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Weak Measurement Protocols for Majorana Bound State Identification

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We propose a continuous weak measurement protocol testing the nonlocality of Majorana bound states through current shot noise correlations. The experimental setup contains a topological superconductor island with three normal-conducting leads weakly coupled to different Majorana states. Putting one lead at finite voltage and measuring the shot noise correlations between the other two (grounded) leads, devices with true Majorana states are distinguished from those without by strong current correlations. The presence of true Majorana states manifests itself in unusually high noise levels or the near absence of noise, depending on the chosen device configuration. Monitoring the noise statistics amounts to a weak continuous measurement of the Majorana qubit and yields information similar to that of a full braiding protocol, but at much lower experimental effort. Our theory can be adapted to different platforms and should allow for the clear identification of Majorana states.

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Introduction.—Throughout the past decade, the quest for stable realizations of Majorana bound states (MBSs) has become a major theme in condensed matter physics [1–8]. A fully manipulable MBS would pave the way to disruptive developments, both in fundamental science and as a building block for a new generation of quantum hardware [9–17]. While initial proposals focused on realizations as end states in topological semiconductor quantum wires, the quest for the Majorana state has led to the recent discovery of various alternative material platforms [18–23]. In all of these, evidence for Majorana states has been reported on the basis of tunneling spectroscopy or related *local* probes; see, e.g., Refs. [7,24–38]. However, in spite of promising signatures, more mundane explanations, such as Andreev bound states representing pairs of "fake" Majorana states, cannot be ruled out, and the interpretation of the experiments continues to be debated; see Refs. [39–58]. In view of this situation, various forms of diagnostics transcending tunneling spectroscopy have been proposed [59–96]. Basically, these fall into two categories, local probes corroborating evidence for the presence of genuine Majorana states albeit still containing potential loopholes or compelling probes, such as braiding protocols, which, however, do not seem to be a realistic option in the immediate future.

In this Letter, we suggest a new type of diagnostic experiment. The strategy will be to access the information stored nonlocally in a set of at least three MBSs through the *statistical fluctuations* of tunneling current probes. As we shall show below, this yields information comparable to that of a full-fledged braiding protocol, but at much lower experimental effort. In fact, the hardware required to perform the measurement is not much different from that currently in operation and should be realizable for the proposed Majorana platforms with present-day technology. We note that statistical fluctuations of tunneling current probes have also been investigated in other studies of topological systems; see, e.g., Refs. [97–99].

Before turning to a more detailed discussion, let us sketch the idea of the approach. Consider the schematic representation of Fig. 1, where the dots represent MBSs supported on a floating mesoscopic superconductor (see the right panels of Fig. 1 for more realistic layouts). Suppose that we measure the tunneling current $I_1(t)$ flowing in response to a voltage bias applied at the wire connecting to MBS γ_1 relative to a grounded wire connecting to γ_0. This current is sensitive to the state of the qubit operator $\sigma_1 \equiv i\gamma_1\gamma_0$ [13,14]. Monitoring the current over short intervals of time, a weak measurement [100,101] is effectively performed, continuously steering the qubit into a state defined by the current readout. Now assume that the current $I_2(t)$ through terminal 2 is recorded as well. This readout couples to $\sigma_2 \equiv i\gamma_2\gamma_0$, and the tendency to alter this operator, noncommuting with σ_1, implies incompatible readouts. Its observable consequence is pronounced in current cross-correlations, which, we will demonstrate, represent a unique signature in that they are qualitatively distinct from the noisy current in the presence of Andreev bound states, or other low energy quasiparticle (poisoning) excitations. More specifically, our prime observable of interest is the current cross-correlation

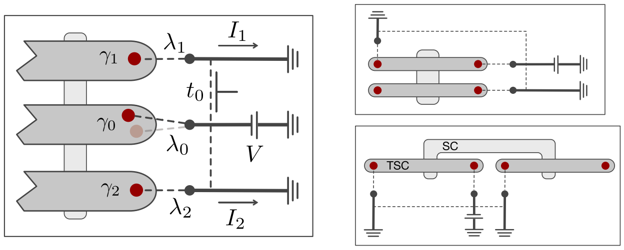

FIG. 1. Setup for probing Majorana bound states in a system of topological quantum wires. (Left panel) Three of the Majorana states (dots) on a Majorana-Cooper box with three topological hybrid nanowires connected by a superconducting backbone [14,15] are tunnel coupled to normal-conducting leads. The schematic on the left indicates that one of the leads ($\alpha = 0$) is biased with a voltage V and acts as a source of electrons into the grounded drain leads ($\alpha = 1$, 2). Tunable tunnel couplings t_0 introduce a direct link between the source and drain leads. Andreev states are distinguished from genuine Majorana states as pairs of MBSs γ_α^i (with $i = 1, 2$) centered close to the tunnel interface (see the faded dot, representing an $i = 2$ state in the wire 0). The cross-correlation shot noise amplitude S_{12} of the currents I_1 and I_2 [Eq. (1)] unambiguously distinguishes between the two types of states. (Right panels) The same experiment can be carried out on a wide range of possible device layouts. The two schematics on the right give examples for additional realistic geometries using only two nanowires.

$$S_{12} = \int dt \langle\langle I_1(t) I_2(0) \rangle\rangle, \tag{1}$$

where $\langle\langle AB \rangle\rangle = \langle AB \rangle - \langle A \rangle \langle B \rangle$. We will analyze this quantity in both the presence and the absence of tunneling bridges (see the vertical dashed lines in Fig. 1) between the electrodes connecting to the island. This additional structure, which can be controlled during an experiment via gate electrodes, gives us sufficient information to distinguish MBSs from the competing cases mentioned above. This is because the noise profile probes the presence of an underlying Pauli algebra, which is a unique characteristic of the Majorana system (alternatively diagnosed in a more elaborate braiding protocol).

Model.—We describe the setup of Fig. 1 by the now standard [66,74–76] Hamiltonian $H = H_C + H_{\mathrm{leads}} + H_T + H_{\mathrm{ref}}$ for a "Majorana-Cooper box." Here $H_C = E_C(N - n_g)^2$ defines the charging energy $E_C = 2e^2/C$ associated with $N = -i\partial_\phi$ Cooper pairs on the floating island (ϕ is the phase of the superconductor). We consider Coulomb valley conditions defined by a back-gate parameter n_g close to an integer value. The normal-conducting leads $\alpha = 0$, 1, 2 are modeled by a Hamiltonian H_{leads}, with electron annihilation operators $c_{\alpha,k}$ for momentum k and density of states $\nu_\alpha = \nu$ assumed to be equal for simplicity. The local tunneling between the Majorana box and the leads is described by

$$H_T = \sum_{\alpha=0,1,2} \sum_{j=1}^{N_\alpha} \lambda_\alpha^j c_\alpha^\dagger \gamma_\alpha^j e^{-i\phi/2} + \mathrm{H.c.}, \tag{2}$$

where $c_\alpha = \sum_k c_{\alpha,k}$. Here γ_α^j represent the low energy box states at terminal α. Representing them by the Majorana operators $\gamma_\alpha^j = (\gamma_\alpha^j)^\dagger$, $\{\gamma_\alpha^j, \gamma_{\alpha'}^{j'}\} = 2\delta_{\alpha\alpha'}\delta_{jj'}$ tunnel coupled by amplitudes λ_α^j, this modeling includes the cases $N_\alpha = 1$ of a genuine Majorana state and $N_\alpha > 1$, where Andreev states described as pairs of spatially overlapping Majorana states [67] compromise the system. Taking note that Majorana states carry no charge, the operator $e^{-i\phi/2}$ in Eq. (2) accounts for the removal of an island electron charge upon tunneling. Finally, the reference arms in Fig. 1 are modeled by $H_{\mathrm{ref}} = \sum_{\alpha=1,2} t_{0,\alpha} c_\alpha^\dagger c_0 + \mathrm{H.c.}$, with the gate-tunable tunneling amplitude $t_{0,\alpha}$. With V denoting the voltage bias applied to the source lead ($\alpha = 0$) and Δ the superconducting gap on the island, we consider the parameter regime ($e = k_B = \hbar = 1$ throughout) $|\lambda|, V \ll E_C, \Delta$ at low temperatures $T \ll V$. In this case, transport through the island is dominated by cotunneling processes, and second-order perturbation theory in the λ_α^j yields the effective Hamiltonian $H_C + H_T \to \tilde{H}_T$ with

$$\tilde{H}_T = \frac{1}{2} \sum_{\alpha \neq \alpha'} O_{\alpha\alpha'} c_\alpha^\dagger c_{\alpha'} + \mathrm{H.c.},$$
$$O_{\alpha\alpha'} = i \sum_{j=1}^{N_\alpha} \sum_{j'=1}^{N_{\alpha'}} t_{1,\alpha\alpha'}^{jj'} \gamma_\alpha^j \gamma_{\alpha'}^{j'}, \tag{3}$$

where $t_{1,\alpha\alpha'}^{jj'} \simeq i\lambda_\alpha^j (\lambda_{\alpha'}^{j'})^* / E_C$. Where possible, we use the simplified notation $t_{1,\alpha\alpha'}^{jj'} = t_1$ and $t_{0,\alpha} = t_0$ throughout. The results discussed below are all perturbative to leading order in the dimensionless tunnel conductances $g_0 \equiv 2\pi\nu^2 |t_0|^2$ and $g_1 \equiv 2\pi\nu^2 |t_1|^2$ characterizing the different connectors between leads. We assume these to be tuned to $g_0 \ll 1$ and $g_1 \ll 1$, conditions that can be checked by designated calibrating measurements.

Qualitative discussion.—If the wires host single Majorana states $N_\alpha = 1$, the projection to the quantized charge sector implies the parity constraint $\gamma_0\gamma_1\gamma_2\gamma_3 = \pm 1$, where the presence of the disconnected Majorana state γ_3 is required to define a complete system of wire-end states [74]. The ground state then is doubly degenerate and defines a qubit with the above Pauli operators $\sigma_{1,2}$ [14,15]. To lowest order in perturbation theory in the tunneling amplitudes, the average currents flowing through the drain leads are given by

$$\langle I_{\alpha=1,2} \rangle = (g_0 + g_1 + 2g_i \langle \sigma_\alpha \rangle) V, \tag{4}$$

where the interference factor $g_i \equiv 2\pi\nu^2 \mathrm{Re}(t_0^* t_1)$ couples to the measured eigenvalue of the respective Pauli operator σ_α

[12,13,66]. In a way made more rigorous below, the recording of the *simultaneously flowing* currents I_α in a measurement of the cross-correlation S_{12} amounts to a continuous weak measurement of σ_1 and σ_2. This view implies that the system cannot settle in a pure state because such a state would need to be a simultaneous eigenstate of σ_1 and σ_2. The observable consequences of this frustration are persistent fluctuations of I_1 and I_2, quantified by S_{12}, Eq. (1). Below we will show how this principle implies a positive cross-correlation $S_{12} \sim F\bar{I}$, where $\bar{I}$ is the average current and F a Fano factor of $O(1)$. As discussed below, this should be compared to parametrically smaller results proportional to higher powers of the tunneling conductance characterizing noninteracting electrons in the tunneling limit [102]. The origin of stronger correlations in the present system is the coupling of transport to a Pauli algebra which effectively conditions the currents in the arms 1 and 2 to each other.

Counting statistics.—We next derive an efficient formalism to compute the cross-correlation S_{12} and related statistical signatures of transport. The first step is to integrate over the lead degrees of freedom to obtain a reduced density matrix ρ_t in the Hilbert space corresponding to the Majorana operators γ_α^j. While this object by itself is not too informative, the statistics of the charge Q_α transmitted in time $t \in [\tau_0, \tau_0 + \tau]$ through the terminals $\alpha = 1$, 2 is obtained by introducing counting field factors $e^{\pm i f(t)\chi_\alpha/2}$ into the hopping amplitudes λ_α^j and t_0, where $f(t) = 1$ in the time interval of observation and $f(t) = 0$ otherwise, the quantities $\chi_{1,2}$ are constant counting fields, and the sign factor refers to counting fields on the forward or backward time evolution in $\rho_t = e^{-iHt}\rho_0 e^{iHt}$. Defining $z_\alpha = \exp(i\chi_\alpha)$, the density matrix $\rho_\tau(z_1, z_2)$ then depends on the counting parameters z_α, and all cumulants of the charges $Q_{1,2}$ are obtained by taking derivatives [101,103,104],

$$\langle\langle Q_1^n Q_2^m \rangle\rangle = (z_1\partial_{z_1})^n (z_2\partial_{z_2})^m|_{z=1} \ln \mathrm{Tr}\rho_\tau(z_1, z_2). \quad (5)$$

The evolution equation governing $\rho_t = \rho_t(z_1, z_2)$ is given by [105]

$$\begin{aligned}\dot{\rho}_t = &-i[H_q, \rho_t] + 2\pi\nu^2 T[\mathcal{D}_{12}(\rho_t) + \mathcal{D}_{21}(\rho_t)] \\ &+ 2\pi\nu^2 V \sum_{\alpha=1,2} [(z_\alpha - 1)(O_{\alpha 0} + t_{0,\alpha})\rho_t(O_{\alpha 0}^\dagger + t_{0,\alpha}^*) \\ &- (z_\alpha - 1)O_{\alpha 0}\rho_t O_{\alpha 0}^\dagger + \mathcal{D}_{\alpha 0}(\rho_t)],\end{aligned} \quad (6)$$

where the superoperators

$$\mathcal{D}_{\alpha\alpha'}(\rho) = \frac{z_\alpha}{z_{\alpha'}} O_{\alpha\alpha'}\rho O_{\alpha\alpha'}^\dagger - \frac{1}{2}\{O_{\alpha\alpha'}^\dagger O_{\alpha\alpha'}, \rho\} \quad (7)$$

act as Lindbladians generalized for the counting parameters $z_{1,2}$, $z_0 \equiv 1$, and $O_{\alpha\alpha'}$ describes the electron transfer from lead $\alpha' \to \alpha$; see Eq. (3). The coherent evolution in Eq. (6) is generated by the effective Hamiltonian

$$\begin{aligned}H_q = &-\nu^2\Lambda \sum_{\alpha=1,2} (t_{0,\alpha}^* O_{\alpha 0} + \mathrm{H.c.}) - \frac{\nu^2\Lambda}{2} \sum_{\alpha<\alpha'} \{O_{\alpha\alpha'}^\dagger, O_{\alpha\alpha'}\} \\ &+ \nu^2 V \ln(\Lambda/2V) \sum_{\alpha=1,2} [O_{\alpha 0}, O_{\alpha 0}^\dagger],\end{aligned} \quad (8)$$

where $\Lambda \gg V$ is the bandwidth of the leads.

True Majorana case.—In spite of its complicated looking appearance, Eq. (6) can be solved, at least to the linear order in V relevant to us. We first note that, in the absence of counting parameters $z_1 = z_2 = 1$, the stationary solution approaches the isotropic limit $\rho_0 = \frac{1}{2}\mathbb{I}_2$ at a timescale $1/\Gamma$. The rate $\Gamma = 2g_1 V$ equals twice the average current flowing through the contact to MBS γ_1, indicating that the latter sets the timescale for the loss of information about the initial states. Generalizing to the case of finite counting fields, we obtain $(z = \frac{1}{2}z_1 + \frac{1}{2}z_2 - 1)$ [105]

$$\ln \mathrm{Tr}\rho_\tau(z_1, z_2) = \frac{\Gamma\tau}{2}\left(-1 + \frac{2g_0 + g_1}{g_1}z + \sqrt{(1+z)^2 + \frac{8g_i^2 z^2}{g_1^2}}\right). \quad (9)$$

This result yields the full counting statistics to order V. Specifically, the stationary limit of the current $\bar{I} = \langle I_\alpha \rangle$ through lead $\alpha = 1$, 2 is given by $\bar{I} = \tau^{-1}\partial_{z_\alpha} \ln \mathrm{Tr}\rho = (g_0 + g_1)V$. This result is independent of σ_α and hence in stark contrast to Eq. (4). It reflects the fact that the continuous weak measurement of two noncommuting Pauli operators has eradicated information about the qubit state and sent the system to a fully mixed state. However, at the same time, one generically encounters an increased level of shot noise cross-correlations $S_{12} = \tau^{-1}\partial_{z_1,z_2}^2 \ln \mathrm{Tr}\rho = F\bar{I}$. Here F is the positive Fano factor, which at $T = 0$ and in the limit $\Gamma\tau \gg 1$ is obtained from Eq. (9) as

$$F = \frac{2g_i^2}{g_1(g_0 + g_1)}. \quad (10)$$

The most important message conveyed by this result is that $F \sim 1$, parametrically exceeding $|F| \sim g_0$ in the noninteracting limit [102]. Also notice that the $O(1)$ contribution to the zero temperature Fano factor vanishes identically for pinched-off reference arms $t_0 \to 0$. This is because the continuous measurement of $I_\alpha \propto |t_0 + t_1\sigma_\alpha|^2$ no longer couples to two noncommuting variables $I_{\alpha,t_0\to 0} \propto |t_1|^2$, and the mechanism of large fluctuations no longer operates. For finite but low temperatures, thermal correlations produce a nonvanishing result for $t_0 \to 0$ with, however, a very small Fano factor $|F| \sim T/V \ll 1$. This discussion shows how a comparison of cross-correlations with and without reference arms in one experimental setup will produce qualitatively different results, signifying the presence of a Pauli algebra. Furthermore, we note that small hybridizations of the MBSs on the box do not affect this qualitative picture [106].

Andreev bound states.—We next discuss how the transport statistics change if at least one of the wires contains an Andreev bound state $N_\alpha > 1$. For definiteness, consider the case $N_0 = 2$, $N_1 = N_2 = 1$ without reference arms, where the source wire harbors an Andreev instead of a Majorana state. We now need to differentiate between tunneling amplitudes, where $t^{jj'}_{1,\alpha\alpha'}$ for $\alpha = 0$, and where $j = 1, 2$ refers to the couplings between the source lead and the two MBSs constituting the Andreev state. The resulting formulas of S_{12} are more cumbersome. For example, under the simplifying assumption $|t^{jj'}_{1,10}| = |t^{jj'}_{1,20}|$ and $\mathrm{Im}(t^{11*}_{1,10}t^{12}_{1,10}) = -\mathrm{Im}(t^{11*}_{1,20}t^{12}_{1,20})$, we obtain [105]

$$F = \frac{[\mathrm{Im}(t^{11*}_{1,10}t^{12}_{1,10})]^2}{(|t^{11}_{1,10}|^2 + |t^{12}_{1,10}|^2)^2}. \tag{11}$$

Except for fine tuned choices, we always have $|F| \sim 1$, as in the true Majorana case with reference arms. This high noise level again originates in the noncommuting nature of the operators O_{10} and O_{20} (although they do not realize a Pauli algebra anymore).

With these results at hand, we propose a protocol to distinguish true vs fake Majorana states; see Table I for a summary. For true MBSs without reference arms $t_0 = 0$, the Fano factor $|F| = O(T/V, g_{1,2})$ is parametrically smaller than the values $|F| = O(1)$ predicted in their presence $t_0 \neq 0$. If the *source terminal* is coupled to an Andreev bound state, strong cross-correlations with $|F| \sim 1$, regardless of the presence or absence of reference arms, are observed. This insensitivity of the noise level to the presence of the link clearly signals the presence of an Andreev state coupled to the source terminal. However, the protocol is blind to the presence of such states in the drain leads; see the third row of Table I. It must therefore be repeated with the role of source and drain interchanged, which amounts to a different choice of bias voltages. On top of that, two more control measurements must be performed likewise by a variation of the gate or bias voltage. (a) To exclude false interpretations based on the measurement $|F| \ll 1$ due to the accidental fine-tuning of parameters [e.g., $\mathrm{Im}(t^{11*}_{1,10}t^{12}_{1,10})$ in Eq. (11)], the protocol should be repeated several times with different values of the gate potentials regulating the tunneling amplitudes. (b) We repeat that all of the above results hold to leading order in the tunnel conductances g_α. To check for the presence of corrections in these parameters, one may repeat the protocol for a sequence of gradually diminishing conductances (adjustable by gate voltage). In the cases labeled ~ 1 in Table I, this will leave the Fano factor parametrically unchanged, while for $\ll 1$ a suppression $\sim g_\alpha$ is predicted.

Quasiparticle poisoning.—The transient in- and out-tunneling of quasiparticles through MBSs represents a source of decoherence and noise which, if sufficiently strong, might compromise the interpretation of the zero frequency noise correlators S_{12}. For completeness, we therefore summarize a protocol [14] geared to the characterization of quasiparticle poisoning processes. Consider both $t_{0,1} = \lambda^j_1 = 0$ such that lead 1 remains decoupled. The current I_2 in Eq. (4) then depends on the state of the MBSs through the expectation value of σ_2 (or, more generally, that of an operator O_{20} if Andreev bound states are present). Beyond a timescale $\tau_{\mathrm{proj}} \simeq (g_0 + g_1)/(4g_i)^2 V$ [105], the measurement of I_2 becomes projective, and a weakly fluctuating result defined by one of the two values $\langle\sigma_2\rangle \to \pm 1$ in Eq. (4) is approached. However, quasiparticle tunneling accidentally switching the state $\sigma_2 \to -\sigma_2$ will cause discrete jumps $\bar{I}_2 \to \bar{I}_2 \pm 4g_i V$ in the readout. This should allow for a detection of quasiparticle induced decoherence.

TABLE I. Qualitative behavior of the Fano factor $|F|$ for true vs fake Majorana states. The key observable distinguishing between the two cases is the large value $|F|$ in the absence of reference arms $t_0 = 0$ for Andreev bound states. Since the protocol diagnoses only Andreev states coupled to the source lead $\alpha = 0$, experiments have to be repeated for different choices of the source and drain leads.

	$t_0 \neq 0$	$t_0 = 0$
True Majorana states ($N_0 = 1$, $N_{1,2} = 1$)	~ 1	$\ll 1$
Andreev bound states ($N_0 = 2$, $N_{1,2} = 1, 2$)	~ 1	~ 1
Andreev bound states ($N_0 = 1$, $N_{1,2} = 2$)	~ 1	$\ll 1$

Large fluctuations.—Finally, it is interesting to relate the strong cross-correlation amplitudes indicative for the presence of noncommuting operator states to the rare event statistics of current flow. To understand this point, consider the probability distribution of the currents I_1 and I_2, obtained from the generating function $\rho_\tau(z_1, z_2)$ in Eq. (9),

$$P(I_1, I_2) = -\int \frac{dz_1}{2\pi z_1}\frac{dz_2}{2\pi z_2}\mathrm{Tr}\rho_\tau(z_1, z_2) z_1^{-I_1\tau} z_2^{-I_2\tau}. \tag{12}$$

Focusing on the tails of the distribution $I_\alpha \gg \bar{I}$, a straightforward saddle-point approximation stabilized by $\Gamma\tau \gg 1$ [105] yields

$$P(I_1, I_2) \overset{I_\alpha \gg \bar{I}}{\simeq} \prod_\alpha \left(\frac{\bar{I}}{I_\alpha}\right)^{I_\alpha \tau}. \tag{13}$$

These tails decay exponentially, but much slower than for a Gaussian distribution. This reflects the fact that the simultaneous measurement of noncommuting operators triggers rare fluctuations stronger than those caused by the superposition of uncorrelated fluctuations [101].

Conclusions.—We have proposed an experimental diagnostic for MBSs which, much as a braiding protocol, probes the commutation relations of a Majorana algebra, but which should be experimentally feasible at drastically lower experimental effort. The approach is based on monitoring the statistics of tunnel currents in response to changes of a few easily accessible system parameters, the gate-controlled tunneling contacts into the system. The comparatively easy variability of these parameters in one experimental run defines a structured pattern of quantitative predictions, the "true Majorana case" being identified by a multitude of testable conditions (as opposed to just one signal in tunneling spectroscopy data). We therefore believe that the experiment would yield a definite fingerprint. Conceptually, it amounts to a continuous weak measurement, the most direct approach to probing the presence of noncommuting operators. Since the measurement outcome qualitatively depends on the underlying operator algebra, the recording of transport statistics as summarized in Table I represents compelling evidence for the presence of a Majorana qubit. While we expect the qualitative distinction between MBSs and Andreev bound states to display a high level of parameter tolerance, it would be rewarding to study nonequilibrium noise for microscopically more refined models. As with conventional quantum devices, the added information sitting in statistical fluctuations would provide a higher level of realistically accessible information on topological quantum wires than that provided by dc transport probes.

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[106] The result (10) displays a high degree of robustness with respect to finite coupling terms $\sim i\gamma_1\gamma_0$ and $\sim i\gamma_2\gamma_0$ in the effective Hamiltonian H_q. We find a slightly higher degree of sensitivity with respect to the term $\varepsilon_{12} i\gamma_1\gamma_2$. However, F stays of order unity as long as $\varepsilon_{12}^2 \lesssim \Lambda^2[\nu^2 \mathrm{Re}(t_0^* t_1)]^2$ is fulfilled. We note that in our proposed geometries γ_1 and γ_2 are hosted by different topological wires such that we expect the corresponding hybridization $|\varepsilon_{12}|$ to be sufficiently small.