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Abstract

In this paper we study the functioning of representative democracy when politicians are better informed than the electorate about conditions relevant for policy choice. We consider a model with two states of the world. The distribution of voters' preferred policies shifts with the state. The two candidates are both completely office-motivated but differ in state-dependent quality. Voters have some information about the state but candidates are better informed. If voters' information is unknown to the candidates when they take positions and sufficiently accurate then candidates will, in refined equilibrium, reveal their information by converging to the most likely median. If voters' information is not sufficiently accurate then there is polarization and the candidates' information is not revealed to the voters. We also show that if voters' information is known to the candidates then they will never reveal their information to the voters. The candidates will either pander by converging on the median that is most likely given only the voters' information or be polarized. With respect to welfare, if voters are well informed then they all prefer that their information is unknown to the candidates. However, if voters are not well informed then it is the other way around, all voters prefer that their information is known by the candidates.

Keywords: Electoral Competition, Uncertainty, Information, Candidate Quality.
JEL Classification: D72, D82.

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1 Introduction

It is a reasonable assumption that politicians are generally better informed than the electorate about conditions relevant for policy choice. They usually have staff to help them receive and process information and sometimes have access to information that is not public, for example information related to national security. Furthermore, they have much stronger incentives than voters to be well informed because their careers depend on how they do as policy makers. Therefore it is highly relevant to study the consequences of this informational asymmetry for the functioning of representative democracy. For example, will politicians' policy positions reveal their information to the electorate such that voters can make a better informed choice in the voting booth? And, if so, will the revealing policy positions be optimal for the voters?

In this paper we set up and analyze a game theoretic model of electoral competition where candidates are better informed than voters. We consider an election with one issue, a continuum of voters, and two candidates who simultaneously announce credible positions. There are two states of the world and the distribution of voters' preferred policies shifts with the state. The candidates are purely office-motivated, their only objective is to maximize the probability of winning. Both candidates and voters receive a signal about the true state, but the candidates' signal is more informative. The candidates' signal is unknown to the voters. With respect to the signal of the voters we consider both the case where it is unknown to candidates and the case where candidates observe it. When we assume that the voters' signal is unknown to the candidates it should not be taken to mean that candidates do not receive the information of the voters, after all they are voters themselves. Rather, it should be interpreted as a situation where candidates take positions relatively early in the campaign where there is considerable uncertainty about what information voters will have on election day.

The voters do not only care about policy, they also care about candidate quality. One candidate has a quality advantage in one state and the other candidate has a quality advantage in the other state, for example because they differ in experience and skills. Furthermore, there is a stochastic element in voters' evaluation of candidates. Suppose, for example, that the two candidates have announced the same position and that the voters know the true state. Then the high quality candidate wins with a probability that is greater than one half but smaller than one. It is the difference in state-dependent quality that creates the central strategic aspect of the model: The candidate with a quality advantage in the state that is most likely given the candidates' signal has an incentive to try to reveal the signal while the candidate with a quality disadvantage has an incentive to try not to reveal it.

We solve the model for Perfect Bayesian Equilibria satisfying some refinement criteria. For equilibria where candidates' positions reveal their information to the voters (*revealing equilibria*) our refinement criterion is based on the notion of unprejudiced beliefs (Bagwell and Ramey (1991)) which was developed precisely for a signalling game with two senders. For *non-revealing equilibria* we introduce a monotonicity condition on voters' beliefs (after showing that the Intuitive Criterion (Cho and Kreps (1987)) does not have any bite in this model). Our equilibrium results are informally summarized in the table below. The two rows represent the two different cases we consider (voters' signal unknown or known to candidates). The two columns represent different levels of accuracy of the voters' information. In the first row the voters' signal is "quite informative", in the second row it is "not too informative". These informal characterizations will, of course, be made precise later on.

	Voters' signal "quite informative"	**Voters' signal "not too informative"**
Case 1: Voters' signal unknown to candidates	*Revealing Eq. (convergence to most likely median)*	*Non-Revealing Eq. (polarization)*
Case 2: Voters' signal known to candidates	*Non-Revealing Eq. (pandering)*	*Non-Revealing Eq. (pandering or polarization)*

In the first case, candidates reveal their information if the signal of the voters is sufficiently informative. They do so by converging to the most likely median (note that, since candidates are better informed than voters, the most likely median given the candidates' signal is also the most most likely median given both signals). This is the outcome we would get if the candidates' information was directly observable to the voters. If voters' information is not sufficiently informative then candidates will be polarized, more precisely each candidate will always announce the median position of the state in which he has a quality advantage. Thus the voters cannot infer the candidates' information. The intuition behind the results is as follows. Each candidate can either "tell the truth" by announcing the median of the most likely state or "lie" by announcing the median of the other state (here we disregard all other positions, in our analysis we of course consider all positions). The candidate who has a quality advantage in the most likely state has no incentive to lie, so he will always tell the truth. The disadvantaged candidate clearly has an incentive to lie. On the other hand, voters have some information of their own. Thus, even though they cannot infer what the candidates' information is if the disadvantaged candidate lies, they are more likely to believe the advantaged candidate. And then he will have both a quality and a policy advantage. So clearly there is a trade-off for the disadvantaged candidate. When the voters' signal is more informative then it is relatively more expensive for the disadvantaged candidate to lie. This is why we get revelation when voters' information is "quite informative" and non-revelation when it is "not too informative".

In the second case, the candidates' information is never revealed to the voters in (refined) equilibrium. There always exists an equilibrium where both candidates announce the median that is most likely given only the voters' signal. This is what we call pandering. When the voters' signal is not too accurate then polarization is also possible. The reason why revelation is not possible in this case is the following. Suppose both candidates tell the truth in equilibrium, i.e., announce the most likely median. When candidates know voters' information then their expectation about voters' belief if they deviate does not depend on their own information. Therefore the expected belief of the voters if the disadvantaged candidate lies is the same for both values of the candidates' signal. This belief must, of course, put a probability greater than or equal to one half on one of the states. And then it follows that when the candidate with a quality advantage in this state is disadvantaged given the candidates' signal, then he will lie. Because if he tells the truth his probability of winning will be less than one half, if he lies it will be at least one half. So we cannot have a revealing equilibrium in this case.

Following the equilibrium analysis of the two cases we compare them with respect to the welfare of the voters, more precisely with respect to each voter's ex ante expected utility. We show that when voters' information is quite accurate then all voters are better off in Case 1, i.e., when candidates do not observe the signal of the voters before taking positions. When the information of the voters is not too accurate then it is the other way around, all voters are better off in Case 2. This result follows from the equilibrium analysis and the following Pareto ranking of strategy profiles: Revelation (convergence to the most likely median) dominates pandering which dominates polarization. This ranking of strategy profiles is quite intuitive. When the candidates converge on the most likely median voters can infer candidates' information and the candidates' positions are optimal to the voters (from an ex ante perspective). When candidates pander voters cannot infer the information of the candidates but the candidates' choice of positions are at least responsive to the signal of the voters. In the polarization profile voters again cannot infer candidates' information and the candidates' positions does not reflect any of the information in the game.

The paper is organized as follows: In Section 2 we briefly review some related literature. Then, in Section 3, we set up the model. Section 4 and 5 contain our equilibrium results for the two cases and in Section 6 we compare the cases with respect to welfare. Finally, we discuss and conclude in Section 7.

2 Related Literature

Two immediately related papers are Schultz (1996) and Martinelli (2001). They ask the same general question as we do but they make different assumptions. In Schultz (1996) candidates are policy-motivated and fully informed about the state of the world. Voters are uninformed. Thus voters only receive information from the candidates' credible positions. There is revelation in (refined) equilibrium if at least one of the candidates has policy preferences that are sufficiently similar to the preferences of the median voter. In any revealing equilibrium there is convergence to the median policy of the true state of the world.

Martinelli (2001) considers a model where both candidates and voters receive private information about the state of the world but candidates are better informed than voters. The main result is that a revealing equilibrium always exists. If candidates are completely policy-motivated then they do not converge in revealing equilibria. But if office-motivation is sufficiently strong then there is convergence.

Several other papers study models in which politicians are better informed than the electorate. In both Alesina and Cukierman (1990) and Harrington (1993) policy is decided after the election and voters are uncertain about the candidates' policy preferences. Therefore, earlier policy decisions by the incumbent reveal information to the voters about what he will do if reelected. That induces the incumbent (who wants to be reelected) to distort his policy choice. In Alesina and Cukierman he does so by choosing a noisy policy instrument, in Harrington it is done by choosing a policy that is more likely to be well received.

Roemer (1994) considers a model where two policy motivated candidates (parties) are better informed about how the economy works than the electorate. Candidates announce both policies and theories of the economy, voters update their beliefs based only on announced theories. In equilibrium there is convergence to the median with respect to policy but divergence with respect to theory.

In Cukierman and Tommasi (1998) the incumbent is better informed than voters about how different policies map into outcomes. Voters update beliefs based on the incumbents (credible) policy announcement and votes for reelection if his announcement is preferred to the expected policy of the challenger. The main insight is that relatively extreme right wing policies are more likely to be implemented by a left wing incumbent (and vice versa) because of credibility issues.

Our model is also related to the literature on candidate quality/valence. Among the contributions to this literature are Ansolabehere and Snyder (2000), Groseclose (2001) and Aragones and Palfrey (2002, 2005). These papers all analyze models of electoral competition where candidates differ in quality such that if they announce sufficiently similar policy positions then each voter votes for the candidate of highest quality. There is no uncertainty about who the high quality candidate is. This is fundamentally different from our model where no candidate has an ex ante quality advantage because quality is state-dependent.

Krasa and Polborn (2009a, 2009b) analyze models of electoral competition where candidates' abilities are policy dependent (see also Schofield (2003) for a model with an additive, policy dependent valence term). For example, some candidates may be better at running a small government while other candidates are better at running a big government. When candidates have different abilities it is shown that, even though candidates are completely office motivated, there will be policy divergence in equilibrium because candidates play to their own strengths. The idea of policy dependent candidate abilities is obviously related to the idea of state-dependent candidate quality that we present in this paper. However, the ideas are also clearly distinct. In our model voters' evaluation of candidates' characteristics does not directly depend on their policy positions. If, for example, voters know the true state and candidates' positions are close then the choice of the voters does not depend on where the candidates' positions are in the policy space.

3 The Model

We consider a one issue election. The policy space X is some closed interval (bounded or unbounded) on the real axis. There are two purely office-motivated candidates, i.e., their only objective is to maximize the probability of winning. The candidates simultaneously announce credible policy positions before the election.

The electorate consists of a continuum of voters (indexed by i). The voters have utility functions over the policy space. The utility functions depend on the state of the world ω which can be either L or H. The utility function of voter i is

$$u_i(x|\omega) = -|x - x_i^*(\omega)|,$$

where $x_i^*(\omega)$ is the preferred policy of voter i in state ω. The preferred policies of the voters in state L are distributed according to some distribution function F_L with unique median $x_{m_L}^*$. We assume that, for each voter i,

$$x_i^*(H) = x_i^*(L) + D,$$

where $D > 0$. Thus the distribution of preferred policies in state H is given by F_H defined by

$$F_H(x) = F_L(x - D).$$

Obviously, the median in state H is $x_{m_H}^* = x_{m_L}^* + D$.

Besides policy, voters also care about candidate quality which is state-dependent. One candidate, *Candidate L*, has a quality advantage in the L state while the other candidate, *Candidate H*, has a quality advantage (of equal size) in the H state.

On top of this there is a symmetric stochastic element to each voter's candidate preference. These two features are modelled the following way. Suppose Candidate L has announced the policy x^L and that Candidate H has announced x^H. Then voter i's utility of voting for Candidate L is

$$U_i((L, x^L)|\omega) = \left\{ \begin{array}{ll} u_i(x^L|L) + \gamma + \delta & \text{if } \omega = L \\ u_i(x^L|H) + \delta & \text{if } \omega = H \end{array} \right\},$$

where $\gamma > 0$ is a parameter and, for some parameter $\sigma > 0$, δ is drawn from the uniform distribution on the interval $[-\frac{1}{2\sigma}, \frac{1}{2\sigma}]$. Note that the realized value of δ is the same for all voters, independent of the state of the world, and unknown to the candidates when they announce positions. Voter i's utility of voting for Candidate H is

$$U_i((H, x^H)|\omega) = \left\{ \begin{array}{ll} u_i(x^H|L) & \text{if } \omega = L \\ u_i(x^H|H) + \gamma & \text{if } \omega = H \end{array} \right\}.$$

Each voter votes for the candidate providing the highest expected utility. So if voter i believes that the probability of state L is μ_L then he votes for Candidate L if and only if

$$\mu_L(u_i(x^L|L)+\gamma+\delta)+(1-\mu_L)(u_i(x^L|H)+\delta) > \mu_L u_i(x^H|L)+(1-\mu_L)(u_i(x^H|H)+\gamma).$$

This is equivalent to

$$\delta > \mu_L(u_i(x^H|L) - u_i(x^L|L) - \gamma) + (1 - \mu_L)(u_i(x^H|H) - u_i(x^L|H) + \gamma).$$

If we plug in the policy utility function of the voter then this inequality becomes

$$\begin{aligned} \delta > \mu_L(|x^L - x_i^*(L)| - |x^H - x_i^*(L)| - \gamma) \\ + (1 - \mu_L)(|x^L - x_i^*(H)| - |x^H - x_i^*(H)| + \gamma). \quad (1) \end{aligned}$$

Both candidates and voters receive signals that are correlated with the state of the world. The two candidates both receive the signal $\omega^C \in \{l, h\}$ and the voters all receive the signal $\omega^V \in \{l, h\}$. The signals are distributed according to

$$\Pr(\omega^C = l|L) = \Pr(\omega^C = h|H) = \theta^C$$

and

$$\Pr(\omega^V = l|L) = \Pr(\omega^V = h|H) = \theta^V,$$

where $\frac{1}{2} < \theta^V < \theta^C \leq 1$. Thus the candidates' signal is more informative than the signal of the voters. We also assume that the signals are independent conditional on the state of the world. So we have, for example,

$$\Pr(\omega^C = l, \omega^V = l|L) = \Pr(\omega^C = l|L) \cdot \Pr(\omega^V = l|L) = \theta^C\theta^V.$$

The assumption about conditional independence implies that the candidates' signal only gives them information about the voters' signal because it gives them information about the state of the world.

Both candidates and voters have the prior $\Pr(L) = \Pr(H) = \frac{1}{2}$. So if candidates and voters use only their own signal to update their beliefs about the state of the world then candidates' beliefs are given by

$$\Pr(L|\omega^C = l) = \Pr(H|\omega^C = h) = \theta^C$$

and voters' beliefs are given by

$$\Pr(L|\omega^V = l) = \Pr(H|\omega^V = h) = \theta^V.$$

If updating is based on both signals then beliefs are given by

$$\Pr(L|\omega^C = l, \omega^V = l) = \Pr(H|\omega^C = h, \omega^V = h) = \frac{\theta^C\theta^V}{\theta^C\theta^V + (1-\theta^C)(1-\theta^V)}$$

and

$$\Pr(L|\omega^C = l, \omega^V = h) = \Pr(H|\omega^C = h, \omega^V = l) = \frac{\theta^C(1-\theta^V)}{\theta^C(1-\theta^V) + (1-\theta^C)\theta^V}.$$

We assume that the signal of the candidates is unknown to the voters. With respect to the voters' signal we consider both the case where it is unknown to candidates and the case where candidates observe it. When we assume that the voters' signal is unknown to the candidates it should not be taken to mean that candidates do not receive the information that voters do. Because candidates are, of course, also voters. Instead it should be interpreted as a situation where candidates have to announce policy positions relatively early in the campaign and that they do not know what information voters will receive between then and election day.

Throughout the paper we assume that

$$\gamma + D < \frac{1}{2\sigma}.$$

This condition implies that if the distance between the candidates positions is at most D then, no matter what voters believe about the state, each candidate has a strictly positive probability of winning the election. It simplifies our analysis considerably because it ensures that, in all situations we need to consider, the realization of δ matters. Alternatively we could have assumed that δ is drawn from some distribution with full support on $\mathbb{R}$, for example the normal distribution. But this would only complicate the analysis of the model without adding substantial insight.

We complete this section by presenting the timelines of the two different election games we consider. When the signal of the voters is unknown to the candidates the timeline is as follows:

1. The candidates receive the signal ω^C and then simultaneously announce policy positions.

2. The voters observe the candidates' positions and receive the signal ω^V. The value of δ is realized. The voters cast their votes.

3. The winning candidate enacts his announced position.

When the signal of the voters is known to the candidates the timeline is:

1. The candidates receive the signal ω^C and *candidates and voters* receive the signal ω^V. The candidates simultaneously announce policy positions.

2. The value of δ is realized. The voters cast their votes.

3. The winning candidate enacts his announced position.

The first situation we refer to as Case 1, the second situation we refer to as Case 2.

4 Analysis of Case 1

4.1 Equilibrium

A strategy profile for the candidates consists of a pair of policy positions for each candidate, one position for each value of the candidates' signal. Thus it can be written

$$(x^L(l), x^L(h)), (x^H(l), x^H(h)),$$

where $x^i(\omega^C)$ is the position of Candidate i when the value of the signal is ω^C.

The belief functions of the voters depend on the candidates' positions and the voters' signal. We make the assumption that all voters have the same belief function. The voters' belief about the probability of state L is written

$$\mu_L(x^L, x^H, \omega^V).$$

Each candidate's objective is to maximize the probability of winning for each value of ω^C given the other candidate's strategy, the belief function of the voters, the distribution of the voters' signal and the distribution of δ. The following lemma shows that the median voter decides the outcome of the election.

Lemma 4.1 (The Median Voter Decides the Outcome) *Suppose that, given the candidates' positions, the voters' signal, and the realization of δ, the median voter strictly prefers Candidate L (H). Then a strict majority of voters strictly prefers Candidate L (H).*

Proof. Let x^L and x^H be the positions of the candidates. Suppose the median voter strictly prefers Candidate L, i.e.,

$$\delta > \mu_L(|x^L - x^*_{m_L}| - |x^H - x^*_{m_L}| - \gamma) + (1-\mu_L)(|x^L - x^*_{m_H}| - |x^H - x^*_{m_H}| + \gamma).$$

We then have to show that, for each voter i in a strict majority,

$$\delta > \mu_L(|x^L - x^*_i(L)| - |x^H - x^*_i(L)| - \gamma) + (1-\mu_L)(|x^L - x^*_i(H)| - |x^H - x^*_i(H)| + \gamma).$$

Suppose $x^L \leq x^H$ (the other case is analogous). It then suffices to show that the inequality above holds for all voters i with $x^*_i(L) \leq x^*_{m_L}$ (this is only a weak majority but by a simple continuity argument the inequality also holds for voters with a preferred point slightly to the right of the median).

Pick a voter i with $x^*_i(L) \leq x^*_{m_L}$. The inequality is satisfied if

$$|x^L - x^*_i(L)| - |x^H - x^*_i(L)| \leq |x^L - x^*_{m_L}| - |x^H - x^*_{m_L}|$$

and

$$|x^L - x^*_i(H)| - |x^H - x^*_i(H)| \leq |x^L - x^*_{m_H}| - |x^H - x^*_{m_H}|.$$

These inequalities are straightforward to verify.

The proof of the statement when the median voter strictly prefers Candidate H is analogous.□

By the lemma and inequality (1) it follows that if $(x^H(l), x^H(h))$ is the strategy of Candidate H then the problem of Candidate L when the candidates have received the signal ω^C is

$$\max_x \Pr_{(\omega^V|\omega^C),\delta}[\delta > \mu_L(x, x^H(\omega^C), \omega^V)((|x - x^*_{m_L}| - |x^H(\omega^C) - x^*_{m_L}| - \gamma) + \\ (1 - \mu_L(x, x^H(\omega^C), \omega^V))(|x - x^*_{m_H}| - |x^H(\omega^C) - x^*_{m_H}| + \gamma)].$$

Similarly, if $(x^L(l), x^L(h))$ is the strategy of Candidate L then the problem of Candidate H when the candidates have received the signal ω^C is

$$\max_x \Pr_{(\omega^V|\omega^C),\delta}[\delta < \mu_L(x^L(\omega^C), x, \omega^V)((|x^L(\omega^C) - x^*_{m_L}| - |x - x^*_{m_L}| - \gamma) + \\ (1 - \mu_L(x^L(\omega^C), x, \omega^V))(|x^L(\omega^C) - x^*_{m_H}| - |x - x^*_{m_H}| + \gamma)].$$

Then we are ready to define our notion of equilibrium. It is that of Perfect Bayesian Equilibrium with the extra condition that all voters have the same belief function.

Definition 4.2 (Equilibrium) *An equilibrium consists of candidate strategies,*

$$(\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h)),$$

and a voter belief function about the probability of state L,

$$\hat{\mu}_L(x^L, x^H, \omega^V),$$

such that the following two conditions hold.

1. *For each value of the candidates' signal, each candidate's position maximizes his probability of winning given the other candidates position, the belief function of the voters, the distribution of the voters' signal, and the distribution of δ.*

2. *The belief function is consistent with Bayes' rule on the equilibrium path. I.e., if $\hat{x}^L(l) \neq \hat{x}^L(h)$ or $\hat{x}^H(l) \neq \hat{x}^H(h)$ then, for all ω^C, ω^V,*

$$\mu_L(\hat{x}^L(\omega^C), \hat{x}^H(\omega^C), \omega^V) = \Pr(L|\omega^C, \omega^V).$$

And if $\hat{x}^L(l) = \hat{x}^L(h) = \hat{x}^L$ and $\hat{x}^H(l) = \hat{x}^H(h) = \hat{x}^H$ then, for all ω^V,

$$\mu_L(\hat{x}^L, \hat{x}^H, \omega^V) = \Pr(L|\omega^V).$$

An equilibrium where the positions of the candidates reveal their information to the voters, i.e., at least one of the candidates announces different positions for different values of their signal, is called a *revealing equilibrium*. An equilibrium where each candidate announces the same position for both values of their signal is called a *non-revealing equilibrium*.

4.2 Revealing Equilibria

We will first introduce a refinement condition that puts restrictions on out-of-equilibrium beliefs in revealing equilibria. Essentially we use the concept of unprejudiced beliefs introduced by Bagwell and Ramey (1991). This concept has previously been used in a model of electoral competition by Schultz (1996).

The idea behind the concept of unprejudiced beliefs is the following: Suppose $(\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h))$ are the candidate strategies in some revealing equilibrium and that voters observe some disequilibrium-pair (x^L, x^H). Then voters consider, for each value of ω^C, whether it takes only one or both candidates to deviate in order to generate (x^L, x^H). If this is different for the two values of ω^C then voters will assume that the true value is the one requiring only one deviation to generate (x^L, x^H). For example, if the disequilibrium-pair satisfies $x^L = \hat{x}^L(l)$,

$x^L \neq \hat{x}^L(h)$, and $x^H \neq \hat{x}^H(h)$ then voters will assume that $\omega^C = l$. When voters have formed their belief about ω^C they simply use Bayesian updating on this and their own signal to form their belief about the state of the world. In the model of Bagwell and Remy (and the model of Schultz) senders are fully informed and the receiver has no private information. So the Bayesian updating part is our (straightforward) addition to the concept of unprejudiced beliefs.

The precise definition of unprejudiced equilibria, i.e., equilibria with unprejudiced beliefs, is given below. It basically says that if one candidate's equilibrium strategy is revealing and the other candidate deviates to an out-of-equilibrium position then voters will believe the non-deviating candidate. This is a slightly different but equivalent formulation of the idea described above.

Definition 4.3 (Unprejudiced Revealing Equilibrium) *Consider a revealing equilibrium* $(\hat{x}^L(l), \hat{x}^L(h))$, $(\hat{x}^H(l), \hat{x}^H(h))$, $\hat{\mu}_L$. *It is* unprejudiced *if the following two conditions are satisfied.*

1. *Suppose* $\hat{x}^L(l) \neq \hat{x}^L(h)$. *Then, for all* $x \neq \hat{x}^H(l), \hat{x}^H(h)$ *and* ω^V,

$$\begin{aligned} \hat{\mu}_L(\hat{x}^L(l), x, \omega^V) &= \hat{\mu}_L(\hat{x}^L(l), \hat{x}^H(l), \omega^V) \text{ and} \\ \hat{\mu}_L(\hat{x}^L(h), x, \omega^V) &= \hat{\mu}_L(\hat{x}^L(h), \hat{x}^H(h), \omega^V). \end{aligned}$$

2. *Suppose* $\hat{x}^H(l) \neq \hat{x}^H(h)$. *Then, for all* $x \neq \hat{x}^L(l), \hat{x}^L(h)$ *and* ω^V,

$$\begin{aligned} \hat{\mu}_L(x, \hat{x}^H(l), \omega^V) &= \hat{\mu}_L(\hat{x}^L(l), \hat{x}^H(l), \omega^V) \text{ and} \\ \hat{\mu}_L(x, \hat{x}^H(h), \omega^V) &= \hat{\mu}_L(\hat{x}^L(h), \hat{x}^H(h), \omega^V). \end{aligned}$$

Our first result shows that in any unprejudiced revealing equilibrium the candidates converge on the median position of the state they believe to be most likely, i.e., $x^*_{m_L}$ if $\omega^C = l$ and $x^*_{m_H}$ if $\omega^C = h$.

Theorem 4.4 (Unprejudiced Revealing Equilibria: Strategies) *In any unprejudiced revealing equilibrium the candidate strategies are*

$$(\hat{x}^L(l), \hat{x}^L(h)) = (\hat{x}^H(l), \hat{x}^H(h)) = (x^*_{m_L}, x^*_{m_H}).$$

Proof. Let $(\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h))$ be the candidate strategies in an unprejudiced revealing equilibrium. At least one of the candidates must announce different policies in the two states. Suppose $\hat{x}^L(l) \neq \hat{x}^L(h)$ (the case $\hat{x}^H(l) \neq \hat{x}^H(h)$ is analogous). Assume that $\hat{x}^H(l) \neq x^*_{m_L}$ and consider an $\omega^C = l$ deviation by Candidate H to a position $x \neq \hat{x}^H(h)$ with

$$0 \leq x - x^*_{m_L} < |\hat{x}^H(l) - x^*_{m_L}|$$

(if $\hat{x}^H(h) \neq x^*_{m_L}$ simply let $x = x^*_{m_L}$). This deviation will increase the median voter's utility of voting for Candidate H at least as much in state L as it will decrease it in state H. And since the voters' beliefs are still given by Bayesian updating based on both signals it follows that candidate H's probability of winning is higher after the deviation, which is a contradiction. Therefore we must have $\hat{x}^H(l) = x^*_{m_L}$. Similarly we get $\hat{x}^H(h) = x^*_{m_H}$. And then we can use the same argument for Candidate L to get $\hat{x}^L(l) = x^*_{m_L}$ and $\hat{x}^L(h) = x^*_{m_H}$. □

Our next step is to find the set of parameter values for which an unprejudiced revealing equilibrium exists. The following result shows that we have existence if and only if the voters' signal is sufficiently informative. The proof is in Appendix A.

Theorem 4.5 (Unprejudiced Revealing Equilibria: Existence) *There exists an unprejudiced revealing equilibrium if and only if*

$$\theta^V \geq \theta^*_R,$$

where

$$\theta^*_R = \frac{1}{2}(1 + \frac{\gamma}{\gamma + D}).$$

The reasoning behind the result is as follows. Because beliefs are unprejudiced we only have to consider deviations to the least likely median, i.e., to $x^*_{m_H}$ when $\omega^C = l$ and to $x^*_{m_L}$ when $\omega^C = h$. So, loosely speaking, we have an equilibrium if neither candidate can profitably "lie" to the electorate. It is easy to define the belief function such that it is never profitable for the advantaged candidate (Candidate L when $\omega^C = l$, Candidate H when $\omega^C = h$) to lie. So we have an equilibrium if and only if we can define a belief function such that, for each value of ω^C, it is not profitable for the disadvantaged candidate to lie by announcing the least likely median. This can only be done for sufficiently high values of θ^V because lying is relatively more costly the more informative the voters' signal is.

It is worth noting that the cut-off value θ^*_R does not depend on θ^C, i.e., the accuracy of the candidates information has no influence on how accurate voters' information has to be in order to make candidates reveal their information[1]. While a higher θ^C makes the disadvantaged candidate worse off in equilibrium it also makes it less attractive to lie because it makes the probability of "being caught" higher (a higher θ^C makes it more likely that $\omega^V = \omega^C$ from the viewpoint of the candidates). And it turns out that these two effects cancel out.

[1]Note, however, that since we assume $\theta^C > \theta^V$ we must have $\theta^C > \theta^*_R$ for an unprejudiced revealing equilibrium to exist.

θ^*_R is obviously increasing in γ. So if the difference-in-quality parameter γ increases then the electorate has to be better informed in order to make the candidates reveal their information. For higher γ it is relatively more costly (in terms of probability of winning) for the disadvantaged candidate to reveal the state. So when γ increases the new cut-off value of θ^V must make it more costly for the disadvantaged candidate not to reveal, i.e., it must be higher. We also see that when the difference in candidate quality vanishes then there exists a revealing equilibrium no matter how little information the electorate has ($\lim_{\gamma \to 0} \theta^*_R = \frac{1}{2}$).

Also note that θ^*_R is decreasing in D, the distance between the medians of the two states. So when the median positions of the two states are further apart then a revealing equilibrium exist for less informed electorates. The reason is that a higher D makes it relatively more costly to lie for the disadvantaged candidate. The good news from this observation is that when the state of the world really matters for policy choice (i.e., D is high) then it takes less voter information to make the candidates reveal their information by converging to the most likely median.

4.3 Non-Revealing Equilibria

In non-revealing equilibria each candidate announces the same position for both values of ω^C. Thus the strategy profile of a non-revealing equilibrium can simply be written

$$(\hat{x}^L, \hat{x}^H),$$

where $\hat{x}^i$ is the position of candidate i. We first consider equilibria with no restrictions on voters' out-of-equilibrium beliefs. We limit our attention to equilibria where the position of each candidate is between the medians of the two states of the world. This is justified by the following observation: Suppose $(\hat{x}^L, \hat{x}^H) \notin [x^*_{m_L}, x^*_{m_H}]^2$ is the strategy profile of a non-revealing equilibrium. Then so is the strategy profile where, for each i, $\hat{x}^i$ is replaced by $x^*_{m_L}$ if $\hat{x}^i < x^*_{m_L}$ and by $x^*_{m_H}$ if $x^*_{m_H} < \hat{x}^i$. Thus we have that if there exists a non-revealing equilibrium where at least one candidate is not positioned between the two medians then there exists a "corresponding" equilibrium with strategy profile in $[x^*_{m_L}, x^*_{m_H}]^2$. The observation is proved in Appendix A.

In the following theorem we find all non-revealing equilibria where the position of each candidate is between the medians of the two states of the world. The proof is in Appendix A.

Theorem 4.6 (Non-Revealing Equilibria: Strategies and Existence) *Let $(\hat{x}^L, \hat{x}^H) \in [x^*_{m_L}, x^*_{m_H}]^2$. Then the following statements hold.*

1. *If* $\max\{\hat{x}^L - x^*_{m_L}, x^*_{m_H} - \hat{x}^H\} < \gamma$ *then* $(\hat{x}^L, \hat{x}^H)$ *is the strategy profile of a non-revealing equilibrium if and only if*
$$\theta^V \leq \frac{1}{2}(1 + \sqrt{\frac{\gamma - \max\{\hat{x}^L - x^*_{m_L}, x^*_{m_H} - \hat{x}^H\}}{(\gamma + (\hat{x}^H - \hat{x}^L))(2\theta^C - 1)}}).$$

2. *If* $\max\{\hat{x}^L - x^*_{m_L}, x^*_{m_H} - \hat{x}^H\} \geq \gamma$ *then* $(\hat{x}^L, \hat{x}^H)$ *is the strategy profile of a non-revealing equilibrium if and only if*
$$\hat{x}^L - \hat{x}^H = \gamma.$$

Briefly described, the arguments of the proof are as follows. If $\max\{\hat{x}^L - x^*_{m_L}, x^*_{m_H} - \hat{x}^H\} < \gamma$ then any deviation x by candidate L (H) gives the lowest probability of winning when $\hat{\mu}_L(x, \hat{x}^H, \omega^V) = 0$ ($\hat{\mu}_L(\hat{x}^L, x, \omega^V) = 1$) for both values of ω^V. Therefore $(\hat{x}^L, \hat{x}^H)$ are equilibrium strategies precisely if candidate L cannot profitably deviate to $x^*_{m_H}$ when $\omega^C = h$ and $\hat{\mu}_L(x^*_{m_H}, \hat{x}^H, \omega^V) = 0$ and candidate H cannot profitably deviate to $x^*_{m_L}$ when $\omega^C = l$ and $\hat{\mu}_L(\hat{x}^L, x^*_{m_L}, \omega^V) = 1$. These conditions lead directly to the cut-off value for θ^V. When $\max\{\hat{x}^L - x^*_{m_L}, x^*_{m_H} - \hat{x}^H\} \geq \gamma$ there is at least one candidate who can always win with a probability equal or arbitrarily close to $\frac{1}{2}$ by deviating (no matter how we specify out-of-equilibrium beliefs). Thus it must be the case that this candidate always wins with a probability of at least $\frac{1}{2}$ in equilibrium. This is only possible if $\hat{x}^L - \hat{x}^H = \gamma$.

An immediate consequence of the theorem is that a non-revealing equilibrium with candidate positions in $[x^*_{m_L}, x^*_{m_H}]$ always exists. If $D < \gamma$ then it follows from the first statement that there exists an equilibrium with $\hat{x}^L = x^*_{m_H}$ and $\hat{x}^H = x^*_{m_L}$ for all θ^C, θ^V. If $\gamma \leq D$ then it follows from the second statement that, for all values of θ^C and θ^V, there exists an equilibrium with $\hat{x}^L - \hat{x}^H = \gamma$.

The abundance of non-revealing equilibria makes it natural to ask if some of them can be eliminated by a suitable refinement condition. In signalling games the most commonly used refinement condition is the Intuitive Criterion (Cho and Kreps (1987)). In games where there are only two sender-types (t_1 and t_2) the content of the criterion is that if a deviation from equilibrium could be profitable only for t_i then, upon observing this deviation, receivers should believe with certainty that the sender is of this type. In order to make the Intutive Criterion fit our set-up we need to make some minor adjustments. We say that a non-revealing equilibrium in our model satisfies the Intuitive Criterion if the following restrictions on out-of-equilibrium beliefs hold. Consider a non-revealing equilibrium $(\hat{x}^L, \hat{x}^H)$, $\hat{\mu}_L$ and a deviation by Candidate L to some x. Suppose that we can make the deviation profitable (by changing the out-of-equilibrium beliefs) if and only if $\omega^C = l$

$(\omega^C = h)$. Then we must have

$$\begin{array}{rcl}\hat{\mu}_L(x, \hat{x}^H, l) & \geq & \Pr(L|\omega^C = l, \omega^V = l), \hat{\mu}_L(x, \hat{x}^H, h) \geq \Pr(L|\omega^C = l, \omega^V = h) \\ (\hat{\mu}_L(x, \hat{x}^H, l) & \leq & \Pr(L|\omega^C = h, \omega^V = l), \hat{\mu}_L(x, \hat{x}^H, h) \leq \Pr(L|\omega^C = h, \omega^V = h)).\end{array}$$

This means that voters should believe that state L (H) is at least as likely as it would be if they learned that $\omega^C = l$ $(\omega^C = h)$ and then did Bayesian updating based on this and their own signal. In other words, we do not allow voters to believe that $\omega^C = h$ $(\omega^C = l)$ is possible at all. Analogous restrictions should hold in out-of-equilibrium situations where Candidate H deviates.

It turns out that (our version of) the Intuitive Criterion does not eliminate any of the non-revealing equilibria from Theorem 4.6. The argument is rather straightforward. Consider a deviation by candidate L to some position x (the argument for deviations by candidate H is completely analogous). The maximum probability of winning that candidate L can achieve by this deviation when we allow out-of-equilibrium beliefs to be changed is independent of the value of ω^C (because we get the maximum probability of winning by letting $\mu_L(x, \hat{x}^H, \omega^V) = 0$ or $\mu_L(x, \hat{x}^H, \omega^V) = 1$ for both values of ω^V and thus the conditional distribution of ω^V given ω^C does not matter). In all equilbria candidate L wins with probability $p \geq \frac{1}{2}$ when $\omega^C = l$ and probability $1-p$ when $\omega^C = h$. So if x could be a profitable deviation when $\omega^C = l$ then the same is true when $\omega^C = h$. Thus the equilibrium belief function does not violate the Intuitive Criterion if, for all $x \neq \hat{x}^L$,

$$\hat{\mu}_L(x, \hat{x}^H, l) \leq \Pr(L|\omega^C = h, \omega^V = l), \hat{\mu}_L(x, \hat{x}^H, h) \leq \Pr(L|\omega^C = h, \omega^V = h).$$

All equilibrium strategies considered in the first statement of the theorem are supported by belief functions satisfying these conditions (we have $\hat{\mu}_L(x, \hat{x}^H, \omega^V) = 0$ for all $x \neq \hat{x}^L$ and both values of ω^V). This is not true for the equilibrium strategies considered in the second part of the theorem. But in this case each candidate wins with probability $\frac{1}{2}$ in equilibrium for both values of ω^C and thus the Intuitive Criterion does not put any restrictions at all on out-of-equilibrium beliefs.

One way of eliminating many of the non-revealing equilibria is to impose a monotonicity condition on the voter belief function. The content of our condition is the following. If one candidate moves to a position that is at least as close to $x^*_{m_L}$ $(x^*_{m_H})$ and not closer to $x^*_{m_H}$ $(x^*_{m_L})$ then μ_L $(1-\mu_L)$ does not decrease. For example, if $x^*_{m_L} \leq x < y \leq x^*_{m_H}$ then a move from y to x by one of the candidates will not make voters believe that state L is less likely.

Definition 4.7 (Monotone Belief Function) *A voter belief function μ_L is said to be* monotone *if the following condition holds. Suppose that, for some $x, y \in X$,*

$$|x - x^*_{m_L}| \leq |y - x^*_{m_L}| \quad \textit{and} \quad |x - x^*_{m_H}| \geq |y - x^*_{m_H}|.$$

Then, for all $z \in X$, $\omega^V \in \{l, h\}$,

$$\mu_L(x, z, \omega^V) \geq \mu_L(y, z, \omega^V) \quad \text{and} \quad \mu_L(z, x, \omega^V) \geq \mu_L(z, y, \omega^V).$$

There are no directly state-dependent costs for the candidates that can justify this condition. Nevertheless, it does seem appealing for voters to think that if one candidate moves closer to, e.g., $x^*_{m_L}$ and not closer to $x^*_{m_H}$ then it is not less likely that $\omega^C = l$ and thus state L is not less likely to be the true state.

It is worth noting that restricting attention to monotone equilibria (i.e., equilibria with monotone belief functions) does not change our result on existence of unprejudiced revealing equilibria. More precisely, the conclusion from Theorem 4.5 still holds if we require revealing equilibria to be both unprejudiced and monotone. This follows easily by verifying that the equilibrium belief function used in the proof is monotone.

In the following result we find the candidate positions that are possible in monotone non-revealing equilibria.

Theorem 4.8 (Monotone Non-Revealing Equilibria: Strategies) *The candidate positions in any monotone non-revealing equilibrium must satisfy*

$$\hat{x}^L \leq x^*_{m_L} \quad \text{and} \quad x^*_{m_H} \leq \hat{x}^H.$$

Proof. Suppose $\hat{x}^L > x^*_{m_L}$. Consider a deviation by Candidate L to

$$x = \left\{ \begin{array}{c} x^*_{m_L} \text{ if } \hat{x}^L - x^*_{m_H} \leq D \\ x^*_{m_H} - (\hat{x}^L - x^*_{m_H}) \text{ if } \hat{x}^L - x^*_{m_H} > D \end{array} \right\}.$$

This deviation will increase the median voter's utility of voting for Candidate L at least as much in state L as it will decrease it in state H. Furthermore, the deviation will, by monotonicity of the belief function, not make voters believe that state L is less likely. Therefore it is easy to see that the deviation is profitable when $\omega^C = l$. Thus we must have $\hat{x}^L \leq x^*_{m_L}$ and by symmetry it follows that $x^*_{m_H} \leq \hat{x}^H$.□

When the monotonicity condition is imposed on non-revealing equilibria we get that such equilibria exist if and only if the accuracy of the voters' signal is below some cut-off value. From the first statement in Theorem 4.6 we already know the cut-off value for monotone non-revealing equilibria with $\hat{x}^L = x^*_{m_L}$ and $\hat{x}^H = x^*_{m_H}$ (the belief function used in the proof satisfies the monotonicity condition for these positions). Furthermore, we can show that if some pair of positions is the strategy profile of a monotone non-revealing equilibrium then so is $(x^*_{m_L}, x^*_{m_H})$. Suppose $(\hat{x}^L, \hat{x}^H)$ is the strategy profile of a monotone non-revealing equilibrium. Since

$\hat{x}^L \leq x^*_{m_L}$ and $x^*_{m_H} \leq \hat{x}^H$ it is easy to see that the strategy profile can be supported by a monotone belief function $\hat{\mu}_L$ satisfying Bayes' rule on the equilibrium path and

$$\begin{aligned} \hat{\mu}_L(\hat{x}^L, x, \omega^V) &= 1 \text{ for all } x \neq \hat{x}^H, \omega^V = l, h; \\ \hat{\mu}_L(x, \hat{x}^H, \omega^V) &= 0 \text{ for all } x \neq \hat{x}^L, \omega^V = l, h. \end{aligned}$$

And then it is straightforward to check that $(x^*_{m_L}, x^*_{m_H})$ can be supported by a monotone belief function $\tilde{\mu}_L$ satisfying Bayes' rule on the equilibrium path and

$$\begin{aligned} \tilde{\mu}_L(x^*_{m_L}, x, \omega^V) &= 1 \text{ for all } x \neq x^*_{m_H}, \omega^V = l, h; \\ \tilde{\mu}_L(x, x^*_{m_H}, \omega^V) &= 0 \text{ for all } x \neq x^*_{m_L}, \omega^V = l, h. \end{aligned}$$

Thus we have the following corollary.

Corollary 4.9 (Monotone Non-Revealing Equilibria: Existence) *There exists a monotone non-revealing equilibrium if and only if*

$$\theta^V \leq \theta^*_N,$$

where

$$\theta^*_N = \frac{1}{2}\left(1 + \sqrt{\frac{\gamma}{(\gamma + D)(2\theta^C - 1)}}\right).$$

For any $\theta^V \leq \theta^*_N$ there exists a monotone non-revealing equilibrium with

$$\hat{x}^L = x^*_{m_L} \quad \text{and} \quad \hat{x}^H = x^*_{m_H}.$$

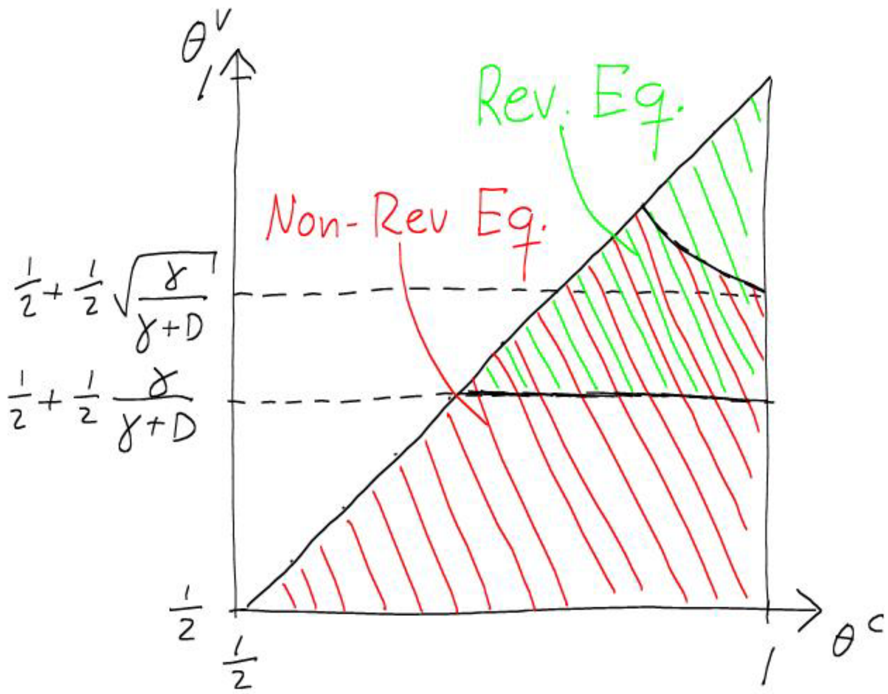
From Theorem 4.5 and Corollary 4.9 it follows that, for all parameter values, either an unprejudiced revealing equilibrium or a monotone non-revealing equilibrium exists. We also see that for some parameter values both types of equilibria exist.

Corollary 4.10 (Existence of Refined Equilibria) *We have that*

$$\theta^*_R = \frac{1}{2}\left(1 + \frac{\gamma}{\gamma + D}\right) < \frac{1}{2}\left(1 + \sqrt{\frac{\gamma}{(\gamma + D)(2\theta^C - 1)}}\right) = \theta^*_N.$$

So for all parameter values there exists either an unprejudiced revealing equilibrium or a monotone non-revealing equilibrium. When $\theta^C > \theta^*_R$ both types of equilibria exist for all θ^V's in an interval of non-zero length.

The following figure sums up our results on existence of refined revealing and non-revealing equilibria.



5 Analysis of Case 2

5.1 Equilibrium

In this case a strategy profile for the candidates consists of four policy positions for each candidate, one position for each possible combination of the candidates' signal and the voters' signal. We let $x^i(\omega^C, \omega^V)$ denote the position of Candidate i when the candidates' signal is ω^C and the voters' signal is ω^V.

As in the previous case the voters observe the position of each candidate and the value of their own signal, so their belief function about the probability of state L can again be written

$$\mu_L(x^L, x^H, \omega^V).$$

We again assume that all voters share the same belief function and thus the result that the median voter decides the outcome of the election (Lemma 4.1) is also valid here.

Our notion of equilibrium is completely analogous to the one used in the previous case. An equilibrium consists of a strategy profile and a voter belief function. The strategy of each candidate maximizes his probability of winning given the other candidates strategy, the belief function of the voters, and all available information. On the equilibrium path the belief function is consistent with Bayes rule. The only thing that is different from the previous case is that the candidates now know the value of the voters' signal so they do not have to rely on the distribution of ω^V given their own signal.

We divide equilibria into three types. An equilibrium is said to be *fully revealing* if, for each value of ω^V, there is at least one candidate who reveals the value of ω^C. I.e., for each ω^V there exists at least one i such that the strategy of Candidate i satisfies

$$\hat{x}^i(l, \omega^V) \neq \hat{x}^i(h, \omega^V).$$

An equilibrium is said to be *partially revealing* if at least one candidate reveals the value of ω^C for one value of ω^V while neither candidate reveals ω^C for the other value of ω^V. For example, we could have

$$\hat{x}^i(l, l) \neq \hat{x}^i(h, l) \quad \text{for at least one } i$$

and

$$\hat{x}^i(l, h) = \hat{x}^i(h, h) \quad \text{for both } i.$$

In such an equilibrium voters infer the true value of ω^C when $\omega^V = l$ but they do not when $\omega^V = h$. The final type of equilibria are *non-revealing equilibria* where the candidates never reveal the true value of ω^C, i.e., their strategies satisfy

$$\hat{x}^i(l, \omega^V) = \hat{x}^i(h, \omega^V) \quad \text{for both } i \text{ and both values of } \omega^V.$$

5.2 Fully and Partially Revealing Equilibria

We will again restrict attention to equilibria with belief functions that are unprejudiced. Thus we assume that if one candidate's equilibrium strategy reveals the value of ω^C for some ω^V then an out-of equilibrium deviation by the other candidate does not change voters' belief. We can formulate the condition as follows.

1. Suppose $\hat{x}^L(l, \omega^V) \neq \hat{x}^L(h, \omega^V)$ for some ω^V. Then, for all $x \neq \hat{x}^H(l, \omega^V), \hat{x}^H(h, \omega^V)$,

$$\begin{aligned} \hat{\mu}_L(\hat{x}^L(l, \omega^V), x, \omega^V) &= \hat{\mu}_L(\hat{x}^L(l, \omega^V), \hat{x}^H(l, \omega^V), \omega^V) \text{ and} \\ \hat{\mu}_L(\hat{x}^L(h, \omega^V), x, \omega^V) &= \hat{\mu}_L(\hat{x}^L(h, \omega^V), \hat{x}^H(h, \omega^V), \omega^V). \end{aligned}$$

2. Suppose $\hat{x}^H(l, \omega^V) \neq \hat{x}^H(h, \omega^V)$ for some ω^V. Then, for all $x \neq \hat{x}^L(l, \omega^V), \hat{x}^L(h, \omega^V)$,

$$\begin{aligned} \hat{\mu}_L(x, \hat{x}^H(l, \omega^V), \omega^V) &= \hat{\mu}_L(\hat{x}^L(l, \omega^V), \hat{x}^H(l, \omega^V), \omega^V) \text{ and} \\ \hat{\mu}_L(x, \hat{x}^H(h, \omega^V), \omega^V) &= \hat{\mu}_L(\hat{x}^L(h, \omega^V), \hat{x}^H(h, \omega^V), \omega^V). \end{aligned}$$

As in the previous case it is fairly straightforward to show that if the value of ω^C is revealed to the voters in equilibrium then each candidate's position must be the most likely median (see Theorem 4.4). So if $\hat{x}^i(l, \omega^V) \neq \hat{x}^i(h, \omega^V)$ for at least one i then we must have

$$\hat{x}^L(l, \omega^V) = \hat{x}^H(l, \omega^V) = x^*_{m_L} \quad \text{and} \quad \hat{x}^L(h, \omega^V) = \hat{x}^H(h, \omega^V) = x^*_{m_H}.$$

Thus, in any unprejudiced fully revealing equilibrium the candidates always converge on the most likely median.

It turns out that when the voter belief function is required to be unprejudiced then neither fully nor partially revealing equilibria exist. Suppose we have an equilibrium which is revealing for $\omega^V = l$. Then, for this value of ω^V, both candidates must be at $x^*_{m_L}$ if $\omega^C = l$ and at $x^*_{m_H}$ if $\omega^C = h$. Two necessary conditions for equilibrium are that candidate H cannot profitably deviate to $x^*_{m_H}$ when $\omega^C = l$ and that candidate L cannot profitably deviate to $x^*_{m_L}$ when $\omega^C = h$. It is easy to see that the first condition implies $\hat{\mu}_L(x^*_{m_L}, x^*_{m_H}, l) > \frac{1}{2}$ and that the second condition implies $\hat{\mu}_L(x^*_{m_L}, x^*_{m_H}, l) < \frac{1}{2}$, which is a contradiction. So we conclude that when voters' information is known to the candidates and their belief function is required to be unprejudiced then it is never possible for voters to infer the candidates' information from their positions.

5.3 Non-Revealing Equilibria

In non-revealing equilibria the candidates' positions do not depend on the value of ω^C, only (possibly) on ω^V. Therefore an equilibrium strategy profile can be written

$$(\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h)),$$

where, for example, $\hat{x}^L(l)$ is the position of candidate L when $\omega^V = l$. Note the difference from Case 1 where $\hat{x}^L(l)$ denoted the equilibrium position of Candidate L when $\omega^C = l$.

Analogously to Case 1 we can find all non-revealing equilibria where positions are between the two medians (restricting attention to such equilibria is justified by the same argument as in Case 1, see p. 14). Since we are primarily interested in non-revealing equilibria with monotone belief functions we defer the result to Appendix B.

The definition of monotone belief functions is exactly as in Case 1 (see Definition 4.7). With the condition of monotonicity we get that in any non-revealing equilibrium we must have

$$\hat{x}^L(l) \leq x^*_{m_L} \quad \text{and} \quad x^*_{m_H} \leq \hat{x}^H(h).$$

Because if, for example, $\hat{x}^L(l) > x^*_{m_L}$ then candidate L could win with a higher probability by moving towards $x^*_{m_L}$ when $\omega^V = l$ (see the proof of Theorem 4.8 for more details, the argument is analogous).

Then we are ready to formulate our main result on monotone non-revealing equilibria. The proof is in Appendix A.

Theorem 5.1 (Mon. Non-Rev. Equilibria: Strategies and Existence)

1. *$((\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h))) \in [x^*_{m_L}, x^*_{m_H}]^4$ is the strategy profile of a monotone non-revealing equilibrium if and only if*

$$\hat{x}^L(l) = x^*_{m_L} \quad \textit{and} \quad \hat{x}^H(h) = x^*_{m_H}$$

and

$$\theta^V \leq \frac{1}{2}(1 + \frac{\gamma}{\gamma + \max\{\hat{x}^H(l) - x^*_{m_L}, x^*_{m_H} - \hat{x}^L(h)\}}).$$

2. *If $((\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h))) \notin [x^*_{m_L}, x^*_{m_H}]^4$ is the strategy profile of a monotone non-revealing equilibrium then so is the profile where, for each i and ω^V, $\hat{x}^i(\omega^V)$ is replaced by $x^*_{m_L}$ if $\hat{x}^i(\omega^V) < x^*_{m_L}$ and by $x^*_{m_H}$ if $x^*_{m_H} < \hat{x}^i(\omega^V)$.*

From the result we immediately see that there always exists a monotone non-revealing equilibrium with

$$\hat{x}^L(l) = \hat{x}^H(l) = x^*_{m_L} \quad \text{and} \quad \hat{x}^L(h) = \hat{x}^H(h) = x^*_{m_H},$$

i.e., an equilibrium where the candidates converge on the median that is most likely given only the voters' signal. This type of candidate behavior we refer to as *pandering*. When θ^V is not too high then the candidate not favored by the voters' signal (Candidate L when $\omega^V = h$, Candidate H when $\omega^V = l$) need not be close to the median that is most likely given this signal. For example, the strategies

$$\hat{x}^L(l) = \hat{x}^L(h) = x^*_{m_L} \quad \text{and} \quad \hat{x}^H(l) = \hat{x}^H(h) = x^*_{m_H}$$

are possible in (monotone) equilibrium if

$$\theta^V \leq \frac{1}{2}(1 + \frac{\gamma}{\gamma + D}).$$

So when the accuracy of the voters' signal is below this cut-off value both pandering and full polarization is possible. And of course a lot of other strategy profiles which involve some pandering and some polarization are also possible.

6 Welfare

In this section we will do a welfare comparison of different strategy profiles. A strategy profile consists of a position for each candidate for each combination of values of ω^V and ω^C. Obviously, only strategy profiles where positions do not depend on the value of ω^V are relevant for Case 1. We will compare the profiles with repect to the *ex ante expected utility* that each voter obtains. When comparing the strategy profiles we will assume that the belief of the voters is given by Bayesian updating and that they vote based on expected utility. So in our comparison we assume that voters behave as they would if the strategy profile was part of an equilibrium. Eventually, we will use the welfare comparison of strategy profiles to do a welfare comparison of Case 1 and 2. More precisely, we will compare the refined equilibria of the two cases for fixed parameter values. Furthermore, we will use the welfare comparisons of strategy profiles to show that, within each case, more voter information is welfare improving.

We will focus on the three main types of candidate behavior that we have encountered in refined equilibria: Revelation, polarization, and pandering. Let us briefly recall what we mean by these three terms. Revelation means that, for both values of the voters' signal ω^V, the strategy profile is given by

$$\hat{x}^L(l,\omega^V) = \hat{x}^H(l,\omega^V) = x^*_{m_L} \text{ and } \hat{x}^L(h,\omega^V) = \hat{x}^H(h,\omega^V) = x^*_{m_H}.$$

Polarization means that, for both values of ω^V,

$$\hat{x}^L(l,\omega^V) = \hat{x}^L(h,\omega^V) = x^*_{m_L} \text{ and } \hat{x}^H(l,\omega^V) = \hat{x}^H(h,\omega^V) = x^*_{m_H}.$$

Finally, pandering means that, for both values of ω^C,

$$\hat{x}^L(\omega^C,l) = \hat{x}^H(\omega^C,l) = x^*_{m_L} \text{ and } \hat{x}^L(\omega^C,h) = \hat{x}^H(\omega^C,h) = x^*_{m_H}.$$

The voting decisions of the voters, and thus the outcome of the election, depend on the voters' beliefs about the state (we assume that they maximize expected utility given their beliefs). Therefore, in order to calculate the ex ante expected utility of each of the three strategy profiles for each voter, we must specify the beliefs of the voters. As mentioned above, we assume that voters are Bayesians. This means that they form their belief based on the values of ω^V and ω^C if the strategy profile is revealing, otherwise (i.e., for the pandering and polarization profiles) they form their belief based only on the value of ω^V. This is of course identical to what the beliefs would be in equilibrium. Therefore, our comparison of strategy profiles makes it straightforward to do a comparison of equilibria.

With the above assumption on voters' beliefs, a strategy profile s defines an outcome map $o_s : (\omega^C, \omega^V, \delta) \mapsto (w, x^w)$, where $w \in \{L, H\}$ is the winning candidate and x^w is the policy position of this candidate. Let $U_i((j, x^j)|\omega)$ denote the utility of voter i when candidate j with position x^j is elected and the state is ω. Given a strategy profile s the utility of voter i in state ω as a function of $(\omega^C, \omega^V, \delta)$ can then be written $U_i(o_s(\omega^C, \omega^V, \delta)|\omega)$. So for given values of ω^C, ω^V, and δ the expected utility for voter i of the strategy profile s is

$$\sum_{\omega=L,H} \Pr(\omega|\omega^C, \omega^V) U_i(o_s(\omega^C, \omega^V, \delta)|\omega).$$

Thus voter i's *ex ante expected utility* of the strategy profile s can be written

$$E_{(\omega^C,\omega^V),\delta}[\sum_{\omega=L,H} \Pr(\omega|\omega^C, \omega^V) U_i(o_s(\omega^C, \omega^V, \delta)|\omega)],$$

where the expectation is with respect to the ex ante distribution of (ω^C, ω^V) and the distribution of δ. In more detail the ex ante expected utility is

$$\sum_{\substack{\omega^C=l,h \\ \omega^V=l,h}} \Pr(\omega^C, \omega^V) \int_{-\frac{1}{2\sigma}}^{\frac{1}{2\sigma}} [\sum_{\omega=L,H} \Pr(\omega|\omega^C, \omega^V) U_i(o_s(\omega^C, \omega^V, \delta)|\omega)] \sigma d\delta.$$

Note that the ex ante distribution of (ω^C, ω^V) is given by

$$\Pr(l,l) = \Pr(h,h) = \frac{1}{2}(\theta^C\theta^V + (1-\theta^C)(1-\theta^V))$$

and

$$\Pr(l,h) = \Pr(h,l) = \frac{1}{2}(\theta^C(1-\theta^V) + (1-\theta^C)\theta^V).$$

First, we compare strategy profiles with respect to policy only, i.e., we consider only the policy part of the voters' utility functions. We let d_i denote the distance of voter i from the median voter, i.e.,

$$d_i = |x_i^*(L) - x_{m_L}^*| = |x_i^*(H) - x_{m_H}^*|.$$

The proof is in Appendix A.

Theorem 6.1 (Welfare Comparison: Policy Utility) *With respect to ex ante expected policy utility (and for fixed parameter values), the following ranking of strategy profiles holds for all voters:*

$$\textit{Revelation} \succeq \textit{Pandering} \succeq \textit{Polarization}.$$

For voters i with $d_i < D$ the ranking is strict, for voters with $d_i \geq D$ the three strategy profiles all give the same ex ante expected policy utility.

Second, we compare strategy profiles with respect to ex ante expected total utility. The δ-term in the utility of voting for candidate L is included in the total utility. Note that the result (and all previous results) does not depend on whether the δ-term is added to the utility of voting for candidate L, subtracted from the utility of voting for candidate H, or something in between. More precisely, all of our analysis would remain unchanged if we had instead let each voter i's utilities of voting for candidate L and H be

$$U_i((L, x^L)|\omega) = \left\{ \begin{array}{ll} u_i(x^L|L) + \gamma + \alpha\delta & \text{if } \omega = L \\ u_i(x^L|H) + \alpha\delta & \text{if } \omega = H \end{array} \right\}$$

and

$$U_i((H, x^H)|\omega) = \left\{ \begin{array}{ll} u_i(x^H|L) - (1-\alpha)\delta & \text{if } \omega = L \\ u_i(x^H|H) + \gamma - (1-\alpha)\delta & \text{if } \omega = H \end{array} \right\},$$

where $\alpha \in [0, 1)$. The proof of the theorem below is in Appendix A.

Theorem 6.2 (Welfare Comparison: Total Utility) *With respect to ex ante expected total utility (and for fixed parameter values), the following ranking of strategy profiles holds for all voters:*

$$\textit{Revelation} \succ \textit{Pandering} \succ \textit{Polarization.}$$

From the results above we see that, with respect to both policy and total utility, there is a clear Pareto ranking of the three types of candidate behavior. Revelation dominates pandering which dominates polarization. This ranking is quite intuitive. In the revelation profile voters learn all available information and both candidates are positioned at the most likely median (note that, since $\theta^C > \theta^V$, the most likely median given ω^C is also the most likely median given both ω^C and ω^V). When candidates pander voters do not learn the value of ω^C and the candidates converge on the most likely median given only the value of ω^V. Therefore it is not surprising that, from an ex ante perspective where all combinations of (ω^C, ω^V) are considered, all voters are better off with revelation than with pandering. (Note that some voters may of course prefer pandering over revelation for specific values of ω^C and ω^V. For example, voters with $x_i^*(L), x_i^*(H) > x_{m_H}^*$ will obviously get a higher policy utility from pandering than from revelation when $(\omega^C, \omega^V) = (l, h)$). When candidates are polarized voters have the same information (the value of their own signal) as when they are pandering. But the candidates' positions do not reflect this information, each candidate always announces the same position. This is bad for the voters and therefore polarization is dominated by pandering.

We can use the results above to do a welfare comparison of Case 1 and 2. Is it optimal for the voters that their information is not available to the candidates when they take positions or is it the other way around? We do this welfare comparison the following way. First of all we restrict attention to refined equilibria where all positions are between the two medians. Furthermore, we disregard the "asymmetric" equilibria of Case 2 where the distance between $\hat{x}^L(h)$ and $x^*_{m_H}$ is different from the distance between $\hat{x}^H(l)$ and $x^*_{m_L}$[2]. Then, with these restrictions, for each case and each set of parameter values select the equilibrium that is optimal with respect to the ex ante expected total utility of each voter. The outcome of this selection is presented in the following table.

	$\theta^V \geq \theta^*_R$	$\theta^V < \theta^*_R$
Case 1	Revelation	Polarization
Case 2	Pandering	Pandering

(Note that equilibria in Case 2 that involve some polarization and some pandering are, like polarization, dominated by pandering. The proof of this claim is similar to the proof that pandering dominates polarization). We then compare the two cases by comparing the optimal equilibria for fixed parameter values. We see that if voters are sufficiently well informed about the state of the world ($\theta^V \geq \theta^*_R$) then Case 1 is optimal. But if they are not then Case 2 is optimal. Note that this result remains the same if we compare the cases with respect to ex ante expected policy utility (except for the fact that with respect to policy utility some voters are indifferent between the two cases so we have a weak rather than strong Pareto ranking). Also note that if we instead of comparing optimal equilibria say that Case 1 is better than Case 2 if any equilibrium of Case 1 dominates any equilibrium of Case 2 (and vice versa) then we get a weaker but similar result. Case 1 dominates Case 2 when only revelation is a refined equilibrium of Case 1, i.e., when $\theta^V > \theta^*_N$. Case 2 dominates Case 1 when only polarization is a refined equilibrium of Case 1, i.e., when $\theta^V < \theta^*_R$. When both revelation and polarization are refined equilibria of Case 1, i.e., when $\theta^*_R \leq \theta^V \leq \theta^*_N$, a comparison is not possible.

Finally, we show that, in each of the two cases, voters are better off with respect to total utility if they have more accurate information, i.e., if θ^V is higher. More precisely we will show that if we compare the optimal equilibrium for two different values of θ^V (while holding all other parameters fixed) then the equilibrium of

[2] These equilibria could be included in the welfare comparison of the two cases in the following natural way. For each asymmetric refined equilibrium consider a lottery where there is probability $\frac{1}{2}$ of ending up in this equilibrium and probability $\frac{1}{2}$ of ending up in its "mirror image" (i.e., the equilibrium where the distance between the position of Candidate L (H) and $x^*_{m_H}$ ($x^*_{m_L}$) when $\omega^V = h$ (l) is equal to the distance between the position of Candidate H (L) and $x^*_{m_L}$ ($x^*_{m_H}$) when $\omega^V = l$ (h) in the original equilibrium). Calculate each voter's expected utility of this lottery. Finally, compare with symmetric equilibria and similar lotteries between other asymmetric equilibria. With this way of including asymmetric equilibria in the welfare comparison all results remain the same.

the situation where voters are better informed will Pareto dominate the other equilibrium (note that we use the same restrictions on equilibria as above). To see this, first note that for a fixed (symmetric) strategy profile the ex ante expected total utility of each voter is increasing in θ^V. This is quite obvious and can easily be checked. With this observation the conclusion easily follows from the table above. In Case 2 pandering is always the optimal equilibrium and thus the conclusion is trivial. In Case 1 the optimal equilibrium is polarization for low values of θ^V (below θ_R^*) and revelation for high values of θ^V (above θ_R^*). And since revelation dominates polarization we are done. Note that if we use policy utility instead of total utility we get the same result. Also note that, in both cases and with respect to both policy and total utility, we can strengthen the result above so that we do not only consider the optimal equilibrium for each set of parameter values. Let $\theta_1^V > \theta_2^V$ (and, of course, $\theta^C > \theta_1^V$). In Case 1 it is easy to see that each equilibrium of the θ_1^V-situation Pareto dominates all equilibria of the θ_2^V-situation. This is not true in Case 2, but we can still strengthen our first result. Let $S_{\theta_i^V}$, $i = 1, 2$, be the set of all equilibrium strategy profiles of the θ_i^V-situation. Then it is easily seen that all profiles in $S_{\theta_2^V} \setminus S_{\theta_1^V}$ are Pareto dominated by each profile in $S_{\theta_1^V}$. So all strategy profiles that are part of a equilibrium in the θ_2^V-situation but not in the θ_1^V-situation are Pareto dominated by each equilibrium strategy in the θ_1^V-situation.

7 Discussion

We have analyzed a model of electoral competition under uncertainty where candidates are better informed than voters. Candidates were assumed to be office motivated and to differ only in state-dependent quality. The office-motivation creates an electoral pressure for the candidates to converge on the median they believe to be most likely and thus reveal their information. However, because of the difference in quality there is always one candidate who also has an incentive to choose a policy position such that voters cannot infer the information of the candidates. This is the central strategic aspect of the model.

If voters' information is unknown to the candidates when they take positions then, in refined equilibrium, candidates will reveal their information by converging on the most likely median if the voters' information is sufficiently accurate. If the information of the voters is not sufficiently accurate then each candidate will announce the median position of the state in which he has a quality advantage. Thus the candidates will be polarized and the voters are not able to infer the candidates' information.

We also considered the situation where candidates know voters' information when they take positions. In that case a refined equilibrium where the candidates' positions reveal their information does not exist. Thus voters always have to rely solely on their own information when deciding who to vote for. Candidates will either pander by converging on the median of the state that is most likely given only the voters' signal or be polarized.

With respect to welfare, we saw that, ex ante, it is optimal for all voters that candidates reveal their information by converging on the most likely median. Furthermore, pandering is better than polarization. Therefore, when voters are well informed it is better for them that candidates do not know their information when they take positions. But when voters are poorly informed then it is the other way around, they are all better off if candidates know which signal they have recieved.

A number of interesting insights about the functioning of representative democracy when candidates are better informed than voters follow from our analysis. First, our results show that policy motivated candidates are not necessary for electoral competition to be inefficient with respect to making all information available to the electorate. As we have seen, differences in state-dependent quality can make fully office motivated candidates play non-revealing strategies. However, if voters are reasonably well informed and candidates are not certain about voters' information when they take positions then electoral competition is efficient. So our results clearly point to the importance of an informed electorate and conditions that make candidates take positions at a point in time where they are not certain about what information voters will have on election day.

It is also interesting to note that if the quality difference between the candidates (measured by the parameter γ) increases then it takes more accurate voter information to get existence of a revealing refined equilibrium. So an election with two candidates who have substantially different skills (high γ) is more likely to lead to a bad policy choice than an election with two reasonably similar candidates (low γ).

Finally note that the optimal equilibrium of the case where candidates know voters' information when they take positions is always pandering. Therefore it is somewhat counterintutive that this case is optimal when the electorate is not too well informed. But this is true, of course, because the optimal refined equilibrium outcome of the other case (voters' information not known to candidates) is better than pandering when voters are well informed and worse when they are poorly informed. So in this respect we have the counterintuitive result that pandering is optimal if and only if the electorate not well informed.

8 References

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9 Appendix A

Proof of Theorem 4.5.

First we show that $\theta^V \geq \theta^*_R \Rightarrow$ existence.

Consider the strategies

$$(\hat{x}^L(l), \hat{x}^L(h)) = (\hat{x}^H(l), \hat{x}^H(h)) = (x^*_{m_L}, x^*_{m_H}).$$

Furthermore, consider a belief function $\hat{\mu}_L$ that is consistent with Bayes rule when $x^L = x^H = x^*_{m_L}$ or $x^L = x^H = x^*_{m_H}$, unprejudiced, and satisfies

$$\hat{\mu}_L(x^*_{m_L}, x^*_{m_H}, l) = 1,\ \mu_L(x^*_{m_L}, x^*_{m_H}, h) = 0,$$

and

$$\hat{\mu}_L(x^*_{m_H}, x^*_{m_L}, l) = \mu_L(x^*_{m_H}, x^*_{m_L}, h) = \frac{1}{2}.$$

We claim that these strategies and such a belief function constitutes an equilibrium when $\theta^V \geq \theta^*_R$. To prove this we have to check the optimality of each candidate's strategy. First consider the strategies when $\omega^C = l$. Since the belief function is unprejudiced it follows easily that neither candidate can gain by deviating to some $x \neq x^*_{m_H}$ (such a deviation will not change the voters' belief). Thus we just have to check that neither candidate can profitably deviate to $x^*_{m_H}$. In equilibrium Candidate L wins with probability

$$\Pr(\omega^V = l|\omega^C = l) \Pr[\delta > (1 - 2\frac{\theta^C\theta^V}{\theta^C\theta^V + (1-\theta^C)(1-\theta^V)})\gamma]$$
$$+ \Pr(\omega^V = h|\omega^C = l) \Pr[\delta > (1 - 2\frac{\theta^C(1-\theta^V)}{\theta^C(1-\theta^V) + (1-\theta^C)\theta^V})\gamma],$$

which is equal to

$$\frac{1}{2} + (2\theta^C - 1)\gamma\sigma.$$

Thus Candidate H wins with probability

$$\frac{1}{2} - (2\theta^C - 1)\gamma\sigma.$$

If Candidate L deviates to $x^*_{m_H}$ then his probability of winning is $\frac{1}{2}$ so this deviation is never profitable for him. If Candidate H deviates to $x^*_{m_H}$ then his probability of winning is

$$\Pr(\omega^V = l|\omega^C = l)\Pr[\delta < -(D+\gamma)] + \Pr(\omega^V = h|\omega^C = l)\Pr[\delta < \gamma + D],$$

which is equal to

$$\frac{1}{2} + (1 - 2(\theta^C\theta^V + (1-\theta^C)(1-\theta^V)))(\gamma + D)\sigma.$$

So the deviation is not profitable if (and only if)

$$\frac{1}{2} + (1 - 2(\theta^C\theta^V + (1-\theta^C)(1-\theta^V)))(\gamma + D)\sigma \leq \frac{1}{2} - (2\theta^C - 1)\gamma\sigma.$$

This inequality is equivalent to

$$\theta^V \geq \theta^*_R.$$

By symmetry it follows that if neither candidate has a profitable deviation when $\omega^C = l$ then this is also the case when $\omega^C = h$. Thus the equilibrium conditions are satisfied if $\theta^V \geq \theta^*_R$.

Finally we show that existence $\Rightarrow \theta^V \geq \theta^*_R$.

Suppose there exists an unprejudiced revealing equilibrium. We know from Theorem 4.4 that the candidate strategies must be

$$(\hat{x}^L(l), \hat{x}^L(h)) = (\hat{x}^H(l), \hat{x}^H(h)) = (x^*_{m_L}, x^*_{m_H}).$$

Two necessary conditions for equilibrium are that Candidate H cannot gain by deviating to $x^*_{m_H}$ when $\omega^C = l$ and that Candidate L cannot gain by deviating to $x^*_{m_L}$ when $\omega^C = h$. Let $\hat{\mu}_L$ be the equilibrium belief function and define

$$\hat{\mu}^l_L = \hat{\mu}_L(x^*_{m_L}, x^*_{m_H}, l) \text{ and } \hat{\mu}^h_L = \hat{\mu}_L(x^*_{m_L}, x^*_{m_H}, h).$$

Then the necessary conditions can be written

$$\begin{aligned}
&\Pr(\omega^V = l|\omega^C = l)(\frac{1}{2\sigma} - \hat{\mu}^l_L(\gamma + D) + (1-\hat{\mu}^l_L)(\gamma + D))\sigma \\
&+ \Pr(\omega^V = h|\omega^C = l)(\frac{1}{2\sigma} - \hat{\mu}^h_L(\gamma + D) + (1-\hat{\mu}^h_L)(\gamma + D))\sigma \leq \frac{1}{2} - (2\theta^C - 1)\gamma\sigma
\end{aligned}$$

and

$$\begin{aligned}
&\Pr(\omega^V = h|\omega^C = h)(\frac{1}{2\sigma} - (1-\hat{\mu}^h_L)(\gamma + D) + \hat{\mu}^h_L(\gamma + D))\sigma \\
&+ \Pr(\omega^V = l|\omega^C = h)(\frac{1}{2\sigma} - (1-\hat{\mu}^l_L)(\gamma + D) + \hat{\mu}^l_L(\gamma + D))\sigma \leq \frac{1}{2} - (2\theta^C - 1)\gamma\sigma.
\end{aligned}$$

Thus it suffices to show that if both the two inequalities above are satisfied then we have $\theta^V \geq \theta_R^*$. By adding the two inequalities and a bit of algebra we get

$$(\hat{\mu}_L^l - \hat{\mu}_L^h)(2(\theta^C\theta^V + (1-\theta^C)(1-\theta^V)) - 1)(\gamma + D) \geq (2\theta^C - 1)\gamma.$$

Thus we see that $\hat{\mu}_L^l > \hat{\mu}_L^h$ and then it follows that

$$(2(\theta^C\theta^V + (1-\theta^C)(1-\theta^V)) - 1)(\gamma + D) \geq \frac{(2\theta^C - 1)\gamma}{(\hat{\mu}_L^l - \hat{\mu}_L^h)} \geq (2\theta^C - 1)\gamma.$$

Rearranging this inequality we get $\theta^V \geq \theta_R^*$. $\square$

Proof of observation on page 14.

Let $(\hat{x}^L, \hat{x}^H)$, $\hat{\mu}_L$ be a non-revealing equilibrium with $(\hat{x}^L, \hat{x}^H) \notin [x_{m_L}^*, x_{m_H}^*]^2$. Define the strategy profile $(\tilde{x}^L, \tilde{x}^H) \in [x_{m_L}^*, x_{m_H}^*]^2$ by

$$\tilde{x}^i = \left\{ \begin{array}{lll} x_{m_L}^* & \text{if} & \hat{x}^i < x_{m_L}^* \\ \hat{x}^i & \text{if} & x_{m_L}^* \leq \hat{x}^i \leq x_{m_H}^* \\ x_{m_H}^* & \text{if} & x_{m_H}^* < \hat{x}^i \end{array} \right\} \text{ for } i = L, H.$$

Define $\tilde{\mu}_L$ by

$$\begin{array}{rcl} \tilde{\mu}_L(\tilde{x}^L, \tilde{x}^H, \omega^V) & = & \hat{\mu}_L(\hat{x}^L, \hat{x}^H, \omega^V), \\ \tilde{\mu}_L(\tilde{x}^L, x, \omega^V) & = & \hat{\mu}_L(\hat{x}^L, x, \omega^V) \text{ for all } x \neq \tilde{x}^H, \omega^V \in \{l, h\}, \\ \tilde{\mu}_L(x, \tilde{x}^H, \omega^V) & = & \hat{\mu}_L(x, \hat{x}^H, \omega^V) \text{ for all } x \neq \tilde{x}^L, \omega^V \in \{l, h\}. \end{array}$$

We claim that $(\tilde{x}^L, \tilde{x}^H)$, $\tilde{\mu}_L$ is a non-revealing equilibrium. Obviously, Bayes rule is satisfied on the equilibrium path. To check that neither candidate has a profitable deviation first define

$$D_i = dist(\hat{x}^i, [x_{m_L}^*, x_{m_H}^*]), \ i = L, H.$$

Let $\hat{P}_i^{Eq}$ denote Candidate i's equilibrium probability of winning in the original equilibrium. It is straightforward to check that Candidate i's probability of winning in the new situation is

$$\hat{P}_i^{Eq} + (D_i - D_j)\sigma, \ j \neq i.$$

Let $\hat{P}_i(x)$ denote Candidate i's probability of winning when deviating to $x \neq \hat{x}^i$ in the original equilibrium. It is straightforward to check that Candidate i's probability of winning by deviating to x in the new situation is

$$\hat{P}_i(x) - D_j\sigma, \ j \neq i.$$

So a deviation by i to $x \neq \hat{x}^i$ is not profitable if

$$\hat{P}_i(x) \leq \hat{P}_i^{Eq} + D_i\sigma.$$

This is obviously true since $(\hat{x}^L, \hat{x}^H)$, $\hat{\mu}_L$ is an equilibrium. If $\tilde{x}^i \neq \hat{x}^i$ we also have to consider deviations to $\hat{x}^i$. After such a deviation the belief of the voters' is exactly as when they observe $(\tilde{x}^L, \tilde{x}^H)$, i.e., it is given by Bayesian updating based on the value of ω^V. And since the deviating candidate is moving from $x^*_{m_L}$ to $\hat{x}^i < x^*_{m_L}$ or from $x^*_{m_H}$ to $\hat{x}^i > x^*_{m_H}$ it follows that such a deviation cannot be profitable. □

Proof of Theorem 4.6.

1. Consider a strategy profile $(\hat{x}^L, \hat{x}^H)$ and a belief function $\hat{\mu}_L$ satisfying

$$\begin{aligned}
\hat{\mu}_L(\hat{x}^L, \hat{x}^H, l) &= \theta^V \text{ and } \hat{\mu}_L(\hat{x}^L, \hat{x}^H, h) = 1 - \theta^V; \\
\hat{\mu}_L(\hat{x}^L, x, \omega^V) &= 1 \text{ for all } x \neq \hat{x}^H, \omega^V = l, h; \\
\hat{\mu}_L(x, \hat{x}^H, \omega^V) &= 0 \text{ for all } x \neq \hat{x}^L, \omega^V = l, h.
\end{aligned}$$

This belief function satisfies Bayes rule on the equilibrium path and no other belief function makes it less profitable for either candidate to deviate. Thus it suffices to show that $(\hat{x}^L, \hat{x}^H)$, $\hat{\mu}_L$ is an equilibrium if and only if the condition on θ^V from the theorem is satisfied. It is easy to see that no candidate can profitably deviate if and only if Candidate H cannot profitably deviate to $x^*_{m_L}$ when $\omega^C = l$ and Candidate L cannot profitably deviate to $x^*_{m_H}$ when $\omega^C = h$. (If $\hat{x}^H = x^*_{m_L}$ ($\hat{x}^L = x^*_{m_H}$) we just have to check that Candidate H (L) cannot profitably deviate to a position very close to $x^*_{m_L}$ ($x^*_{m_H}$).)

When $\omega^C = l$ Candidate H's equilibrium probability of winning is

$$\frac{1}{2} - (2\theta^C - 1)(2\theta^V - 1)^2(\gamma + (\hat{x}^H - \hat{x}^L))\sigma.$$

By deviating to $x^*_{m_L}$ Candidate H wins with probability

$$\frac{1}{2} - (\gamma - (\hat{x}^L - x^*_{m_L}))\sigma.$$

(If $\hat{x}^H = x^*_{m_L}$ then he can win with a probability that is arbitrarily close to this number by deviating to a position x that is sufficiently close to $x^*_{m_L}$). Thus the deviation is not profitable if and only if

$$\frac{1}{2} - (2\theta^C - 1)(2\theta^V - 1)^2(\gamma + (\hat{x}^H - \hat{x}^L))\sigma \geq \frac{1}{2} - (\gamma - (\hat{x}^L - x^*_{m_L}))\sigma,$$

which is equivalent to

$$\theta^V \leq \frac{1}{2}(1 + \sqrt{\frac{\gamma - (\hat{x}^L - x^*_{m_L})}{(\gamma + (\hat{x}^H - \hat{x}^L))(2\theta^C - 1)}}).$$

Analogously we get that Candidate L does not have a profitable deviation if

$$\theta^V \leq \frac{1}{2}(1 + \sqrt{\frac{\gamma - (x^*_{m_H} - \hat{x}^H)}{(\gamma + (\hat{x}^H - \hat{x}^L))(2\theta^C - 1)}}).$$

Thus we have an equilibrium if and only if θ^V is below the minimum of the two cut-off values above. The minimum is precisely the cut-off value from the statement.

2. First note that if $(\hat{x}^L, \hat{x}^H)$ is an equilibrium strategy profile and $\hat{x}^L - \hat{x}^H \neq \gamma$ then each candidate will, for one value of ω^C, win with a probability strictly lower than $\frac{1}{2}$. Also note that at least one of the candidates has a deviation that will give him a win with probability $\frac{1}{2}$ no matter what the value of ω^C is and no matter what voters believe if they observe this deviation: If $\hat{x}^L - x^*_{m_L} > \gamma$ then candidate H can deviate to $\hat{x}^L - \gamma$, if $x^*_{m_H} - \hat{x}^H > \gamma$ then candidate L can deviate to $\hat{x}^H + \gamma$. Thus $(\hat{x}^L, \hat{x}^H)$ with $\hat{x}^L - \hat{x}^H \neq \gamma$ cannot be an equilibrium strategy profile.

Then consider $(\hat{x}^L, \hat{x}^H)$ with $\hat{x}^L - \hat{x}^H = \gamma$ and a voter belief function $\hat{\mu}_L$ satisfying

$$\begin{aligned}
\hat{\mu}_L(\hat{x}^L, \hat{x}^H, l) &= \theta^V \text{ and } \hat{\mu}_L(\hat{x}^L, \hat{x}^H, h) = 1 - \theta^V;\\
\hat{\mu}_L(\hat{x}^L, x, \omega^V) &= 0 \text{ for all } x < \hat{x}^H, \omega^V = l, h;\\
\hat{\mu}_L(\hat{x}^L, x, \omega^V) &= 1 \text{ for all } x > \hat{x}^H, \omega^V = l, h;\\
\hat{\mu}_L(x, \hat{x}^H, \omega^V) &= 0 \text{ for all } x < \hat{x}^L, \omega^V = l, h;\\
\hat{\mu}_L(x, \hat{x}^H, \omega^V) &= 1 \text{ for all } x > \hat{x}^L, \omega^V = l, h.
\end{aligned}$$

In this situation each candidate wins with probability $\frac{1}{2}$ for each value of ω^C. If a candidate deviates then he wins with a probability that is strictly smaller than $\frac{1}{2}$. Therefore we have an equilibrium.

Proof of Theorem 5.1.

1. The necessity of first condition follows from monotonicity (see the remarks above the theorem). And then the necessity of the second condition follows from part 1. of the result in Appendix B. To show that the conditions are sufficient we have to show that $((x^*_{m_L}, \hat{x}^L(h)), (\hat{x}^H(l), x^*_{m_H}))$ can be supported as an equilibrium by a *monotone* belief function. Let $\hat{\mu}_L$ be a monotone belief function satisfying the following conditions (none of these conditions violate monotonicity).

$$\begin{aligned}
\hat{\mu}_L(x^*_{m_L}, \hat{x}^H(l), l) &= \theta^V \text{ and } \hat{\mu}_L(\hat{x}^L(h), x^*_{m_H}, h) = 1-\theta^V; \\
\hat{\mu}_L(x^*_{m_L}, x, l) &= 1 \text{ for all } x < \hat{x}^H(l); \\
\hat{\mu}_L(x^*_{m_L}, x, l) &= \theta^V \text{ for all } x > \hat{x}^H(l); \\
\hat{\mu}_L(\hat{x}^L(h), x, h) &= 1-\theta^V \text{ for all } x \neq x^*_{m_H}; \\
\hat{\mu}_L(x, x^*_{m_H}, h) &= 1-\theta^V \text{ for all } x < \hat{x}^L; \\
\hat{\mu}_L(x, x^*_{m_H}, h) &= 0 \text{ for all } x > \hat{x}^L; \\
\hat{\mu}_L(x, \hat{x}^H(l), l) &= \theta^V \text{ for all } x \neq x^*_{m_L}.
\end{aligned}$$

It is easy to see that if Candidate H cannot profitably deviate to $x^*_{m_L}$ when $\omega^V = l$ (if $\hat{x}^H(l) \neq x^*_{m_L}$) and Candidate L cannot profitably deviate to $x^*_{m_H}$ when $\omega^V = h$ (if $\hat{x}^H(h) \neq x^*_{m_H}$) then neither candidate has any profitable deviations. Straightforward calculations show that neither of these two deviations are profitable if θ^V is below the upper bound stated in the theorem.

2. Suppose $((\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h))) \notin [x^*_{m_L}, x^*_{m_H}]^4$ is the strategy profile of a monotone non-revealing equilibrium. Consider the profile where, for each i and ω^V, $\hat{x}^i(\omega^V)$ is replaced by $x^*_{m_L}$ if $\hat{x}^i(\omega^V) < x^*_{m_L}$ and by $x^*_{m_H}$ if $x^*_{m_H} < \hat{x}^i(\omega^V)$. Denote this profile $((\tilde{x}^L(l), \tilde{x}^L(h)), (\tilde{x}^H(l), \tilde{x}^H(h)))$. Because of monotonicity we must have $\tilde{x}^L(l) = x^*_{m_L}$ and $\tilde{x}^H(h) = x^*_{m_H}$. Let $\tilde{\mu}_L$ be a monotone belief function satisfying the same conditions as $\hat{\mu}_L$ in the proof of the first statement (with $\hat{x}^L(h)$ and $\hat{x}^H(l)$ replaced by $\tilde{x}^L(h)$ and $\tilde{x}^H(l)$). We then claim that if Candidate i can profitably deviate from $\tilde{x}^i(\omega^V)$ when the voters' belief is given by $\tilde{\mu}_L$ then the same deviation would also be profitable in the original equilibrium, which is a contradiction. Candidate H has a profitable deviation from $\tilde{x}^H(\omega^V)$ when the voters' belief is given by $\tilde{\mu}_L$ if and only if it is profitable for him to deviate to $x^*_{m_L}$ when $\omega^V = l$. Suppose that this is the case. Using notation similar to that in the proof of the observation on page *14*, Candidate H's probability of winning from announcing $\tilde{x}^H(l)$ when $\omega^V = l$ is

$$\hat{P}^{Eq}_H(l) + (D_H - D_L)\sigma.$$

Candidate H's probaility of winning after deviating to $x^*_{m_L}$ is at most

$$\hat{P}_H(x^*_{m_L}, l) - D_L\sigma.$$

Thus we see that if the deviation is profitable then we must have

$$\hat{P}_H(x^*_{m_L}, l) > \hat{P}^{Eq}_H(l),$$

which means that in the original equilibrium a deviation by Candidate H to $x^*_{m_L}$ is profitable, which is a contradiction. Analogously we also get a contradiction if we assume that Candidate L can profitably deviate from $\tilde{x}^L(h)$ to $x^*_{m_H}$ when $\omega^V = h$. Thus $((\tilde{x}^L(l), \tilde{x}^L(h)), (\tilde{x}^H(l), \tilde{x}^H(h)))$, $\tilde{\mu}_L$ is a monotone non-revealing equilibrium. □

Proof of Theorem 6.1.

First we show that the ex ante expected policy utility of the median voter satisfies the ranking in the theorem. Then we show that if the ranking holds for the median voter then it holds for all voters.

The median voter's ex ante expected policy utility from the revelation strategy profile is:

$$\begin{aligned}
& \Pr(\omega^C = l, \omega^V = l)(\Pr(L|l,l)0 + \Pr(H|l,l)(-D)) \\
& + \Pr(\omega^C = h, \omega^V = h)(\Pr(H|h,h)0 + \Pr(L|h,h)(-D)) \\
& + \Pr(\omega^C = l, \omega^V = h)(\Pr(L|l,h)0 + \Pr(H|l,h)(-D)) \\
& + \Pr(\omega^C = h, \omega^V = l)(\Pr(H|h,l)0 + \Pr(L|h,l)(-D)) \\
= \; & \Pr(\omega^C = \omega^V)(\Pr(L|l,l)0 + \Pr(H|l,l)(-D)) \\
& + \Pr(\omega^C \neq \omega^V)(\Pr(L|l,h)0 + \Pr(H|l,h)(-D)) \\
= \; & (\theta^C\theta^V + (1-\theta^C)(1-\theta^V))(\frac{(1-\theta^C)(1-\theta^V)}{\theta^C\theta^V + (1-\theta^C)(1-\theta^V)}(-D)) \\
& +(\theta^C(1-\theta^V) + (1-\theta^C)\theta^V)(\frac{\theta^V(1-\theta^C)}{\theta^C(1-\theta^V) + (1-\theta^C)\theta^V}(-D)) \\
= \; & -(1-\theta^C)D.
\end{aligned}$$

The median voter's ex ante expected policy utility from the pandering profile is

$$\begin{aligned}
& \Pr(\omega^C = l, \omega^V = l)(\Pr(L|l,l)0 + \Pr(H|l,l)(-D)) \\
& + \Pr(\omega^C = h, \omega^V = h)(\Pr(H|h,h)0 + \Pr(L|h,h)(-D)) \\
& + \Pr(\omega^C = l, \omega^V = h)(\Pr(L|l,h)(-D) + \Pr(H|l,h)0) \\
& + \Pr(\omega^C = h, \omega^V = l)(\Pr(H|h,l)(-D) + \Pr(L|h,l)0) \\
= \ & \Pr(\omega^C = \omega^V)(\Pr(L|l,l)0 + \Pr(H|l,l)(-D) \\
& + \Pr(\omega^C \neq \omega^V)(\Pr(L|l,h)(-D) + \Pr(H|l,h)0) \\
= \ & (\theta^C\theta^V + (1-\theta^C)(1-\theta^V))(\frac{(1-\theta^C)(1-\theta^V)}{\theta^C\theta^V + (1-\theta^C)(1-\theta^V)}(-D)) \\
& +(\theta^C(1-\theta^V) + (1-\theta^C)\theta^V)(\frac{\theta^C(1-\theta^V)}{\theta^C(1-\theta^V) + (1-\theta^C)\theta^V}(-D)) \\
= \ & -(1-\theta^V)D,
\end{aligned}$$

which is lower than the utility from the revelation profile since $\theta^C > \theta^V$.

Let δ^*_{pol} denote the δ-value for which the median voter switches his vote when the candidates are polarized and $\omega^V = l$ (the δ-value for which the median voter switches his vote when $\omega^V = h$ is then $-\delta^*_{pol}$). The ex ante expected policy utility from the polarization profile can then be written

$$\begin{aligned}
& \Pr(\omega^C = l, \omega^V = l)(\Pr(\delta < \delta^*_{pol})\Pr(L|l,l)(-D) + \Pr(\delta > \delta^*_{pol})\Pr(H|l,l)(-D)) \\
& + \Pr(\omega^C = h, \omega^V = h)(\Pr(\delta > -\delta^*_{pol})\Pr(H|h,h)(-D) + \Pr(\delta < -\delta^*_{pol})\Pr(L|h,h)(-D)) \\
& + \Pr(\omega^C = l, \omega^V = h)(\Pr(\delta < -\delta^*_{pol})\Pr(L|l,h)(-D) + \Pr(\delta > -\delta^*_{pol})\Pr(H|l,h)(-D)) \\
& + \Pr(\omega^C = h, \omega^V = l)(\Pr(\delta > \delta^*_{pol})\Pr(H|h,l)(-D) + \Pr(\delta < \delta^*_{pol})\Pr(L|h,l)(-D)) \\
= \ & \Pr(\omega^C = \omega^V)(\Pr(\delta < \delta^*_{pol})\Pr(L|l,l)(-D) + \Pr(\delta > \delta^*_{pol})\Pr(H|l,l)(-D)) \\
& + \Pr(\omega^C \neq \omega^V)(\Pr(\delta > \delta^*_{pol})\Pr(H|h,l)(-D) + \Pr(\delta < \delta^*_{pol})\Pr(L|h,l)(-D)) \\
= \ & \theta^C\theta^V \Pr(\delta < \delta^*_{pol})(-D) + (1-\theta^C)(1-\theta^V)\Pr(\delta > \delta^*_{pol})(-D) \\
& +\theta^C(1-\theta^V)\Pr(\delta > \delta^*_{pol})(-D) + \theta^V(1-\theta^C)\Pr(\delta < \delta^*_{pol})(-D).
\end{aligned}$$

Since $\delta^*_{pol} = (1-2\theta^V)(D+\gamma)$ we have

$$\begin{aligned}
\Pr(\delta > \delta^*_{pol}) &= (\frac{1}{2\sigma} + (2\theta^V - 1)(D+\gamma))\sigma \\
&= \frac{1}{2} + (2\theta^V - 1)(D+\gamma)\sigma.
\end{aligned}$$

Thus the ex ante expected utility becomes

$$D((2\theta^V - 1)^2(D+\gamma)\sigma - \frac{1}{2}).$$

Since $D + \gamma < \frac{1}{2\sigma}$, this number is strictly smaller than

$$D((2\theta^V - 1)^2\frac{1}{2} - \frac{1}{2}) = -2\theta^V(1 - \theta^V)D,$$

which is strictly smaller than $-(1 - \theta^V)D$. Thus the ranking of profiles in the theorem is satisfied for the median voter.

Then consider a voter with $d_i = |x_i^*(L) - x_{m_L}^*| = |x_i^*(H) - x_{m_H}^*| \in (0, D)$. For such a voter the ex ante expected policy utility of the three profiles are

$$-(1 - \theta^C)D - d_i\theta^C,$$

$$-(1 - \theta^V)D - d_i\theta^V,$$

and

$$D((2\theta^V - 1)^2(D + \gamma)\sigma - \frac{1}{2}) - d_i((2\theta^V - 1)^2(D + \gamma)\sigma + \frac{1}{2}).$$

It is then straightforward to check that the ranking of the theorem is satisfied for all voters with $d_i \in (0, D)$.

For voters with $d_i \geq D$ the ex ante policy utility of all three equilibria are equal to $-d_i$. So for such voters all equilibria are equally good with respect to policy. □

Proof of Theorem 6.2.

By Theorem 6.1 it suffices to show that the ranking from the theorem holds with respect to ex ante expected *quality utility* ("the γ and δ terms"). With respect to quality all voters have identical preferences, so we do not need to distinguish between different voters.

We first show that revelation gives voters a strictly higher ex ante expected quality utility than pandering. In both of these strategy profiles there is always policy convergence. Thus the voting decision of each voter is based solely on quality utility. When candidates reveal their information then, for each combination of values of ω^C, ω^V, and δ, voters take the optimal decision given all available information in the game. When candidates are pandering voters cannot infer the value of ω^C and thus, for each combination of values of ω^C and ω^V, there are values of δ such that voters do not make the optimal choice (i.e., the optimal choice had kney known both ω^C and ω^V). Therefore revelation must give a higher ex ante expected quality utility.

Finally, we show that pandering dominates polarization with respect to quality utility. Since there is always policy convergence in the pandering profile it follows that, for each combination of values of ω^C, ω^V, and δ, voters take the optimal decision with respect to quality utility under the constraint that they do not know

ω^C. In the polarization profile there is always policy divergence and therefore, for each combination of values of ω^C and ω^V, there are values of δ such that the median voter (who decides the outcome of the election) does not make the optimal choice with respect to quality utility under the constraint that he does not know ω^C. Therefore pandering dominates polarization with respect to ex ante expected quality utility. $\square$

10 Appendix B

Theorem 10.1 (Non-Rev. Equil. in Case 2: Strategies and Existence) *Let* $((\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h))) \in [x^*_{m_L}, x^*_{m_H}]^4$.

1. *If* $\max\{\hat{x}^L(l) - x^*_{m_L}, x^*_{m_H} - \hat{x}^H(h)\} < \gamma$ *then* $(\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h)))$ *is the strategy profile of a non-revealing equilibrium if and only if*

$$\theta^V \leq \frac{1}{2}(1 + \min\{\frac{\gamma - (\hat{x}^L(l) - x^*_{m_L})}{\gamma + (\hat{x}^H(l) - \hat{x}^L(l))}, \frac{\gamma - (x^*_{m_H} - \hat{x}^H(h))}{\gamma + (\hat{x}^H(h) - \hat{x}^L(h))}\}).$$

2. *If* $\hat{x}^L(l) - x^*_{m_L} \geq \gamma$, $x^*_{m_H} - \hat{x}^H(h) < \gamma$ *then* $(\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h)))$ *is the strategy profile of a non-revealing equilibrium if and only if*

$$\hat{x}^L(l) - \hat{x}^H(l) = \gamma$$

   and

$$\theta^V \leq \frac{1}{2}(1 + \frac{\gamma - (x^*_{m_H} - \hat{x}^H(h))}{\gamma + (\hat{x}^H(h) - \hat{x}^L(h))}).$$

3. *If* $\hat{x}^L(l) - x^*_{m_L} < \gamma$, $x^*_{m_H} - \hat{x}^H(h) \geq \gamma$ *then* $(\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h)))$ *is the strategy profile of a non-revealing equilibrium if and only if*

$$\hat{x}^L(h) - \hat{x}^H(h) = \gamma$$

   and

$$\theta^V \leq \frac{1}{2}(1 + \frac{\gamma - (\hat{x}^L(l) - x^*_{m_L})}{\gamma + (\hat{x}^H(l) - \hat{x}^L(l))}).$$

4. *If* $\min\{\hat{x}^L(l) - x^*_{m_L}, x^*_{m_H} - \hat{x}^H(h)\} \geq \gamma$ *then* $(\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h)))$ *is the strategy profile of a non-revealing equilibrium if and only if*

$$\hat{x}^L(l) - \hat{x}^H(l) = \hat{x}^L(h) - \hat{x}^H(h) = \gamma.$$

Proof.

1. Consider a belief function $\hat{\mu}_L$ satifying the following conditions:

$$\begin{aligned}
\hat{\mu}_L(\hat{x}^L(l), \hat{x}^H(l), l) &= \theta^V \text{ and } \hat{\mu}_L(\hat{x}^L(h), \hat{x}^H(h), h) = 1 - \theta^V; \\
\hat{\mu}_L(\hat{x}^L(h), x, h) &= 1 \text{ for all } x \neq \hat{x}^H(h); \\
\hat{\mu}_L(\hat{x}^L(l), x, l) &= 1 \text{ for all } x \neq \hat{x}^H(l); \\
\hat{\mu}_L(x, \hat{x}^H(h), h) &= 0 \text{ for all } x \neq \hat{x}^L(h); \\
\hat{\mu}_L(x, \hat{x}^H(l), l) &= 0 \text{ for all } x \neq \hat{x}^L(l).
\end{aligned}$$

This belief function supports the strategy profile as an equilibrium if and only if Candidate L cannot profitably deviate to $x^*_{m_H}$ when $\omega^V = h$ and Candidate H cannot profitably deviate to $x^*_{m_L}$ when $\omega^V = l$. These conditions are equivalent to the condition on θ^V stated in the theorem. Finally, if the condition from the theorem is not satisfied then it is easy to see that, no matter how we define out-of-equilibrium beliefs, one of the deviations considered above will be profitable.

2. Use the arguments from *1.* when $\omega^V = h$ and the arguments from *4.* when $\omega^V = l$.

3. Use the arguments from *1.* when $\omega^V = l$ and the arguments from *4.* when $\omega^V = h$.

4. Consider a belief function $\hat{\mu}_L$ satifying the following conditions:

$$\begin{aligned}
\hat{\mu}_L(\hat{x}^L(l), \hat{x}^H(l), l) &= \theta^V \text{ and } \hat{\mu}_L(\hat{x}^L(h), \hat{x}^H(h), h) = 1 - \theta^V; \\
\hat{\mu}_L(\hat{x}^L(\omega^V), x, \omega^V) &= 0 \text{ for all } x < \hat{x}^H(\omega^V), \omega^V = l, h; \\
\hat{\mu}_L(\hat{x}^L(\omega^V), x, \omega^V) &= 1 \text{ for all } x > \hat{x}^H(\omega^V), \omega^V = l, h; \\
\hat{\mu}_L(x, \hat{x}^H(\omega^V), \omega^V) &= 0 \text{ for all } x < \hat{x}^L(\omega^V), \omega^V = l, h; \\
\hat{\mu}_L(x, \hat{x}^H(\omega^V), \omega^V) &= 1 \text{ for all } x > \hat{x}^L(\omega^V), \omega^V = l, h.
\end{aligned}$$

Such a belief function supports the strategy profile if the conditions stated in the theorem are satisfied.

If $\hat{x}^L(l) - \hat{x}^H(l) < \gamma$ ($\hat{x}^L(h) - \hat{x}^H(h) < \gamma$) then consider a deviation by Candidate H (L) to $\hat{x}^L(l) - \gamma$ ($\hat{x}^H(h) + \gamma$) when $\omega^V = l$ ($\omega^V = h$). After this deviation he wins with probability $\frac{1}{2}$ no matter what the out-of-equilibrium belief of the voters is. Thus it is a profitable deviation.

If $\hat{x}^L(l) - \hat{x}^H(l) > \gamma$ ($\hat{x}^L(h) - \hat{x}^H(h) > \gamma$) then consider a deviation by Candidate L (H) to $\hat{x}^H(l) + \gamma$ ($\hat{x}^L(h) - \gamma$) when $\omega^V = l$ ($\omega^V = h$). After this deviation he wins with probability $\frac{1}{2}$ no matter what the out-of-equilibrium belief of the voters is. Thus it is a profitable deviation.$\square$