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Automated Market Makers

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Abstract

A new type of Automated Market Makers (AMMs) powered by Blockchain technology keep liquidity on-chain and offer transparent price mechanisms. This innovation is a significant step in the direction of building a more transparent and efficient financial market. This paper explores analytically market mechanisms and shows the conditions when those mechanisms are equivalent. Furthermore, we show that AMM mechanisms inherently create losses for market makers from inefficient prices (dictated by the AMM solutions), however, these mechanisms work well for assets with low volatility. We further analytically explore the losses and quantify them. The paper ends by discussing the design of efficient decentralized exchange compared to traditional Central Limited Order Books (CLOBs) and highlights the former’s potential regarding decentralized finance.

Keywords: blockchain, decentralized exchanges, automated liquidity providers, auction, mechanism design.

Introduction

Traditional financial exchanges offer regulated assets with predefined settlement mechanisms of up to three days. These inefficiencies are the byproduct of the respective regulations that drives slow reconciliation on traditional markets. With the invention of blockchain technology (Nakamoto 2008), a distributed ledger allows the creation of an asset together with the reconciliation process of transferring these assets between users without a central party and without delays. It is argued that blockchain technology creates new opportunities for the growth of trade-processing thanks to its ability to establish a single source of trust between untrusted parties and also by removing the intermediaries (Chiu and Shang 2019; Egelund-Müller et al. 2017; Nofer et al. 2017). Some of the most attractive aspects of crypto assets include their ability to provide traditional financial services in a faster, more transparent manner, and the fact that they are cost efficient and independent of a centralized service provider (Ross and Jensen 2019). In particular, exchanging assets in an efficient and secure way has sparked a number of innovative blockchain and market solutions that offer paths toward efficient and secure trades. We follow (Rai 2017) and (Gupta 2018) and formulate the following research question:

- Do Automated Market Makers (AMM) enabled by blockchain technologies offer more efficient financial markets or price discovery mechanisms?

We consider the AMM models to be one of the more innovative and promising solutions and we analyze their properties with respect to price changes relative to the primary markets for crypto assets.

Designing markets that result in the efficient allocation of goods and services is by no means an easy task. In the ideal first best world, all trades where the buyer’s willingness to pay is higher than the seller’s willingness to accept will happen. It is, however, well-known that the existence of market power, asymmetric information and transaction costs, such as basic searching and matching costs, prevent first best allocation (Vulkan et al. 2013). One of the most applied market solutions is the so-called double auction, whereby many buyers and sellers submit bids and asks, and prices are set either by discrete or continuous clearing. All traditional financial markets are dominated by the double auction with continuous clearing, also referred to as the Central Limit Order Book (CLOB). This very simple auction requires deep liquidity and trading activity to create an efficient price formation. As opposed to discrete clearing, each order is processed separately, and the resulting price changes impact each order separately. The size of the individual order and the existing order books determine the fluctuation in prices. Larger orders are typically

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traded outside of these exchanges, e.g., on off-exchange matching or dark pool markets. Also, the inherent nature of the queuing system for orders has resulted in front-running from high-frequency trading. Despite these shortcomings and the fact that liquidity is a much greater challenge for crypto assets compared to traditional financial markets, the CLOB solution is by far the most common trading institution for crypto assets. Furthermore, the CLOB solutions are typically implemented in a centralized manner, which is more susceptible to cyber-attacks as a single point of failure (Gandal et al. 2018).

As blockchain technologies have become more popular (Buterin 2014), a growing number of innovative alternatives to traditional financial market solutions have evolved (Beck et al. 2017; Glaser 2017; Rossi et al. 2019). As the ownership of these crypto assets essentially represents ownership of a private encryption key that allows transactions with these assets, the existence of crypto assets relies entirely on self-governed blockchain and the very limited regulatory protection. One solution to this fundamental security challenge is to avoid single points of failure by developing decentralized market solutions. One of these innovations is to trade assets directly via means of smart contracts in a so-called “non-custodian” on-chain manner, i.e., through decentralized exchanges (DEXs). The main advantages of DEXs are that information is transparent, there is no single point of failure, and the users maintain the custody of their own assets. However, due to the nature of blockchain, designing an efficient, fast and fair DEX is not as straightforward as simply copying the centralized alternatives. The main aim of this research is to provide a brief overview of these institutions, investigate, in detail, a particular class of these exchanges, known as automated market makers, and discuss the trade-off between security and market efficiency. The paper concludes by outlining a future research and development agenda in terms of designing efficient and secure DEXs.

Types of Decentralized Exchanges

Efficient markets are closely linked to liquidity as defined by the degree to which an asset or security can be quickly bought or sold on the market at a price that reflects its true value. As such, liquidity is a goal in market design. A trading venue can use mechanism design to incentivize liquidity. DEXs are the ideal market place to offer instantaneous settlement and transfer of value. Two of the approaches to providing liquidity in DEXs are:

- Order book based liquidity providers
- Automated liquidity providers

The order book liquidity provider model is similar to that of centralized CLOB exchanges. In these models, users submit their bids/asks - price-quantity bids - to the open order books, while the CLOB protocol continuously matches and executes orders. Based on the architecture of the order book, decentralized CLOB exchanges fall into the following two categories: on-chain order books and off-chain order books.

**On-chain order books:** With on-chain order books, all orders and their verifications are submitted on the blockchain. In other words, the users must pay for each update to the order book and wait for the network to reach consensus. This results in a less censored and more trustworthy exchange as everyone has access to the orders. However, it has the disadvantages of lower speed and higher transaction costs. Examples of this model include Bitshares (Schuh and Larimer 2017) and Stellar (Mazieres 2016).

**Off-chain order books:** With off-chain order books, all orders are handled in a centralized manner with only the final confirmation of the transactions being enforced by a smart contract on blockchain. This results in improved performance and lower costs, although it requires more trust. In addition, as the order book is not confirmed each time, it may contain incorrect information. Examples of this model include Hallex (Hallgren et al. 2017) and 0x (Warren and Bandeali 2017).

**Automated Market Maker (AMM):** Automated market makers or liquidity providers are algorithmic agents or smart contracts that automatically provide liquidity on electronic markets. They offer a new way of producing liquidity by swapping two tokenized assets (e.g., tokenized oil versus tokenized gold). AMM exchanges overcome the concept of an order book. Instead of setting prices by demand and supply, an AMM pools liquidity together and sets prices by way of a deterministic pricing formula. Therefore, it eliminates

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the need for counterparties (buyers or sellers). Examples of this model include Bancor (Hertzog et al. 2017), Uniswap (Adams 2019), and Kybernetwork (Luu 2017).

As mentioned, the vast majority of crypto assets are traded through traditional CLOB exchanges as opposed to alternative trading venues such as AMM. However, the straightforward copying of the CLOB institution from traditional financial markets is challenging. Although the decentralization of the order books addresses security challenges, it comes at the cost of lower transaction speed\(^2\), which again challenges liquidity. On the other hand, CLOB is a complete market mechanism that sets prices and selects winners as opposed to AMMs that require arbitrageurs to remove price differences through arbitrage.

**On the Efficiency of Automated Market Makers**

In AMM models, the price is determined by deterministic formulas based on the supply and demand of the assets. As a result, these models rely on the arbitrageurs to level the price with the other markets. In other words, when there is a price difference, the arbitrageurs start buying/selling assets from the smart contract, which changes the supply and demand of an asset and levels the price. However, this results in inefficiency for liquidity providers as they endure losses (or opportunity costs relative to the true market price that the arbitrageurs utilize). In this section, review two of these models; the constant product model and the swap token model, and then demonstrate the loss endured by the liquidity providers in these two AMM models.

**Constant Product Model**

In the constant product model, the liquidity pools consist of all liquidity providers’ assets in a smart contract that can be traded with any counterparty. A trading pair consists of two pools of assets in tokenized form in a smart contract, the total value of which is the product of the balances of its two pools. Any transaction (either buy or sell) changes the balances of the pools (and hence the total value of the contract). The main aim of the constant product model is to ensure that the total value of the contract is the same before and after each transaction. Formally, let \( \alpha \) and \( \beta \) be two tokenized assets, and let the pools consist of \( B_\alpha > 0 \) and \( B_\beta > 0 \) token, as the balances of each pool. The value of the contract is defined as \( B_\alpha \times B_\beta = k \). Consider a transaction which sends \( \Delta_\beta > 0 \) of the \( \beta \) tokens to the contract and receives \( \Delta_\alpha > 0 \) of \( \alpha \) tokens, i.e. buying \( \alpha \) tokens. Then the value of the contract must change in such a way that:

\[
(B_\alpha - \Delta_\alpha) \times (B_\beta + \Delta_\beta) = k.
\]  

(1)

Figure 1 illustrates the constant product model.

---

**Example 1.** Let \( \alpha \) tokens be any tokenized asset and the \( \beta \) tokens be tokenized dollars and \( B_\alpha = 100 \) and \( B_\beta = 200 \). This sets the price (exchange rate) of each \( \alpha \) tokens at 2$. Now consider trading 1$ in this model. By Equation (1), we must have \( 100 \times 200 = (100 - \Delta_\alpha) \times (200 + 1) \), which results in \( \Delta_\alpha = 0.49 \). After this transaction, the total value of the contract is the same, while the price of each \( \alpha \) token increases to 2.02$ as there are fewer \( \alpha \) tokens in the contract compared to the initial setup.

\(^2\) However, new innovations in consensus mechanisms such as proof-of-stake and second layers such as lightning network aim to address low transaction speed.
For illiquid tokenized assets, this AMM model may function as the primary price signal. For more liquid assets, the primary market venues function sets prices. As the prices in this AMM model only change based on the supply and demand of the assets, an arbitrage opportunity is created. If the prices in the primary market change, but the prices in the AMM model remain unchanged, the arbitrageurs can buy tokens at a lower price or sell at a higher price (until the prices are leveled). This arbitrage is directly translated into an opportunity cost or loss for the liquidity providers as some parts of the liquidity are sold at a lower price (or bought at a higher price). In the following, let the price of α tokens in a reference market be $P^\alpha_\beta$, and the price in the contract be $P^\alpha_\beta$ such that $P^\alpha_\beta = \xi \times P^\alpha_\beta$. The following proposition quantifies the liquidity providers’ exact loss compared to the case in which they did not provide liquidity to the pool.

**Proposition 1.** Let $\xi > 0$ be such that $P^\alpha_\beta = \xi \times P^\alpha_\beta$. Then the percentage of loss resulting from providing liquidity to the pool is $\left(\frac{2/J}{1+\xi} - 1\right) \times 100$.

**Proof.** Let $B'_\alpha = B_\alpha - \Delta_\alpha$ and $B'_\beta = B_\beta + \Delta_\beta$, be the new balances. Note that $P^\alpha_\beta = \frac{B_\beta}{B_\alpha}$. Therefore, as $B_\alpha \times B_\beta = k$ we have $P^\alpha_\beta = \frac{k}{B_\alpha}$, which results in $B_\alpha = \sqrt{\frac{k}{P^\alpha_\beta}}$ and $B_\beta = \sqrt{k \times P^\alpha_\beta}$. By Equation (1), it is required that $B'_\alpha \times B'_\beta = k$. Therefore, $B'_\alpha = \sqrt{\frac{k}{P^\alpha_\beta}} = \frac{B_\alpha}{\sqrt{\xi}}$. Similarly, we have $B'_\beta = \sqrt{\xi} B_\beta$. The total value of the pool (in terms of β tokens) after arbitragers have made changes to the price is $T' = B'_\alpha \times P^\alpha_\beta + B'_\beta$. However, in the case of holding the assets, the total value of the pool would be $T = B_\alpha \times P^\alpha_\beta + B_\beta$. This results in $\frac{T'-T}{T} \times 100 = \left(\frac{2/J}{1+\xi} - 1\right) \times 100$. •

Figure 2 presents the losses resulting from changes in the price. The blue curve represents the decrease in price of the α tokens (hence increase in the price of β tokens) with respect to their initial price, while the red curve represents the decrease in the price of β tokens (hence increase in the price of α tokens) as the initial price. Based on this Figure, any change in the price (either an increase or decrease) results in losses for the liquidity providers.

![Figure 2. The loss resulting from providing liquidity to the constant product model](image)

**Swap Tokens**

Swap tokens represent another type of AMM model. In such a model, there is a pair of tokens that can be exchanged and each is supported by a reserve. To exchange the two token an intermediary token is created which facilitates the swap between the two tokens. More precisely, let α and β be two tokenized assets. A liquidity provider produces a “swap token” called αβ tokens, which has supporting reserves of both α and β tokens. The αβ tokens can be converted to either α or β tokens, based on a pricing formula that depends on the balance of α and β tokens in the reserve of αβ tokens. This allows the αβ tokens to act as a decentralized exchange that can automatically convert between its two reserves. This process only depends on the balance of α and β in the reserve of αβ tokens, and hence the price is discovered without a need for any other parties. Let αβ tokens be supported by $B_\alpha$ of α tokens and $B_\beta$ of β tokens and $S_{\alpha\beta}$ be the total

\[ S_{\alpha\beta} = \frac{\alpha_\beta}{\sqrt{\frac{B_\alpha}{B_\beta}}} \]

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supply of $\alpha \beta$ tokens. Let $p_a^{\alpha \beta} (p_{\beta}^{\alpha \beta})$ be the price of each unit of $\alpha \beta$ tokens in terms of $\alpha$ ($\beta$) tokens. At any point in time, each of the $\alpha \beta$ tokens must maintain a ratio between their total value (Supply $\times$ Price) with their reserve. This ratio is called reserve ratio, which is denoted by $RR_\alpha$ ($RR_\beta$) for $\alpha$ ($\beta$) tokens (such that $RR_\alpha + RR_\beta = 100\%$), i.e., $RR_\alpha = \frac{a}{s_\alpha \times p_{\alpha}^{\alpha \beta}}$. By fixing the reserve ratio, the price of each $\alpha \beta$ token can be determined by either of its reserves. Therefore, the price of each $\alpha \beta$ token in terms of $\alpha$ tokens is:

$$p_a^{\alpha \beta} = \frac{B_\alpha}{S_{\alpha \beta} \times RR_\alpha}.$$  \hspace{1cm} (2)

From this setup, two formulas can be derived (Rosenfeld 2016); one for calculating the amount of $\alpha \beta$ tokens that can be received by paying either $\alpha$ or $\beta$ tokens, and another one for calculating the amount of $\alpha$ or $\beta$ tokens that can be received by paying $\alpha \beta$ tokens. Formally, a transaction trading $\Delta_\beta$ of $\beta$ tokens for $\Delta_\alpha$ of $\alpha$ tokens is achieved by first converting the $\Delta_\beta$ to $\alpha \beta$ tokens and then converting the $\alpha \beta$ tokens to $\alpha$ tokens using the following formulas:

$$\# \alpha \beta \text{ tokens} = S_{\alpha \beta} \times \left( (1 + \frac{\Delta_\beta}{B_\beta})^{RR_\beta} - 1 \right)$$  \hspace{1cm} (3)

$$\Delta_\alpha = B_{\alpha} \times \left( 1 - \left( 1 - \frac{\# \alpha \beta \text{ tokens}}{S_{\alpha \beta}} \right)^{\frac{1}{RR_\alpha}} \right)$$  \hspace{1cm} (4)

The new balances after this transaction will be $B_{\alpha}' = B_{\alpha} - \Delta_\alpha$ and $B_{\beta}' = B_{\beta} + \Delta_\beta$.

**Example 2.** Let $\alpha$ tokens be any tokenized asset and the $\beta$ tokens be tokenized dollars. Let the supply of $\alpha \beta$ tokens be 1000 ($S_{\alpha \beta} = 1000$) and the $\alpha \beta$ tokens be backed by 100 $\alpha$ tokens ($B_\alpha = 100$) with a reserve ratio of 20% ($RR_\alpha = 20\%$), and by 2005 ($B_\beta = 200$) with a reserve ratio of 80% ($RR_\beta = 80\%$). With this setup, each $\alpha \beta$ token is worth $p_a^{\alpha \beta} = \frac{1000}{1000 \times 0.2} = 0.5$ $\alpha$ tokens and $p_{\beta}^{\alpha \beta} = \frac{200}{1000 \times 0.8} = 0.25$$. Therefore, every $\alpha$ token equals 0.5$. Consider buying some $\alpha$ tokens with 1$. By Equation (3), the number of received $\alpha \beta$ tokens would be 3.99, which gives 1.98 of $\alpha$ tokens using Equation (4). After this exchange, $B_\alpha = 98.02$ and $B_\beta = 201$, and by Equation (2), we have $p_a^{\alpha \beta} = 0.49$ and $p_{\beta}^{\alpha \beta} = 0.251$, which implies that every $\alpha$ token is worth 0.513$. This reflects the fact that as the number of $\alpha$ tokens in the contract has decreased and the number of dollars tokens has increased. As a result, the value of each $\alpha$ token has increased in dollars.

Similar to the constant product model, when there is a gap between the price of a reference market and the contract prices, the swap tokens rely on the arbitrageurs to level the price. Using the same notation as in Proposition 1, we get the following proposition:

**Proposition 2.** Let $\xi > 0$ be such $p_{\beta}^{\alpha \beta} = \xi \times p_{\beta}^{\alpha \beta}$, and $RR_\alpha = RR_\beta = 50\%$. Then the percentage of loss resulting from providing liquidity to the pool is $\left( \frac{\xi \sqrt{\xi}}{1 + \xi} - 1 \right) \times 100$.

**Proof.** As the price of $\alpha$ tokens in the contract is lower, the arbitrageurs will buy $\Delta_\alpha$ of $\alpha$ tokens by spending $\Delta_\beta$ of $\beta$ tokens. Therefore, first we have to convert $\Delta_\beta$ tokens to get $A$ $\alpha \beta$ tokens, and then convert these $A$ tokens to get $\Delta_\alpha$ of $\alpha$ tokens. Using Equation (3), $A = S_{\alpha \beta} \times \left( (1 + \frac{\Delta_\beta}{B_\beta})^{RR_\beta} - 1 \right)$. Then by Equation (4) and these $A$ tokens we have $\Delta_\alpha = B_{\alpha} \times \left( 1 - \left( 1 - \frac{A}{S_{\alpha \beta} + A} \right)^{\frac{1}{RR_\alpha}} \right)$. Substituting $A$ into this equation yields:

$$\Delta_\alpha = B_{\alpha} - \frac{B_{\alpha}}{\left( B_\beta + \Delta_\beta \right)^{\frac{1}{RR_\alpha}}}.$$  \hspace{1cm} (5)
Using Equation (2), we have:

\[
\frac{p_a^\alpha}{p_a^\beta} = \frac{\frac{B_\beta}{\sum a_{\beta} \times RR_\beta}}{\frac{B_\alpha}{\sum a_{\alpha} \times RR_\alpha}} = \frac{B_\beta}{\sum a_{\alpha} \times RR_\alpha} = \frac{B_\beta}{B_\alpha \times RR_\beta}
\]

Therefore, as \( p_a^\alpha = \xi \times p_a^\beta \) the previous equation and the fact that the reserve ratios are fixed, we have \( \frac{\frac{\sum a_{\alpha} \times RR_\beta}{\sum a_{\beta} \times RR_\alpha}}{\frac{\sum a_{\alpha} \times RR_\alpha}{\sum a_{\beta} \times RR_\beta}} = \frac{p_a^\alpha}{p_a^\beta} \). Replacing the value of \( \Delta_a \) from Equation (5) into this equation, and solving for \( \Delta_\beta \) results in

\[
\Delta_\beta = \left( \frac{1}{\xi} \right)^{\frac{RR_\alpha}{RR_\beta}} - B_\beta, \quad \text{and hence } \Delta_\alpha = B_\alpha \left( 1 - \frac{1}{\xi} \right) \text{ and } \Delta_\beta = B_\beta \left( \sqrt{\xi} - 1 \right) .
\]

Therefore, the new balances are \( B_\alpha' = \frac{\sum a_{\alpha} \times RR_\alpha}{\sum a_{\alpha} \times RR_\alpha} \) and \( B_\beta' = B_\beta \left( \sqrt{\xi} - 1 \right) \). The total value of the pool (in terms of \( \beta \) tokens) after the arbitrage equals \( T' = B_\alpha' \times p_a^\alpha + B_\beta' \). However, the total value of the pool in the case of holding the assets (not providing liquidity to the pool) would be \( T = B_\alpha \times p_a^\alpha + B_\beta \). This results in \( T' \times T \times 100 = \sqrt{\frac{\xi}{1 + \xi}} - 1 \times 100 \), which is illustrated in Figure 2. \( \blacksquare \)

From the proof of Proposition 1 and 2 we can conclude the following:

**Corollary 1.** The constant product model and the swap token model are the same when \( RR_\alpha = RR_\beta = 50\% \).

By Corollary 1 and Figure 2, both AMM models result in losses to the market makers when there is a price gap between the AMM prices and other market venues. A closer look reveals that this is not only due to the price difference between the two markets. These results can be generalized to any transaction that is conducted with AMMs. To be more precise, any transaction will affect the relative balance of tokens held in the contract, which in turn affects the price of the tokens. Now we can define the price before the transaction as \( P \), the price after the transaction as \( P' \) and the price factor as \( \xi \) such that \( P' = \xi P \). Therefore, the aforementioned results hold for any transaction. Therefore, AMMs inherently result in losses for the market makers. A mechanism that is used in the AMMs to ensure the liquidity providers remain incentivized is transaction fees. This means that for any transaction conducted in the pool, a part of it is returned to the pool, which over time causes the pool to increase in size, which in turn benefits the liquidity providers in the long run. Another important conclusion derived from Figure 2 is that the losses resulting from these models are not linear with respect to changes in price. Hence, in markets with low volatility, losses are lower, which results in greater profit for the liquidity providers considering the accumulated fees. Therefore, from the perspective of liquidity providers, these models are better suited for assets with low volatility. Despite these shortcomings, the AMM models represent an innovative alternative to the traditional order book models as they eliminate the need for the presence of buyers and sellers at the same time and provide relatively fast trades. The AMM models are composable and allow external users to provide liquidity directly without the need for third parties, therefore, allowing startups to build their AMM models with minimum investment (Beinke et al. 2018).

**Towards an Efficient Decentralized Primary Market**

AMM models represent an innovative use of blockchain technologies and a highly relevant complementary market to the dominant CLOB. However, they are only a complementary or partial solution as they rely entirely on the existence of an efficient primary market. Therefore, we conclude the paper by returning to our basic research question about whether blockchain technologies can offer more efficient financial market and price discovery mechanisms.

From a market design perspective, the vast literature on auction design provides an ideal starting point. The most common aim of an exchange or a double auction is to maximize social welfare as the sum of sellers and buyers. This implies that the commodities must be allocated to those who value them most. In the absence of such a transparent allocatively efficient auction market, the resulting prices will typically function as focal points for trades outside of the auction, potentially attracting all trades. To see this, note that a potential buyer (seller) will not accept a price that is higher (lower) than those on the efficient auction. In light of this simple observation, the mere existence of AMM models indicates that the dominant CLOB solution does not provide such an allocative efficient auction market.
The primary sources of inefficiency are informational asymmetry, market power and transaction costs including search and matching costs. All of these can be reduced to what are the most important criteria for an auction market; attracting traders and avoiding collusion, see e.g., (Klempperer 2002). Let collusion include the security breaches, which allows us to reinterpret what really matters in designing exchanges; minimizing security breaches and maximizing liquidity. While DEXs address security through no single point of failure, an improved market design addresses market efficiency by approximating the efficient equilibrium prices and executing trades at these prices. AMM models simplify the exchange process without leaving the decentralized blockchain infrastructure. However, they rely on efficient prices from elsewhere, which requires compensation to the liquidity providers as analyzed in the previous section.

An alternative market solution is a DEX with discrete clearing as opposed to continuous clearing – a so-called “batch auction”. The frequent batch auction provides all buyers and sellers with the same trading functionalities used for, e.g., market makers in designing exchanges. The development of DEXs that both fully utilize blockchain technologies and address security is a technological bedrock for simplifying the use of auctions. So-called Oracle functionalities used for automating monitoring and collecting information may address informational overload and smart contracts serve as automated trading agents to feed with preference information. In general, because of the desirable properties of auctions, they may pave the way to achieving more efficient financial markets based on blockchain technologies.

**Conclusion**

The development of DEXs that both fully utilize blockchain technologies and facilitate efficient markets is still at an early stage. The initial attempts involve decentralized order book on CLOB exchanges and the swap-based structure built into the AMM models. AMM models provide a different class of solutions that are decentralized, fast and attract liquidity. Although the AMM model overcomes the problem of the need for the presence of buyers and sellers on the market at the same time, the institutions are sensitive to the size of the pool and the trades. Furthermore, AMM models dictate prices that are not linked to the primary markets, e.g., the CLOB exchanges. Consequently, AMM models rely on arbitrageurs to level the price with the outside primary market, which as we showed, results in inefficiency and losses for the liquidity providers. We further show that the most common AMM models are equivalent if the market markers provide equal relative liquidity support to the exchanged crypto assets. Our research concludes that AMMs work well for assets with high liquidity and low price fluctuations. Furthermore, our results indicate that future market mechanisms need to be designed to encourage mean reverting behavior similar to traditional markets where liquidity providers may benefit from a rebate.
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