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**Theory including future not excluded: Formulation of complex action theory II**

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In Ref. [1] we have found errata. They are composed of two parts: one part is for the body, which is also explained in our recently published book [2], while the other part is for the appendix, which is mainly a result of the corrections to Ref. [3]. They do not influence the result of the manuscript. Rather, the latter part provides us a new additional result: the Schrödinger equation described with the Hamiltonian $\hat{H}_B$ has been derived for the future state $|B(t)\rangle$ via the Feynman path integral in the complex action theory.

In the fifth line below Eq. (5.8), where $f(D)f(D)^\dagger$ should have been replaced with $(f(D)f(D)^\dagger)^{-1}$, we have chosen $f(D)$ such that $(f(D)f(D)^\dagger)^{-1} = F(\hat{H}^\dagger)$, which is rewritten as $(f(D)f(D)^\dagger)^{-1} = F(\hat{H}^\dagger)$. However, this relation does not stand, because the left-hand side is Hermitian, while the right-hand side is not Hermitian. Accordingly, the expression $Q' = F(\hat{H}^\dagger)Q$ below Eq. (5.8), which was introduced based on the above relation, has to be corrected. In addition, the next statement, “$F(\hat{H}^\dagger)Q \simeq F(\hat{H}_B^\dagger)Q$ for the restricted subspace,” is not right.

This is because, for any reasonable function $\tilde{h}$ and any state $|A(t)\rangle = \sum_i q_i(t)|\lambda_i\rangle$, one has the Schrödinger equation $i\hbar\frac{d}{dt}|A(t)\rangle = \hat{H}|A(t)\rangle$, the following relation holds for large $t - T_A$: $\tilde{h}(\hat{H})|A(t)\rangle \simeq \tilde{h}(\hat{H}_\text{eff} + iB\Lambda_A)|A(t)\rangle \equiv \tilde{h}(\hat{H}_\text{eff})|\tilde{A}(t)\rangle$, where we have used the automatic Hermiticity mechanism and introduced $|\tilde{A}(t)\rangle \equiv \sum_{i\in A} q_i(t)|\lambda_i\rangle$, $\Lambda_A \equiv \sum_{i\in A} |\lambda_i\rangle\langle\lambda_i|Q$, and another function $\tilde{h}$ such that $\tilde{h}(\text{Re} \lambda_i) = \text{h}(\text{Re} \lambda_i + iB)$. Similarly, the statement “$Q_2 = F(\hat{H}_\text{eff}^\dagger)Q$ for the restricted subspace” given in Eq. (5.6) has to be corrected.

To correct the above points, on behalf of $F(\text{Re} \lambda_i) = |b_i|^2$ and Eq. (5.6), we introduce functions $G$ and $\tilde{G}$ such that $G(\text{Re} \lambda_i + iB) = \tilde{G}(\text{Re} \lambda_i) = b_i$, and express $Q_2$ as follows:

$$Q_2 = \sum_{i\in A} |b_i|^2|\lambda_i\rangle_B B\langle\lambda_i|$$

$$= \sum_{i\in A} G(\hat{H}_\text{eff} + iB\Lambda_A)^\dagger|\lambda_i\rangle_B B\langle\lambda_i|G(\hat{H}_\text{eff} + iB\Lambda_A)$$

$$= \tilde{G}(\hat{H}_\text{eff}^\dagger)Q\Lambda_A \tilde{G}(\hat{H}_\text{eff}),$$

(1)

where, in the second and third equalities, supposing that $\text{Re} \lambda_i$'s are not degenerate, we have used $|\lambda_i\rangle_B = Q|\lambda_i\rangle$, and $B|\lambda_i\rangle G(\text{Re} \lambda_i + iB) = B|\lambda_i\rangle G(\hat{H}_\text{eff} + iB\Lambda_A)$ for $i \in A$. We note that...
\[ Q \Lambda_A = Q \sum_{i \in A} |\lambda_i\rangle \langle \lambda_i|_Q \] is Hermitian, and so is \( Q_2 \). Next we define \( Q' \) by \( Q' \equiv G(\hat{H})^\dagger Q G(\hat{H}) = (P_{G^{-1}})^{-1} P_{G^{-1}} \), where \( P_{G^{-1}} \equiv G(\hat{H})^{-1} P \) diagonalizes \( \hat{H} \): \( (P_{G^{-1}})^{-1} \hat{H} P_{G^{-1}} = P^{-1} \hat{H} P = D \). In addition, we introduce \( |\lambda_i\rangle^{G^{-1}} \equiv G(\hat{H})^{-1} |\lambda_i\rangle \), so that \( |\lambda_i\rangle^{G^{-1}} \) is \( Q' \)-orthogonal, i.e., orthogonal with regard to the proper inner product \( I_Q' \): \( I_Q' (|\lambda_i\rangle^{G^{-1}}, |\lambda_j\rangle^{G^{-1}}) = G^{-1} (|\lambda_i\rangle Q' |\lambda_j\rangle^{G^{-1}}) = \delta_{ij} \).

We use the automatic Hermiticity mechanism for large \( t - T_A \). Then, since \( |A(t)\rangle \) behaves as \( |\tilde{A}(t)\rangle = \sum_{i \in A} a_i(t) |\lambda_i\rangle \), \( Q' \) used in the normalized matrix element \( \langle O \rangle^{A_0}_{Q' \lambda} \) is estimated in the subspace restricted by \( A \) as follows:

\[
Q' \simeq G(\hat{H}_{\text{eff}} + iB \Lambda_A)^\dagger Q \Lambda_A G(\hat{H}_{\text{eff}} + iB \Lambda_A)
= G(\hat{H}_{\text{eff}})^\dagger Q \Lambda_A G(\hat{H}_{\text{eff}})
= Q_2,
\]

where in the last equality we have used Eq. \((1)\). The three sentences “We first point out … replaced with \( |\tilde{A}(t)\rangle \)” below Eq. \((5.8)\) should be replaced with the above argument.

A \( dt \)-dependent normalization factor, say \( \frac{1}{\alpha(dt)} \), should be inserted on the right-hand sides of Eq. \((A.2)\) and of the first line of Eq. \((A.4)\). The following sentence should be inserted after the sentence “\( C \) is an arbitrary … complex plane” below Eq. \((A.2)\): “In addition, \( \alpha(dt) \) is a \( dt \)-dependent normalization factor, which is properly fixed later.” The factor \( \sqrt{\frac{2\pi\theta dt}{m}} \) in the second line of Eq. \((A.4)\) should be deleted. The following sentences should be inserted after the phrase “where … Eq. \((3.7)\)” below Eq. \((A.4)\): “Here we have taken \( \alpha(dt) = \sqrt{\frac{2\pi\theta dt}{m}} \) so that both sides of Eq. \((A.4)\) correspond to each other in the vanishing limit of \( dt \). Then Eq. \((A.4)\) is reduced to \( |\psi(t + dt)\rangle = e^{-\frac{i}{\hbar} \hat{H} dt} |\psi(t)\rangle \).” The next sentence, “Thus we have found that … Eq. \((A.2)\),” below Eq. \((A.4)\) should be deleted with “Thus we have derived the Schrödinger equation and found that … Eq. \((A.2)\).” The following sentence should be added after the above replaced sentence: “Such a derivation of the Schrödinger equation is well known in the real action theory \((4)\).” Factors \( \frac{1}{\alpha(dt)^2} \), \( \frac{1}{\alpha(-dt)^2} \), and \( \frac{1}{\alpha(-dt)} \) should be inserted on the right-hand side of the equation in the second sentence of the last paragraph of the appendix, on the right-hand sides of Eqs \((A.5)\) and \((A.6)\), respectively. The second sentence below Eq. \((A.6)\), “Indeed, \( \hat{H}_B \) is given … \( \hat{H}^\dagger \),” should be replaced with “Indeed, we obtain the Schrödinger equation \( |B(t - dt)\rangle = e^{\frac{i}{\hbar} \hat{H}_B dt} |B(t)\rangle \), where \( \hat{H}_B \) is given … \( \hat{H}^\dagger \).”

References