Conductance-Matrix Symmetries of a Three-Terminal Hybrid Device

G. C. Ménard,1,2 G. L. R. Anselmetti,1,2 E. A. Martinez,1,2 D. Puglia,1,2 F. K. Malinowski,1,2 J. S. Lee,3 S. Choi,4 M. Pendharkar,4 C. J. Palmstrøm,3,4,5 K. Flensberg,1 C. M. Marcus,1,2 L. Casparis,1,2,4 and A. P. Higginbotham1,2,†

1Center for Quantum Devices, Niels Bohr Institute, University of Copenhagen, Universitetsparken 5, 2100 Copenhagen, Denmark
2Microsoft Quantum—Copenhagen, University of Copenhagen, Universitetsparken 5, 2100 Copenhagen, Denmark
3California NanoSystems Institute, University of California, Santa Barbara, California 93106, USA
4Department of Electrical Engineering, University of California, Santa Barbara, California 93106, USA
5Materials Department, University of California, Santa Barbara, California 93106, USA

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We present conductance-matrix measurements of a three-terminal superconductor-semiconductor hybrid device consisting of two normal leads and one superconducting lead. Using a symmetry decomposition of the conductance, we find that antisymmetric components of pairs of local and nonlocal conductances qualitatively match at energies below the superconducting gap, and we compare this finding with symmetry relations based on a noninteracting scattering matrix approach. Further, the local charge character of Andreev bound states is extracted from the symmetry-decomposed conductance data and is found to be similar at both ends of the device and tunable with gate voltage. Finally, we measure the conductance matrix as a function of magnetic field and identify correlated splittings in low-energy features, demonstrating how conductance-matrix measurements can complement traditional single-probe measurements in the search for Majorana zero modes.

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Symmetry relations for quantum transport are often connected to deep physical principles and make strong predictions for comparison with experiment. For instance, the Onsager-Casimir relations [1–3] arise from microscopic reversibility and were central in early studies of quantum-coherent transport [4–6]. Later, predicted departures from these relations due to interaction effects [7–9], which include bias dependence of the effective potentials, were observed in nonlinear transport [10,11]. The introduction of superconducting terminals results in additional symmetries, as conductance occurs via Andreev reflection from electrons to holes, and is invariant under particle-hole conjugation [12]. For a two-terminal normal-superconducting device, the conductance $g(V)$ is a symmetric function of bias voltage $V$ neglecting interaction effects. As shown in a partner theoretical Letter, for multiterminal superconducting devices $g(V)$ need not be symmetric, although a curious relation exists between the antisymmetric components of the local and nonlocal conductances [13]. These predictions have, to our knowledge, not been tested.

Hybrid superconductor-semiconductor nanowire structures have recently become a topic of intense interest [14–19], motivated in part by proposals for achieving topological superconductivity and Majorana zero modes (MZMs) [20,21]. In two-terminal superconductor-semiconductor devices, observed asymmetries in the subgap conductance [22] have been suggested to arise from a dissipative fermionic reservoir, effectively acting as a third lead [23], although, as in the normal-conducting case [3], bias dependence of the self-consistent potential can also cause a deviation from symmetry [24]. Multiterminal superconducting devices are a topic of particular interest, as they can be used for MZM [25–31], Cooper-pair splitter [32,33], and multiterminal Josephson studies [34–38]. In multiterminal superconducting quantum dot devices, bias asymmetries have been observed [39] and a relationship between nonlocal conductance and the bound-state charge has been proposed [40,41].

In this Letter, we report a symmetry analysis of the conductance matrix measured in a three-terminal, superconductor-semiconductor hybrid device. The antisymmetric components for pairs of conductance-matrix elements are found to qualitatively match at energies below the superconducting gap, with quantitative departures that scale with bias voltage and increase in isolated gate-voltage regions. We use the symmetry-decomposed nonlocal conductance to extract the local charge character of states within the superconducting gap as a function of gate voltage, discovering that the charge is approximately equal on both sides of the device. Finally, we measure local and nonlocal conductances at nonzero magnetic field and identify isolated low-energy states with correlated splittings on each end of the wire, using inferred charge as an additional spectroscopic tool for comparison with theory. This work provides new methods for studying the local charge density of subgap bound states and distinguishing...
between topological and trivial states. More generally, it provides a better understanding of the role of symmetry in multiterminal superconducting quantum devices.

Selective area growth (SAG) of InAs nanowires [42,43] is performed by chemical beam epitaxy (CBE) on an InP substrate masked with silicon oxide [44]. The nanowire is half covered by an Al film, which was deposited in situ after CBE growth, and is selectively etched to form a superconducting lead. The device [Fig. 1(a)] consists of a cold-grounded device, and do not find deviations from voltage divider effects using a four-probe measurement on a dilution refrigerator.

Conductance is measured by applying a dc bias voltage $V_{bias} = V_{R(L)}$ and an ac voltage $\delta V_{R(L)}$ at the right (left) terminal with two different ac excitation frequencies $f_R \sim 18$ Hz and $f_L \sim 42$ Hz. The in-phase ac current $\delta I_{R(L)}$ flowing to the right (left) side is measured with the middle superconducting lead grounded. The device is tunnel coupled to the normal leads by adjusting the two tunnel-gate voltages to $V_{gl} = -0.485 \ V$ and $V_{gl} = -1.29 \ V$, such that $g \ll e^2/h$, which also ensures that the applied voltages drop over the tunnel barriers. We have checked for spurious voltage divider effects using a four-probe measurement on the semiconductor with an induced component. The remaining conductance-matrix elements were also measured and exhibit similar features [46].

To further explore the symmetry properties, the conductance traces are decomposed into their symmetric, antisymmetric, and asymmetric components. Figure 1(d) shows $g_{LL}$ as a function of $V_L$ with $V_R = 0$. Several peaks occur symmetrically around zero bias. We assign the two highest energy peaks to coherence peaks, a signature of the Bardeen-Cooper-Schrieffer (BCS) density of states in the proximitized semiconductor with an induced gap $\Delta \sim 250 \ \mu eV$, in agreement with previous observation for similar material systems [43,45]. The other peaks are subgap states with energies $E_0 < \Delta$. Both the high-bias conductance ($V_L > \Delta$) as well as the subgap peak heights are asymmetric in bias (Fig. 1(b)). The nonlocal conductance $g_{LR}$, measured as a function of $V_R$ with $V_L = 0$ in Fig. 1(c), exhibits features corresponding to the peaks in $g_{LL}$. The sign of the peak amplitudes in $g_{RL}$, however, changes with a sign change in bias, indicating a strong odd component.

FIG. 1. (a) Colored scanning electron micrograph of the three-terminal device and schematic of the measurement setup. (b) Left side local conductance $g_{LL}$ as a function of left side bias voltage $V_L$. (c) Left side nonlocal conductance $g_{LR}$ as a function of right side bias voltage $V_R$. (d) Extracted antisymmetric component of the local conductance $g_{LL}^{as}$ with respect to $V_L$. (e) Extracted antisymmetric component of the nonlocal conductance $g_{LR}^{as}$ with respect to $V_R$. The blue dashed line shows $g_{LL}^{as}$ for comparison.
bias-voltage dependence of the effective potentials [3], single-particle scattering into the nominally superconducting lead [13], or inelastic processes within the hybrid region [23]. Above the superconducting gap, departures from symmetry scale smoothly with bias voltage, and we have found they persist even when one lead is completely pinched off, which would seem to favor an explanation based on bias-dependent potentials. In addition, we have found that the observed symmetry departures have relatively little field dependence, suggesting that they are not due to the presence of dissipation from vortices.

Next, we investigate the conductance matrix as a function of gate voltage. Figure 2(a) shows $g_{LL}$ ($g_{LR}$) as a function of $V_p$ and bias $V_L$ ($V_R$). $g_{LL}(V_p)$ makes the assignment of coherence peaks and subgap states clearer. The coherence peaks do not move when changing the gate voltage, while subgap states appear at different bias voltages for different $V_p$. We attribute these subgap states to Andreev bound states (ABSs), although it is not entirely clear where the states are confined. We speculate that confinement could result from a Fermi velocity mismatch due to the presence of dissipation from vortices.

 FIG. 2. (a) Local conductance (left-hand panel) and nonlocal conductance (right-hand panel) as a function of plunger gate $V_p$ and bias voltage. (b) Antisymmetric components of local (left) and nonlocal conductance (right) as a function of plunger gate $V_p$ and bias voltage. The colored lines indicate the location of the cuts for the local (blue) and nonlocal (orange) traces in Fig. 1.

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Symmetry decomposing the datasets as a function of bias voltage, while subgap states appear at different bias voltages for different $V_p$. We attribute these subgap states to Andreev bound states (ABSs), although it is not entirely clear where the states are confined. We speculate that confinement could result from a Fermi velocity mismatch due to the presence of dissipation from vortices.

It is interesting to note that ABSs with the same slope with respect to $V_p$ have the same sign in $g_{LR}$. Further, where the slope changes sign, i.e., at inflection points, the nonlocal conductance disappears and changes sign as well [Fig. 2(a)]. Reference [13] shows that, for a spectrally isolated subgap state at energy $E_0$, the sign of the nonlocal conductance at bias voltages near $V_0 = E_0/e$ is generally related to the state’s local charge density. The symmetric part of the conductance, for a bound state coupled to the left (right) leads at rate $\Gamma_{L(R)}$ and energy $|E_0| > \Gamma_{L(R)}$, $k_BT$ is

$$g_{RL}^{\text{s}}(V_0) = a q_L q_R,$$

(4)

and the antisymmetric part is

$$g_{RL}^{\text{as}}(V_0) = a n_L q_R \text{sgn}(V_0),$$

(5)

where $q_{L(R)} = u_{L(R)}^2 - v_{L(R)}^2$ is the local charge density, $n_{L(R)} = u_{L(R)}^2 + v_{L(R)}^2$ is the local probability density, and $u_{L(R)}$, $v_{L(R)}$ are the left (right) Bogoliubov–de Gennes amplitudes. The general expression for $a$ is cumbersome, but in the limiting case $\Gamma_{L(R)} \gg k_BT$ it takes the simple form, $a = (2e^2/h)[\Gamma_{L(R)}/(\Gamma_{L(n_L) + \Gamma_{R(n_R)}]}^2].$ Similar expressions also hold for $g_{LR}(V_0)$. As a consequence, there is a proportionality between the symmetry-decomposed nonlocal conductance and the local charge densities, $g_{RL}^{\text{as}}(V_0) \propto q_L q_R$ and $g_{RL}^{\text{as}}(V_0) \propto q_L q_R$.

Proportionality constants can be eliminated by considering the ratio of conductances,

$$Q_L = \frac{g_{RL}^{\text{as}}(V_0)}{g_{RL}^{\text{as}}(V_0)} \text{sgn}(V_0)$$

(6)

$$= \frac{u_L^2 - v_L^2}{u_L^2 + v_L^2}.$$  

(7)

$Q_L$ is a measure of the local charge character; $Q_L = +1$ for a state that is locally electronlike ($u \gg v$), and $Q_L = -1$ for a state that is locally holelike ($v \gg u$).

Motivated by these theoretical relations, we compare in Fig. 3(a) the measured symmetric component of a single nonlocal conductance $g_{RL}$, with the product of the antisymmetric components of both nonlocal conductances,
by extracting peak positions of the ABS resonances from the data. To further explore the charge of subgap states, the character of bound state [orange and green markers in Fig. 3(b)]. Sign changes of $Q_L$ indicate that the state is changing from electronlike to hololelike, or vice versa. We note that, based on Eq. (6), sign changes $g^s_{RL}$ cause sign changes in $Q_L$. Hence, the correlation between the sign of the nonlocal conductance and $dE_0/dV_p$ is consistent with the charge interpretation. Using $g_{LR}$, $Q_R$ can be analogously calculated [open markers in Fig. 3(b)], and it is found to be similar to $Q_L$ for both bound states, within experimental error [46]. The similarity between $Q_L$ and $Q_R$ suggests that the particle and hole wave function components, $u$ and $v$, are rigid on the length scale of the present device, which is unusual for mesoscopic devices where wave functions are often considered to be random and uncorrelated in space.

In theory, when $u$ and $v$ are spatially uniform Ref. [13] predicts that a symmetry relation emerges:

$$g^s_{LL}(V_0)g^s_{LR}(V_0) = 2[g^s_{LR}(V_0)]^2 - [g^s_{RR}(V_0)]^2. \quad (8)$$

A parametric plot of the left-hand side vs the right-hand side of Eq. (8) for all identified peaks in the dataset reveals that these quantities are approximately equal [Fig. 3(c)], supporting the view that the coherence factors are spatially uniform. Indeed, performing a linear fit of the data gives a slope of $0.98 \pm 0.03$ with a small intercept $(0.005)^2 \pm (0.002)^2(c^2/h)^2$, indicating a good general agreement with the relationship predicted by Eq. (8). However, there are regions of gate voltage where systematic deviations from linear behavior are observed, including the previously discussed region $V_p \sim -3.64$ V, where feature crossings are observed in $g_{LR}$.

Having established the nonlocal conductance as a tool for characterizing subgap states, we now apply a magnetic field while measuring the conductance matrix. Figures 4(a) and 4(d) show the evolution of $g_{LL}$ and $g_{RR}$ in a magnetic field $B$ applied parallel to the wire. The ABSs evolve as a function of $B$ as detected from both ends of the device. As the field initially increases, the low-lying states split. The upper-split states are repelled from higher-lying states, and the lower-split states eventually merge at zero energy. For further increases in field, the low-energy states oscillate around zero energy, a signature typically attributed to hybridized MZM in devices comparable in length to the coherence length [17]. The correlated splitting of zero-bias peaks, measured from both wires ends, was proposed as a “smoking gun” signature of MZM [25], but given the presence of strong correlations at zero magnetic field in this device [31], we suggest that this signature is not by itself conclusive.

The nonlocal conductances $g_{LR}$ and $g_{RL}$ [Figs. 4(b) and 4(c)] as a function of field have asymmetric features corresponding to subgap states in the local conductance, and also exhibit several changes in sign. To study the sign of $g_{RL}$ for the low-energy features, peaks in the local conductance $g_{LL}$ are determined [orange line in Fig. 4(c)], and the peak nonlocal conductance $g_{RL}(V_0)$ is extracted at these points. Of particular interest is the behavior at the field $B^* \sim 0.6$ T, where the energy of the oscillating states has a turning point. The nonlocal conductance at $B^*$ is nonzero [Fig. 4(e)], inconsistent with the expected behavior.

The plots in Fig. 3(a) are qualitatively similar, as expected from Eqs. (4) and (5), and suggest that gate dependence of the BCS charge has a dominant effect on the conductance of subgap peaks. To further explore the charge of subgap states, the data are analyzed by extracting peak positions of the ABS resonances from $g_{LL}$. Decompositions of the nonlocal conductance are evaluated at these positions to obtain $g^s_{RL}(V_0)$ and $g^s_{LR}(V_0)$. $Q_L$ is calculated from the peak values for two different subgap states [orange and green lines in Fig. 3(a)]. The local charge character for these states oscillates as a function of $V_p$, highlighting the gate-tunable charge character of bound state [orange and green markers in Fig. 3(b)]. Sign changes of $Q_L$ indicate that the state is changing from electronlike to hololelike, or vice versa. We note that, based on Eq. (6), sign changes $g^s_{LR}$ cause sign changes in $Q_L$. Hence, the correlation between the sign of the nonlocal conductance and $dE_0/dV_p$ is consistent with the charge interpretation. Using $g_{LR}$, $Q_R$ can be analogously calculated [open markers in Fig. 3(b)], and it is found to be similar to $Q_L$ for both bound states, within experimental error [46]. The similarity between $Q_L$ and $Q_R$ suggests that the particle and hole wave function components, $u$ and $v$, are rigid on the length scale of the present device, which is unusual for mesoscopic devices where wave functions are often considered to be random and uncorrelated in space.

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fields corresponding to the subgap states and therefore have vanishing nonlocal conductance for hybridized MZMs, which are chargeless at turning points and therefore have vanishing nonlocal conductance \([13,47]\). At a higher field \(B^{**} \sim 1\ T\), there is a second turning point where the nonlocal conductance is small, consistent with the behavior expected for chargeless, hybridized MZMs \([48]\).

Nonlocal transport gives evidence against a Majorana interpretation at intermediate fields \((B > B^{**})\). Intermediate-field effects in hybrid nanowires, in particular the possibility of quasi-Majorana modes that emerge before the true topological transition \([29,49–52]\), have been discussed at length in the literature. It is, to the best of our knowledge, an open theoretical problem to check if quasi-Majorana modes, or other effects \([53]\), can explain the anomalous charge character that we have inferred near \(B^{*}\). At higher fields \((B > B^{**})\), nonlocal transport does not rule out a topological regime, but an examination of more datasets, and longer device lengths, is required to reach a firm conclusion.

In summary, the observed approximate symmetry relations are consistent with a noninteracting scattering picture and justify the use of symmetry-decomposed conductance to infer the BCS charge. At finite field, we have observed correlated splittings of zero-bias peaks, but find that at intermediate fields their charge character is not consistent with a simple Majorana picture. We anticipate that measurements of the local (on-diagonal) conductance, combined with the insights from the nonlocal (off-diagonal) conductance, will play an important role in substantiating the existence of MZMs.

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L. C. and A. P. H. contributed equally to this work.

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\[\text{FIG. 4.} \ \text{Local conductances } g_{LL} \ (a) \ \text{and } g_{RR} \ (d) \ \text{and nonlocal conductances } g_{LR} \ (b) \ \text{and } g_{RL} \ (c) \ \text{as a function of bias voltage and parallel magnetic field } B. \ \text{The orange curve overlaid on (c) indicates peaks found from the local conductance } g_{LL}. \ (e) \ \text{Peak nonlocal conductance } g_{RL}(V_0) \ (\text{orange}) \ \text{and corresponding bias voltage } V_0, \ (\text{black}) \ \text{as a function of magnetic field } B. \ \text{Magnetic fields corresponding to the subgap states’ energy-turning points, } B^* \ \text{and } B^{**}, \ \text{are indicated.} \]

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\[\text{lucas.casparis@microsoft.com} \]
\[\text{andrew.higginbotham@ist.ac.at} \]

Correction: The article identification number for this Letter was assigned incorrectly and has been fixed.