Rebalancing Unemployment Benefits in a Unionized Labour Market*

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Abstract

The basic trade union model is extended to allow for a more sophisticated unemployment benefit system consisting of two benefit levels, one for short-term and one for long-term unemployed, and a rule determining whether an unemployed is short- or long-term. The purpose of this extension is twofold; to get a more realistic analysis of the actual benefit systems in most countries, and to analyse alternative reforms to the traditional one of changing a uniform benefit level. Reforms that rebalance the benefit rates holding constant either expected utility of an unemployed, aggregate benefit expenditures, or aggregate utility of union members can reduce unemployment.

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1. Introduction

The most established theories today capable of explaining high and involuntary unemployment are those of trade unionism, efficiency wages, and equilibrium search behavior. *All* of these share the prediction that increases in unemployment benefits increase wages and unemployment (see e.g. Oswald 1985, Shapiro & Stiglitz 1984, and Pissarides 1990, respectively). Looking closer into the models, it becomes clear that 'unemployment benefits' are synonymous with a *constant* monetary compensation paid in each period of unemployment and for an *unlimited* duration. This is a quite crude treatment of the actual benefit systems in most countries. Looking at OECD countries, Atkinson & Micklewright (1991) concludes (among other things) that '*UI benefit is paid for a limited duration, and the rate of benefit may decline over time*' [Atkinson & Micklewright (1991) p. 1689]. Two problems might arise when making conclusions from the present models; *(i)* one might get wrong predictions, and *(ii)* one might overlook some possible ways of reforming the benefit systems. This paper concentrates on the second issue but gives also an example of the first. In doing so, we restrict ourselves to the theory of trade unionism. More specifically, we extend the simple monopoly union model of Dunlop (1944), to allow for a more general benefit system consisting of two benefit levels, one for short-term and one for long-term unemployed, and a rule determining whether an unemployed is classified as short- or long-term. For example, the rule might state that an unemployed is long-term if having experienced more than 7 months of unemployment during the last year.

Our first result shows, contrary to conventional wisdom, that increases in the benefit level for short-term unemployed may reduce wages and unemployment,[1] cf. *(i)* above. This may occur because incidence of long-term unemployment is increasing in unemployment itself creating an incentive for unions to reduce unemployment in order to move a larger fraction of unemployed to the short-term benefit level. This effect counteracts (and may overturn) the traditional incentive for wage pressure caused by the increased opportunity cost of employment.

Our main purpose is to analyse reforms that rebalance the benefit levels, cf.

[1] This possibility is well-known from theories of voluntary unemployment. Using 'partial-partial' search models, Mortensen (1977) and Burdett (1979) show that limited benefit duration has important effects on job search incentives implying that; (a) the escape rate from unemployment is increasing towards benefits exhaustion, an effect documented empirically by e.g. Katz & Meyer (1990); (b) a rise in benefits may reduce duration of unemployment because those not currently eligible for benefits have increased incentives for job search. Such individual incentive effects are beyond the scope of this paper.

(ii) above. When doing so, one has to keep something fixed. From a political point of view, there may be many different targets to fix depending on whether policy makers are most concerned with income distribution, government budget, or large groups of voters. Therefore, we look at three possible targets: expected income of an unemployed, aggregate benefit expenditures, and aggregate utility of union members. Independently of which target is kept fixed, our results show that it is possible to reduce wages and unemployment through a rebalancing of the benefit rates that increases the rate of short-term and reduces the rate of long-term unemployed. Numerical exercises based on a simple estimation on Danish data suggest that the wage effect that can be obtained from a (standard) 1% reduction of a uniform benefit rate may be achieved instead by increasing the rate of short-term unemployed by approximately 2%, reducing the rate of long-term by 2%, and using a rule stating that unemployed receive the low rate after 7 months of unemployment. This occurs although the two groups of unemployed are of equal size. Thus, potential efficiency gains from rebalancing the benefit system may have been overlooked in the standard theory.

The paper is organized as follows. Section 2 embeds the generalized unemployment benefit system into a simple monopoly union model. Section 3 derives the result of changing the benefit level of short-term unemployed and states the main theorem of the paper concerning the rebalancing of benefit rates. An increasing relationship between incidence of long-term unemployment and the unemployment rate itself is crucial for the results. Section 4 provides theoretical and empirical evidence for this relationship. Section 5 uses estimates from the previous section to give an impression of the magnitudes of the effects derived in section 3. Section 6 contains concluding remarks.

2. A Simple, Unionized Labour Market

We consider a specific labour market with many identical workers all organized in a trade union. Each worker supplies inelastically one unit of labour in each period. The number of workers and thus the total per period labour supply is normalized to one. Firms demand labour in each period according to a downward sloping demand curve $L(w)$. The time unit is an "employment period", the shortest possible employment spell from hiring to firing, say a week. The (nominal) wage, w, is set unilaterally by the trade union whereas aggregate employment is determined entirely by the firms. The objective of the trade union is to maximize the ex ante expected utility of a representative member or, identically in our setting (see Oswald 1985, 1987), the aggregate utility of the members. Assuming that each individual member's indirect utility function is linear in income (risk neutrality), and that the union has only negligible influence on the general prices at which its members buy goods, the trade union should maximize the per period expected income of a representative member.[2]

In the standard union model it is assumed that all unemployed get the same per period benefit b independently of their unemployment record. This yields an objective function of the union equal to $(1 - u(w))\, w + u(w)\, b$, where $u(w) \equiv 1 - L(w)$. Maximizing this gives the standard result: The wage rate is a simple mark-up over the opportunity cost of employment, i.e. over b. An increase in the benefit level increases unambiguously wages and unemployment.

We consider a more general unemployment benefit system consisting of: a) A *rule* that decides whether an unemployed is short-term or long-term unemployed: A worker who is unemployed in a specific period is classified as long-term if having experienced j or fewer employment periods within the last m preceding periods (that is, $m - j$ or more unemployment periods). b) *Two rates* of unemployment benefit b_1 and b_2 for short-term and long-term unemployed, respectively, where we assume everywhere that $b_1 \geq b_2$. The standard case corresponds to $b_1 = b_2 = b$, where the rule does not matter.

It is of importance for the union how a specific level of unemployment divides into short-term and long-term. We make three assumptions on this division which are justified theoretically as well as empirically in Section 4. First, after a given number of periods with a constant unemployment rate, the long term fraction of unemployed reach a certain level and stays constant as long as the overall unemployment is unchanged. This fraction is the *long-term incidence of unemployment* denoted by the relationship $\phi(u)$. It follows that union members will divide into $1 - u$ employed, $u(1 - \phi(u))$ short-term unemployed, and $u\phi(u)$ long-term unemployed in steady state. Our second assumption is that these fractions are also the long run probabilities for any individual union member of being employed, short-term unemployed, or long-term unemployed, respectively. That is, for any individual worker the best prediction of his status in a period far from now is that he will be employed, short-term unemployed, or long-term unemployed with exactly these probabilities. The third and main assumption is that ϕ is increasing

[2] Since we are concerned with a split of benefit rates which increases risk, the assumption of risk neutrality does not appear as innocent as in the standard model with just one benefit rate. However, we demonstrate that our results are almost unaffected if risk aversion is assumed. Thus, for simplicity we assume risk neutrality in the main exposition.

in u.

We assume that the union is "long sighted" (not discounting the future much) and therefore use the just described long run probabilities in the determination of the representative member's expected utility or income. Thus, the trade union sets w to maximize,

$$\Omega(w, b_1, b_2) \equiv (1 - u(w))\, w + u(w)\left[(1 - \phi(u(w)))\, b_1 + \phi(u(w))\, b_2\right]. \tag{2.1}$$

The square bracket is the (long run) expected income of a union member conditional on being unemployed, or simply "the utility of an unemployed" (as far as the union is concerned). The first order condition, $\Omega_w(w^*, b_1, b_2) = 0$, gives,

$$w^* = \frac{b_1 - \phi(u(w^*))\left[1 + \eta(u(w^*))\right][b_1 - b_2]}{1 - 1/\varepsilon(w^*)}, \tag{2.2}$$

where $\varepsilon(w) \equiv \frac{u'(w)w}{1-u(w)} = -\frac{L'(w)w}{L(w)}$ is the (numerical) wage-elasticity of labour demand and $\eta(u) \equiv \frac{\phi'(u)u}{\phi(u)}$ is the elasticity of the incidence function. We assume that $\varepsilon(w)$ is everywhere larger than one, that the above numerator is positive (which is fulfilled if the difference between b_1 and b_2 is not too large), that the second order condition $\Omega_{ww}(w^*, b_1, b_2) < 0$ is fulfilled (this holds, e.g., when both $L(w)$ and $\phi(u)$ are iso-elastic), and that the above formula determines the optimal wage rate w^* uniquely. The standard result appears when $b_1 = b_2 = b$ revealing that lower b means lower w and thus u. However, the 'average' benefit level is no longer the only policy parameter.

3. Rebalancing Unemployment Benefits

Our interest is in structural reforms which, at an appropriately chosen short-term/long-term rule, rebalance the two rates b_1 and b_2 in a way that keeps some aggregate measure like total benefit expenditures or utility of an unemployed fixed.

It will illuminate the basic incentive effects at work first to consider the simple, non-structural policy experiment of increasing b_1 leaving everything else unchanged, an unambiguous improvement for the unemployed. The formula for w^*, (2.2) above, shows that w^* will fall as b_1 increases, if $1 < \phi(u(w^*))(1 + \eta(u(w^*)))$ or stated differently,[3]

$$\frac{1 - \phi(u^*)}{u^*} < \frac{\phi(u^*)\eta(u^*)}{u^*} = \phi'(u^*). \tag{3.1}$$

[3] This argument uses the second order condition. The total effect of a change in b_1 is determined by the first order condition $\Omega_w(w^*, b_1, b_2) = 0$ and the Implicit Function Theorem

It may be surprising that an improvement in the conditions of the unemployed may imply lower wages and unemployment in an otherwise rather standard union model. There are, however, two opposite incentive effects involved in an increase in b_1: *(i)* It disturbs the balance between employed and unemployed in favour of the latter to which the union unambiguously responds by increasing w in accordance with the standard result. The size of this effect is proportional to the left hand side of the above condition expressing how heavily the short term unemployed, now getting higher benefits, weigh in total unemployment. *(ii)* It disturbs the balance between short-term and long-term unemployed in favour of the first group to which the union responds by attempting to push workers from long-term to short-term unemployment. Since ϕ is increasing in u, this can only be done by lowering w and hence u; the right hand side of the above condition measures the strength of this effect since it indicates *how* increasing ϕ is in u.

Although the condition (3.1) is unlikely to be fulfilled (see Section 5), the above indicates that the effects of changing benefits may be far less pronounced if what is changed is a temporary benefit level b_1 rather than an ever lasting b, as normally presumed in theoretical models. Since most countries have an end to the unemployment benefit period this could be a reason why it has been hard empirically to document large significant effects of changing unemployment benefits.[4] Note, that an increase in b_2 leads unambiguously to an increase in w^*.

The main purpose is to analyse structural reforms of the unemployment benefit system that rebalance the benefit levels b_1 and b_2 keeping fixed either expected utility of an unemployed union member,

$$(1-\phi\left(u\left(w^*\right)\right)) b_1+\phi\left(u\left(w^*\right)\right) b_2=\bar{b}, \tag{3.2}$$

or total aggregate expenditure on unemployment benefits,

$$u(w^*)\left[(1-\phi\left(u\left(w^*\right)\right)) b_1+\phi\left(u\left(w^*\right)\right) b_2\right]=\overline{B}, \tag{3.3}$$

or expected utility of a union member,

$$(1-u\left(w^*\right)) w^*+u\left(w^*\right)\left[(1-\phi\left(u\left(w^*\right)\right)) b_1+\phi\left(u\left(w^*\right)\right) b_2\right]=\overline{\Omega}. \tag{3.4}$$

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giving,

$$\frac{dw^*}{db_1}=-\frac{\Omega_{wb_1}\left(w^*,b_1,b_2\right)}{\Omega_{ww}\left(w^*,b_1,b_2\right)}=-u'\left(w^*\right)\frac{1-\phi\left(u\left(w^*\right)\right)\left(1+\eta\left(u\left(w^*\right)\right)\right)}{\Omega_{ww}\left(w^*,b_1,b_2\right)},$$

which is negative exactly under the stated condition because $\Omega_{ww}<0$.

[4] Atkinson & Micklewright (1991) notes that empirical studies from UK and US indicate that a 10 percentage point increase in the replacement ratio (ratio of benefits to earnings in work) will increase average duration of unemployment by only about one week

Note, that the two first criteria amount to holding certain parts of $\Omega(w^*, b_1, b_2)$ fixed. The policy experiment is to increase b_1 (marginally) and adjust b_2 to fulfill one of the requirements. One has to decide whether these should hold *1)* before adjustment to a new equilibrium (at the old wage and unemployment rate), or *2)* when comparing the old and the new equilibrium. The second case, taking into account all relevant feed-back effects, is definitely of greatest interest and the subject of Theorem 1 below. However, we start with the first case to display the direct incentive effects at work. Since the requirements are then considered at the initial values of w^* and $u^* \equiv u(w^*)$, all three of them imply that b_2 must be decreased as b_1 is increased according to the easily interpretable equation,

$$\frac{db_2}{db_1} = -\frac{1-\phi(u^*)}{\phi(u^*)}. \tag{3.5}$$

Since w^* is determined by $\Omega_w(w^*, b_1, b_2) = 0$, the Implicit Function Theorem and (3.5) yields,

$$\frac{dw^*}{db_1} = -\frac{\Omega_{wb_1}(w^*, b_1, b_2) + \Omega_{wb_2}(w^*, b_1, b_2)\frac{db_2}{db_1}}{\Omega_{ww}(w^*, b_1, b_2)} = \frac{u'(w^*)\,\eta(u^*)}{\Omega_{ww}(w^*, b_1, b_2)} < 0.$$

Thus, if b_1 is increased and b_2 reduced to keep either of the aggregate measures fixed, then the response of the union is to reduce its wage claim. This is best understood by comparing the experiment to the simple one of changing b_1 alone. The only additional element is the reduction in b_2 which exactly eliminates the effect *(i)* of the simple experiment; the trade-off between employed and unemployed is unaffected since the utility of an unemployed is unchanged. The balance between short-term and long-term unemployed is, however, changed in favour of the first leaving only effect *(ii)* from the simple experiment. Therefore the total incentive effect goes unambiguously in the direction of lower wages and unemployment. The size of the effect depends on the elasticity $\eta(u^*)$.

Imposing one of the requirements to hold when comparing equilibria adds some feed-back effects. In the case of fixed expected income of an unemployed, it must be taken into account that the direct effect increases the fraction of unemployed getting the high benefit level making a further reduction in b_2 necessary. This further reduces the wage rate etc. Thus, the total effect is larger than the direct effect. In the case of fixed expenditures on benefits the feed-back effects may go in either direction depending on whether the direct reduction in unemployment reduces or increases expenditure: The reduction in unemployment per se tends to reduce the outlays, but simultaneously a larger fraction of unemployed moves to the high benefit level which tends to increase expenditures. If the net effect is an increase in expenditures then the feed-back effects enforce the result further (and for the same reason as before), whereas a decrease in expenditures results in feed-back effects that dampen the direct effect. For the case of fixed worker welfare the feed-back effect on the requirement (3.4) equals $\Omega_w(w^*, b_1, b_2)\frac{dw^*}{db_1}$. Since $\Omega_w = 0$, there is no feed-back effect in this case and the total effect is therefore identical to the direct effect (this is just an application of the Envelope Theorem). The overall conclusion is that the considered reform reduces unemployment in all cases,

Theorem 3.1. *Under the stated assumptions, a rebalancing of unemployment benefits that increases the benefit level of short-term unemployed and adjusts the benefit level of long-term unemployed (downwards) to keep fixed either* **a)** *the long run expected income of an unemployed, or* **b)** *aggregate expenditures on benefits, or* **c)** *the long run welfare of a union member, will reduce wages and unemployment provided that the equilibrium is stable.*

Proof. Combining $\Omega_w(w^*, b_1, b_2) = 0$ with (3.2), (3.3), and (3.4), respectively, and using Cramer's rule yield the derivative,

$$\frac{dw^*}{db_1} = -\frac{\Omega_{wb_1}(w^*, b_1, b_2)\,\phi(u(w^*)) - (1-\phi(u(w^*)))\,\Omega_{wb_2}(w^*, b_1, b_2)}{\phi(u(w^*))\,D}$$

$\Rightarrow$

$$\frac{dw^*}{db_1} = \frac{u'(w^*)\,\eta(u(w^*))}{D},$$

where D equals,

$$D = \Omega_{ww}(w^*, b_1, b_2) + \frac{u'(w^*)}{u(w^*)}\eta(u(w^*))(b_1 - b_2)\,\Omega_{wb_2}(w^*, b_1, b_2),$$

in case (3.2),

$$D = \Omega_{ww}(w^*, b_1, b_2) + \frac{u'(w^*)}{u(w^*)}\left([\eta(u(w^*)) + 1](b_1 - b_2) - \frac{b_1}{\phi(u(w^*))}\right)\Omega_{wb_2}(w^*, b_1, b_2),$$

in case (3.3), and

$$D = \Omega_{ww}(w^*, b_1, b_2),$$

in case (3.4). Thus $\frac{dw^*}{db_1}$ is negative if D is negative which is required for stability.[5] The downward sloping labour demand curve implies that $\frac{du^*}{db_1}$ is negative. ■

Theorem 3.1 only reports on wage and unemployment effects, not on welfare. We have, however, the following,

Corollary 3.2. *It is possible to find reforms that rebalance benefit levels such that welfare of union members increase and total expenditure on benefits decrease.*

Proof. To keep benefit expenditures fixed amounts to holding the second term in $\Omega = (1 - u(w^*))\, w^* + u(w^*)\, [(1 - \phi(u(w^*)))b_1 + \phi(u(w^*))b_2]$ fixed; Ω is then only affected by changes in the term $(1 - u(w^*))\, w^*$. This increases when w decreases as labour demand is elastic, $\varepsilon(w^*) > 1$. Therefore, Ω increases. By duality the reform that keeps Ω fixed involves lower expenditures. Obviously, it is possible to find reforms in between these extremes both increasing Ω and reducing expenditures. ■

Appendix A.1 demonstrates that the theorem above holds without any changes if it is assumed that union members are risk averse (having indirect utility functions which are concave rather than linear in income). The corollary holds, of course, if workers are not too risk averse, but the Appendix A.1 also shows that it holds for any amount of risk aversion if the initial equilibrium is characterized by a uniform benefit level.

4. The Incidence of Long Term Unemployment

It is an implicit assumption behind the above results that the union cannot manipulate directly who gets unemployed and thus how unemployment is divided into short-term and long-term. We find this realistic when the rule distinguishing between short- and long-term unemployment is "sluggish", that is, of the form "an unemployed is long-term unemployed if he had less than 26 weeks of employment during the forgoing 52 weeks", rather than "an unemployed is long-term unemployed after 26 consecutive weeks of unemployment". Note the difference: both

[5] Note, that one of the requirements together with the wage equation yield two reaction functions in the (w, b_2) space. The condition for stability of an initial (Nash) equilibrium yields and additional constraint on the difference between b_1 and b_2 in case **a)**. The assumptions stated under equation (2.2) ensure that the initial equilibrium is stable in case **b)** and **c)**.

rules imply that an unemployed ends in the long-term category after 26 consecutive weeks of unemployment, but if one has been unemployed for a long time, say 52 weeks, then according to the second rule one week of employment is enough to get back into the short-term category, whereas according to the first rule it takes 26 weeks of employment. Thus, the first rule is much less manipulable.

The main explicit assumptions are on the incidence function ϕ: We have assumed *(i)* that after a number of periods with unemployment rate u, the steady state fraction $\phi(u)$ of long-term unemployment in total unemployment is indeed established, *(ii)* that the terms $(1-u)$, $u(1-\phi(u))$, and $u\phi(u)$ are not only the steady state fractions of employment, short-term unemployment, and long-term unemployment respectively, but also the individual union member's long run probabilities of ending in either of the three categories, and finally and most crucial *(iii)* that ϕ is increasing in u. This section justifies these assumptions theoretically and provides some empirical evidence in favour of the last crucial assumption.

4.1. Theoretical Evidence

The function ϕ depends on the specific *rule* according to which an unemployed is considered long- or short-term unemployed, and on the underlying *unemployment dynamics*. The rules we consider are of the form: To receive unemployment benefit in a considered period one must be unemployed in that period. If, during the m preceding periods, one was employed in j or fewer (unemployed in $m-j$ or more) periods, then one is long term unemployed and receives b_2 in the considered period, otherwise one is short term unemployed and receives b_1. Note, that the special case $j=0$ corresponds to the simple rule that an unemployed is long-term if having experienced m or more consecutive periods of unemployment. To bear in mind the rule dependence what was formerly called $\phi(u)$ is now called $\phi(m,j,u)$.

In what follows assume that the rate of unemployment is constantly $u>0$. To understand the importance of the dynamics of unemployment shares consider first the standard case where every worker has in all periods independently probability u of becoming unemployed. For any worker then the probability of having exactly j employment periods out of m is just the binomial probability of j successes in m trials when the success probability is $1-u$ independently in all trials. The probability of j or fewer employment periods out of m is then $B(m,j,1-u)$, where B is the cumulative distribution function for the binomial distribution. Consider a specific period. For any worker, employed or unemployed in that period, the event of j or fewer employment periods in the m succeeding periods has probability

$B(m, j, 1-u)$. Since there are many workers there are also many unemployed and it follows from the law of large numbers that the fraction $B(m, j, 1-u)$ of these will be long-term unemployed, so $\phi(m, j, u) = B(m, j, 1-u)$. Note first, that the division $\phi(m, j, u)$ of unemployment in long term/short term is indeed established no later than after m periods (in period $m+1$ from now) with a constant unemployment rate, u. Second, it is straightforward that any worker will have probability $1-u$ of being employed, $u\phi(u)$ of being long-term unemployed, and $u(1-\phi(u))$ of being short-term unemployed in period $(m+1)$ from now, independently of the present employment record of the worker. Third, ϕ is increasing in u, since the binomial cumulative distribution function is decreasing in the independent success probability.

In the real world the unemployment risk of an already employed worker is less than that of an already unemployed (see e.g. Layard et.al. 1991 p. 226). Assume therefore that the probability of becoming unemployed in a period is $\alpha(u)$ for a worker who was employed the period before, and $\beta(u)$ for one who was unemployed. Our basic assumptions are that, $0 < \alpha(u) \leq u \leq \beta(u) < 1$, and that $\alpha(\cdot)$ and $\beta(\cdot)$ are strictly increasing in u. These assumptions are realistic but still contain the independent (binomial) special case.[6] In the more general case, it is not obvious that $\phi(m, j, u)$ is increasing in u due to the dependence between periods

In a steady state, flow out of unemployment $(1-\beta(u))\, u$ equals flow into unemployment $(1-u)\, \alpha(u)$ yielding the following relationship,

$$\frac{1-u}{u} = \frac{1-\beta(u)}{\alpha(u)}. \tag{4.1}$$

For illustration we fix $m = 2$, and compute ϕ for the two relevant cases, $j = 0$ and $j = 1$. For this purpose, call the considered period for which we want to compute the incidence of long term unemployment number 3, call the two preceding periods which are decisive for the long term/short term distinction numbers 2 and 1, and the one before that number 0. It is assumed that unemployment has been u ever since period 0. First, let $j = 0$, so we are interested in which fraction of the unemployed in period 3 who were also unemployed in both of periods 1 and 2. Go back to period 0. Here $1-u$ workers are employed each having probability

[6] We could have considered further backward looking unemployment dynamics where the unemployment risk depends on even earlier periods. We abstain from this for simplicity and since empirical research points to that the big impact on unemployment risk comes from recent experience whereas earlier periods are less important, see e.g. Layard et.al. (1991) p. 226.

$\alpha\beta^2$ of getting unemployed in all of the periods 1-3 (α for period 1, and β for each of 2 and 3), while u are unemployed each having probability β^3 of unemployment in periods 1-3. So, a total of $(1-u)\alpha\beta^2 + u\beta^3$ workers are unemployed in all of the periods 1-3. Another expression for this is $u\phi(2,0,u)$, where by multiplying ϕ with u, one goes from measuring as a fraction of the unemployed to measuring as a fraction of *all* workers. So, $u\phi(2,0,u) = (1-u)\alpha(u)\beta^2(u) + u\beta^3(u)$. Dividing on both sides by u, and using (4.1), gives $\phi(2,0,u) = \beta^3(u)$. Since β is strictly increasing in u, so is $\phi(2,0,u)$. To compute $\phi(2,1,u)$ one proceeds the same way starting from either employment or unemployment in period 0, now adding probabilities up over all the lapses that involve one or zero employment periods among periods 1-2, and unemployment in period 3. There are three such lapses and one gets $u\phi(2,1,u) = (1-u)\left[\alpha\beta^2 + (1-\alpha)\alpha\beta + \alpha(1-\beta)\alpha\right] + u\left[\beta^3 + (1-\beta)\alpha\beta + \beta(1-\beta)\alpha\right]$, and then $\phi(2,1,u) = \alpha(u) + \beta(u) - \alpha(u)\beta(u)$. Again, this is increasing in u.

Note from above that the division $\phi(2,j,u)$ of unemployment in long term/short term is established after three periods with unemployment rate u. Under the considered unemployment dynamics, $\phi(m,j,u)$ is established no later than after $(m+1)$ periods (in period $m+2$ from now) with constant unemployment rate u. It is also general that ϕ is increasing in u,

Proposition 4.1. *Under the assumptions mentioned above, the incidence of long term unemployment is strictly increasing in the unemployment rate. Formally: If $\beta(u) > \alpha(u)$, $\alpha'(u) > 0$, and $\beta'(u) > 0$ for all u, then $\phi(m,j,u)$ is strictly increasing in u, independently of m and j $(j \leq m-1)$.*

Proof. : See Appendix A.2.

Finally, since the Markov chain defined by the transition probabilities α and β is irreducible and aperiodic, $1-u$, $u(1-\phi(u))$, and $u\phi(u)$ are also the long run probabilities of an individual worker of ending up in a specific period in either of the categories employed, short-term unemployed, or long-term unemployed.

4.2. Empirical Evidence

Some existing evidence suggests that the incidence of long-term unemployment is increasing in the rate of unemployment. For example, the OECD concludes that '*In general, high-unemployment countries such as Ireland, Italy and Spain typically had the highest incidence of LTU in 1989, with over half of the unemployed* *made up of the long-term unemployed, and low-unemployment countries such as Norway, Finland and Sweden had the lowest.*' [OECD 1992, p.67] where LTU denotes long-term unemployed.[7]

We look at the function $\phi(u)$ for Denmark when the rule states that an unemployed is classified as long-term if having experienced less than x per cent of employment during the last year where $x \in \{10, 20, 30, 40, 50\}$. Let u_t be the annual unemployment rate and let y_t be the fraction of unemployment during a year carried by persons employed less than x per cent of the year; u_t is reported by Statistics Denmark whereas y_t is calculated for the period 1979 to 1996 using data from Statistics Denmark (see Appendix A.3). In Figure 1 we have plotted y_t as function of u_t for each of the five rules described above.

(Figure 1 here)

Figure 1 reveals a clear positive relationship between incidence of long-term unemployment and the unemployment rate independently of which of the five rules is considered (correlation-coefficients are displayed to the right of the curves).[8] Unfortunately, y_t differs from the theoretical $\phi(u)$, since the latter is the steady state incidence after a certain number of periods with a constant u. However, this problem might not be crucial as the data represents average values of 52 periods.

For the numerical exercises in the next section, we will need an estimate of the elasticity η, the percentage change in the incidence of long-term unemployment caused by a one per cent change in the unemployment rate, which is important for the size of the effects reported in Section 3. It is beyond the scope of this paper (relying on a relative small sample) to estimate a long run relationship of $\phi(u)$ in a fully dynamic model of y_t. Instead, we will just rely on a rough estimate of η by simple OLS estimations starting from the simple iso-elastic relationship,

$$y_t = \delta_1 u_t^{\delta_2} \zeta_t, \tag{4.2}$$

where δ_1 and δ_2 are parameters and ζ_t an error term. A direct log-linear estimation gives significant and relative large values of δ_2 but unsatisfactory residuals also when including time trend and lagged values of the variables. Therefore, we have

[7] OECD defines LTU as persons unemployed 12 months or more.

[8] One may note that the rules $x \in \{60, 70, 80, 90\}$ show a similar positive relationship. Obviously, this is not the case for $x = 100$ where $y_t = 1\ \forall t$.

chosen to report the more conservative estimates based on an estimation in log-differences,

$$\log y_t - \log y_{t-1} = \underset{0}{\omega_1} + \underset{+}{\omega_2} (\log u_t - \log u_{t-1}) + \xi_t, \tag{4.3}$$

where the new error term ξ_t equals $\log \zeta_t - \log \zeta_{t-1}$, the coefficient ω_2 is our estimate of the elasticity η, and the labels below the coefficients indicate the expected value and sign, respectively. The results of the estimations are given in Table 1.

Table 1: Estimation of (4.3).

	$x = 10$	$x = 20$	$x = 30$	$x = 40$	$x = 50$
ω_1	0.01 (0.44)	0.00 (0.20)	0.00 (0.05)	0.00 (-0.01)	-0.00 (-0.11)
ω_2	0.57 (3.77)	0.64 (6.04)	0.59 (7.26)	0.50 (8.23)	0.42 (9.30)
R^2	0,49	0,71	0.78	0.82	0.85
Std. Err.	0.0710	0.0494	0.0378	0.0285	0.0212
AR(1)	0.15 (0.70)	0.00 (1.00)	0.01 (0.94)	0.38 (0.54)	0.94 (0.33)
AR(2)	1.12 (0.57)	1.64 (0.44)	1.56 (0.46)	1.11 (0.57)	1.53 (0.47)
Normality	3.55 (0.17)	4.01 (0.13)	4.72 (0.09)	4.61 (0.10)	4.51 (0.11)
ARCH 1	0.58 (0.46)	0.33 (0.58)	0.01 (0.91)	0.10 (0.75)	0.21 (0.66)

Note: Parentheses after estimated parameters are t-values. AR(1) and AR(2) are χ^2-tests for autocorrelated residuals, Normality is a $\chi^2(2)$-test for normally distributed residuals, and ARCH is a F-test for autoregressive conditional heteroscedasticity. Parentheses after dianostics are p-values.

Table 1 confirms the presumption concerning the two parameters; ω_1 is close to zero and insignificant for each rule whereas ω_2 is positive and significant. We will take this as rough evidence for an elasticity η in the interval $[0.42, 0.64]$.[9]

[9] A direct log-linear estimation of (4.2) yields significant estimates in the range $[0.41, 0.76]$. A log-linear estimation including a significant time trend yields significant estimates in the range $[0.38, 0.52]$. Both of these estimations are, however, outperformed by (4.3). Trend and lagged values become insignificant if included in this estimation. Graphs in the Appendix show that the last 3 observations might contribute a lot to the estimates. An estimation of (4.3) excluding these observations yields estimates in the range $[0.38, 0.54]$.

5. Some Numerical Exercises

We now use the estimated elasticities in Table 1 to provide an impression of the possible magnitudes of the effects presented in Section 3. From an initial situation where $b_1 = b_2 = b$, we compare the wage effect from the standard experiment of decreasing the common b with one percent to that of rebalancing b_1 and b_2 at alternative short-term/long-term rules. We are interested in how large a fraction of the wage reducing effect from the standard experiment is obtained by the second type of experiments.

We assume that both the labour demand curve L, and the incidence function ϕ are iso-elastic so that the formula (2.2) for the optimal wage w^* reduces to,

$$w^* = \frac{b_1 - \phi(u(w^*))(1+\eta)(b_1 - b_2)}{1 - \frac{1}{\varepsilon}}. \tag{5.1}$$

To do the calculations below one needs in principle the value of $\phi(u(w^*))$ for the particular rule considered. The estimation in Table 1 did not determine the value of δ_1; thus ϕ is not fully determined. However, for each year the value of ϕ is approximately y_t in the data. In the following we take 1990 as a base year. This is chosen for two reasons: First, both with respect to the rate of unemployment and with respect to the incidence of long-term unemployment year 1990 is in the middle of the sample at any of the rules $x \in \{10, 20, 30, 40, 50\}$ (see Figure 1). Second, in Denmark it was still possible in 1990 to obtain the same level of unemployment benefit for around 10 years which is close to a system of a uniform rate for all unemployed (reforms in the nineties changed that). This is important when comparing reforms that split unemployment benefits with the traditional exercise of changing a common b. In the following exercises, we simply substitute y_{1990} for $\phi(u(w^*))$ for the rule under consideration (see Table A2 in Appendix).

First we consider again the simple experiment of increasing the benefit rate b_1 of the short-term unemployed leaving everything else unchanged. Starting from an initial situation of $b_1 = b_2$, the elasticity in w^* wrt. b_1 is computed from (5.1) above,[10]

$$\rho_{w^* b_1} = \frac{dw^*/w^*}{db_1/b_1} = 1 - \phi(u(w^*))(1+\eta), \tag{5.2}$$

[10] Considering w^* as a function of b_1, $w(b_1)$, implicit differentiation of (5.1) gives $w' = \left[1 - (1+\eta)(\phi' u' w'(b_1 - b_2) + \phi\right] / (1 - \frac{1}{\varepsilon})$. Measuring at $b_1 = b_2$, and multiplying by $\frac{b_1}{w^*}$ on both sides give (5.2).

which is less than the standard elasticity of one. The condition $1-\phi(1+\eta)<0$ for an increase in b_1 to imply a lower wage is violated for all of the rules considered, e.g. for $x=20$ the condition states $1<0.32(1+0.64)$. Thus, increases in unemployment benefits of the short-term unemployed do increase wages and unemployment. However, the effect may well be rather small as indicated by Table 2, obtained by inserting y_{1990} and estimates of η corresponding to the different rules,

Table 2: Wage elasticity with respect to benefit of short-term unemployed.

x :	10	20	30	40	50	s
$\rho_{w^*b_1}$:	0.65	0.47	0.34	0.21	0.11	1

The last column displays the (s)tandard one-to-one effect. The first column shows that wages rise by only 0.65% (65% of the standard effect), following a one per cent increase in the short-term benefit level if short-term unemployed are those who have been employed more than 10% during a year. Moving to the right in the Table reveals that the elasticity declines rapidly as the group entitled to the short-term benefit rate is reduced; e.g. for $x=50$ the wage response is only 11% of the standard effect, although the group entitled to the raise in benefits accounts for 38% of unemployment.

We now turn to structural reforms that rebalance the benefit rates keeping worker welfare Ω fixed. As demonstrated in Section 3, this type of reform involves no feed-back effects. So, when we compute the wage response from a change in one of the benefit rates we can simply proceed as follows: The reaction in the other benefit rate is given by $\frac{db_2}{db_1}=-\frac{1-\phi}{\phi}$, cf. (3.5), and the effect on w^* is then computed from (5.1) taking into account this reaction.[11] It is easiest first to compute the partial elasticity of w^* wrt. a change in b_2. Just as (5.2) was derived, we get (using $b_2/b_1=1$),

$$\rho_{w^*b_2}=\phi(u(w^*))(1+\eta). \tag{5.3}$$

Now, compare a one per cent reduction in the common benefit level to a rebalancing that reduces the benefit level b_2 of long-term unemployed also by one per cent and adjust b_1 upwards accordingly. Thus $db_1=-\frac{\phi}{1-\phi}db_2$, or $\frac{db_1/b_1}{db_2/b_2}=-\frac{\phi}{1-\phi}$,

[11] From the proof of Theorem 3.1, it follows that from an initial situation of $b_1=b_2$, the wage effect of a reform that keeps Ω fixed is the same as the wage effect of a reform that keeps utility of an unemployed fixed. So, the computations to follow cover both types of reform.

where we have used $b_2/b_1 = 1$. The total relative change in the wage rate from this exercise is,

$$\rho_{w^*b_2} + \rho_{w^*b_1}\frac{db_1/b_1}{db_2/b_2} = \frac{\phi(u(w^*))\eta}{1-\phi(u(w^*))},$$

from which we compute,

Table 3: Wage effects of rebalancing benefits when b_2 is decreased by one per cent.

$x:$	10	20	30	40	50	s
dw^*/w^*	-0.16	-0.30	-0.43	-0.55	-0.69	-1

The last column displays the one per cent reduction in the wage of a one per cent reduction of the common benefit level. The other columns show that it is possible to achieve $16\%, 30\%$, and so fourth (depending on the rule) of this effect by reducing only the benefit rate of long-term unemployed by the same amount (1%), and at the same time increase the benefit rate of short-term unemployed to keep overall worker welfare unchanged. Thus, rebalancing of benefits is clearly an interesting alternative to the standard reduction of all benefit rates.

Our last experiment asks how much the benefit rates have to be adjusted in order to obtain a one per cent reduction in the wage rate and still fulfill the requirement of unchanged worker welfare. So, we require that, $-1 = \rho_{w^*b_1}\frac{db_1}{b_1} + \rho_{w^*b_2}\frac{db_2}{b_2}$. If we write db_2/b_2 as $\frac{db_2}{db_1}\frac{db_1}{b_1}\frac{b_1}{b_2}$, use $b_1/b_2 = 1$, and insert from above this gives, $-1 = -\eta\frac{db_1}{b_1}$, or,

$$\frac{db_1}{b_1} = \frac{1}{\eta} \quad \text{, and hence} \quad \frac{db_2}{b_2} = -\frac{1-\phi(u(w^*))}{\phi(u(w^*))\eta}.$$

Table 4 follows by inserting ϕ and η according to the considered reform,

Table 4: Percentage change needed to obtain a 1% change in the wage through rebalancing.

$x:$	10	20	30	40	50	s
db_1/b_1	1.8	1.6	1.7	2.0	2.4	-1
db_2/b_2	-6.1	-3.3	-2.4	-1.8	-1.4	-1

It follows that a one per cent wage reduction that leaves worker welfare unchanged can be obtained in many ways depending on which rule is used to classify unemployed as long-term. For instance, it may be achieved by reducing the rate of long-term by 1.8%, and increasing the rate of short-term by 2.0%, if the rule is $x = 40$ where long-term account for approximately half of the unemployment. This is an attractive alternative, we believe, to the 1% reduction of *both* benefit levels required in the standard model to obtain the same beneficial effect on wage and unemployment.

6. Concluding Remarks

The structure of benefit systems has not received much attention in the large body of literature on trade unions (see e.g. Farber 1986 and Booth 1995). Some papers have analysed the incentive effects of altering the financing of unemployment benefits (e.g. Holmlund & Lundborg 1988, 1989) but, to our knowledge, none have analysed the possibility of restructuring the different benefit rates within the benefit system.[12] This is a short-coming as our results show that a rebalancing may increase efficiency at relatively low costs.

One might question the generality of the conclusions as the results were derived in a simple monopoly union setting. Our results can, however, easily be generalized to a model with (Nash) wage bargaining. We have chosen to abstract from this because of the well known, and in our case difficult, problem of how to define the disagreement point. Furthermore, it is shown in Appendix that the results are nearly identical if workers are risk averse.

The results of our paper support strongly reforms that rebalance benefit rates in favour of short-term unemployed. However, one should be cautious with policy recommendations before taking into consideration other effects of such reforms. An obvious worry concerns the distributional effects in a world of heterogeneous labour; a common reform for the whole labour market may have adverse effects on groups having a high risk of unemployment. On the other hand, the reforms may also have beneficial effects over the cycle from a stabilization point of view. The fact that many unemployed fall down on the low rate during a downturn reduces the wage pressure of the union. Thus, such reforms can also work as 'automatic stabilizers' (through the supply side).

[12] Such issues have been addressed within 'partial-partial' search theory. E.g. Shavell & Weiss (1979) shows under relative mild conditions that the benefit scheme maximizing expected utility of unemployed involves declining benefit level over time when workers act in a self-interested way and total benefit expenditures are fixed.

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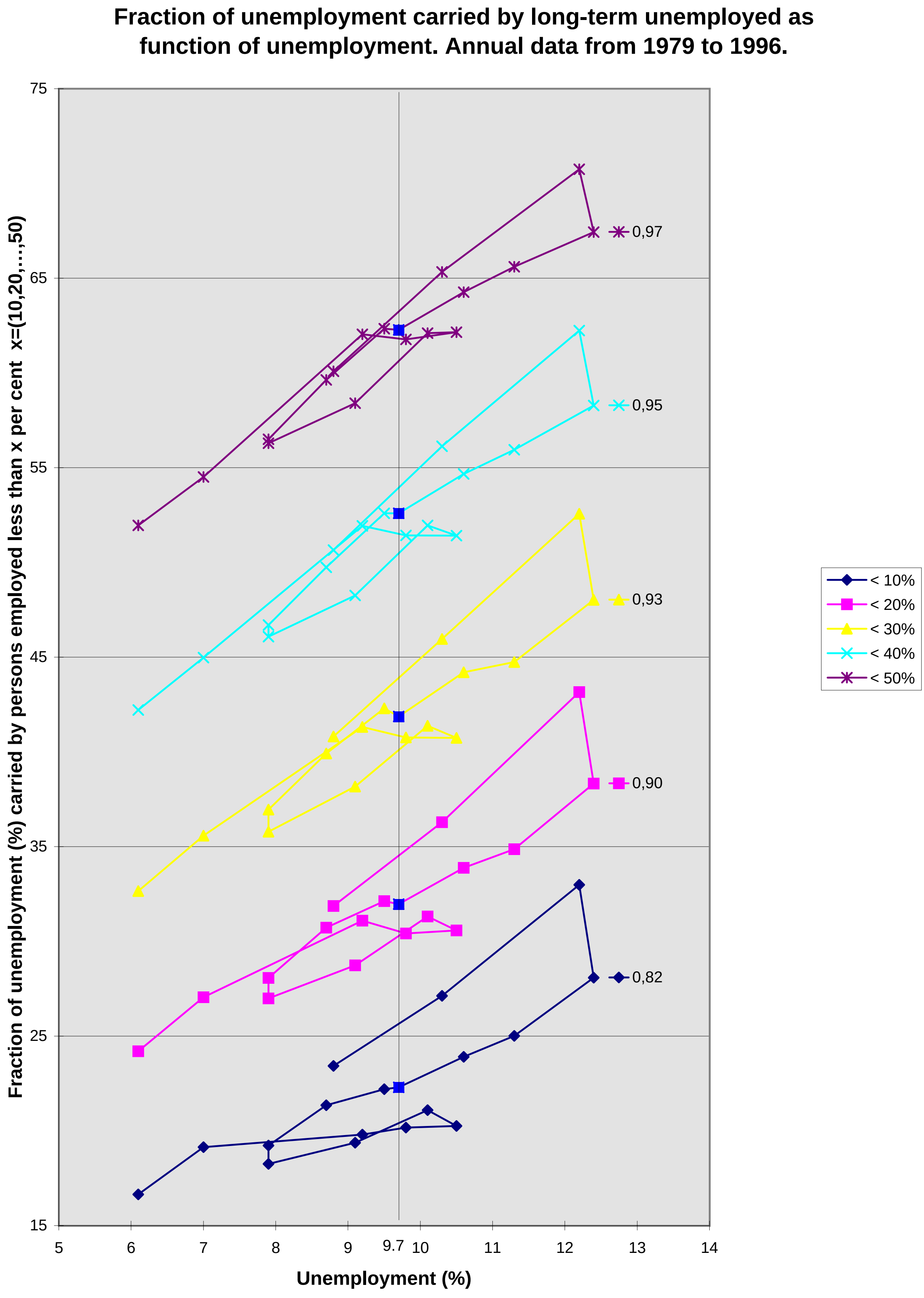
Figure 1



Source: Calculations based on data from Statistics Denmark.

A. Appendix

A.1. Risk aversion

Here, we prove that Theorem 3.1 is unchanged when union members are risk averse. We also show that Corollary 3.2 still holds if the initial equilibrium is characterized by a uniform benefit level, i.e. $b_1 = b_2$. Assume that the flow utility of a union member is characterized by the strictly concave function $v(x)$ where x is the flow income equal to w, b_1, or b_2 depending on the current state of the worker. In this case, the objective function of the union equals

$$\Omega(w, b_1, b_2) = v(w)(1-u) + u[(1-\phi(u))v(b_1) + \phi(u)v(b_2)]$$

$\Rightarrow$

$$\begin{aligned}\Omega_w(w, b_1, b_2) &= v'(w)(1-u(w)) - u'(w)[v(w) - v(b_1)] - \\ &\quad u'(w)[\phi(u(w)) + u(w)\phi'(u(w))][v(b_1) - v(b_2)]\end{aligned}$$

Looking at a rebalancing of the benefit levels that keeps aggregate utility fixed gives

$$\frac{dw^*}{db_1} = \frac{u'(w^*)\eta(u(w^*))v'(b_1)}{\Omega_{ww}(w^*, b_1, b_2)},$$

which is negative as $\Omega_{ww}(w^*, b_1, b_2) < 0$ is the second order condition of the union's problem. Looking instead at a fixed utility of an unemployed, we get

$$\frac{dw^*}{db_1} = \frac{u'(w^*)\eta(u(w^*))v'(b_1)}{\Omega_{ww}(w^*, b_1, b_2) + \frac{u'(w^*)}{u(w^*)}\eta(u(w^*))\frac{v(b_1)-v(b_2)}{v'(b_2)}\Omega_{wb_2}(w^*, b_1, b_2)},$$

which is negative if the model is stable (i.e. the denominator has to be negative). Requiring instead that benefit expenditures are fixed, we get

$$\begin{aligned}\frac{dw^*}{db_1} &= \frac{u'(w^*)\eta(u(w^*))[\phi(u(w^*))v'(b_1) + (1-\phi(u(w^*)))v'(b_2)]}{\Omega_{ww}(w^*, b_1, b_2) + \frac{u'(w^*)}{u(w^*)}\left([\eta(u(w^*)) + 1](b_1 - b_2) - \frac{b_1}{\phi(u(w^*))}\right)\Omega_{wb_2}(w^*, b_1, b_2)} - \\ &\quad \frac{u'(w^*)[1-\phi(u(w^*))][v'(b_1) - v'(b_2)]}{\Omega_{ww}(w^*, b_1, b_2) + \frac{u'(w^*)}{u(w^*)}\left([\eta(u(w^*)) + 1](b_1 - b_2) - \frac{b_1}{\phi(u(w^*))}\right)\Omega_{wb_2}(w^*, b_1, b_2)}.\end{aligned}$$

The first numerator is clearly positive and the assumption $b_1 \geq b_2$ implies that also the second numerator is positive whereas both denominators have to be negative as a stability requirement. Thus, the total effect is unambiguously negative.

To prove Corollary 3.2, we just have to prove that welfare increases when benefit levels are rebalanced keeping benefit expenditures fixed. Using the Envelope Theorem, we have

$$\frac{d\Omega}{db_1} = \frac{\partial\Omega\left(w^*, b_1, b_2\right)}{\partial b_1} + \frac{\partial\Omega\left(w^*, b_1, b_2\right)}{\partial b_2}\frac{db_2}{db_1}$$

$\Rightarrow$

$$\begin{aligned}\frac{d\Omega}{db_1} &= u\left(w^*\right)\left(1-\phi\left(u\left(w^*\right)\right)\right)\left(v'\left(b_1\right)-v'\left(b_2\right)\right)\\ &\quad -v'\left(b_2\right)u'\left(w^*\right)\left[b_1-\phi\left(u\left(w^*\right)\right)\left(1+\eta\left(u\left(w^*\right)\right)\right)\left(b_1-b_2\right)\right]\frac{dw^*}{db_1}\end{aligned}$$

The first term is non-positive because of the assumption $b_1 \geq b_2$. The second term is positive because $\frac{dw^*}{db_1}$ is negative and

$$\left[b_1-\phi\left(u\left(w^*\right)\right)\left(1+\eta\left(u\left(w^*\right)\right)\right)\left(b_1-b_2\right)\right] > 0,$$

which follows from the numerator of the wage equation (2.2). Thus, the total effect is ambiguous. However, if $b_1 = b_2$ in the initial equilibrium then the first term vanishes and the total effect is positive.

A.2. Proof of proposition 4.1

For the sake of this proof let $\gamma(m, j, u)$ be the (steady state) fraction of the *employed* in the considered period who had j or fewer employment periods during the last m periods preceding the considered one. Just like with ϕ above, one can compute $\gamma(2,0,u) = \alpha(u)\beta(u)$, and $\gamma(2,1,u) = 2\alpha(u) - \alpha^2(u)$. Of course, $\phi(2,2,u) = \gamma(2,2,u) = 1$. What is important to note for this proof is that for all j, including $j = m = 2$, $\phi(2,j,u) \geq \gamma(2,j,u)$ (this uses $\beta \geq \alpha$), and for all $j \leq m-1 = 1$, that is, $j = 0$ or 1, both $\phi(2,j,u)$ and $\gamma(2,j,u)$ are strictly increasing in u.

Assume for $m = k-1$ that for all $j \leq m$, $\phi(m,j,u) \geq \gamma(m,j,u)$, and for all $j \leq m-1$, both $\phi(m,j,u\ )$ and $\gamma(m,j,u)$ are strictly increasing in u. We will show that then the same holds for $m = k$. This will finish a proof by induction since the first step is established above.

Now, $u\phi(k,j,u)$ is the fraction of *all* workers who are unemployed in the considered period and employed in j or fewer of the preceding k periods. In the period just before the considered one, that is, the last period of the k preceding ones, call it period -1, each such worker must have been either employed or unemployed. So, each of the currently unemployed workers among the $u\phi(k,j,u)$ must *either* be among those who **1)** were employed in period -1, and had at most $j-1$ employment periods out of the $k-1$ periods just preceding period -1, *or* be among those who **2)** were unemployed in period -1, and had at most j employment periods out of the $k-1$ periods just before -1. In group **1)** there are $(1-u)\gamma(k-1,j-1,u)$ workers each becoming unemployed in the considered period with probability α; in group **2)** there are $u\phi(k-1,j,u)$ workers each becoming unemployed with probability β. Thus, $u\phi(k,j,u) = (1-u)\gamma(k-1,j-1,u)\alpha + u\phi(k-1,j,u)\beta$, where $j \leq k-1$, so $\phi(k-1,j,u)$ is always meaningful (but in case $j = k-1$, it equals one). A similar rewriting can be made for $(1-u)\gamma(k,j,u)$ giving altogether,

$$u\phi(k,j,u) = (1-u)\gamma(k-1,j-1,u)\alpha(u) + u\phi(k-1,j,u)\beta(u),$$

$$(1-u)\gamma(k,j,u) = (1-u)\gamma(k-1,j-1,u)(1-\alpha(u)) + u\phi(k-1,j,u)(1-\beta(u)),$$

holding for all $j \leq k-1$. Dividing on both sides with u and $1-u$ respectively, using (4.1), and rearranging give,

$$\phi(k,j,u) = \gamma(k-1,j-1,u) + \beta(u)\left[\phi(k-1,j,u) - \gamma(k-1,j-1,u)\right],$$

$$\gamma(k,j,u) = \gamma(k-1,j-1,u) + \alpha(u)\left[\phi(k-1,j,u) - \gamma(k-1,j-1,u)\right].$$

By the induction hypothesis, $\phi(k-1,j,u) \geq \gamma(k-1,j,u)$ for all $j \leq k-1$. Since $\gamma(k-1,j,u) \geq \gamma(k-1,j-1,u)$, the square brackets are positive, and then since $\beta(u) \geq \alpha(u)$, we have $\phi(k,j,u) \geq \gamma(k,j,u)$ for all $j \leq k-1$. But it also holds for $j = k$, since then $\phi = \gamma = 1$. Finally differentiate $\phi(k,j,u)$ with respect to u to get,

$$\begin{aligned}\frac{\partial\phi(k,j,u)}{\partial u} &= (1-\beta)\frac{\partial\gamma(k-1,j-1,u)}{\partial u} + \beta\frac{\partial\phi(k-1,j,u)}{\partial u} \\ &\quad + \left[\phi(k-1,j,u) - \gamma(k-1,j-1,u\right]\frac{\partial\beta(u)}{\partial u}.\end{aligned}$$

From the induction hypothesis the square bracket is again positive (possibly zero) and also all of the partial derivatives on the right hand side are positive with at least the one for γ being strictly positive. Since $0 < \beta < 1$, it follows that $\partial\phi(k,j,u)/\partial u$ is strictly positive for all $j \leq k-1$. Similarly can be done for $\partial\gamma(k,j,u)/\partial u$. We have thus established what was required for $m = k$. ■

A.3. Data Construction

Table A1 contains the data obtained from Statistics Denmark. The second column contains the unemployment rate whereas the next 10 columns contain the distribution of unemployed persons during a year on different unemployment spells, i.e. the first displays the number of unemployed persons (in thousands) who have been unemployed between 0 and 10% during a year, the next column displays the number of unemployed who have been unemployed between 10% and 20% during a year, etc. To obtain y_t, we first calculate the number of full-time unemployed corresponding to column Z_1 to Z_{10} using the following formula

$$U_t = \sum_{i=1}^{10} Z_i \left(0.05 + \frac{i-1}{10}\right),$$

where it is assumed that the average length of the spell for a group is the midpoint, e.g. 0.05 for the first group. Now, y_t is calculated for the five different rules $x \in \{10, 20, 30, 40, 50\}$ using the following formulas:

$$y_t|_{x=10} = \frac{Z_{10} * 0.95}{U_t},$$

$$y_t|_{x=20} = \frac{Z_{10} * 0.95 + Z_9 * 0.85}{U_t},$$

$$y_t|_{x=30} = \frac{Z_{10} * 0.95 + Z_9 * 0.85 + Z_8 * 0.75}{U_t},$$

$$y_t|_{x=40} = \frac{Z_{10} * 0.95 + Z_9 * 0.85 + Z_8 * 0.75 + Z_7 * 0.65}{U_t},$$

$$y_t|_{x=50} = \frac{Z_{10} * 0.95 + Z_9 * 0.85 + Z_8 * 0.75 + Z_7 * 0.65 + Z_6 * 0.55}{U_t}.$$

Table A2 contains the values of these variables which are illustrated graphically over time in Figure A1 together with the unemployment rate.

Table A1 *. Unemployment rate and data used for calculating y(t)* .

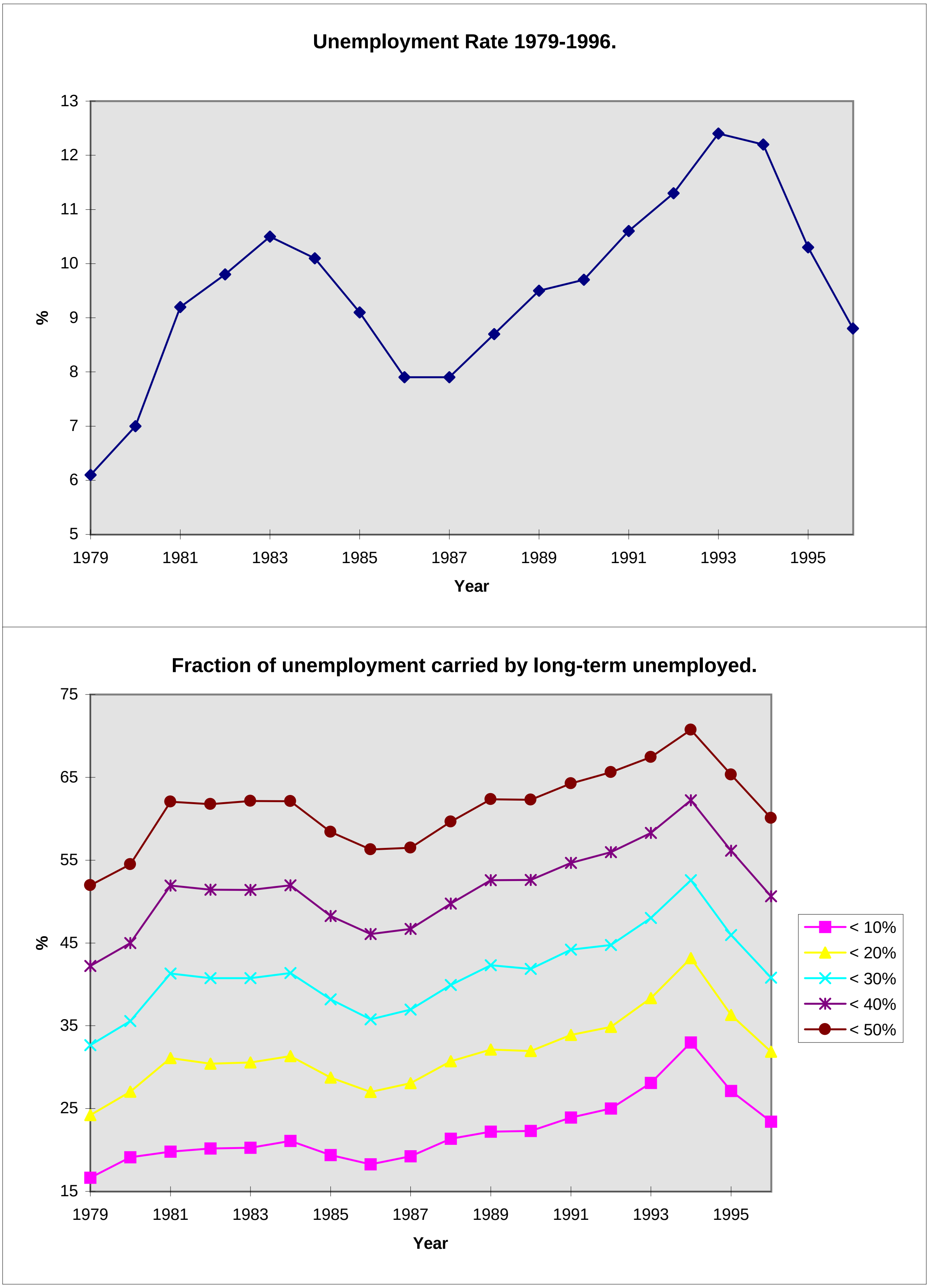
Year	u (%)	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8	Z9	Z10
1979	6.1	199.2	97.8	72.8	51.9	38.7	28.9	24.0	18.4	14.5	28.6
1980	7.0	200.2	100.3	73.2	57.2	45.9	32.0	26.7	21.0	17.2	37.2
1981	9.2	176.1	102.4	76.8	65.5	58.3	44.8	39.8	33.2	32.4	50.8
1982	9.8	183.9	96.2	84.3	74.4	66.4	49.5	43.2	36.3	31.7	55.9
1983	10.5	187.9	90.9	80.9	78.7	81.7	55.5	46.7	38.5	34.5	60.6
1984	10.1	188.3	94.2	83.3	76.1	75.9	51.2	45.2	37.2	33.4	61.6
1985	9.1	212.8	99.4	82.9	76.4	72.0	46.8	39.3	31.9	27.9	51.7
1986	7.9	222.7	97.8	78.3	71.0	60.3	41.3	35.3	26.1	22.9	42.8
1987	7.9	241.0	97.4	79.3	67.9	61.2	40.1	33.7	26.6	23.4	45.5
1988	8.7	199.0	97.0	79.0	71.0	66.0	44.0	37.0	30.0	27.0	55.0
1989	9.5	184.0	96.0	80.0	71.0	70.0	47.0	42.0	36.0	31.0	62.0
1990	9.7	187.0	98.0	83.0	72.0	73.0	48.0	45.0	36.0	31.0	64.0
1991	10.6	179.0	89.0	92.0	72.0	80.0	52.0	48.0	41.0	35.0	75.0
1992	11.3	177.0	95.0	92.0	79.0	80.0	56.0	55.0	42.0	37.0	84.0
1993	12.4	180.0	102.0	93.0	79.0	85.0	58.0	55.0	45.0	42.0	103.0
1994	12.2	191.0	99.0	84.0	68.0	69.0	53.0	51.0	43.0	41.0	119.0
1995	10.3	220.0	102.0	86.0	67.0	63.0	48.0	45.0	37.0	31.0	82.0
1996	8.8	251.4	100.3	85.8	64.9	59.2	42.3	37.3	29.4	24.5	60.8

Note: Z1 to Z10 are measured in thousands.
Source: Statistics Denmark.

Table A2 *. y(t) for different rules* .

Year	x = 10	x = 20	x = 30	x = 40	x = 50
1979	16.6	24.2	32.7	42.2	52.0
1980	19.1	27.1	35.6	45.0	54.5
1981	19.8	31.1	41.3	51.9	62.0
1982	20.2	30.4	40.8	51.4	61.8
1983	20.3	30.6	40.7	51.4	62.2
1984	21.1	31.3	41.4	52.0	62.1
1985	19.4	28.7	38.2	48.3	58.4
1986	18.3	27.0	35.8	46.1	56.3
1987	19.2	28.1	36.9	46.7	56.5
1988	21.3	30.7	39.9	49.7	59.6
1989	22.2	32.1	42.3	52.6	62.3
1990	22.3	32.0	41.9	52.6	62.3
1991	23.9	33.9	44.2	54.7	64.3
1992	25.0	34.9	44.7	55.9	65.6
1993	28.1	38.3	48.0	58.3	67.4
1994	33.0	43.2	52.6	62.2	70.7
1995	27.1	36.3	46.0	56.1	65.3
1996	23.4	31.9	40.8	50.6	60.1

Figure A1



Source: Statistics Denmark and own calculations.