
Theory including future not excluded: Formulation of complex action theory II

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In Ref. [1] we have found errata. They are composed of two parts: one part is for the body, which is also explained in our recently published book [2], while the other part is for the appendix, which is mainly a result of the corrections to Ref. [3]. They do not influence the result of the manuscript. Rather, the latter part provides us a new additional result: the Schrödinger equation described with the Hamiltonian \(\hat{H}_B\) has been derived for the future state \(|B(t)\rangle\) via the Feynman path integral in the complex action theory.

In the fifth line below Eq. (5.8), where \(f(D)f(D)\dagger\) should have been replaced with \((f(D)f(D)\dagger)^{-1}\), we have chosen \(f(D)\) such that \((P\dagger)^{-1}(f(D)f(D)\dagger)^{-1}P = F(\hat{H}\dagger)\), which is rewritten as \((f(D)f(D)\dagger)^{-1} = F(D)\). However, this relation does not stand, because the left-hand side is Hermitian, while the right-hand side is not Hermitian. Accordingly, the expression \(Q' = F(\hat{H}\dagger)Q\) below Eq. (5.8), which was introduced based on the above relation, has to be corrected. In addition, the next statement, “\(F(\hat{H}\dagger)Q \simeq F(\hat{H}'\dagger)Q\) for the restricted subspace,” is not right. This is because, for any reasonable function \(h\) and any state \(\langle \lambda(t) \rangle = \sum_i^A q_i(t)|\lambda_i\rangle\) that obeys the Schrödinger equation \(ih\frac{d}{dt}|\lambda(t)\rangle = \hat{H}|\lambda(t)\rangle\), the following relation holds for large \(t - T_A\):

\(h(\hat{H})|\lambda(t)\rangle \simeq h(\hat{H}_{\text{eff}'} + iB\Lambda_A)|\lambda(t)\rangle \equiv \hat{h}(\hat{H}_{\text{eff}})|\lambda(t)\rangle\),

where we have used the automatic Hermiticity mechanism and introduced \(\hat{A}(t) \equiv \sum_{i \in A} q_i(t)|\lambda_i\rangle\), \(\Lambda_A \equiv \sum_{i \in A} |\lambda_i\rangle\langle\lambda_i|\), and another function \(\hat{h}\) such that \(\hat{h}(\text{Re}\lambda_i) = h(\text{Re}\lambda_i + iB)\). Similarly, the statement “\(Q_2 = F(\hat{H}_{\text{eff}}')Q\) for the restricted subspace” given in Eq. (5.6) has to be corrected.

To correct the above points, on behalf of \(F(\text{Re}\lambda_i) = |b_i|^2\) and Eq. (5.6), we introduce functions \(G\) and \(\tilde{G}\) such that \(G(\text{Re}\lambda_i + iB) = \tilde{G}(\text{Re}\lambda_i) = b_i\), and express \(Q_2\) as follows:

\[Q_2 = \sum_{i \in A} |b_i|^2 |\lambda_i\rangle_B \langle \lambda_i| = \sum_{i \in A} \tilde{G}(\hat{H}_{\text{eff}} + iB\Lambda_A)^\dagger |\lambda_i\rangle_B \langle \lambda_i| G(\hat{H}_{\text{eff}} + iB\Lambda_A) = \tilde{G}(\hat{H}_{\text{eff}})^\dagger Q A \Lambda_A \tilde{G}(\hat{H}_{\text{eff}}),\]

where, in the second and third equalities, supposing that \(\text{Re}\lambda_i\)'s are not degenerate, we have used \(|\lambda_i\rangle_B = Q|\lambda_i\rangle\), and \(b(\lambda_i)G(\text{Re}\lambda_i + iB) = b(\lambda_i)G(\hat{H}_{\text{eff}} + iB\Lambda_A)\) for \(i \in A\). We note that
$Q \Lambda_A = Q \sum_{i \in A} |\lambda_i\rangle \langle \lambda_i|_Q$ is Hermitian, and so is $Q_2$. Next we define $Q'$ by $Q' \equiv G(\hat{H})^\dagger QG(\hat{H}) = (P_{G^{-1}})^{-1}P_{G^{-1}}^{-1}$, where $P_{G^{-1}} \equiv G(\hat{H})^{-1}P$ diagonalizes $\hat{H}$: $(P_{G^{-1}})^{-1} \hat{H}P_{G^{-1}} = P^{-1} \hat{H}P = D$. In addition, we introduce $|\lambda_i\rangle^{G^{-1}} \equiv G(\hat{H})^{-1}|\lambda_i\rangle$, so that $|\lambda_i\rangle^{G^{-1}}$ is $Q'$-orthogonal, i.e., orthogonal with regard to the proper inner product $I_Q'$: $I_Q'(|\lambda_i\rangle^{G^{-1}},|\lambda_j\rangle^{G^{-1}}) \equiv G^{-1}(|\lambda_i\rangle^{Q'}|\lambda_j\rangle^{Q'}) = \delta_{ij}$.

We use the automatic Hermiticity mechanism for large $t - T_A$. Then, since $|A(t)\rangle$ behaves as $|\tilde{A}(t)\rangle \equiv \sum_{i \in A} a_i(t)|\lambda_i\rangle$, $Q'$ used in the normalized matrix element $\langle O \rangle^{Ad}_{Q'}$ is estimated in the subspace restricted by $A$ as follows:

$$
Q' \simeq G(\dot{\hat{H}}_{\text{eff}} + iB\Lambda_A)^\dagger Q \Lambda_A G(\dot{\hat{H}}_{\text{eff}} + iB\Lambda_A) \quad \text{for the restricted subspace}
$$

$$
= \tilde{G}(\dot{\hat{H}}_{\text{eff}})^\dagger Q \Lambda_A \tilde{G}(\dot{\hat{H}}_{\text{eff}})
$$

$$
= Q_2,
$$

(2)

where in the last equality we have used Eq. (1). The three sentences “We first point out … replaced with $|\tilde{A}(t)\rangle$” below Eq. (5.8) should be replaced with the above argument.

A $dt$-dependent normalization factor, say $\frac{1}{\alpha(dt)}$, should be inserted on the right-hand sides of Eq. (A.2) and of the first line of Eq. (A.4). The following sentence should be inserted after the sentence “$C$ is an arbitrary … complex plane” below Eq. (A.2): “In addition, $\alpha(dt)$ is a $dt$-dependent normalization factor, which is properly fixed later.” The factor $\sqrt{\frac{2\pi i\hbar dt}{m}}$ in the second line of Eq. (A.4) should be deleted. The following sentences should be inserted after the phrase “where … Eq. (3.7)” below Eq. (A.4): “Here we have taken $\alpha(dt) = \sqrt{\frac{2\pi i\hbar dt}{m}}$ so that both sides of Eq. (A.4) correspond to each other in the vanishing limit of $dt$. Then Eq. (A.4) is reduced to $|\psi(t + dt)\rangle = e^{-\frac{1}{\hbar}\dot{\hat{H}}dt}|\psi(t)\rangle$. “The next sentence, “Thus we have found that … Eq. (A.2),” below Eq. (A.4) should be replaced with “Thus we have derived the Schrödinger equation and found that … Eq. (A.2).” The following sentence should be added after the above replaced sentence: “Such a derivation of the Schrödinger equation is well known in the real action theory [4].” Factors $\frac{1}{\alpha(dt)^2}$, $\frac{1}{\alpha(-dt)^2}$, and $\frac{1}{\alpha(dt)}$ should be inserted on the right-hand side of the equation in the second sentence of the last paragraph of the appendix, on the right-hand sides of Eqs (A.5) and (A.6), respectively. The second sentence below Eq. (A.6), “Indeed, $\hat{H}_B$ is given … $\hat{H}^\dagger$, should be replaced with “Indeed, we obtain the Schrödinger equation $|B(t - dt)\rangle = e^{\frac{\hbar}{i}\hat{H}_Bdt}|B(t)\rangle$, where $\hat{H}_B$ is given … $\hat{H}^\dagger$. “

References