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Theory including future not excluded: Formulation of complex action theory II

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In Ref. [1] we have found errata. They are composed of two parts: one part is for the body, which is also explained in our recently published book [2], while the other part is for the appendix, which is mainly a result of the corrections to Ref. [3]. They do not influence the result of the manuscript. Rather, the latter part provides us a new additional result: the Schrödinger equation described with the Hamiltonian \( \hat{H}_B \) has been derived for the future state \( |B(t)\rangle \) via the Feynman path integral in the complex action theory.

In the fifth line below Eq. (5.8), where \( f(D)D^\dagger \) should have been replaced with \( (f(D)f(D)^\dagger)^{-1} \), we have chosen \( f(D) \) such that \( (P^\dagger)^{-1}(f(D)f(D)^\dagger)^{-1}P^\dagger = F(\hat{H}^\dagger) \), which is rewritten as \( (f(D)f(D)^\dagger)^{-1} = F(D^\dagger) \). However, this relation does not stand, because the left-hand side is Hermitian, while the right-hand side is not Hermitian. Accordingly, the expression \( Q' = F(\hat{H}^\dagger)Q \) below Eq. (5.8), which was introduced based on the above relation, has to be corrected. In addition, the next statement, “\( F(\hat{H}^\dagger)Q \approx F(\hat{H}_B^\dagger)Q \) for the restricted subspace,” is not right. This is because, for any reasonable function \( \hat{h} \) and any state \( |\Lambda(t)\rangle = \sum_{i} a_{i}(t)|\lambda_{i}\rangle \) that obeys the Schrödinger equation \( i\hbar \frac{d}{dt}|\Lambda(t)\rangle = \hat{H}|\Lambda(t)\rangle \), the following relation holds for large \( t - T_A \):

\[
h(\hat{H})|\Lambda(t)\rangle \approx h(\hat{H}_B + i\Lambda A)|\Lambda(t)\rangle = \hat{h}(\hat{H}_B)|\Lambda(t)\rangle,
\]

where we have used the automatic Hermiticity mechanism and introduced \( |\Lambda(t)\rangle = \sum_{i \in A} |\lambda_{i}\rangle |\lambda_{i}\rangle \), \( \Lambda_A \equiv \sum_{i \in A} |\lambda_{i}\rangle |\lambda_{i}\rangle Q \), and another function \( \hat{h} \) such that \( \hat{h}(\text{Re} \lambda_{i}) = h(\text{Re} \lambda_{i} + iB) \).

Similarly, the statement “\( Q_2 = F(\hat{H}_B^\dagger)Q \) for the restricted subspace” given in Eq. (5.6) has to be corrected.

To correct the above points, on behalf of \( F(\text{Re} \lambda_{i}) = |b_{i}|^{2} \) and Eq. (5.6), we introduce functions \( G \) and \( \tilde{G} \) such that \( G(\text{Re} \lambda_{i} + iB) = \tilde{G}(\text{Re} \lambda_{i}) = b_{i} \), and express \( Q_2 \) as follows:

\[
Q_2 = \sum_{i \in A} |b_{i}|^{2} |\lambda_{i}\rangle_B B \langle \lambda_{i} | = \sum_{i \in A} G(\hat{H}_B + i\Lambda A)^\dagger |\lambda_{i}\rangle_B B \langle \lambda_{i} | G(\hat{H}_B + i\Lambda A) = \tilde{G}(\hat{H}_B)^\dagger Q \Lambda_A \tilde{G}(\hat{H}_B),
\]

where, in the second and third equalities, supposing that \( \text{Re} \lambda_{i} \)'s are not degenerate, we have used \( |\lambda_{i}\rangle_B = Q |\lambda_{i}\rangle \), and \( B |\lambda_{i}\rangle G(\text{Re} \lambda_{i} + iB) = |\lambda_{i}\rangle G(\hat{H}_B + i\Lambda A) \) for \( i \in A \).

We note that
\[ Q \Lambda_A = Q \sum_{i \in A} |\lambda_i\rangle \langle \lambda_i|_Q \] is Hermitian, and so is \( Q_2 \). Next we define \( Q' \) by \[ Q' \equiv G(\hat{H})^\dagger Q G(\hat{H}) = (P_{G^{-1}})^{-1}P_{G^{-1}}, \] where \( P_{G^{-1}} \equiv G(\hat{H})^{-1} P \) diagonalizes \( \hat{H} \) to \( (P_{G^{-1}})^{-1} \hat{H} P_{G^{-1}} = P^{-1} \hat{H} P = D \).

In addition, we introduce \( |\lambda^G_i\rangle \equiv G(\hat{H})^{-1} |\lambda_i\rangle \), so that \( |\lambda^G_i\rangle \) is \( Q' \) - orthogonal, i.e., orthogonal with regard to the proper inner product \( I_Q' \): \[ I_Q' (|\lambda^G_i\rangle,|\lambda^G_j\rangle) = G^{-1} (|\lambda_i\rangle |Q'\rangle |\lambda_j\rangle)^G^{-1} = \delta_{ij}. \]

We use the automatic Hermiticity mechanism for large \( t - T_A \). Then, since \( |A(t)\rangle \) behaves as \( |\tilde{A}(t)\rangle \equiv \sum_{i \in A} a_i(t) |\lambda_i\rangle \), \( Q' \) used in the normalized matrix element \( \langle Q' \rangle^A_{\lambda} \) is estimated in the subspace restricted by \( A \) as follows:

\[ Q' \simeq G(\hat{H}_{\text{eff}} + iB \Lambda_A)^\dagger Q \Lambda_A G(\hat{H}_{\text{eff}} + iB \Lambda_A) \quad \text{for the restricted subspace} \]
\[ = \tilde{G}(\hat{H}_{\text{eff}})^\dagger Q \Lambda_A \tilde{G}(\hat{H}_{\text{eff}}) \]
\[ = Q_2, \quad (2) \]

where in the last equality we have used Eq. (1). The three sentences “We first point out … replaced with \( |\tilde{A}(t)\rangle \)” below Eq. (5.8) should be replaced with the above argument.

A \( dt \) - dependent normalization factor, say \( \frac{1}{\alpha(dt)} \), should be inserted on the right-hand sides of Eq. (A.2) and of the first line of Eq. (A.4). The following sentence should be inserted after the sentence “\( C \) is an arbitrary … complex plane” below Eq. (A.2): “In addition, \( \alpha(dt) \) is a \( dt \) - dependent normalization factor, which is properly fixed later.” The factor \( \sqrt{2\pi \hbar dt/m} \) in the second line of Eq. (A.4) should be deleted. The following sentences should be inserted after the phrase “where … Eq. (3.7)” below Eq. (A.4): “Here we have taken \( \alpha(dt) = \sqrt{2\pi \hbar dt/m} \) so that both sides of Eq. (A.4) correspond to each other in the vanishing limit of \( dt \). Then Eq. (A.4) is reduced to \( |\psi(t + dt)\rangle = e^{-\frac{i}{\hbar} \hat{H} dt} |\psi(t)\rangle \).” The next sentence, “Thus we have found that … Eq. (A.2),” below Eq. (A.4) should be replaced with “Thus we have derived the Schrödinger equation and found that … Eq. (A.2).” The following sentence should be added after the above replaced sentence: “Such a derivation of the Schrödinger equation is well known in the real action theory [4].” Factors \( \frac{1}{\alpha(dt)^2}, \frac{1}{\alpha(-dt)^2}, \) and \( \frac{1}{\alpha(-dt)} \) should be inserted on the right-hand side of the equation in the second sentence of the last paragraph of the appendix, on the right-hand sides of Eqs (A.5) and (A.6), respectively. The second sentence below Eq. (A.6), “Indeed, \( \hat{H}_B \) is given … \hat{H}^\dagger \),” should be replaced with “Indeed, we obtain the Schrödinger equation \( |B(t - dt)\rangle = e^{\frac{i}{\hbar} \hat{H}_B dt} |B(t)\rangle \), where \( \hat{H}_B \) is given … \hat{H}^\dagger \).”

References