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On a CFT limit of planar γ_i-deformed $\mathcal{N}=4$ SYM theory

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ABSTRACT

We show that an integrable four-dimensional non-unitary field theory that was recently proposed as a certain limit of the γ_i-deformed $\mathcal{N}=4$ SYM theory is incomplete and not conformal – not even in the planar limit. We complete this theory by double-trace couplings and find conformal one-loop fixed points when admitting respective complex coupling constants. These couplings must not be neglected in the planar limit, as they can contribute to planar multi-point functions. Based on our results for certain two-loop planar anomalous dimensions, we propose tests of integrability.

1. Introduction

In a recent paper [1], a certain limit is applied to the γ_i-deformation of $\mathcal{N}=4$ SYM theory [2], and the authors claim that the resulting non-unitary theory is conformal in the planar limit where the number N of colors is sent to infinity.

In this letter, we point out that the Lagrangian given in [1] is incomplete and does not define a CFT – not even in the planar limit. We complete the theory by the missing couplings that are required for renormalizability. They are quartic and have double-trace color structures and non-trivial β-functions.[1] Although double-trace couplings are apparently subleading in the large-N expansion, they must not be neglected in the planar limit [6]. Planar multi-point correlation functions for infinitely many composite single-trace operators depend on these couplings and are hence sensitive to the β-functions. We show this explicitly by determining to two-loop order the planar anomalous dimensions of several single-trace operators composed of two scalar fields. Moreover, we give an example for a planar four-point correlation function of further operators that depends on one of the double-trace couplings. Allowing for complex coupling constants in this model, whose single-trace part is already non-unitary, we find (one-loop) fixed points.

2. The proposed theory

In [1], the authors propose to apply the following limit to the γ_i-deformation

$$\gamma_i \to i\infty\,, \quad \lambda \to 0 \quad \text{with} \quad \xi_i = \sqrt{\lambda}\,\mathrm{e}^{-\frac{i}{2}\gamma_i} = \text{const.} \tag{1}$$

and focus on the special case of only a single non-vanishing coupling constant $\xi = \xi_3$. They claim that in the planar limit the Lagrangian of the resulting theory of two interacting complex scalars is given by equation (1) in their paper, which in our notation and conventions reads

$$\mathcal{L} = \mathrm{tr}\Big(-\partial^\mu\bar{\phi}_1\partial_\mu\phi^1 - \partial^\mu\bar{\phi}_2\partial_\mu\phi^2 + \frac{\xi^2}{N}\bar{\phi}_1\bar{\phi}_2\phi^1\phi^2\Big)\,. \tag{2}$$

This Lagrangian follows immediately when applying the limit (1) to the single-trace part of the action of the γ_i-deformation as written e.g. in [7].

3. Running double-trace couplings

The Lagrangian (2) is incomplete since certain divergences in correlation functions of composite (single-trace) operators cannot be absorbed by renormalizing these operators – not even in the planar limit. These divergences have to be canceled by terms for quartic double-trace couplings.

At one-loop order, the counter terms are determined by contracting two copies of the quartic scalar single-trace vertex of (2).

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[1] Quartic double-trace couplings emerged earlier in other contexts, e.g. in the effective potential of the worldvolume theory of interacting D3-branes [3], in non-supersymmetric orbifold theories [4] where they were related to tachyons in the dual string theory [5].

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For fields that transform in the adjoint representation of the global $SU(N)$ group, the only terms that have to be added to (2) read[2]

$$\mathcal{L}_{\mathrm{dt}} = -\frac{\xi^2}{N^2}\Big[\sum_{i\le j=1}^{2}(Q_{ij}^{ij}+\delta Q_{ij}^{ij})\,\mathrm{tr}(\bar\phi_i\bar\phi_j)\,\mathrm{tr}(\phi^i\phi^j) + (\tilde Q+\delta\tilde Q)\,\mathrm{tr}(\bar\phi_1\phi_2)\,\mathrm{tr}(\phi^1\bar\phi^2)\Big], \quad (3)$$

where Q_{ii}^{ii}, Q_{12}^{12} and $\tilde Q$ with $i=1,2$ denote the respective tree-level couplings. The counter terms δQ_{ii}^{ii}, δQ_{12}^{12} and $\delta\tilde Q$ are easily determined in $D=4-2\varepsilon$ dimensions and lead to the following one-loop β-functions in the planar limit

$$\begin{aligned}\beta_{Q_{ii}^{ii}} &= 2\varepsilon\delta Q_{ii}^{ii} = (1+4(Q_{ii}^{ii})^2)\frac{\xi^2}{(4\pi)^2},\\ \beta_{Q_{12}^{12}} &= 2\varepsilon\delta Q_{12}^{12} = 2(1-Q_{12}^{12})^2\frac{\xi^2}{(4\pi)^2},\\ \beta_{\tilde Q} &= 2\varepsilon\delta\tilde Q = 2(1-\tilde Q)^2\frac{\xi^2}{(4\pi)^2}. \end{aligned}\quad (4)$$

The β-functions (4) are non-vanishing for $Q_{ii}^{ii}=Q_{12}^{12}=\tilde Q=0$, showing that the theory given by the single-trace Lagrangian (2) alone is neither complete nor conformal. The double-trace couplings (3) cannot even be neglected in the planar limit: their counter terms are required to renormalize certain planar Feynman diagrams. This is exemplified below.

The β-functions for Q_{12}^{12} and $\tilde Q$ have a (one-loop) fixed point at $Q_{12}^{12}=1$ and $\tilde Q=1$, respectively, such that their running can be avoided. The β-functions for Q_{ii}^{ii}, however, do not have fixed points for real tree-level couplings Q_{ii}^{ii}. In fact, the β-functions for Q_{ii}^{ii} follow immediately when applying the limit (1) to our result [7], which we have obtained together with J. Fokken and which shows that the γ_i-deformation is not a CFT. In contrast to the γ_i-deformation, a theory with the single-trace Lagrangian (2) is not unitary. Hence, let us be bold and even allow complex coupling constants. In this case, we can choose the imaginary values $Q_{ii}^{ii}=\pm\frac{i}{2}$ that are their one-loop fixed-point values.

4. Effect on the planar spectrum

As we have already mentioned before, the double-trace couplings must not be neglected in the planar limit since their counter terms are required to renormalize planar correlation functions. Together with J. Fokken, we have already pointed this out in our calculation [8] of the anomalous dimensions of the composite operators $\mathcal{O}^{ii}=\mathrm{tr}(\phi^i\phi^i)$ in the γ_i-deformation.

Here, we determine to two-loop order the planar anomalous dimensions of all composite operators (states) built from two scalar fields that receive contributions from the double-trace couplings and counter terms (3). The affected states are $\mathcal{O}^{ii}=\mathrm{tr}(\phi^i\phi^i)$, $\mathcal{O}^{12}=\mathrm{tr}(\phi^1\phi^2)$, $\tilde{\mathcal{O}}=\mathrm{tr}(\bar\phi_1\phi^2)$ and their complex conjugates. The results for their anomalous dimensions read

$$\begin{aligned}\gamma^{\varrho}_{\mathcal{O}^{ii}} &= 4Q_{ii}^{ii}\frac{\xi^2}{(4\pi)^2} - 2\frac{\xi^4}{(4\pi)^4} - 2\varrho\beta_{Q_{ii}^{ii}}\frac{\xi^2}{(4\pi)^2},\\ \gamma^{\varrho}_{\mathcal{O}^{12}} &= 2(Q_{12}^{12}-1)\frac{\xi^2}{(4\pi)^2} - \varrho\beta_{Q_{12}^{12}}\frac{\xi^2}{(4\pi)^2},\\ \gamma^{\varrho}_{\tilde{\mathcal{O}}} &= 2(\tilde Q-1)\frac{\xi^2}{(4\pi)^2} - \varrho\beta_{\tilde Q}\frac{\xi^2}{(4\pi)^2}, \end{aligned}\quad (5)$$

where the first line also follows from [8] in the limit (1). The parameter ϱ captures the scheme dependence that starts at two-loop order and vanishes whenever the respective β-function is zero. The scheme labeled by ϱ is defined by applying minimal subtraction using $\xi_\varrho=\xi\,\mathrm{e}^{\frac{\varrho}{2}\varepsilon}$ as coupling constant. For example, we have $\varrho=0$ in the DR scheme but $\varrho=-\gamma_{\mathrm{E}}+\log 4\pi$ in the $\overline{\mathrm{DR}}$ scheme.[3]

At the fixed-point values $Q_{ii}^{ii}=\pm\frac{i}{2}$, $Q_{12}^{12}=\tilde Q=1$, the scheme dependence vanishes as expected, and we find that only the first of the above anomalous dimensions is non-vanishing but complex and reads

$$\gamma^{\varrho}_{\mathcal{O}^{ii}} = \pm 2i\frac{\xi^2}{(4\pi)^2} - 2\frac{\xi^4}{(4\pi)^4}. \quad (6)$$

5. Effect on planar multi-point correlation functions

Not only the two-point functions and hence anomalous dimensions (5) depend on the double-trace couplings (3) in the planar limit. Also, planar higher-point functions and hence also OPEs of operators built from more than two fields are sensitive to these couplings.

As an example, we consider the four-point correlation function of the following operators each built from three scalars

$$\begin{aligned}&\mathrm{tr}(\phi_1\phi_2^2)(x_1), \qquad \mathrm{tr}(\phi_1\bar\phi_2^2)(x_2),\\ &\mathrm{tr}(\bar\phi_1\phi_2^2)(x_3), \qquad \mathrm{tr}(\bar\phi_1\bar\phi_2^2)(x_4). \end{aligned}\quad (7)$$

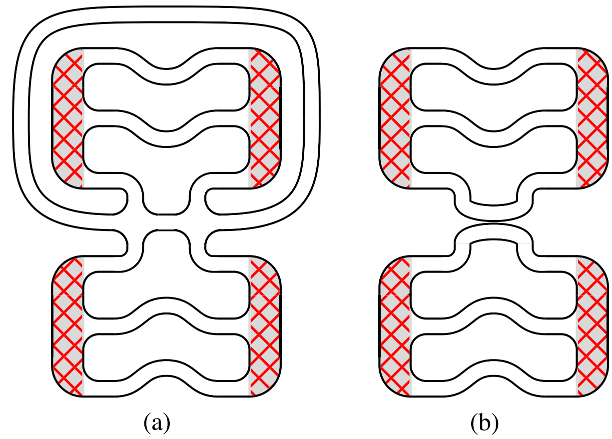

Fig. 1. The planar four-point function of the operators (7) receives a UV divergent contribution associated with a one-loop vertex correction. The divergence is canceled by the second diagram that contains the counter term of the respective double-trace coupling (3). The four-point function is shown in double-line notation and the operators are depicted as criss-crossed areas.

At order $\mathcal{O}(\xi^4)$ and when double-trace couplings are disregarded, the diagrams contain two quartic scalar single-trace vertices. There is one such diagram that is planar and contains a UV divergence that cannot be absorbed by renormalizing the operators (7). It contains a loop formed by the two direct connections of these two vertices and is shown in Fig. 1(a). Its UV divergence is associated with the one-loop correction of a quartic scalar double-trace vertex and has to be canceled by a respective counter-term diagram that also contributes in the planar limit. The latter diagram is shown in Fig. 1(b) and contains the respective counter-term coupling from the double-trace Lagrangian (3).[4] The result is therefore sensitive to the β-function of that coupling. This is another indication that

[2] For $U(N)$ fields, further couplings have to be considered [7].

[3] In the present case, the DR scheme coincides with the MS scheme.

[4] For non-vanishing tree-level double-trace couplings, also a third diagram with two double-trace couplings exists. It does not affect the argument.

one must not neglect the double-trace couplings – not even in the planar limit.

Diagrams similar to those in Fig. 1 exist for all correlation functions of composite single-trace operators in which the total charge of a subset of these operators matches the charge of a single-trace factor in a double-trace coupling. Likewise, a composite operator $\mathcal{O}^{ii}$, $\mathcal{O}^{12}_{12}$, $\tilde{\mathcal{O}}$ or a complex conjugate thereof can occur in the OPE of two single-trace operators whose total charge matches its charge. Via the anomalous dimension for that operator taken from (5), such an OPE is then sensitive to the respective β-function. The planar correlation functions of the field theory are hence widely affected by the double-trace couplings (3).

6. Conclusions

In this letter, we have explicitly shown that the model proposed in [1] is neither complete nor conformal – not even in the planar limit. We have shown how to complete the model in the planar limit by including the required double-trace couplings and their counter terms. Admitting complex coupling constants, we could find conformal fixed points for all induced couplings at one-loop order.

It would be very interesting to determine whether the fixed points persist at higher loop orders and at finite N, i.e. when terms beyond the planar limit that are subleading in the large-N expansion are taken into account.

Finally, the model opens the possibility for very interesting studies concerning integrability in a simplified setup which are of high relevance for integrability in $\mathcal{N}=4$ SYM theory and in its deformations. For instance, the very interesting question whether integrability is connected to conformal invariance can be investigated by studying the spectrum of the operators $\mathcal{O}^{ii}$ and $\mathcal{O}^{12}_{12}$ or $\tilde{\mathcal{O}}$ at the conformal fixed points and away from them. In particular, it should be analyzed whether the different behaviors (non-vanishing and vanishing) of their anomalous dimensions (5) at the fixed points can be reproduced in the integrability-based approach. This analysis should also be extended to both classes of multi-point functions: those that are sensitive and those that are not sensitive to the breakdown of conformal invariance.

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