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Theory including future not excluded: Formulation of complex action theory II

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In Ref. [1] we have found errata. They are composed of two parts: one part is for the body, which is also explained in our recently published book [2], while the other part is for the appendix, which is mainly a result of the corrections to Ref. [3]. They do not influence the result of the manuscript. Rather, the latter part provides us a new additional result: the Schrödinger equation described with complex action theory.

In the fifth line below Eq. (5.8), where \( f(D)D^\dagger \) should have been replaced with \( (f(D)f(D)^\dagger)^{-1} \), we have chosen \( f(D) \) such that \( (P^\dagger)^{-1}(f(D)f(D)^\dagger)^{-1}P^\dagger = F(\bar{H}^\dagger) \), which is rewritten as \( (f(D)f(D)^\dagger)^{-1} = F(D^\dagger) \). However, this relation does not stand, because the left-hand side is Hermitian, while the right-hand side is not Hermitian. Accordingly, the expression \( Q' = F(\bar{H}^\dagger)Q \) below Eq. (5.8), which was introduced based on the above relation, has to be corrected. In addition, the next statement, “\( F(\bar{H}^\dagger)Q \simeq F(\bar{H}_{\text{eff}}^\dagger)Q \) for the restricted subspace,” is not right.

This is because, for any reasonable function \( \bar{h} \) and any state \( |A(t)\rangle = \sum_i a_i(t)|\lambda_i\rangle \) that obeys the Schrödinger equation \( i\hbar \frac{d}{dt} |A(t)\rangle = \bar{H}|A(t)\rangle \), the following relation holds for large \( t - T_A \):

\[
h(\bar{H})|A(t)\rangle \simeq h(\bar{H}_{\text{eff}} + iB\Lambda_A)|A(t)\rangle \equiv h(\bar{H}_{\text{eff}})|\bar{A}(t)\rangle,
\]

where we have used the automatic Hermiticity mechanism and introduced \( |\bar{A}(t)\rangle \equiv \sum_{i\in A} a_i(t)|\lambda_i\rangle \), \( \Lambda_A \equiv \sum_{i\in A} |\lambda_i\rangle\langle \lambda_i|Q \), and another function \( \bar{h} \) such that \( \bar{h}(\text{Re} \lambda_i) = h(\text{Re} \lambda_i + iB) \). Similarly, the statement “\( Q_2 = F(\bar{H}_{\text{eff}}^\dagger)Q \) for the restricted subspace” given in Eq. (5.6) has to be corrected.

To correct the above points, on behalf of \( F(\bar{H}_{\text{eff}}) = |b_i|^2 \) and Eq. (5.6), we introduce functions \( \bar{G} \) and \( \bar{G} \) such that \( G(\text{Re} \lambda_i + iB) = \bar{G}(\text{Re} \lambda_i) = b_i \), and express \( Q_2 \) as follows:

\[
Q_2 = \sum_{i\in A} |b_i|^2 |\lambda_i\rangle_B \langle \lambda_i| = \sum_{i\in A} G(\bar{H}_{\text{eff}} + iB\Lambda_A)^\dagger |\lambda_i\rangle_B \langle \lambda_i|G(\bar{H}_{\text{eff}} + iB\Lambda_A)
\]

\[
= \bar{G}(\bar{H}_{\text{eff}}^\dagger)Q\Lambda_A\bar{G}(\bar{H}_{\text{eff}}),
\]

where, in the second and third equalities, supposing that \( \text{Re} \lambda_i \)'s are not degenerate, we have used \( |\lambda_i\rangle_B = Q|\lambda_i\rangle \), and \( G(\text{Re} \lambda_i + iB) = b_i \langle \lambda_i|G(\bar{H}_{\text{eff}} + iB\Lambda_A) \) for \( i \in A \). We note that
\[ Q\Lambda_A = Q \sum_{i \in A} |\lambda_i\rangle \langle \lambda_i|_Q \] is Hermitian, and so is \( Q_2 \). Next we define \( Q' \) by \( Q' \equiv G(\hat{H})^\dagger G(\hat{H}) = (P_{G^{-1}})^{-1}P_{G^{-1}}^{-1}P_{G^{-1}} \), where \( P_{G^{-1}} \equiv G(\hat{H})^{-1}P \) diagonalizes \( \hat{H} \). \( (P_{G^{-1}})^{-1} \hat{H}P_{G^{-1}} = P^{-1} \hat{H}P = D \).

In addition, we introduce \( |\lambda_i\rangle^{G^{-1}} \equiv G(\hat{H})^{-1}|\lambda_i\rangle \), so that \( |\lambda_i\rangle^{G^{-1}} \) is \( Q' \)-orthogonal, i.e., orthogonal with regard to the proper inner product \( I_Q' \): \( I_Q'(|\lambda_i\rangle^{G^{-1}},|\lambda_j\rangle^{G^{-1}}) = G^{-1} (\langle \lambda_i|Q'\rangle|\lambda_j\rangle^{G^{-1}} = \delta_{ij} \).

We use the automatic Hermiticity mechanism for large \( t - T_A \). Then, since \( |\tilde{A}(t)\rangle \equiv \sum_{i \in A} a_i(t)|\lambda_i\rangle \), \( Q' \) used in the normalized matrix element \( \langle Q'\rangle_{\tilde{A}D} \) is estimated in the subspace restricted by \( A \) as follows:

\[
Q' \approx G(\hat{H}_{\text{eff}} + iB\Lambda_A)^\dagger Q\Lambda_A G(\hat{H}_{\text{eff}} + iB\Lambda_A) \quad \text{for the restricted subspace}
\]

\[
= \tilde{G}(\hat{H}_{\text{eff}})^\dagger Q\Lambda_A \tilde{G}(\hat{H}_{\text{eff}})
\]

\[
= Q_2,
\]

where in the last equality we have used Eq. (1). The three sentences “We first point out ... replaced with \( |\tilde{A}(t)\rangle \)” below Eq. (5.8) should be replaced with the above argument.

A \( dt \)-dependent normalization factor, say \( \frac{1}{\alpha(dt)} \), should be inserted on the right-hand sides of Eq. (A.2) and of the first line of Eq. (A.4). The following sentence should be inserted after the sentence “C is an arbitrary ... complex plane” below Eq. (A.2): “In addition, \( \alpha(dt) \) is a \( dt \)-dependent normalization factor, which is properly fixed later.” The factor \( \sqrt{\frac{2\pi i\hbar dt}{m}} \) in the second line of Eq. (A.4) should be deleted. The following sentences should be inserted after the phrase “where ... Eq. (3.7)” below Eq. (A.4): “Here we have taken \( \alpha(dt) = \sqrt{\frac{2\pi i\hbar dt}{m}} \) so that both sides of Eq. (A.4) correspond to each other in the vanishing limit of \( dt \). Then Eq. (A.4) is reduced to \( |\psi(t + dt)\rangle = e^{-\frac{i}{\hbar} \hat{H}dt} |\psi(t)\rangle \).” The next sentence, “Thus we have found ... Eq. (A.2),” below Eq. (A.4) should be replaced with “Thus we have derived the Schrödinger equation and found that ... Eq. (A.2).” The following sentence should be added after the above replaced sentence: “Such a derivation of the Schrödinger equation is well known in the real action theory [4].” Factors \( \frac{1}{\alpha(dt)^s}, \frac{1}{\alpha(-dt)^t}, \) and \( \frac{1}{\alpha(-dt)} \) should be inserted on the right-hand side of the equation in the second sentence of the last paragraph of the appendix, on the right-hand sides of Eqs (A.5) and (A.6), respectively. The second sentence below Eq. (A.6), “Indeed, \( \hat{H}_B \) is given ... \hat{H}^\dagger \),” should be replaced with “Indeed, we obtain the Schrödinger equation \( |B(t - dt)\rangle = e^{\frac{i}{\hbar} \hat{H}_B dt} |B(t)\rangle \), where \( \hat{H}_B \) is given ... \hat{H}^\dagger \).”

References