\hat{g}}{2} \mathcal{W}^{A \mu}_{\nu} \mathcal{W}^{B \mu}_{\nu}.
\]

The introduction of field space metrics in the kinetic terms reflects the geometry of the field space due to the power-counting expansion. These metrics are the core conceptual difference of the relation between Lagrangian parameters, compared to the SM, in the Ward identities we derive. The field spaces defined by these metrics are curved; see Refs. [19–21]. The background-field gauge fixing relies on the basis independent transformation properties of \( g_{AB} \) and \( h_{IJ} \) and the fields, under background-field gauge transformations (\( \delta \hat{F} \)) with infinitesimal local gauge parameters \( \delta \alpha^A(x) \) given by

\[
\delta \hat{\phi}^I = -\delta \alpha^A \hat{\gamma}^A_{IJ} \hat{\phi}^J,
\]

\[
\delta \hat{W}^{A \mu}_{\nu} = -\frac{1}{2} (\partial^\mu \delta \hat{\phi}^I + \delta \hat{\phi}^I \hat{\gamma}^A_{IJ} \partial^\nu \hat{\phi}^J) \delta \hat{\gamma}^A_{IJ},
\]

\[
\delta \hat{h}_{IJ} = \hat{h}_{KL} \delta \hat{\gamma}^A_{IJ} \hat{\gamma}^A_{KL} + \hat{h}_{IK} \delta \hat{\gamma}^A_{IJ} \hat{\gamma}^A_{KL},
\]

\[
\delta g_{AB} = \hat{g}_{CB} \hat{\gamma}^C_{DA} \delta \hat{\gamma}^D_{IJ} + \hat{g}_{AC} \hat{\gamma}^C_{DB} \delta \hat{\gamma}^D_{IJ},
\]

\[
\delta \hat{\gamma}^A_{IJ} = \hat{\gamma}^A_{IJ} \delta \hat{\alpha}^A,
\]

\[
\delta \hat{f}_i = \hat{\Lambda}_i^A \delta \hat{\alpha}^A,
\]

where we have left the form of the transformation of the fermion fields implicit. Here \( i, j \) are flavor indices. The background-field gauge invariance of the generating functional, i.e.,

\[
\frac{\delta Z[\hat{F}, J]}{\delta \alpha^A} = 0,
\]

is established by using these gauge transformations in conjunction with the linear change of variables on the quantum fields.

The generating functional of connected Green’s functions is given by

\[
W[\hat{F}, J] = -i \log Z[\hat{F}, J],
\]

where \( J = \{ J^A_I, J^I_I, J^I_J \} \). As usual the effective action is the Legendre transform

\[
\Gamma[\hat{F}, \hat{\phi}] = W[\hat{F}, J] - \int dx^4 J \cdot \hat{F}\big|_{\hat{F}_{\text{fix}}}.
\]

\(^2\)The explicit forms of \( g_{AB} \) and \( h_{IJ} \) are basis dependent. The forms of the corrections for the Warsaw basis at \( \mathcal{L}(6) \) are given in Ref. [16].
Here our notation is chosen to match Ref. [22]. S-matrix elements are constructed via [22–24]

\[ \Gamma^{\text{full}}[\tilde{F},0] = \Gamma[\tilde{F},0] + i \int d^4x L^\text{BF}_{\text{eff}}. \]  

The last term in Eq. (11) is a gauge-fixing term for the background fields, formally independent from Eq. (6), and introduced to define the propagators of the background fields.

Finally, we define a generating functional of connected Green’s functions \( W_c[J] \) as a further Legendre transform [24],

\[ W_c[J] = \Gamma^{\text{full}}[\tilde{F}] + i \int d^4x \left[ \sum_{\tilde{F}} \tilde{J}_{\tilde{F}} \tilde{F} + \sum_f (\tilde{J}_f + \tilde{J}_f^f) \right], \]

with \( \tilde{F} = \{ \mathcal{W}_A, \phi^f \} \) and

\[ i\tilde{J}_{f^f} = -\frac{\delta\Gamma^{\text{full}}}{\delta f^f}, \quad i\tilde{J}_f = -\frac{\delta\Gamma^{\text{full}}}{\delta f}, \quad i\tilde{J}_{\tilde{F}} = \frac{\delta\Gamma^{\text{full}}}{\delta \tilde{F}}. \]

### III. WEAK EIGENSTATE WARD IDENTITIES

The BFM Ward identities follow from the invariance of \( \Gamma[\tilde{F},0] \) under background-field gauge transformations,

\[ \frac{\delta\Gamma[\tilde{F},0]}{\delta \tilde{d}B} = 0. \]

In position space, the identities are

\[ 0 = (\partial^\mu \delta B^A \partial^\nu \delta B^B) \frac{\delta\Gamma}{\delta \tilde{d}B^A} \frac{\delta\Gamma}{\delta \tilde{d}B^B} - \frac{\delta B^A}{\delta \tilde{d}B^B} \frac{\delta B^B}{\delta \tilde{d}B^A} \frac{\delta\Gamma}{\delta \tilde{d}B^A} \frac{\delta\Gamma}{\delta \tilde{d}B^B}. \]

For some \( n \)-point function Ward identities, the background fields are set to their vacuum expectation values (vevs). This is defined through the minimum of the classical action \( S \), where the scalar potential is a function of \( H^H \), which we denote as \( \langle \rangle \). For example, the scalar vev defined in this manner is through \( \sqrt{2\langle H^H \rangle} \equiv \bar{v}_T \), and explicitly \( \langle \phi^f \rangle \) with an entry set to the numerical value of the vev does not transform via \( \tilde{\gamma}_{A} \).

A direct relation follows between the tadpoles (i.e., the one-point functions \( \delta\Gamma/\delta\phi^f \)) and, \( \langle \phi^f \rangle \), given by

\[ 0 = \partial^\mu \frac{\delta\Gamma}{\delta \tilde{d}B^A} \frac{\delta B^A}{\delta \tilde{d}B^B} \frac{\delta\Gamma}{\delta \tilde{d}B^B} - \frac{\delta B^A}{\delta \tilde{d}B^B} \frac{\delta B^B}{\delta \tilde{d}B^A} \frac{\delta\Gamma}{\delta \tilde{d}B^A} \frac{\delta\Gamma}{\delta \tilde{d}B^B}. \]

### IV. MASS-EIGENSTATE WARD IDENTITIES

The mass-eigenstate SM Ward identities in the BFM are summarized in Ref. [15]. The transformation of the gauge fields, gauge parameters, and scalar fields into mass eigenstates in the SMEFT is

\[ \mathcal{V}^{A,L} = \sqrt{g^{AB}} U_{BC} \mathcal{A}^{C,L}, \]

\[ \bar{\alpha}^A = \sqrt{g^{AB}} U_{BC} \bar{\alpha}^C, \]

\[ \phi^f = \sqrt{h^{IK}} V_{KL} \tilde{\phi}^L. \]
VI. PHOTON IDENTITIES

The Ward identities for the two-point functions involving the photon are given by

\[
0 = \partial^\mu \frac{\delta^2 \Gamma}{\delta \hat{A}^{\mu} \delta \hat{A}^\nu}, \quad 0 = \partial^\mu \frac{\delta^2 \Gamma}{\delta \hat{A}^{\mu} \delta \Phi^I}.
\]

Using the convention of Ref. [15] for the decomposition of the vertex function

\[
-ib_{\mu} \hat{\phi}(k, -k) = (-g_{\mu \nu} k^2 + k_{\mu} k_{\nu} + g_{\mu \nu} M_{\nu}^2) \delta_{\nu \nu},
\]

\[
+ \left( -g_{\mu \nu} + \frac{k_{\mu} k_{\nu}}{k^2} \right) \Sigma^\nu \delta^\nu \delta_{\nu L} - \frac{k_{\mu} k_{\nu}}{k^2} \Sigma^\nu \delta_{\nu L},
\]

an overall normalization factors out of the photon two-point Ward identities compared to the SM, and

\[
\Sigma^\nu_{L, \text{SMEFT}}(k^2) = 0, \quad \Sigma^\nu_{T, \text{SMEFT}}(0) = 0.
\]

The latter result follows from analyticity at \( k^2 = 0 \).

VII. \( \mathcal{W}^\pm, Z \) IDENTITIES

Directly, one finds the identities

\[
0 = \partial^\mu \frac{\delta^2 \Gamma}{\delta \hat{A}^{\mu} \delta \hat{A}^\nu} - \hat{M}_Z \frac{\delta^2 \Gamma}{\delta \Phi^I \delta \hat{A}^\nu}, \quad 0 = \partial^\mu \frac{\delta^2 \Gamma}{\delta \hat{W}^{\mu \nu} \delta \hat{W}^{\rho \sigma}} + i \hat{M}_W \frac{\delta^2 \Gamma}{\delta \hat{W}^{\mu \nu} \delta \hat{W}^{\rho \sigma}}.
\]

and

\[
0 = \partial^\mu \frac{\delta^2 \Gamma}{\delta \hat{W}^{\mu \nu} \delta \hat{A}^\nu} \pm i \hat{M}_W \frac{\delta^2 \Gamma}{\delta \hat{W}^{\mu \nu} \delta \hat{A}^\nu}, \quad 0 = \partial^\mu \frac{\delta^2 \Gamma}{\delta \hat{W}^{\mu \nu} \delta \hat{W}^{\rho \sigma}} + i \hat{M}_W \frac{\delta^2 \Gamma}{\delta \hat{W}^{\mu \nu} \delta \hat{W}^{\rho \sigma}}.
\]

These identities have the same structure as in the SM. The main differences are the factors multiplying the tadpole...
terms. By definition, the vev is defined as \( \sqrt{2 \langle H^2 \rangle} \equiv \tilde{v}_T \). The substitution of the vev leading to the \( Z \) boson mass in the SMEFT (\( \hat{M}_Z \)) absorbs a factor in the scalar mass-eigenstate transformation matrix as \( \sqrt{2 \langle H^2 \rangle} = \sqrt{2 \langle H^2 \rangle} V^{-1} Y H \). If a scheme is chosen so that \( \delta \Gamma / \delta \tilde{\phi}^i \) vanishes, then transformation to the mass-eigenstate basis of the one-point function \( \delta \Gamma / \delta \tilde{\phi}^i \) is still vanishing in each equation above. One way to tackle tadpole corrections is to use the Fleischer-Jegerlehner (FJ) tadpole scheme; for discussion see Refs. [26,27].

VIII. A, Z IDENTITIES

The mapping of the SM Ward identities for \( \Gamma_{AZ} \) in the BFM given in Ref. [15] to the SMEFT is

\[
0 = \partial^\mu \frac{\delta^2 \Gamma}{\delta A^i \delta \tilde{Z}^\mu}.
\]

As an alternative derivation, the mapping between the mass eigenstate (\( Z, A \)) fields in the SM and the SMEFT (\( \tilde{Z}, \tilde{A} \)) reported in Ref. [28] directly follows from Eq. (27). Input parameter scheme dependence drops out when considering the two-point function \( \Gamma_{AZ} \) in the SM mapped to the SMEFT and a different overall normalization factors out. One still finds \( \Sigma_{L.SMEFT}^A \delta^2 \Gamma \) = 0 and, as a consequence of analyticity at \( k^2 = 0 \), \( \Sigma_{L.SMEFT}^A \delta^2 \Gamma(0) = 0 \). This result has been used in the BFM calculation reported in Refs. [29,30].

IX. CONCLUSIONS

We have derived Ward identities for the SMEFT, constraining both the perturbative and power-counting expansions. The results presented already provide a clarifying explanation to some aspects of the structure of the SMEFT that has been determined at tree level. The utility of these results is expected to become clear as studies of the SMEFT advance to include subleading corrections.

ACKNOWLEDGMENTS

We acknowledge support from the Carlsberg Foundation, the Villum Fonden, and the Danish National Research Foundation (Grant No. DNRF91) through the Discovery center. We thank W. Dekens, A. Manohar, G. Passarino, and P. Stoffer for discussions and/or comments on the draft, and P. van Nieuwenhuizen for his AQFT notes.

APPENDIX: NOTATION

The metric forms and transformations to \( \mathcal{L}^{(6)} \) in the Warsaw basis are explicitly [6,31]
In the mass-eigenstate basis, the transformed generators are

$$\gamma^I_{1,J} = \frac{\hat{g}_2}{2\sqrt{2}} \begin{bmatrix} 0 & 0 & i & -1 \\ 0 & 0 & -1 & -i \\ -i & 1 & 0 & 0 \\ 1 & i & 0 & 0 \end{bmatrix},$$

$$\gamma^I_{2,J} = \frac{\hat{g}_2}{2\sqrt{2}} \begin{bmatrix} 0 & 0 & -i & -1 \\ 0 & 0 & i & -1 \\ i & 1 & 0 & 0 \\ 1 & -i & 0 & 0 \end{bmatrix},$$

$$\gamma^I_{3,J} = \frac{\hat{g}_2}{2} \begin{bmatrix} 0 & 0 & 0 & 0 \\ c^2_\delta - s^2_\delta & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$  

$$\gamma^I_{4,J} = \hat{e} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$  

The generating functional for connected Green’s functions formulation. An alternative approach is to derive the Ward identities in terms of the generating functional for connected Green’s functions, $W_c$. The noninvariance of $L_{GF}^{BF}$ under background-field gauge transformations leads to

$$\frac{\delta W_c}{\delta a^B} = i \int d^4x \frac{\delta}{\delta a^B} L_{GF}^{BF}.$$

We choose the gauge-fixing term for the background fields

$$L_{GF}^{BF} = -\frac{1}{2\xi} \langle g_{AB} \rangle G^A G^B,$$

$$G^X = \partial_a W^{X,a} + \frac{\xi}{2} \langle g^{X,a} \rangle \langle \phi^A \rangle \langle \phi^B \rangle \langle h_{I,K} \rangle \frac{\delta}{\delta a^B} \langle \phi^L \rangle .$$

The variation of the gauge-fixing term with respect to the background-gauge parameter is

$$\frac{\delta}{\delta a^B} L_{GF}^{BF} = \frac{1}{\xi} \langle g_{AD} \rangle \left( \frac{\Box A}{\partial a^B} + i \partial^a \delta_a^A \frac{\delta W_c}{\delta J_{\psi^L}} + \frac{\xi}{2} \langle g^{AE} \rangle \frac{\delta}{\delta J_{\psi^L}} \langle h_{I,K} \rangle \frac{\delta}{\delta a^B} \langle \phi^L \rangle \right) G^D_{\psi^L},$$

where

$$G^D_{\psi^L} = \frac{1}{\xi} \langle g_{BD} \rangle \Box G^D_{\psi^L}.$$

Consider the difference between the vev defined by $\langle \rangle$ and an alternate vev denoted by $\langle \phi^A \rangle$ where the minimum of the action still dictates the numerical value, but in addition $\langle \phi^A \rangle$ transforms as $\delta \langle \phi^A \rangle = \hat{\gamma}^I_{1,J} \langle \phi^A \rangle$. Replacing all instances of $\langle \rangle$ in the above equations with this expectation value, and related transformation properties on the modified metrics, one finds

$$\frac{\delta}{\delta a^B} L_{GF}^{BF} = \frac{1}{\xi} \langle g_{BD} \rangle \Box G^D_{\psi^L}.$$

The two results coincide for on-shell observables; for further discussion this point, and tadpole schemes, see Ref. [24]. We postpone a detailed discussion of these two approaches to a future publication.
WARD IDENTITIES FOR THE STANDARD MODEL EFFECTIVE … PHYS. REV. D 101, 013005 (2020)