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Theory including future not excluded: Formulation of complex action theory II

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In Ref. [1] we have found errata. They are composed of two parts: one part is for the body, which is also explained in our recently published book [2], while the other part is for the appendix, which is mainly a result of the corrections to Ref. [3]. They do not influence the result of the manuscript. Rather, the latter part provides us a new additional result: the Schrödinger equation described with the Hamiltonian \(\hat{H}_B\) has been derived for the future state \(|B(t)\rangle\) via the Feynman path integral in the complex action theory.

In the fifth line below Eq. (5.8), where \(f(D)f(D)^\dagger\) should have been replaced with \((f(D)f(D)^\dagger)^{-1}\), we have chosen \(f(D)\) such that \((P^\dagger)^{-1}(f(D)f(D)^\dagger)^{-1}P^\dagger = F(\hat{H}^\dagger)\), which is rewritten as \((f(D)f(D)^\dagger)^{-1} = F(D)^\dagger\). However, this relation does not stand, because the left-hand side is Hermitian, while the right-hand side is not Hermitian. Accordingly, the expression \(Q' = F(\hat{H}^\dagger)Q\) below Eq. (5.8), which was introduced based on the above relation, has to be corrected. In addition, the next statement, “\(F(\hat{H}^\dagger)Q \simeq F(\hat{H}_{\text{eff}}^\dagger)Q\) for the restricted subspace,” is not right. This is because, for any reasonable function \(h\) and any state \(|A(t)\rangle = \sum_i a_i(t)|\lambda_i\rangle\) that obeys the Schrödinger equation \(i\hbar \frac{d}{dt}|A(t)\rangle = \hat{H}|A(t)\rangle\), the following relation holds for large \(t - T_A\): 
\[
h(\hat{H})|A(t)\rangle \simeq h(\hat{H}_{\text{eff}} + iB\Lambda_A)|A(t)\rangle = \hat{h}(\hat{H}_{\text{eff}})|A(t)\rangle,
\]
where we have used the automatic Hermiticity mechanism and introduced \(|\tilde{A}(t)\rangle = \sum_{i \in A} a_i(t)|\lambda_i\rangle\), \(\Lambda_A = \sum_{i \in A} |\lambda_i\rangle\langle\lambda_i|_A\), and another function \(\tilde{h}\) such that \(\tilde{h}(\text{Re } \lambda_i) = h(\text{Re } \lambda_i + iB)\). Similarly, the statement “\(Q_2 = F(\hat{H}_{\text{eff}})Q\) for the restricted subspace” given in Eq. (5.6) has to be corrected.

To correct the above points, on behalf of \(F(\text{Re } \lambda_i) = |b_i|^2\) and Eq. (5.6), we introduce functions \(G\) and \(\tilde{G}\) such that \(G(\text{Re } \lambda_i + iB) = \tilde{G}(\text{Re } \lambda_i) = b_i\), and express \(Q_2\) as follows:

\[
Q_2 = \sum_{i \in A} |b_i|^2 |\lambda_i\rangle_B \langle \lambda_i|_B
\]
\[
= \sum_{i \in A} G(\hat{H}_{\text{eff}} + iB\Lambda_A)^\dagger |\lambda_i\rangle_B \langle \lambda_i|_B G(\hat{H}_{\text{eff}} + iB\Lambda_A)
\]
\[
= \tilde{G}(\hat{H}_{\text{eff}})^\dagger Q_1 \Lambda_A \tilde{G}(\hat{H}_{\text{eff}}) ,
\]

where, in the second and third equalities, supposing that \(\text{Re } \lambda_i\)'s are not degenerate, we have used \(|\lambda_i\rangle_B = Q|\lambda_i\rangle\), and \(b(\text{Re } \lambda_i + iB) = b(\text{Re } \lambda_i)G(\hat{H}_{\text{eff}} + iB\Lambda_A)\) for \(i \in A\). We note that
We use the automatic Hermiticity mechanism for large \( t \) and then we have found that \( \hat{A}(t) \) behaves as
\[ |\tilde{A}(t)\rangle = \sum_{i(t) \in A} a_i(t) |\lambda_i\rangle, \]
\( Q' \) used in the normalized matrix element \( \langle Q' | A \rangle \) is estimated in the subspace restricted by \( A \) as follows:
\[
Q' \simeq G(\hat{H}_\text{eff} + iB\Delta_A)^{\dagger} Q\Delta_A G(\hat{H}_\text{eff} + iB\Delta_A)
= \tilde{G}(\hat{H}_\text{eff})^{\dagger} Q\Delta_A \tilde{G}(\hat{H}_\text{eff})
= Q_2,
\]
where in the last equality we have used Eq. (1). The three sentences “We first point out … replaced with \( |\tilde{A}(t)\rangle \)” below Eq. (5.8) should be replaced with the above argument.

A \( dt \)-dependent normalization factor, say \( \frac{1}{\alpha(dt)} \), should be inserted on the right-hand sides of Eq. (A.2) and of the first line of Eq. (A.4). The following sentence should be inserted after the sentence “\( C \) is an arbitrary … complex plane” below Eq. (A.2): “In addition, \( \alpha(dt) \) is a \( dt \)-dependent normalization factor, which is properly fixed later.” The factor \( \sqrt{2\pi i\frac{\hbar dt}{m}} \) in the second line of Eq. (A.4) should be deleted. The following sentences should be inserted after the phrase “where … Eq. (3.7)” below Eq. (A.4): “Here we have taken \( \alpha(dt) = \sqrt{\frac{2\pi i\hbar dt}{m}} \) so that both sides of Eq. (A.4) correspond to each other in the vanishing limit of \( dt \). Then Eq. (A.4) is reduced to
\[ |\psi(t + dt)\rangle = e^{-\frac{i}{\hbar} \hat{H} dt} |\psi(t)\rangle. \]

The next sentence, “Thus we have found that … Eq. (A.2),” below Eq. (A.4) should be replaced with “Thus we have derived the Schrödinger equation and found that … Eq. (A.2).” The following sentence should be added after the above replaced sentence: “Such a derivation of the Schrödinger equation is well known in the real action theory [4].” Factors \( \frac{1}{\alpha(dt)^2}, \frac{1}{\alpha(-dt)^2}, \) and \( \frac{1}{\alpha(-dt)} \) should be inserted on the right-hand side of the equation in the second sentence of the last paragraph of the appendix, on the right-hand sides of Eqs (A.5) and (A.6), respectively. The second sentence below Eq. (A.6), “Indeed, \( \hat{H}_B \) is given … \hat{H}^\dagger, \)” should be replaced with “Indeed, we obtain the Schrödinger equation
\[ |B(t - dt)\rangle = e^{\frac{\xi}{\hbar} \hat{H}_B dt} |B(t)\rangle, \]
where \( \hat{H}_B \) is given … \hat{H}^\dagger, \)”.

References