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Theory including future not excluded:
Formulation of complex action theory II

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In Ref. [1] we have found errata. They are composed of two parts: one part is for the body, which is also explained in our recently published book [2], while the other part is for the appendix, which is mainly a result of the corrections to Ref. [3]. They do not influence the result of the manuscript. Rather, the latter part provides us a new additional result: the Schrödinger equation described with the Hamiltonian $\hat{H}_B$ has been derived for the future state $|B(t)\rangle$ via the Feynman path integral in the complex action theory.

In the fifth line below Eq. (5.8), where $f(D)^{-1}$ should have been replaced with $(f(D)f(D)^\dagger)^{-1}$, we have chosen $f(D)$ such that $(\tilde{f}(D)^{-1})(f(D)f(D)^\dagger)^{-1}P = F(\hat{H}^\dagger)$, which is rewritten as $(f(D)f(D)^\dagger)^{-1} = F(D)^\dagger$. However, this relation does not stand, because the left-hand side is Hermitian, while the right-hand side is not Hermitian. Accordingly, the expression $Q' = F(\hat{H}^\dagger)\hat{Q}$ below Eq. (5.8), which was introduced based on the above relation, has to be corrected. In addition, the next statement, “$F(\hat{H}^\dagger)\tilde{Q} \simeq F(\hat{H}_B^\dagger)\hat{Q}$ for the restricted subspace,” is not right. This is because, for any reasonable function $\tilde{f}$ and any state $|A(t)\rangle = \sum_i a_i(t)|\lambda_i\rangle$ that obeys the Schrödinger equation $i\hbar \frac{d}{dt}A(t) = \hat{H}|A(t)\rangle$, the following relation holds for large $t - T_A$: $h(\hat{H})|A(t)\rangle \simeq h(\hat{H}_{eff} + iB\Lambda_A)|A(t)\rangle \equiv \hat{h}(\hat{H}_{eff})|\lambda_i\rangle$, where we have used the automatic Hermiticity mechanism and introduced $|\lambda_i\rangle \equiv \sum_{i \in A} a_i(t)|\lambda_i\rangle$, $\Lambda_A \equiv \sum_{i \in A} |\lambda_i\rangle\langle\lambda_i|Q$, and another function $\tilde{h}$ such that $\tilde{h}(\text{Re} \lambda_i) = h(\text{Re} \lambda_i + iB)$. Similarly, the statement “$Q_2 = F(\hat{H}_{\text{eff}}^\dagger)\hat{Q}$ for the restricted subspace” given in Eq. (5.6) has to be corrected.

To correct the above points, on behalf of $F(Re\lambda_i) = |b_i|^2$ and Eq. (5.6), we introduce functions $G$ and $\tilde{G}$ such that $G(Re\lambda_i + iB) = \tilde{G}(Re\lambda_i) = b_i$, and express $Q_2$ as follows:

\[
Q_2 = \sum_{i \in A} |b_i|^2 |\lambda_i\rangle_B \langle \lambda_i| = \tilde{G}(\hat{H}_{\text{eff}}^\dagger + iB\Lambda_A)^\dagger |\lambda_i\rangle_B \langle \lambda_i|G(\hat{H}_{\text{eff}} + iB\Lambda_A) = \tilde{G}(\hat{H}_{\text{eff}}^\dagger)\hat{Q}_A \tilde{G}(\hat{H}_{\text{eff}}),
\]

where, in the second and third equalities, supposing that $Re \lambda_i$’s are not degenerate, we have used $|\lambda_i\rangle_B = Q|\lambda_i\rangle$, and $b(\lambda_i)G(Re\lambda_i + iB) = b(\lambda_i)G(\hat{H}_{\text{eff}} + iB\Lambda_A)$ for $i \in A$. We note that...
\[ Q A_A = Q \sum_{i \in A} |\lambda_i\rangle \langle \lambda_i|_Q \] is Hermitian, and so is \( Q_2 \). Next we define \( Q' \) by \( Q' \equiv G(\hat{H})^\dagger QG(\hat{H}) = (P_{G^{-1}})^{-1}P_{G^{-1}} \), where \( P_{G^{-1}} \equiv G(\hat{H})^{-1}P \) diagonalizes \( \hat{H} \): \( (P_{G^{-1}})^{-1} \hat{H} P_{G^{-1}} = P^{-1}\hat{H} P = D \). In addition, we introduce \( |\lambda_i\rangle^{G^{-1}} \equiv G(\hat{H})^{-1}|\lambda_i\rangle \), so that \( |\lambda_i\rangle^{G^{-1}} \) is \( Q' \)-orthogonal, i.e., orthogonal with regard to the proper inner product \( I_Q' : I_Q'(\langle \lambda_i\rangle^{G^{-1}}, |\lambda_j\rangle^{G^{-1}}) = G^{-1}(\langle \lambda_i\rangle^{Q'}|\lambda_j\rangle^{G^{-1}}) = \delta_{ij} \). We use the automatic Hermiticity mechanism for large \( t - T_A \). Then, since \( |A(t)\rangle \) behaves as \( |A(t)\rangle = \sum_{i \in A} a_i(t)|\lambda_i\rangle \), \( Q' \) used in the normalized matrix element \( \langle O' A \rangle_Q \) is estimated in the subspace restricted by \( A \) as follows:

\[
Q' \simeq G(\hat{H}_{\text{eff}} + iB A_A)^\dagger Q A_A G(\hat{H}_{\text{eff}} + iB A_A) \quad \text{for the restricted subspace}
\]

\[
= \tilde{G}(\hat{H}_{\text{eff}})^\dagger Q A_A \tilde{G}(\hat{H}_{\text{eff}})
\]

\[
= Q_2,
\]

where in the last equality we have used Eq. (1). The three sentences “We first point out … replaced with \( |\tilde{A}(t)\rangle \)” below Eq. (5.8) should be replaced with the above argument.

A \( dt \)-dependent normalization factor, say \( \frac{1}{\alpha(dt)} \), should be inserted on the right-hand sides of Eq. (A.2) and of the first line of Eq. (A.4). The following sentence should be inserted after the sentence “\( C \) is an arbitrary … complex plane” below Eq. (A.2): “In addition, \( \alpha(dt) \) is a \( dt \)-dependent normalization factor, which is properly fixed later.” The factor \( \sqrt{2\pi\hbar d t / m} \) in the second line of Eq. (A.4) should be deleted. The following sentences should be inserted after the phrase “where … Eq. (3.7)” below Eq. (A.4): “Here we have taken \( \alpha(dt) = \sqrt{2\pi\hbar d t / m} \) so that both sides of Eq. (A.4) correspond to each other in the vanishing limit of \( dt \). Then Eq. (A.4) is reduced to \( |\psi(t + dt)\rangle = e^{-\frac{i}{\hbar}A A_A A_B} |\psi(t)\rangle \).”

The next sentence, “Thus we have found that … Eq. (A.2),” below Eq. (A.4) should be replaced with “Thus we have derived the Schrödinger equation and found that … Eq. (A.2).” The following sentence should be added after the above replaced sentence: “Such a derivation of the Schrödinger equation is well known in the real action theory [4].” Factors \( \frac{1}{\alpha(dt)^2}, \frac{1}{\alpha(-dt)^2}, \text{and } \frac{1}{\alpha(-dt)} \) should be inserted on the right-hand side of the equation in the second sentence of the last paragraph of the appendix, on the right-hand sides of Eqs (A.5) and (A.6), respectively. The second sentence below Eq. (A.6), “Indeed, \( \hat{H}_B \) is given … \( \hat{H}^Y \),” should be replaced with “Indeed, we obtain the Schrödinger equation \( |B(t - dt)\rangle = e^{-\frac{i}{\hbar}H_B d t} |B(t)\rangle \), where \( \hat{H}_B \) is given … \( \hat{H}^Y \).”

References