Constraint reordering for iterative multi-body simulation with contact
Andrews, Sheldon; Erleben, Kenny; Teichmann, Marek

Publication date:
2017

Document Version
Peer reviewed version

Citation for published version (APA):
Constraint reordering for iterative multi-body simulation with contact

Sheldon Andrews¹,², Kenny Erleben³, Paul G. Kry¹, Marek Teichmann²

¹ McGill University
Montreal, Canada
sheldon.andrews@mail.mcgill.ca
kry@cs.mcgill.ca

² CM Labs Simulations, Inc.
Montreal, Canada
marest@cm-labs.com

³ University of Copenhagen
Copenhagen, DK
kenny@di.ku.dk

Abstract

Multi-body simulations with contact are non-smooth systems and wrought with discontinuities which arise due to non-interpenetration and frictional constraints. Linear systems are used for applications where real-time performance is a concern, such as interactive training or video games, which gives rise to a linear complementarity problem (LCP). A common mathematical formulation [5] of the LCP for the velocity-level equations of motion is

\[
JM^{-1}J^T \Delta \lambda + J(v + \Delta M^{-1}f_{ext}) = w
\]

where \( J \in \mathbb{R}^{m \times n} \) is the Jacobian matrix encoding the non-penetration and friction constraints, \( M \in \mathbb{R}^{n \times n} \) is the generalized mass matrix, \( v \) and \( f_{ext} \in \mathbb{R}^n \) are the generalized velocities and external forces of simulation bodies, respectively, \( \lambda \in \mathbb{R}^m \) are Lagrange multipliers representing the non-interpenetration normal forces and tangential frictional forces of each contact. The box constraints defined by \( z_{hi} \) and \( z_{lo} \) contain the lower and upper bound, respectively, of the normal and frictional impulses.

The unilateral and discontinuous nature of the system in Eq.(1) is problematic for many numerical solvers. Previous work has solved the LCP using simplex based pivoting methods, such as Lemke’s or the block pivoting approach by Judice and Pires [6]. These methods are able to provide exact solutions to the LCP, but are computationally infeasible for more than several hundred contact constraints. Iterative methods are more prolific for simulation of [5]. Since the simulations involve only two or three frictional contacts, all possible permutations of constraint equation ordering can easily be evaluated (i.e. there are 720 and 362880 permutations, respectively, for the capsule and three sphere body example). The default constraint ordering is such that if \( j \equiv (i \mod 3) \) is zero, it corresponds to the non-interpenetration constraint of the \( j \)th contact, and rows \( i + 1 \) and \( i + 2 \) are the associated friction equations.

Our own experiments verify that the constraint order affects rate of convergence. We highlight this by simulating the examples shown in Fig. 1 using the complementarity formulation of [5]. Since the simulations involve only two or three frictional contacts, all possible permutations of constraint equation ordering can easily be evaluated (i.e. there are 720 and 362880 permutations, respectively, for the capsule and three sphere body example). The default constraint ordering is such that if \( j \equiv (i \mod 3) \) is zero, it corresponds to the non-interpenetration constraint of the \( j \)th contact, and rows \( i + 1 \) and \( i + 2 \) are the associated friction equations.

The convergence plots for the default, best, and worst orderings are show in Fig. 2. The best ordering converges in less than 20 iterations for both examples. However, for the worst ordering, the error remains high even...
after 25 iterations. Furthermore, as indicated by Fig. 3, there is a large variation in the number of iterations required to reach a reasonable error threshold.

Motivated by these results, our work investigates strategies to accelerate the convergence of iterative solvers for multibody simulation by reordering of the constraint equations. We present an analysis of the following strategies:

- Solving constraint equations in a randomized order;
- Re-ordering constraint equations by heuristics based on the complementarity error and the effective mass;
- Grouping constraint equations and solving for several variables at once by a blockwise PGS.

We investigate the viability of each strategy for a number of rigid body simulation scenarios involving frictional contact and develop heuristics that allow automatic re-ordering, and grouping, of constraint equations to improve solver performance.

References


