A non-integrable quench from AdS/dCFT

de Leeuw, Marius; Kristjansen, Charlotte; Vardinghus, Kasper E.

Published in:
Physics Letters B

DOI:
10.1016/j.physletb.2019.134940

Publication date:
2019

Document version
Publisher's PDF, also known as Version of record

Physics Letters B 798 (2019) 134940

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

A non-integrable quench from AdS/dCFT

Marius de Leeuw [a], Charlotte Kristjansen [b,*], Kasper E. Vardinghus [b]

[a] School of Mathematics & Hamilton Mathematics Institute, Trinity College Dublin, Dublin, Ireland
[b] Niels Bohr Institute, Copenhagen University, Blegdamsvej 17, 2100 Copenhagen Ø, Denmark

ARTICLE INFO

Article history:
Received 15 July 2019
Received in revised form 3 September 2019
Accepted 11 September 2019
Available online 17 September 2019
Editor: N. Lambert

Keywords:
AdS/CFT correspondence
Defect CFT
Probe branes
One-point functions
Matrix product states
Quantum quenches

ABSTRACT

We study the matrix product state which appears as the boundary state of the AdS/dCFT set-up where a probe D7 brane wraps two two-spheres stabilized by fluxes. The matrix product state plays a dual role, on one hand acting as a tool for computing one-point functions in a domain wall version of $\mathcal{N}=4$ SYM and on the other hand acting as the initial state in the study of quantum quenches of the Heisenberg spin chain. We derive a number of selection rules for the overlaps between the matrix product state and the eigenstates of the Heisenberg spin chain and in particular demonstrate that the matrix product state does not fulfil a recently proposed integrability criterion. Accordingly, we find that the overlaps can not be expressed in the usual factorized determinant form. Nevertheless, we derive some exact results for one-point functions of simple operators and present a closed formula for one-point functions of more general operators in the limit of large spin-chain length.

1. Introduction

Exact results for overlaps between states in integrable spin chains have important applications in the calculation of correlation functions in supersymmetric gauge theories as well as in the study of quantum quenches in statistical physics. Recently, especially overlaps between Bethe eigenstates and matrix product states have attracted attention. From the point of view of the AdS/dCFT correspondence, overlaps between Bethe eigenstates and specific matrix product states encode information about one-point functions in domain wall versions of $\mathcal{N}=4$ SYM theory [1–5] and in statistical physics the same matrix product states play the role of the initial state of a quantum quench [6–8].

Interestingly, all spin chain states $|\Psi\rangle$ for which it has been possible to write the overlap with the Bethe eigenstates in a closed form have been characterized by being annihilated by the entire tower of parity odd conserved charges of the chain. Furthermore, for all of these cases the annihilation of the state by the odd charges could be used to show that the overlaps with Bethe eigenstates were only non-vanishing for Bethe states with paired roots[1] and finally the overlaps took a factorized form with the Gaudin norm matrix, G [9,10], playing a prominent role. More precisely, for Bethe states with paired roots the determinant of the Gaudin matrix factorizes as[2]

$$\det G = \det G_+ \det G_-, \tag{1}$$

and the normalized overlap takes the (schematic) form

$$\frac{\langle \Psi | \mathbf{u}\rangle}{\langle \mathbf{u} | \mathbf{u}\rangle^{1/2}} = \prod_i f(u_i)\sqrt{\frac{\det G_+}{\det G_-}}. \tag{2}$$

These observations lead the authors of [11] to suggest that matrix product states should be denoted as integrable when annihilated by all odd charges of the spin chain and in that case would play a role analogous to that of the integrable boundary states of Zamolodnikov for continuum quantum field theories [12]. Furthermore, in [13] integrable matrix product states were related

* Corresponding author.
E-mail address: kristjan@nbi.dk (C. Kristjansen).

[1] States with paired roots are states for which the roots take the form $\{u_i, -u_i\}\bigcup S_u$, where $q_{2n+1}(u) = 0$ for $u \in S_u$. For the SU(2) Heisenberg spin chain that we consider in the present letter, $S_u = \emptyset$, but for spin chains with nested Bethe ansätze such as the SU(3) or the SO(6) spin chain there can be a single root a zero [3,5].

[2] For a detailed explanation of how this happens for a model with a nesting we refer to [3].

https://doi.org/10.1016/j.physletb.2019.134940

to novel types of solutions to the twisted Boundary Yang-Baxter equations, carrying extra internal degrees of freedom.

Note, however, that the notion of integrability of a matrix product state has (so far) not been used neither to prove the existence of, nor to derive a closed expression for the overlaps with the Bethe eigenstates. Furthermore, it is not excluded that a matrix product state which is not integrable in the sense above could have a closed formula describing its overlaps with the Bethe eigenstates and finally the integrability criterion only directly applies to spin chains for which the conserved charges can be defined to have a specific parity.

The approach of using matrix product states in the calculation of one-point functions in defect versions of $\mathcal{N}=4$ SYM theory was introduced in [1,2] where the field theory was taken to have gauge groups of different rank, $U(N)$ and $U(N-k)$, on the two sides of a co-dimension one defect [14,15]. This domain wall set-up has a dual string theoretical description as a D3-D5 probe brane system where the D5 probe has geometry $AdS_4\times S^2$ and where there are k units of magnetic flux through the S^2 [14,16,17]. In this case it was possible to find a closed expression of the form (2) for the one-point functions of all scalar operators which involved finding a closed expression for the overlap between a matrix product state and a Bethe eigenstate of the SO(6) spin chain [5].

The approach was pursued for a different D3-D5 based defect version of $\mathcal{N}=4$ SYM in [18], namely that constructed from the β-deformed theory. These investigations did not reveal a closed formula for the one-point functions. In this case, neither the Hamiltonian, nor the higher conserved charges of the associated integrable spin have a definite parity and thus the integrability criterion above does not immediately apply.

There exists another AdS/dCFT set-up which is very similar to the D3-D5 probe brane system and which also leads to a domain wall version of $\mathcal{N}=4$ SYM theory, namely a D3-D7 probe brane system, likewise with background gauge field flux. The D3-D7 probe brane set-up comes in two different versions corresponding to two different probe brane embeddings with respectively SO(5) and SO(3)×SO(3) symmetry [19–21]. In the SO(5) symmetric case the matrix product state of relevance for the computation of scalar one-point functions belongs to the integrable class in the sense above [5]. We note, however, that at the present moment a closed expression for the one-point functions is not known [4].

In this paper we will study the matrix product state that encodes the one-point functions of the SO(3)×SO(3) symmetric D3-D7 probe brane system and show that as opposed to its above mentioned relatives it does not qualify as an integrable boundary state. In accordance with this we find that the one-point functions can not be written in the form of (2) and no indication of an alternative closed formula in terms of determinants was observed. Nevertheless, we are still able to extract non-trivial exact information about the one-point functions of the corresponding dCFT.

Let us mention that very recently matrix product states have made their appearance in the calculation of three-point functions in $\mathcal{N}=4$ SYM theory involving two determinant operators and one single trace non-protected operator [22]. This is very natural as the dual string theory computation is very similar to the one required for the computation of one-point functions [23] with the parameter describing the background gauge-field flux being replaced by the angular momentum of a giant graviton.

Our letter is organized as follows. In section 2 we introduce the relevant matrix product state and sketch its role in the calculation of one-point functions. We shall be brief regarding this point and refer to [24,25] for details. Subsequently, in section 3, we investigate the action of the simplest odd charge on the matrix product state and derive a number of selection rules for the one-point functions of the corresponding dCFT. In section 4 we present a few exact results for one-point functions of simple operators and, in particular, we quantify the deviation of the results from the formula (2). Finally, in section 5 we present a closed formula for the one-point functions in the limit of large-L, where L is the number of fields in the operator considered, respectively the length of the spin chain involved. Section 6 contains our conclusion.

2. The matrix product state

The classical equations of motion of $\mathcal{N}=4$ SYM admit a fuzzy funnel solution where for $x_3>0$ the six scalar fields take the values [21]

$$\begin{aligned}\phi_i^{cl}(x)&=-\frac{1}{x_3}\left(t_i^{k_1}\otimes\mathbb{1}_{k_2}\right)\oplus 0_{N-k_1k_2}\quad\text{for}\quad i=1,2,3,\\ \phi_i^{cl}(x)&=-\frac{1}{x_3}\left(\mathbb{1}_{k_1}\otimes t_{i-3}^{k_2}\right)\oplus 0_{N-k_1k_2}\quad\text{for}\quad i=4,5,6,\end{aligned}\tag{3}$$

while the fermionic fields as well as the gauge fields vanish, and where for $x_3<0$ all fields carry a $\mathrm{U}(N-k_1k_2)$ representation and vanish in the classical limi. Here, the matrices $t_i^{k_a}$ constitute a k_a-dimensional irreducible representation of SU(2). This solution realizes a domain wall which separates a region ($x_3<0$) where the field theory has gauge group $\mathrm{U}(N-k_1k_2)$ from a region ($x_3>0$) where the theory has gauge group $\mathrm{U}(N)$, broken by the vevs. We shall be interested in studying the tree-level one-point functions in the SU(2) sub-sector of conformal operators, built from the complex fields Z and X defined by

$$X=\phi_1+\mathbb{i}\phi_4,\qquad Z=\phi_2+\mathbb{i}\phi_5,\tag{4}$$

and described by a certain eigenstate $|\{u_i\}\rangle\equiv|\mathbf{u}\rangle$ of the integrable Heisenberg spin chain where the u_i are the corresponding Bethe roots [26]. Already in [21] a closed expression for the overlap of the vacuum state with the Bethe eigenstates was found and matched to a string theory result. This match between gauge and string theory was recently extended to the next to leading order in [27]. Here we will deal with excited states corresponding to non-protected operators in the field theory. Computing the tree-level value of the one point functions, which amounts to inserting the classical values for the fields in the expressions for the conformal operators, can be implemented by means of the following matrix product state

$$\langle\mathrm{MPS}_{(k_1,k_2)}(\alpha)|=\mathrm{tr}\prod_{n=1}^{L}\Big(\langle\uparrow|_n\otimes T_1^{(k_1,k_2)}(\alpha)+\langle\downarrow|_n\otimes T_2^{(k_1,k_2)}(\alpha)\Big),\tag{5}$$

where

$$T_i^{(k_1,k_2)}(\alpha)=t_i^{k_1}\otimes\mathbb{1}_{k_2}+\alpha\mathbb{1}_{k_1}\otimes t_i^{k_2}.\tag{6}$$

The introduction of the parameter α allows us to write the commutation relation for the T matrices as

$$\left[T_i^{(k_1,k_2)}(\alpha),T_j^{(k_1,k_2)}(\beta)\right]=\mathbb{i}\varepsilon_{ijk}T_k^{(k_1,k_2)}(\alpha\beta).\tag{7}$$

The parameter α also allows us to interpolate between various models. The case $\alpha=\pm\mathbb{i}$ will be relevant for the computation of the D3-D7 one-point functions, while the cases $\alpha=0,\pm1$ are related to the D3-D5 probe brane matrix product state.

More precisely, the one-point functions of interest for the D3-D7 brane case can be expressed as

$$\langle \mathcal{O}_L \rangle = \left(\frac{8\pi^2}{\lambda}\right)^{\frac{L}{2}} L^{-\frac{1}{2}} \frac{C_{k_1,k_2}}{x_3^L}, \tag{8}$$

where

$$C_{k_1,k_2} = \frac{\langle \mathbf{u} \,|\text{MPS}_{(k_1,k_2)}(\alpha = \mathbb{i})\rangle}{\langle \mathbf{u} \,|\, \mathbf{u} \rangle^{\frac{1}{2}}}. \tag{9}$$

The case $\alpha = 0$ is trivially related to the D3-D5 probe brane matrix product state but all other cases differ from the latter in a crucial manner. In particular, one can easily convince oneself, that as opposed to what was the case in the D3-D5 set-up [2] there is no recursive relation which connects matrix product states with different bond dimensions to each other. We notice, however, that by setting $k_1 = 1$ or $k_2 = 1$ we recover the matrix product state of relevance for the D3-D5 probe brane set-up [1,2].

3. Integrability test and selection rules

Using the explicit expression for the simplest odd charge, Q_3, of the SU(2) Heisenberg spin chain (with periodic boundary conditions)

$$Q_3 = \sum_{n=1}^{L} [P_{n,n+1}, P_{n+1,n+2}], \tag{10}$$

where P is the permutation operator, one finds that

$$Q_3|\text{MPS}_{(k_1,k_2)}(\alpha)\rangle \neq 0, \tag{11}$$

for $L \geq 12$ and for all values of α, k_1 and k_2 except the trivial ones where the matrix product state is related to the matrix product state of the D3-D5 probe brane set-up. Hence, the state (5) does not belong to the class of matrix product states denoted as integrable and nothing prevents Bethe eigenstates with unpaired roots from having a non-vanishing overlap with this state. Indeed, one easily finds by explicit computation examples of Bethe eigenstates with un-paired roots and with non-vanishing overlap with the matrix product state (5). Such Bethe eigenstates are first encountered for $L = 12, M = 6$, where M is the number of excitations. Furthermore, even for Bethe states with only paired roots explicit computations of overlaps have not revealed a closed formula for the overlaps. However, one can still derive a number of exact results. In order to do so it is useful to start by deriving a set of selection rules. First, we notice that in order for a Bethe state to have a non-vanishing overlap with the matrix product state (5) it needs to have an even length and an even number of excitations. This result follows from the SU(2) algebra having the following automorphisms [1]

$$U t_1 U^{-1} = t_1, \quad U t_{2,3} U^{-1} = -t_{2,3}, \tag{12}$$

$$V t_3 V^{-1} = t_3, \quad V t_{1,2} V^{-1} = -t_{1,2}, \tag{13}$$

with U and V unitary matrices, which naturally lift to the algebra of the $T_i^{(k_1,k_2)}$ so that for inst.

$$(U_1 \otimes U_2)\, T_1^{(k_1,k_2)}(\alpha)\, (U_1^{-1} \otimes U_2^{-1}) = T_1^{(k_1,k_2)}(\alpha). \tag{14}$$

Secondly, we have the relation

$$(\mathbb{1} \otimes V)\, T_{1,2}^{(k_1,k_2)}(\alpha)\, (\mathbb{1} \otimes V^{-1}) = T_{1,2}^{(k_1,k_2)}(-\alpha), \tag{15}$$

and in addition there exists an invertible matrix S, a so-called shuffle matrix, which interchanges the factors in the direct matrix product, i.e.

$$S\,(\mathbb{1} \otimes t_i)\, S^{-1} = t_i \otimes \mathbb{1}, \tag{16}$$

which for $k_1 = k_2 = k$ implies

$$S\, T_{1,2}^{(k,k)}(\alpha)\, S^{-1} = \alpha\, T_{1,2}^{(k,k)}(1/\alpha). \tag{17}$$

Together these relations give for $k_1 = k_2 = k$

$$\begin{aligned} \text{Tr}\,(T_{s_1}(\alpha) \ldots T_{s_L}(\alpha)) &= \alpha^L\, \text{Tr}\,(T_{s_1}(1/\alpha)\ \ldots T_{s_L}(1/\alpha)) \\ &= \text{Tr}\,(T_{s_1}(-\alpha) \ldots T_{s_L}(-\alpha)), \end{aligned}$$

with $s_i \in \{1, 2\}$. Thus for $\alpha = \pm\mathbb{i}$ we need that $\mathbb{i}^L = 1$, i.e. $L/4 \in \mathbb{N}$ in order for the overlap not to vanish.

Finally, we are only interested in cyclically symmetric Bethe eigenstates, i.e. Bethe eigenstates with total momentum zero, as only such states can represent a single trace operator of $\mathcal{N} = 4$ SYM. We note, however, that due to the cyclicity of the matrix product state, its overlap with Bethe eigenstates is vanishing for non-cyclic states.

4. Exact results

As explained in [1] the coordinate space Bethe ansatz provides an explicit expression for the Bethe eigenstates which is useful for the calculation of the overlaps. More precisely, we have for the overlap of the matrix product state with a Bethe eigenstate with M excitations

$$\langle \text{MPS}|\, \mathbf{u}\, \rangle = \mathcal{N} \sum_{\sigma \in S_M} A_\sigma \sum_{1 \leq n_1 < \ldots < n_M \leq L} \prod_{j=1}^{M} x_{\sigma_j}^{n_j} \langle \text{MPS}|\{n_i\}\rangle, \tag{18}$$

where A_σ is a product of two particle scattering matrices corresponding to the permutation σ and where

$$x_j = \frac{u_j + \frac{\mathbb{i}}{2}}{u_j - \frac{\mathbb{i}}{2}}. \tag{19}$$

Furthermore,

$$\langle \text{MPS}|\{n_i\}\rangle = \text{tr}(T_1 \ldots T_1 T_2 T_1 \ldots T_1 T_2 T_1 \ldots T_1) \tag{20}$$

where the M generators of type T_2 are located at the sites $n_1, \ldots, n_M$. Finally $\mathcal{N}$ is a normalization constant in the form of a phase which we will choose so that the one-point function coefficient C_{k_1,k_2} is real and positive. For details we refer to [1]. We note that due to the tensor structure in the matrix product state all trace factors of the type (20), even that corresponding to the vacuum, involve binomial sums [27].

By means of the relation (18) we can evaluate the overlap between the matrix product state and the two-excitation state for any value of α. For $k_1 = k_2 = 2$ the resulting trace factor (20) can be simplified and evaluated explicitly[3]

$$\begin{aligned} \langle \text{MPS}|\{n_i\}\rangle = &\left[\left(\frac{\alpha+1}{\alpha-1}\right)^{\sum_i (-1)^i n_i} \left(\frac{\alpha-1}{2}\right)^L \right. \\ &\left. + \left(\frac{\alpha-1}{\alpha+1}\right)^{\sum_i (-1)^i n_i} \left(\frac{\alpha+1}{2}\right)^L \right] \\ &\times \frac{2}{(\alpha^2-1)^{\frac{M}{2}}} \sum_{m=0}^{\frac{M}{2}} \alpha^{2m} \\ &\times \sum_{\substack{A \subset \{1,2,\ldots,M\} \\ |A| = 2m}} (-1)^{\sum_i A_i} (-1)^{\sum_i n_{A_i}}. \end{aligned} \tag{21}$$

[3] Here we are excluding the cases $\alpha = \pm 1$. For a discussion of these, see below.

From, this we can deduce the result for the overlap for $k_1 = k_2 = M = 2$, which reads

$$\frac{\langle \text{MPS}_{2,2}(\alpha)|\mathbf{u}\rangle}{\langle \mathbf{u}|\mathbf{u}\rangle^{1/2}} = \frac{u\sqrt{u^2+\frac{1}{4}}}{2^{L-1}}\sqrt{\frac{L}{L-1}}\times \left[\frac{\alpha\left((\alpha-1)^{L-1}+(\alpha+1)^{L-1}\right)}{u^2+\frac{1}{4\alpha^2}} - \frac{(\alpha-1)^{L-1}-(\alpha+1)^{L-1}}{u^2+\frac{\alpha^2}{4}}\right]. \tag{22}$$

For $\alpha = \mathbb{i}$ this reduces to

$$\frac{\langle \text{MPS}_{2,2}(\pm i)|\mathbf{u}\rangle}{\langle \mathbf{u}|\mathbf{u}\rangle^{1/2}} = \frac{1}{2^{\frac{L}{2}-2}}\sqrt{\frac{L}{L-1}}\frac{u\sqrt{u^2+\frac{1}{4}}}{u^2-\frac{1}{4}}. \tag{23}$$

Furthermore, for $\alpha = 0$ the overlap is proportional to the overlap with the D3-D5 matrix product state for $k = 2$. Finally for $\alpha = \pm 1$ the tensor product of the two two-dimensional SU(2) representations decomposes as $3 \oplus 1$ and the overlap becomes proportional to the overlap with the D3-D5 matrix product state for $k = 3$.

For four excitations the result is non-zero even when the rapidities are unpaired. It can be computed exactly, but it is very lengthy. For the special where the rapidities are paired, the result drastically simplifies

$$\frac{\langle \text{MPS}_{2,2}(\pm\mathbb{i})|\mathbf{u}\rangle}{\langle \mathbf{u}|\mathbf{u}\rangle^{1/2}} = \frac{1}{2^{\frac{L}{2}-2}}\left[\prod_{i=1}^{2}\frac{u_i\sqrt{u_i^2+\frac{1}{4}}}{u_i^2-\frac{1}{4}}\right]\sqrt{\frac{\det G^+}{\det G^-}} + \frac{L}{2^{\frac{L}{2}+3}\sqrt{\det G}}\left[\prod_{i=1}^{2}\frac{u_i}{(u_i^2-\frac{1}{4})^2\sqrt{u_i^2+\frac{1}{4}}}\right]\times \frac{(u_1^2-u_2^2)^2(-1-4u_1^2-4u_2^2+48u_1^2u_2^2)}{(1+(u_1-u_2)^2)(1+(u_1+u_2)^2)\left(u_1^2u_2^2-\frac{1}{16}\right)}. \tag{24}$$

Here, in the first line we have singled out the “integrable” piece of the overlap, cf. eqn. (2). We notice that the remaining part of the expression is subdominant in the limit $L \to \infty$ behaving as $\mathcal{O}(\frac{1}{L})$ relative to the first term. Moreover, we see that even for states with paired rapidities, the overlap can not be written in the form of (2). We have not been able to find any determinant type formula which reproduces this result.

It is likewise possible to find the overlaps (23) and (24) for higher values of k_1 and k_2 but unlike what was the case for the D3-D5 set-up there does not seem to exist a recursion relation that relates the overlaps for different values of k_1 and k_2 which can be traced back to the lack of a recursive relation between the matrix product states corresponding to different values of (k_1, k_2). For $M = 2$ and general values of k_1 and k_2 we find

$$\frac{\langle \text{MPS}_{2,2}(\pm i)|\mathbf{u}\rangle}{\langle \mathbf{u}|\mathbf{u}\rangle^{1/2}} = u\sqrt{u^2+\frac{1}{4}}\sqrt{\frac{L}{L-1}}\Bigg\{ \tag{25}$$

$$\sum_{n=-\frac{k_1}{2}}^{\frac{k_1}{2}}\sum_{m=-\frac{k_2-1}{2}}^{\frac{k_2-1}{2}}\left[n^2-\frac{k_1^2}{4}\right]\frac{(\alpha m+n+\frac{1}{2})^{L-1}}{(\alpha m+n)^2+u^2}+ \tag{26}$$

$$\sum_{m=-\frac{k_2}{2}}^{\frac{k_2}{2}}\sum_{n=-\frac{k_1-1}{2}}^{\frac{k_1-1}{2}}\left[m^2-\frac{k_2^2}{4}\right]\alpha^L\frac{(\alpha^{-1}n+m+\frac{1}{2})^{L-1}}{(\alpha^{-1}n+m)^2+u^2}\Bigg\}. \tag{27}$$

As in [1,27] it is possible to extract the leading k_1, k_2 limit of the overlap, a quantity which is of relevance for comparison with the string theory side, cf. [28,21].

For two excitations we find

$$\frac{\langle \text{MPS}_{2,2}(\alpha)|\mathbf{u}\rangle}{\langle \mathbf{u}|\mathbf{u}\rangle^{1/2}} = \frac{2^{1-L}}{L(L-2)(L-3)}\sqrt{\frac{L}{L-1}}u\sqrt{u^2+\frac{1}{4}}\times \left[(\alpha^2+1)\frac{(k_1+\alpha k_2)^L-(k_1-\alpha k_2)^L}{\alpha}+ L\frac{(k_1-\alpha^3k_2)(k_1-\alpha k_2)^{L-1}-(k_1+\alpha^3k_2)(k_1+\alpha k_2)^{L-1}}{\alpha}\right]. \tag{28}$$

This greatly simplifies when $\alpha = \mathbb{i}$ and we get

$$\frac{\langle \text{MPS}_{2,2}(\mathbb{i})|\mathbf{u}\rangle}{\langle \mathbf{u}|\mathbf{u}\rangle^{1/2}} = \frac{2^{1-L}(k_1^2+k_2^2)}{(L-2)(L-3)}\sqrt{\frac{L}{L-1}}u\sqrt{u^2+\frac{1}{4}}\times \left[(k_1-ik_2)^{L-2}-(k_1+ik_2)^{L-2}\right].$$

Notice that the one-point function scales as k^L, whereas in the D3-D5 set-up the leading k behaviour is of order k^{L-1} for $M = 2$. In fact, sending $k_1 \to \infty$ while keeping k_2 finite should yield the usual D3-D5 result. Indeed, imposing this limit, we see that (28) vanishes and we find that the subleading term reproduces the results from [1].

5. The large-L limit

The large-L limit of the overlaps is of relevance both for studying quantum quenches in the thermodynamical limit and for comparing with semi-classical string theory in the AdS/dCFT set-up [2]. In order to study the large-L limit of the overlaps, we notice that provided the trace factor $\langle \text{MPS}|\{n_i\}\rangle$ in eqn. (18) is exponential in the n_i the sum over the n_i becomes geometrical and can be carried out explicitly. We observe that the leading L contribution originates from terms in the sum over n_i for which paired rapidities are next to each other with each pair giving rise to a factor of L. In particular, for M excitations, states with all roots paired have an overlap with the matrix product state which behaves as $2^{2-\frac{L}{2}}L^{M/2}$ for large-L, whereas states with fewer paired roots have overlaps which scale with a lower power of L.

We can use the observation above to facilitate the extraction of the large-L behaviour of the overlaps for finite M. This limit is known as the zero density limit in statistical mechanics and a few examples of calculations of overlaps in this limit exists, see f.inst. [29,30]. In our case, to determine the pre-factor of the leading L term for finite M we can truncate the sum over permutations in (18) to only those which keep the paired roots next to each other. For M excitations and for states with only paired roots this reduces the number of permutations in the sum from $M!$ to $2^{M/2}(M/2)!$. Based on explicit computations of the overlaps between the matrix product state and Bethe states with $M = 2, 4, 6$ we find the following expression for the large-L contribution to the one-point function for states with finite M, $k_1 = k_2 = 2$ and only paired roots.

$$\frac{\langle \text{MPS}_{2,2}(\pm\mathbb{i})|\mathbf{u}\rangle}{\langle \mathbf{u}|\mathbf{u}\rangle^{1/2}} = \frac{1}{2^{\frac{L}{2}-2}}\left\{\prod_{i=1}^{M/2}\frac{u_i\sqrt{u_i^2+\frac{1}{4}}}{u_i^2-\frac{1}{4}}+\mathcal{O}\left(\frac{1}{L}\right)\right\}. \tag{29}$$

We notice that for $M = 2$ there is no $\mathcal{O}\left(\frac{1}{L}\right)$ correction term, cf. eqn. (23).

6. Summary and conclusion

With the present investigations we have completed the analysis of the integrability structure of matrix product states of relevance for one-point functions in defect versions of (non-deformed) $\mathcal{N}=4$ SYM based on probe-brane set-ups with fluxes. The supersymmetric D3-D5 probe brane set-up lead to a matrix product state fulfilling the integrability criterion of [11] and a closed formula for all scalar one-point functions of the dCFT could be derived [5]. The two non-supersymmtric D3-D7 probe brane set-ups have different behaviours. In the SO(5) symmetric case the relevant matrix product state is integrable in the sense that it is annihilated by all the odd charges of the SO(6) spin chain but a closed expression for the one-point functions has so far not been found. For the D3-D7 set-up with SO(3)×SO(3) symmetry, studied here, we found that the matrix product state did not fulfil the proposed integrability criterion. We mention, however, that for this case, as for the D3-D5 probe brane set-up, we still have a complete match between one-point functions of chiral primaries computed in respectively string and gauge theory to two leading orders in a double scaling limit [27].[4] Generalizing the string computation to non-protected operators constitutes an interesting open problem in both models.

Despite the lack of integrability indicators we were able to extract from our data a closed formula for the large-L limit of the one-point functions of operators corresponding to Bethe eigenstates with paired roots. The study of the large-L limit of the overlaps was facilitated by the observation that only a subset of the permutations appearing in the expression for coordinate space Bethe eigenfunctions would contribute in this limit. This observation may prove useful for the study of the large-L limit of other similar overlap problems.

Finally, let us mention that while the considerations in the present paper are mainly relevant for tree-level one-point functions, the perturbative framework for calculating one-point functions at higher loop order for the here considered SO(3)×SO(3) symmetric defect version of $\mathcal{N}=4$ SYM was set up in [27], generalizing the ideas of [31,32]. In particular, using the framework of [27] it is possible to compute the one-loop correction to the one-point functions given in eqns. (23) and (24) and to test if tree-level and one-loop results are related via a simple flux factor as it was the case for the D3-D5 probe brane set-up [33].

Acknowledgements

We thank Asger Ipsen, Georgios Linardopoulos, Matthias Volk, Matthias Wilhelm and Konstantin Zarembo for useful discussions. Furthermore, we thank Jan Ambjørn for giving us access to his computer system. C.K. and K.E.V. were supported by DFF-FNU through grant number DFF-4002-00037. MdL was supported by SFI and the Royal Society for funding under grants UF160578 and RGF\EA\180167.

References

[1] M. de Leeuw, C. Kristjansen, K. Zarembo, One-point functions in defect CFT and integrability, J. High Energy Phys. 08 (2015) 098, https://doi.org/10.1007/JHEP08(2015)098, arXiv:1506.06958.
[2] I. Buhl-Mortensen, M. de Leeuw, C. Kristjansen, K. Zarembo, One-point functions in AdS/dCFT from matrix product states, J. High Energy Phys. 02 (2016) 052, https://doi.org/10.1007/JHEP02(2016)052, arXiv:1512.02532.
[3] M. de Leeuw, C. Kristjansen, S. Mori, AdS/dCFT one-point functions of the SU(3) sector, Phys. Lett. B 763 (2016) 197, https://doi.org/10.1016/j.physletb.2016.10.044, arXiv:1607.03123.
[4] M. de Leeuw, C. Kristjansen, G. Linardopoulos, One-point functions of non-protected operators in the SO(5) symmetric D3-D7 dCFT, J. Phys. A 50 (25) (2017) 254001, https://doi.org/10.1088/1751-8121/aa714b, arXiv:1612.06236.
[5] M. De Leeuw, C. Kristjansen, G. Linardopoulos, Scalar one-point functions and matrix product states of AdS/dCFT, Phys. Lett. B 781 (2018) 238–243, https://doi.org/10.1016/j.physletb.2018.03.083, arXiv:1802.01598.
[6] M. Mestyán, B. Bertini, L. Piroli, P. Calabrese, Exact solution for the quench dynamics of a nested integrable system, J. Stat. Mech. 1708 (8) (2017) 083103, https://doi.org/10.1088/1742-5468/aa7df0, arXiv:1705.00851.
[7] L. Piroli, E. Vernier, P. Calabrese, B. Pozsgay, Integrable quenches in nested spin chains I: the exact steady states, arXiv:1811.00432.
[8] L. Piroli, E. Vernier, P. Calabrese, B. Pozsgay, Integrable quenches in nested spin chains II: fusion of boundary transfer matrices, arXiv:1812.05330.
[9] M. Gaudin, Diagonalisation d'une Classe d'Hamiltoniens de Spin, J. Phys. France 37 (1976) 1087.
[10] V.E. Korepin, Calculation of norms of Bethe wave functions, Commun. Math. Phys. 86 (1982) 391.
[11] L. Piroli, B. Pozsgay, E. Vernier, What is an integrable quench?, Nucl. Phys. B 925 (2017) 362, https://doi.org/10.1016/j.nuclphysb.2017.10.012, arXiv:1709.04796.
[12] S. Ghoshal, A.B. Zamolodchikov, Boundary S-matrix and boundary state in two-dimensional integrable quantum field theory, Int. J. Mod. Phys. A 9 (1994) 3841, https://doi.org/10.1142/S0217751X94001552, Erratum: Int. J. Mod. Phys. A 9 (1994) 4353, arXiv:hep-th/9306002.
[13] B. Pozsgay, L. Piroli, E. Vernier, Integrable matrix product states from boundary integrability, arXiv:1812.11094.
[14] A. Karch, L. Randall, Open and closed string interpretation of SUSY CFT's on branes with boundaries, J. High Energy Phys. 06 (2001) 063, https://doi.org/10.1088/1126-6708/2001/06/063, arXiv:hep-th/0105132.
[15] D. Gaiotto, E. Witten, Supersymmetric boundary conditions in N=4 super Yang-Mills theory, J. Stat. Phys. 135 (2009) 789–855, https://doi.org/10.1007/s10955-009-9687-3, arXiv:0804.2902.
[16] N.R. Constable, R.C. Myers, O. Tafjord, The noncommutative bion core, Phys. Rev. D 61 (2000) 106009, https://doi.org/10.1103/PhysRevD.61.106009, arXiv:hep-th/9911136.
[17] N.R. Constable, R.C. Myers, O. Tafjord, Non-abelian brane intersections, J. High Energy Phys. 06 (2001) 023, https://doi.org/10.1088/1126-6708/2001/06/023, arXiv:hep-th/0102080.
[18] E. Widén, One-point functions in β-deformed $\mathcal{N}=4$ SYM with defect, J. High Energy Phys. 11 (2018) 114, https://doi.org/10.1007/JHEP11(2018)114, arXiv:1804.09514.
[19] R.C. Myers, M.C. Wapler, Transport properties of holographic defects, J. High Energy Phys. 12 (2008) 115, https://doi.org/10.1088/1126-6708/2008/12/115, arXiv:0811.0480.
[20] O. Bergman, N. Jokela, G. Lifschytz, M. Lippert, Quantum hall effect in a holographic model, J. High Energy Phys. 10 (2010) 063, https://doi.org/10.1007/JHEP10(2010)063, arXiv:1003.4965.
[21] C. Kristjansen, G.W. Semenoff, D. Young, Chiral primary one-point functions in the D3-D7 defect conformal field theory, J. High Energy Phys. 01 (2013) 117, https://doi.org/10.1007/JHEP01(2013)117, arXiv:1210.7015.
[22] Y. Jiang, S. Komatsu, E. Vescovi, Structure constants in $\mathcal{N}=4$ SYM at finite coupling as worldsheet g-function, arXiv:1906.07733.
[23] A. Bissi, C. Kristjansen, D. Young, K. Zoubos, Holographic three-point functions of giant gravitons, J. High Energy Phys. 06 (2011) 085, https://doi.org/10.1007/JHEP06(2011)085, arXiv:1103.4079.
[24] M. de Leeuw, A.C. Ipsen, C. Kristjansen, M. Wilhelm, Introduction to integrability and one-point functions in $\mathcal{N}=4$ SYM and its defect cousin, in: Les Houches Summer School: Integrability: From Statistical Systems to Gauge Theory les Houches, France, June 6–July 1, 2016, 2017, arXiv:1708.02525.
[25] M. de Leeuw, One-point functions in AdS/dCFT, J. Phys. A, Math. Theor. (2019), in press, arXiv:1908.03444.
[26] J.A. Minahan, K. Zarembo, The Bethe ansatz for $\mathcal{N}=4$ super Yang-Mills, J. High Energy Phys. 03 (2003) 013, https://doi.org/10.1088/1126-6708/2003/03/013, arXiv:hep-th/0212208.
[27] A.G. Grau, C. Kristjansen, M. Volk, M. Wilhelm, A quantum check of non-supersymmetric AdS/dCFT, J. High Energy Phys. 01 (2019) 007, https://doi.org/10.1007/JHEP01(2019)007, arXiv:1810.11463.
[28] K. Nagasaki, S. Yamaguchi, Expectation values of chiral primary operators in holographic interface CFT, Phys. Rev. D 86 (2012) 086004, https://doi.org/10.1103/PhysRevD.86.086004, arXiv:1205.1674.
[29] J. De Nardis, B. Wrouters, M. Brockmann, J.-S. Caux, Variational solution for the interaction quench in the Lieb-liniger Bose gas, arXiv:1308.4310v1.
[30] P. Le Doussal, P. Calabrese, The KPZ equation with flat initial condition and the directed polymer with one free end, J. Stat. Mech. (2012) 06001, https://doi.org/10.1088/1742-5468/2012/06/P06001, arXiv:1204.2607.

[4] The similar computation has not been carried out for the SO(5) symmetric set-up.

[31] I. Buhl-Mortensen, M. de Leeuw, A.C. Ipsen, C. Kristjansen, M. Wilhelm, One-loop one-point functions in gauge-gravity dualities with defects, Phys. Rev. Lett. 117 (23) (2016) 231603, https://doi.org/10.1103/PhysRevLett.117.231603, arXiv:1606.01886.
[32] I. Buhl-Mortensen, M. de Leeuw, A.C. Ipsen, C. Kristjansen, M. Wilhelm, A quantum check of AdS/dCFT, J. High Energy Phys. 01 (2017) 098, https://doi.org/10.1007/JHEP01(2017)098, arXiv:1611.04603.
[33] I. Buhl-Mortensen, M. de Leeuw, A.C. Ipsen, C. Kristjansen, M. Wilhelm, Asymptotic one-point functions in gauge-string duality with defects, Phys. Rev. Lett. 119 (26) (2017) 261604, https://doi.org/10.1103/PhysRevLett.119.261604, arXiv:1704.07386.