Metastable Nonextremal Antibranes

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We find new and compelling evidence for the metastability of supersymmetry-breaking states in holographic backgrounds whose consistency has been the source of ongoing disagreements in the literature. As a concrete example, we analyze anti-D3 branes at the tip of the Klebanov-Strassler throat. Using the blackfold formalism we examine how temperature affects the conjectured metastable state and determine whether and how the existing extremal results generalize when going beyond extremality. In the extremal limit we exactly recover the results of Kachru, Pearson, and Verlinde, in a regime of parameter space that was previously inaccessible. Away from extremality we uncover a metastable black Neveu-Schwarz five-brane (NS5) state that disappears near a geometric transition where black anti-D3 branes and black NS5 branes become indistinguishable. This is remarkably consistent with complementary earlier results based on the analysis of regularity conditions of backreacted solutions. We therefore provide highly nontrivial evidence for the metastability of antibranes in noncompact throat geometries since we find a consistent picture over different regimes in parameter space.

DOI: 10.1103/PhysRevLett.122.181601

Introduction.—An understanding of controlled supersymmetry-breaking (SUSY) breaking in string theory is arguably one of the most important goals in order to progress string phenomenology. Thanks to holography, the use of controlled SUSY breaking in string backgrounds would improve our understanding of strongly coupled phenomena in field theories with broken SUSY. The latter is an important issue independent of the conjecture that string theory is the quantum theory underlying our physical world.

One of the canonical methods for SUSY breaking in string theory employs antibranes in warped throats. The study of this mechanism has a long history, starting with the original papers [1,2] whose motivation was the holographic description of dynamical breaking of supersymmetry. Since then, antibranes have become an indispensable tool to break SUSY in various contexts. The applications beyond holography include de Sitter [3], inflationary model building [4], and the construction of nonextremal black hole microstates [5]. For some of these applications additional complications due to the compactness of extra dimensions arise, which we will not address. Instead, we restrict to the study of antibranes in the noncompact KS throat.

The motivation for that is twofold. On one hand, the KS throat is a prime example of a top-down holographic background with broken conformal symmetries that exhibits confinement. Studying SUSY breaking in this context is therefore a concrete example of the ultimate goal to get a computational handle on QCD-like field theories in the absence of SUSY. On the other hand, for string phenomenology it has been argued that the KS throat at least constitutes a local description of the more elusive compact throat geometries.

In this Letter we present entirely novel arguments for the highly debated metastability of SUSY-breaking states in these throats and, at the same time, demonstrate how to extend the analysis to situations that incorporate finite temperature effects in the holographic dual, offering new methods to study phase transitions in this context.

Our candidate metastable states are described by $D^3$’s at the tip of the throat. They can decay because the surrounding 3-form fluxes induce delocalized $D3$ charges out of which $D3$’s can nucleate and annihilate with the $\overline{D3}$’s. This process can be described in terms of brane polarization in which the $D3$’s “puff” into a spherical NS5 wrapping a contractible $S^2$ inside the $S^3$ cycle [2]. As the NS5 moves over the $S^3$, it changes the sign of the $D3$ charges it carries, effectively mediating the brane-flux decay. To find a metastable state the NS5 needs to find a balance between the $H_3$-flux force that wants to push the NS5 over the $S^3$.
and the force of its own “weight” doing the opposite. In the 
probe limit, KPV found that such a balance of forces 
exists whenever the ratio $p/M$ is small enough [2]. Here, $p$

denotes the number of $D3$’s and $M$ the quantum of 3-form 
flux piercing the $S^3$.

The existence of this KPV state has been refuted in 
various works starting with the investigations of Ref. [6].
The problem, found at the time, arises when trying to go 
beyond the probe limit and investigate what happens once 
the branes backreact. In particular Ref. [6], and many 
subsequent works [7], found that the backreacted geometry 
had singular 3-form fluxes in such a way that it would cause 
immediate brane-flux decay [8]. As a response, Ref. [9] 
argued that the singularity is renormalized in such a way 
that does not affect stability when $p = 1$, which is a 
case that is not amenable to a supergravity analysis. 
References [10,11] argued that metastability can also be 
retained when $p \gg 1$ since the observed singularities 
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cannot be proven to exist once one backreacts spherical 
NS5’s instead of pointlike $D3$’s. In fact, all proofs of

Several studies [12–15] have investigated the effect of
adding temperature to the $D3$’s. Most of these works 
were motivated by the would-be singularity in the 3-form fluxes.
Whether or not a singularity can be cloaked by a horizon 
that arises when moving away from extremality is believed 
to be an important criterion for deciding the fate of 
singularities [16]. Although strong indications were found 
that one should not worry about singularities at all [9–11],
it remains an outstanding problem to understand what 
happens when the $D3$’s are at finite temperature. If the state 
would destabilize infinitesimally away from extremality, 
it would be a sign of being gapless, which is not wanted. In 
this Letter we present additional evidence that this does not 
happen and provide new, previously inaccessible, quantitative 
results that support the picture of Refs. [10,11].

Our approach.—From the NS5 viewpoint $D3$-NS5 
polarization is most naturally described in the supergravity 
regime where $g_s M P \gg 1$ (see, e.g., Ref. [17]). This involves 
a daunting task: solving the type IIB supergravity equations 
to find $D3$-NS5 bound state solutions wrapping an $S^3$ in 
the presence of the fluxes of the KS background. In this Letter, 
we attack this problem using blackfold techniques [18–20].

In the blackfold formalism, problems of the above type 
are treated systematically by setting up a scheme of 
matched asymptotic expansions, where the solution is 
approximated in a far zone by the background solution of interest (here the KS background) and in a near zone 
by a uniform flat-space $p$-brane solution (here the $D3$-NS5 
bound state). This scheme is possible when the characteristic 
length scales of the near-zone solution, denoted collectively 
by $r_b$, are hierarchically smaller than the characteristic 
length scales $R$ of world volume inhomogeneities, and 

the characteristic length scales $L$ of the background. The 
regime $r_b \ll R, L$ is a regime of long-wavelength 
extrapolations. For a general discussion of blackfolds and the approximations they entail we refer the reader to Ref. [20] and references therein. The specifics of the regimes 
of interest in our context are summarized below.

A key component of the above analysis reformulates part 
of the supergravity equations as an effective world volume theory. In the case at hand, this is a supergravity-derived 
6d effective theory that describes the long-wavelength 
properties of NS5 branes with dissolved 3-brane charge. 
In what follows, we focus exclusively on the leading-order 
equations of the 6d theory (blackfold equations). At the 
very least, these equations pose necessary conditions for 
the existence of the long-sought supergravity solution in the 
above regime.

**Forced blackfold equations.**—The leading order blackfold equations for black branes in the presence of general external fluxes in (super)gravity were obtained in Ref. [20].

We consider $D3$-NS5 branes at the tip of the KS throat, 
where in appropriate units the string frame metric is [21]

$$ds^2 = g_s M b_0^2 \epsilon_3^2 [ds^2(M_{0123}) + d\Omega_3^2 + dr^2 + r^2 d\Omega_2^2].$$

Here, $M_{0123}$ is Minkowski space in the directions 0123, 
$d\Omega_2^2$ is the metric element of the round unit n sphere, 
$b_0^2 \approx 0.93266$, and $\epsilon_3$ the string scale. There is also a 
Ramond-Ramond (RR) field strength $F_3 = dB_2 = 2M r_3^{-2} Vol(S^3)$ across the $S^3$ and a 7-form flux $H_7 = dB_6$
dual to $H_3 = dB_2$,

$$H_7 = \frac{1}{g_s} \star H_3 = -\frac{\epsilon_3^4}{g_s} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge F_3. \quad (2)$$

The dilaton is constant and the self-dual 5-form field strength vanishes at the tip.

The boosted $D3$-NS5 bound state solution in flat space, 
which forms the seed of our blackfold expansion, is well 
known (see, e.g., Refs. [22,23]). The following thermody-

namic data of this solution (in the Einstein frame) are

$$T_{ab} = C \left[ r_0^2 \left( u_a u_b - \frac{1}{2} \gamma_{ab} \right) - r_0^2 \sin^2 \theta \sinh^2 \alpha \gamma_{ab} 
- r_0^2 \cos^2 \theta \sinh^2 \alpha \gamma_{ab} \right], \quad (3)$$

and the charge currents

$$J_2 = C r_0^2 \sin^2 \alpha \sin \theta \cos \theta \nu \wedge \nu, \quad (4)$$

$$J_4 = C r_0^2 \sin \alpha \cosh \alpha \sin \theta \star (\nu \wedge \nu), \quad (5)$$

$$J_6 = C r_0^2 \sin \alpha \cosh \alpha \cos \theta \star 1, \quad (6)$$

where $C = [1/(2\pi)^5 g_s^2 \epsilon_3^8]$ and $\gamma_{ab}$ is the induced metric on 
the five brane world volume (parametrized by $\sigma^\alpha$ with

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The Hodge dual of $\gamma_{ab}$ is $\ast$ and $\hat{\gamma}_{ab}$ is a projector onto the directions of the dissolved $D3$-brane charge inside the five brane. General boosts or rotations of the $D3$-NS5 bound state solution are expressed in terms of the velocity timelike unit vector $u^a$ and the spacelike orthonormal vectors $v^a$, $w^a$. In terms of these vectors $\hat{\gamma}_{ab} = \gamma_{ab} - v_a v_b - w_a w_b$. The electric currents $J_4$, $\dot{\theta}$ express the $D3$, NS5 currents of the solution while $J_2$ is a consequence of the nonzero $C_3$ field of the solution. $r_0$ is the Schwarzschild radius. In the extremal limit, $r_0 \to 0$, $\alpha \to \infty$ while the combination $r_0^2 e^{2\alpha}$ is kept fixed. The parameter $\theta$ controls how much $D3$ brane charge is dissolved inside the NS5 brane.

The general effective blackfold equations of the $D3$-NS5 brane in the presence of 3-form NSNS/RR fluxes, constant dilaton and vanishing $1, 5$-form RR fluxes are [20]

$$
\nabla_a T^{\mu\nu} = \frac{g_0^{-1}}{6!} H^{\mu a_1 \ldots a_6} J_{a_1 \ldots a_6} + \frac{1}{2!} F_{(3)}^{\mu a_1 a_2} J_{2a_1 a_2}
$$

$$
+ \frac{3}{4!} H_{3}^{\mu a_1 a_2} C_{a_1 a_2} J_{4a_1 a_2},
$$

$$
\partial \ast J_2 + H_3 \ast \ast J_4 = 0,
$$

$$
d \ast J_4 - \ast \dot{\theta} \wedge F_3 = 0, \quad d \ast \dot{\theta} = 0.
$$

After eliminating $\tan \theta$, it is trivial to check that the resulting equation for $\psi$ coincides with the Euler-Lagrange equations of the DBI action obtained in Ref. [2] by $S$ duality of the $D5$ brane. In this manner, we recover the equations of motion of the KPV effective action from supergravity. A more general relation between the extremal blackfold equations and DBI equations can be derived along the lines of Ref. [24] (see also Ref. [25]).

The maximum value $p^*$ that allows a metastable vacuum has the following meaning in the blackfold language. Since the NS5 branes at extremality have a nonvanishing Hagedorn local temperature $T_H = (1/2\pi) \sqrt{|C/|Q_5|/\sqrt{\cos \theta}}$, one can show that $p^*$ is very close to the point $\tilde{p}$ where this temperature takes its maximum possible value. The latter occurs when $\cos \tilde{\theta} = 1$, i.e. when the 3-brane charge is depleted. From Eq. (11) we deduce that at that point $\tilde{\theta} \approx 0.7506$ obeying the equation $\cot \tilde{\theta} = \frac{\tilde{p}}{p_0}$. Then, Eq. (10) gives

$$
\frac{\tilde{p}}{p_0} = \tilde{\psi} - \frac{1}{2} \sin(2\tilde{\psi}) \approx 0.08,
$$

also noted in Ref. [2].

**Nonextremal static configurations.**—It is straightforward to repeat the above exercise for non-extremal configurations at finite $\alpha$. We continue to focus on the same ansatz for the world volume fields, now restricting to time-independent profiles. The nonextremal, static version of Eq. (10) is exactly the same as before. On the other hand, when expressed in terms of $\alpha$ Eq. (11) becomes

$$
\cot \tilde{\psi} = \frac{\frac{\tilde{p}}{p_0} (\coth \alpha + \tan \theta)}{2 \cos^2 \theta + (\sinh \alpha)^{-2}}.
$$

This equation can be written alternatively in terms of the local temperature $T = (1/2\pi r_0 \cosh \alpha)$, or the local density $\rho = 2\pi p_0 r_0 \cosh \alpha$, or any other quantity that characterises the deviation from extremality. We refer the reader to Sec. I in Supplemental Material [26] for a derivation of the equations of motion from an effective potential (using in part results of Refs. [27,28]). In what follows, we will make use of an effective potential $V_S$ that keeps the entropy $S$ fixed.

**Results for nonextremal configurations.**—Consider first the regime $\rho/M < p^*/M \approx 0.08$, where the extremal solutions have a meta-stable vacuum. In Fig. 1 we show how the effective potential $V_S$ changes as we vary the entropy $S$ for a fixed value of $p/M$. We observe two interesting new features. First, as soon as $S$ is turned on, a new unstable vacuum emerges (black dots on the right plot of Fig. 1) near the North pole, $\psi \approx 0$. For sufficiently small...
values of $S$ there are three extrema: two unstable and one metastable. Second, as we increase $S$ further the new unstable extremum comes closer to the metastable vacuum and the two merge at a critical value of the entropy $S^*$, which is a function of $p/M$. Above this value the metastable vacuum is lost. The new unstable state represents a fat black NS5 with a highly pinched $\mathbb{R}^3 \times S^5$ horizon geometry that resembles a black $D3$. Instead, the metastable state starts life near extremality as a thin black NS5 with $\mathbb{R}^3 \times S^2 \times S^3$ horizon topology. At the merger the metastable black NS5 turns effectively into a black $D3$. The picture of a merger driven by horizon geometry is reinforced by the following observation.

A quantitative measure of the “fatness” of a black NS5 wrapping an $S^2$ is provided by the ratio $d = 2\sqrt{p r_0} / [\sqrt{M} \sin(\psi)]$ [29], where $r_0 \equiv \sqrt{C r_0} / \sqrt{Q_5}$ is dimensionless. The ratio $d$, which is a natural function of $p/M$ and the equilibrium $\psi$, compares the scale $2\sqrt{g_s M r_0 \ell_s}$, associated to the Schwarzschild radius and the scale of the $S^2$ wrapped by the NS5 world volume $\sqrt{g_s M \ell_s} \sin(\psi)$. As an illustration, on the left plot in Fig. 2 we see how $d$ behaves in the unstable branch (blue color) and the metastable branch (orange color). Recall that $\alpha \to \infty$ represents extremality. The unstable branch has visibly higher values of $d$, expressing the dominance of the Schwarzschild radius. The metastable branch captures a thin black NS5 with small values of $d$. The merger occurs at a value of $d$ notably close to 1.

On the right plot of Fig. 2 we show how $d$ at the merger point behaves as a function of $p/M$. Remarkably, the ratio remains effectively constant, near the value 0.89 over a significant range. It deviates slightly from this value in the vicinity of the upper bound of $p/M$, where effects from the second unstable state (already visible in KPV [2]) become important. The characteristically weak dependence of $d$ on $p/M$ is a clear signal that the properties of the merger point are closely tied to the properties of the horizon geometry. Finally, by increasing $p/M$ further, above the critical value $p^*/M \approx 0.08$, we observe the complete loss of the metastable vacuum exactly as in the extremal KPV analysis [2]. The unstable vacuum in the vicinity of the North pole, however, remains, even above $p^*/M$, and constitutes the single vacuum of the nonextremal static blackfold equations.

Discussion.—The results of the blackfold analysis are consistent in the regime of large $g_s p$ when $N_5 \ll g_s M \sin^2 \psi$ and $\sqrt{p/M} \ll \sqrt{g_s M \sin^2 \psi}$ (see Sec. II of Supplemental Material [26]). For sufficiently large $M$ our calculations are therefore valid everywhere except for a small region around the North pole. At extremality we recover the results of KPV [2] in a regime very different from theirs [$p \sim O(1)$]. Moving away from extremality was so far impossible. Our analysis reveals two novel features of black $D3$-NS5 branes: a new unstable state near the North pole, and the
persistence of the metastable state for small enough horizon radius. For a critical value of the horizon radius the metastable state and the new unstable state merge. Above this critical value, the metastable state is lost. Remarkably, these two features coincide with what was anticipated in Ref. [10] based on a very complementary viewpoint that extracted features from would-be exact solutions of NS5 branes. A no-go-theorem, based on singularities, implies that the black NS5, if it exists, has a maximum horizon radius until it disappears. A black D3 also escapes the nogo, but was deemed unphysical in Ref. [10], since it does not persist in the extremal limit.

We have now shown that when the nogo’s of Refs. [10,11] do not apply we find go’s. Together with the consistency of the extremal limit, this constitutes strong evidence for the existence of the metastable states, since they have now been argued for in complementary regimes. Our analysis should impact the understanding of finite temperature effects in confining cascading gauge theories. We leave a study of this aspect to future work.

We thank F.F. Gautason and J.P. van der Schaar for useful discussions. The work of T. V. R. is supported by the FWO odysses Grant No. G.0.E52.14N and the C16/16/005 grant of the KULeuven. We acknowledge support from the European Science Foundation HoloGrav Network. The work of V. N. is supported in part by a grant of the Independent Research Fund Denmark (Grant No. DFF-6108-00340). J. A. is partly supported by the Netherlands Organisation for Scientific Research (NWO).


Our definition of $d$ contains a rescaling of $r_0$ in such a way that we recover the known anti-$D3$ energy in the extremal limit, given by $2g^{-1}$, where we work in conventions for which the warp factor at the tip has been absorbed in a rescaling of the Minkowskian coordinates. That explains, in particular, the factor of 2 in the definition of $d$. 
