Theory including future not excluded

Formulation of complex action theory II (vol 2013, 023B04, 2013)

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In Ref. [1] we have found errata. They are composed of two parts: one part is for the body, which is also explained in our recently published book [2], while the other part is for the appendix, which is mainly a result of the corrections to Ref. [3]. They do not influence the result of the manuscript. Rather, the latter part provides us a new additional result: the Schrödinger equation described with the Hamiltonian \( \hat{H}_B \) has been derived for the future state \(|B(t)\rangle\) via the Feynman path integral in the complex action theory.

In the fifth line below Eq. (5.8), where \( f(D)\) should have been replaced with \( (f(D)f(D)^\dagger)^{-1} \), we have chosen \( f(D) \) such that \( (P^\dagger)^{-1}(f(D)f(D)^\dagger)^{-1}P^\dagger = F(\hat{H}_B^\dagger) \), which is rewritten as \( (f(D)f(D)^\dagger)^{-1} = F(D)^\dagger \). However, this relation does not stand, because the left-hand side is Hermitian, while the right-hand side is not Hermitian. Accordingly, the expression \( Q' = F(\hat{H}_B^\dagger)Q \) below Eq. (5.8), which was introduced based on the above relation, has to be corrected. In addition, the next statement, “\( F(\hat{H}_B^\dagger)Q \simeq F(\hat{H}_B^\dagger)Q \) for the restricted subspace,” is not right. This is because, for any reasonable function \( \hat{h} \) and any state \(|A(t)\rangle = \sum_i a_i(t)|\lambda_i\rangle \) that obeys the Schrödinger equation \( i\hbar \frac{d}{dt}|A(t)\rangle = \hat{H}|A(t)\rangle \), the following relation holds for large \( t \rightarrow T_A \): \( \hat{h}(\hat{H})|A(t)\rangle \simeq \hat{h}(\hat{H}_B + iB\Lambda_A)|A(t)\rangle \equiv \hat{h}(\hat{H}_{eff})|\tilde{A}(t)\rangle \), where we have used the automatic Hermiticity mechanism and introduced \(|\tilde{A}(t)\rangle \equiv \sum_{i\in A} |\lambda_i\rangle|\lambda_i\rangle \); \( \Lambda_A \equiv \sum_{i\in A} |\lambda_i\rangle\langle \lambda_i| \); and another function \( \hat{\tilde{h}} \) such that \( \hat{\tilde{h}}(\text{Re} \lambda_i) = h(\text{Re} \lambda_i + iB) \). Similarly, the statement “\( Q_2 = \tilde{F}(\hat{H}_{eff})\tilde{Q} \) for the restricted subspace” given in Eq. (5.6) has to be corrected.

To correct the above points, on behalf of \( F'(\text{Re} \lambda_i) = |b_i|^2 \) and Eq. (5.6), we introduce functions \( G \) and \( \tilde{G} \) such that \( G(\text{Re} \lambda_i + iB) = \tilde{G}(\text{Re} \lambda_i) = b_i \), and express \( Q_2 \) as follows:

\[
Q_2 = \sum_{i\in A} |b_i|^2 |\lambda_i\rangle_B \langle \lambda_i| = \sum_{i\in A} G(\hat{H}_{eff} + iB\Lambda_A)^\dagger |\lambda_i\rangle_B \langle \lambda_i|G(\hat{H}_{eff} + iB\Lambda_A) = \tilde{G}(\hat{H}_{eff})\tilde{Q} \Lambda_A \tilde{G}(\hat{H}_{eff}),
\]

where, in the second and third equalities, supposing that \( \text{Re} \lambda_i 's \) are not degenerate, we have used \(|\lambda_i\rangle_B = Q|\lambda_i\rangle\), and \( b(\lambda_i)|G(\text{Re} \lambda_i + iB) = b(\lambda_i)|G(\hat{H}_{eff} + iB\Lambda_A) \) for \( i \in A \). We note that...
$Q A = Q \sum_{i \in A} |\lambda_i\rangle \langle \lambda_i|_Q$ is Hermitian, and so is $Q_2$. Next we define $Q'$ by $Q' \equiv G(\hat{H})^* Q G(\hat{H}) = (P_{G^{-1}})^{-1} P_{G^{-1}}$, where $P_{G^{-1}} \equiv G(\hat{H})^{-1} P$ diagonalizes $\hat{H}$: $(P_{G^{-1}})^{-1} \hat{H} P_{G^{-1}} = P^{-1} \hat{H} P = D$. In addition, we introduce $|\lambda_i\rangle^{G^{-1}} \equiv G(\hat{H})^{-1} |\lambda_i\rangle$, so that $|\lambda_i\rangle^{G^{-1}}$ is $Q'$-orthogonal, i.e., orthogonal with respect to the proper inner product $I_Q$: $I_Q (|\lambda_i\rangle^{G^{-1}}, |\lambda_j\rangle^{G^{-1}}) \equiv G^{-1} (|\lambda_i\rangle |\lambda_j\rangle^{G^{-1}}) = \delta_{ij}$.

We use the automatic Hermiticity mechanism for large $t - T_A$. Then, since $|\tilde{A}(t)\rangle \equiv \sum_{i \in A} a_i(t) |\lambda_i\rangle$, $Q'$ used in the normalized matrix element $\langle \tilde{O} | A \tilde{Q} | Q \rangle$ is estimated in the subspace restricted by $A$ as follows:

$$Q' \simeq G(\hat{H}_{eff} + iB A) G(\hat{H}_{eff} + iB A)$$

for the restricted subspace

$$= \tilde{G}(\hat{H}_{eff}) G(\hat{H}_{eff})$$

where in the last equality we have used Eq. (1). The three sentences “We first point out … replaced with $|\tilde{A}(t)\rangle$” below Eq. (5.8) should be replaced with the above argument.

A $dt$-dependent normalization factor, say $\frac{1}{\alpha(dt)}$, should be inserted on the right-hand sides of Eq. (A.2) and of the first line of Eq. (A.4). The following sentence should be inserted after the sentence “$C$ is an arbitrary … complex plane” below Eq. (A.2): “In addition, $\alpha(dt)$ is a $dt$-dependent normalization factor, which is properly fixed later.” The factor $\sqrt{\frac{2\pi i \hbar dt}{m}}$ in the second line of Eq. (A.4) should be deleted. The following sentences should be inserted after the phrase “where … Eq. (3.7)” below Eq. (A.4): “Here we have taken $\alpha(dt) = \sqrt{\frac{2\pi i \hbar dt}{m}}$ so that both sides of Eq. (A.4) correspond to each other in the vanishing limit of $dt$. Then Eq. (A.4) is reduced to $|\psi(t + dt)\rangle = e^{-\frac{i}{\hbar} \tilde{H} dt} |\psi(t)\rangle$.” The next sentence, “Thus we have found that … Eq. (A.2),” below Eq. (A.4) should be replaced with “Thus we have derived the Schrödinger equation and found that … Eq.(A.2).” The following sentence should be added after the above replaced sentence: “Such a derivation of the Schrödinger equation is well known in the real action theory [4].” Factors $\frac{1}{\alpha(dt)^2}$, $\frac{1}{\alpha(-dt)}$, and $\frac{1}{\alpha(-dt)}$ should be inserted on the right-hand side of the equation in the second sentence of the last paragraph of the appendix, on the right-hand sides of Eqs (A.5) and (A.6), respectively. The second sentence below Eq. (A.6), “Indeed, $\tilde{H}_B$ is given … $\tilde{H}_B^\dagger$,” should be replaced with “Indeed, we obtain the Schrödinger equation $|B(t - dt)\rangle = e^{\frac{i}{\hbar} \tilde{H}_B dt} |B(t)\rangle$, where $\tilde{H}_B$ is given … $\tilde{H}_B^\dagger$.”

References