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Published in:
CERME9 - Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education

Publication date:
2015

Media and milieus for complex numbers: An experiment with Maple based text

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Based on the notion of institutionally conditioned relationship to an organisation of knowledge and practice, we present our first design and implementation of a self-study module on complex numbers in an introductory mathematics course for non-mathematics majors. The basic idea is to develop an "interactive text" in the computer algebra system "Maple", designed to create a reasonably self-sustaining dialectics of media and milieus for students to learn about and work with complex numbers. We discuss some of the obstacles and constraints met in a first implementation of such a text, as well as hypothesis for future implementations.

Keywords: Media-milieus, Maple, self-study, interactive text.

INTRODUCTION

Mathematics departments and auditoria are among the last strongholds for chalkboards. Sfard (2014) reflects on the reasons why lectures continue to be a common teaching format in undergraduate mathematics education and hypothesizes that "watching a mathematician in action and imitating his moves while also trying to figure out the reasons for the strange things he is doing may be the only way to come to grips with [*mathematical objects*]" (p. 202). She also points out that this "coming to grips" can be initiated but by no means accomplished through watching; it requires solitary "thinking" (or "self-communication", in the terminology of Sfard). However, the importance of individual study in advanced mathematics is indeed upstream our current "world of incessant chatter, where everybody talks to everybody else and where educators preach collaborative learning" (p. 202).

This necessity of individual work could, in turn, be related to and in part explained by the primacy of *written discourse* in post-elementary mathematics, a phenomenon which appears already in the context of school algebra and which breaks with the usual role of written text as merely a formalized version of spoken language:

> It is important to notice that in the algebraic symbol manipulations, this relationship between oral and written work is reversed: writing comes first and orality is just a "secondary" accompaniment of the written algebraic formulations, which are furthermore not always easy to "oralize". Contrary to our mental habits, written algebraic symbolism is not a derivation of oral language: it is the source, the manifestation and the touchstone of algebraic "thinking". (Bosch, 2012, p. 7)

Certainly, the primacy of individual written work is fully compatible with the ostensive functions of chalkboard lectures and also with other common formats of university teaching such as exercise tutorials. It does not imply that other more interactive forms of teaching cannot be very useful. But given the crucial role of individual writing in advanced mathematics, the proliferation of online courses, and the increasing pressure on universities to deliver cost effective teaching, the following question is increasingly urgent for university mathematics education: to what extent can cost-intensive face-to-face teaching be dispensed with? What kinds of "real time" interaction with teachers and fellow students are necessary, if any?

These questions arose in a very practical and rather abrupt form for the first author, as he prepared to teach a first year course on calculus for a mixed public of science students: due to a mismatch of administrative rules and the calendar around Christmas, the course had to do with 8 instead of the usual 9 weeks of teaching. Given the density of topics to be covered in the course, spanning from review of secondary level calculus over linear differential equations to surface and multiple integrals, the decision was made to select one week of the normal teaching programme, in which complex numbers and basic complex functions are introduced – and replace it with "self-study". That did not mean that complex numbers should be any less part of the course as they continued to appear in other topics, in exam questions etc. It also did not mean that no support was to be given for students work with complex numbers; only would normal teaching time not be set aside for it, neither in lectures taught by the first author, or in class sessions taught by instructors.

Our basic idea for dealing with this situation was to create an "interactive" text based on the computer algebra system *Maple 17*. In this paper, we present our first designs and observations, as well as a theoretical framework, which helps to shape our work with the task and to situate it in the wider problem area, which was outlined in the first paragraphs above. We stress that this is on-going work, and we present neither a fully developed design, nor a systematic empirical study of its functioning. Thus, the points of the paper are, mainly, a set of theoretically sharpened design ideas.

THEORETICAL BACKGROUND: MEDIA AND MILIEUS

To render the quandaries outlined in the introduction more precise, we have found it useful to model them within the framework of the anthropological theory of the didactic (see, for instance, Chevallard, 1999; Winsløw, 2011, for more detailed introductions). In the most general formulation, we consider an organization O of mathematical practice and knowledge, a generic student x within an institution I, and we are interested in the conditions and constraints for establishing a given relationship, denoted $R_I(x,O)$, between x and O. Among the most immediate conditions are the *media* and *milieus* (cf. Chevallard, 2009) made available by the institution to establish and develop $R_I(x,O)$. According to the original definitions,

> The word *media* designates any system of representation of a part of the natural or social world in view of a certain public: the lecture of a professor of mathematics, a treatise on chemistry, a televised news programme, a regional or national newspaper, an Internet site, etc., are in this sense media systems. A *milieu* is understood in a sense close to that of *adidactic* in the theory of didactic situations. In fact, we designate as *milieu* any system that can be regarded as *devoid of intention* in terms of the answers it can bring, implicitly or explicitly, to a given question. By contrast (...) media are in general motivated by certain intentions, for instance, the intention to "inform". Naturally, a media can also, with regard to some particular question, be considered a milieu and used as such. (Chevallard, 2009, p. 344).

Medias and milieus in the university context

Adidactical situations in the university context do not always take the form of teacher initiated student work within a classroom setting, as in the primary school settings investigated by Brousseau (1997). The individual work of students referred to in the introduction must be considered an important form of *adidactical situation* if this notion should apply to central learning situations in the university setting. Thus, when students study a textbook and encounter an inference they do not follow, they are supposed to consider the text as a milieu that resources and constrains their efforts to fill in the gap. Similarly, an exercise to be solved is part of a milieu to which the students can apply and adapt their own relevant knowledge, and thus develop their relationship with the mathematical organizations to be acquired.

Consider a generic student x of the mathematics course *MatIntro* at the University of Copenhagen (I) whose relation to the mathematical practices and theories related to complex numbers ($O_{\mathbb{C}}$) we wish to study, and which of course have to be specified further. Without further assumption about x, we may have reason to stipulate a number of conditions and constraints related to x, in view of developing $R_I(x,O_{\mathbb{C}})$ according to the aims of I; in particular, we may assume that the students has established certain relations $R_I(x,O_i)$ to other mathematical organizations O_i in some way related to $O_{\mathbb{C}}$, as a result of being a student in secondary school (I') and university (I). In the example, such assumptions concern the detailed practice and theory previously encountered by students in relation to relevant domain such as arithmetic and basic algebra, including polynomial equations, vectors in the plane and exponential functions. Such assumptions can be based on a study of media and milieus through which many students are likely to have established their relation to O_i – for instance textbooks (media), tasks devolved to students (part of milieu) and computer algebra systems (which can function as kind of a milieu for students).

Chevallard (2009, p. 345) insists that "the existence of a vigorous (and rigorous) dialectics between media and milieus is a crucial condition to avoid that the study process is reduced to an uncritical copying of elements of answers which are scattered in institutions and society". In fact, even in the situation of lectures referred to in the introduction, this dialectics is possible: while the lecturer is of course, basically, acting as a medium, he may use the blackboard to create a milieu and let the students observe how he interacts with it, as when he says: "Let's see what happens if we replace *z* by $re^{i\theta}$ in these identities", and subsequently calculates in *real time* (with the possibility of committing errors, of volunteering students contributing at least orally, etc.). This way, students may observe "the mathematician in action" against a milieu, and students should try to follow and even anticipate the moves, which bring about the solution to whatever problem is at stake – but in the secure position of individual "thinking", without being exposed to the responsibility of completing all required actions in public, as the lecturer.

Even in the most traditional university teaching of mathematics, one can find rigorous systems of media and milieus. The main problem is that of their being vigorous: to what extent do students develop a critical and autonomous relationship to the "answers" found in the media proposed by the institution? Do students interact with milieus in which these answers are related to meaningful questions?

CONTEXT AND DESIGN

We now return to the somewhat special task of "teaching" first year students the basics of complex numbers without disposing of any regular teaching time. Focusing on this context as a case, the aim of this paper is to discuss design principles for creating a *rigorous and vigorous system of media and milieus for students' self-study*, using a computer algebra system, i.e. a mono-media rather than a multi-media design. And, since *vigour* is also an empirical quality, we end by a few observations of the first experiments with our design.

We first specify some conditions and constraints, which the design was based on.

Conditions and constraints from the context

The course *MatIntro* caters to a number of different BSc study programmes in science, including pure and applied mathematics, and its contents are thus the result of adapting to the needs of these. The reasons for sharing this and other first year courses among study programmes are in part financial, but also the flexibility it gives for students to change programme without having to take new basic courses. On the other hand, this arrangement leaves little room for the teachers in charge to change the contents of the course. The "self-study" material developed for complex numbers thus had to be relatively neutral in relation to its specific use in science disciplines.

The design was experimented in a run of a version of the course with about 250 students from biochemistry, chemistry and nano-science. There was no face-to-face teaching assigned to the design, and only very limited human resources available for interacting in other ways with the students, for instance providing feedback to exercise work. The only exception was what the lecturer (the first author) could use his own time during the course period. The design should thus provide students with media, which they could access on their own, and milieus in which they could both acquire and validate adequate relationships to the subject.

Another important condition for the solution to be developed is that *MatIntro* uses the advanced computer algebra system (CAS) Maple substantially, including weekly "labs". Maple functions as a learning resource in the course and as a tool supplementing and enhancing paper and pencil techniques. The focus is on insight rather than on computing results. The course grade is based on weekly hand-ins requiring Maple and two multiple-choice tests, in which no electronic tools are allowed.

As with many other CAS, work in *Maple* takes place in a window where input and output appear in consecutive lines, much as in a word processor window with the crucial exception that the latter has *only* input. The contents of the window can be saved as a file (called a *Maple sheet*), for later use and development. Students less familiar with *Maple* often use *Maple* sheets found on the Internet (including the manufacturer's own support pages) in order to get ideas for how to solve a given problem. In *MatIntro* and similar courses, teachers regularly publish Maple sheets on the course web page, to demonstrate *Maple* techniques and other mathematical points to students. Such sheets are typically short and sketchy, and the idea is that students may use them (or parts of them) in their work with related tasks.

Design

The basic idea of our design was to create an interactive introduction to complex numbers as a set of Maple sheets to be used for self-study by the students. 'Interactive' means that the document serves both as media and milieu and 'self-study' means that Sfard's 'observing a mathematician in action' (e.g., in a lecture) is replaced by 'observing Maple in action' through embedded input-output turns.

The choice of a course theme (complex numbers) for self-study was motivated by two considerations. Firstly, this subject is somewhat isolated in the course syllabus, which makes the experiment less risky for the course as such. Secondly, it seems particularly suited to exhibit *Maple in action*. The course goals focus on a small number of connected techniques within complex arithmetic, algebra of low degree polynomials, and basic properties of the complex exponential function. These are linked only to relatively simple Maple routines; moreover, complex numbers is a new area to most students and as already said, is relatively isolated in the course. So it could be conceivably developed as a kind of "Maple world".

Traditional use of Maple in teaching is as a *milieu* in which certain computations, visualisations and experiments can be performed using an input-output scheme. This milieu becomes operational only as a supplement to media and other elements of milieus that students interact with during lectures, exercise solving etc. The students must navigate in this juxtaposition of different media and milieus to build a set of relationships $R_I(x,O)$ as demanded from *I*. The common formats of teaching gives various support to the student, and in particular the 'mathematician in action' is not just part of a media system for mathematical content but also demonstrates how to use milieus (which could be Maple experiments) to take on various concrete tasks. If "live" (blackboard or otherwise) manipulations of mathematical objects are to be replaced by a simple Maple sheet, it must reproduce some of the same experience of purposeful, reasoned instruction, complemented with work in explorative milieus. One added potential (difficult to realize!) is that students could take on a more active role as compared with the common situation in lectures.

In our design, we used Maple to create a media-milieu dialectics in three ways: as a generator of *dynamic text* (media with embedded milieu), as a generator of *drills for techniques to solve specific tasks* with *feedback to students' solution proposals* (milieu with embedded media), and to *explore and exhibit phenomena* (media with embedded milieu). In order to explain these options, we name the main components of a Maple sheet:

- *Maple text*: non-executable text, essentially as in a word processor;
- *Maple input* : executable text;
- *Maple output* : results of executions;
- *Maple Components*: programmable interactive components (code is not shown) including sliders, buttons, Math Containers for in- and output, and other items.

The user or author of a sheet may hide or show Maple input, to produce different appearances of the worksheet. With the input hidden, the worksheet may read as a simple textbook (media) with its mixture of text, formulae, calculations and graphics; it is invisible that the latter are in fact Maple output. When the Maple input is revealed one sees the embedded milieu, which can be acted on.

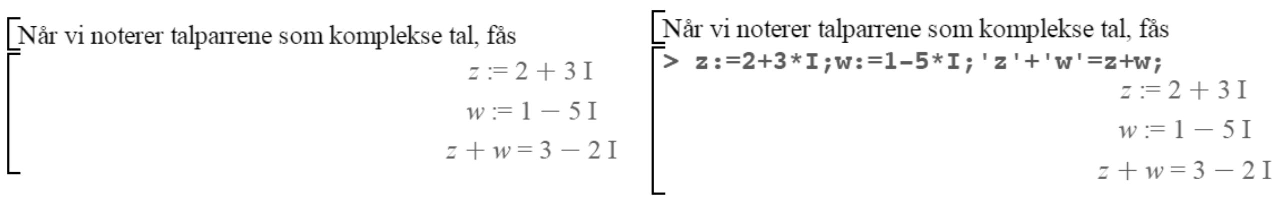

Figure 1: An excerpt of a Maple sheet with text, input (hidden in left version) and output

Here is a simple example (cf. Figure 1 which shows a short excerpt of an introductory Maple sheet for the course): if we have defined two complex numbers by `>z:=2+3*I;w:=1-5*I;` a statement of their sum should be coded `>'z'+'w'=z+w;` rather than `>'z'+'w'=3-2*I;`. In both cases the output as part of the text reads $z + w = 3 - 2\,I$, but the first option will preserve the correctness of the second part, even if changes are effected in the first. Thus, '`=z+w;`' is "Maple in action", while '`=3-2*I;`' is just communicating the sheet author's knowledge.

The example illustrates how media-milieu dialectics is used explicitly in the design. When Maple is the primary source of mathematical content the Maple in- and outputs are a-didactical and the dialectics is rigorous so that the user may act in this milieu (here, mainly re-assign z and w or change the operation). Conversely, no non-Maple manipulation, such as '`=3-2*I;`' can be negotiated by the user and therefore may lead to imitation rather than construction of knowledge.

Milieus are embedded more explicitly into the sheet by using Maple applets. The most basic way is a *drill*, aimed at education a specific technique. Computation of reciprocals of complex numbers serves as an example. It consists of six components (see Figure 2): a button "new number", an output container exhibiting a complex number, an input container to be filled by the student, an erase button, a true-false button, and an output container displaying either "true" or "false".

When "new number" is clicked, Maple exhibits a complex number randomly chosen from a pre-set range (with more than 400 different numbers). Thus Maple serves two purposes. It generates a variation of tasks of identical type and provides feedback to the student's performance with a technique to solve the task.

Math containers are also used to let the students investigate phenomena from a more theoretical point. The geometry of complex multiplication by a real factor is explored by means of a "new number" button, a slider to choose the real factor and a plot container exhibiting the geometric effect of multiplication. Similar milieus are offered to explore the geometric meaning of multiplication by purely imaginary factors and the general case. The student is asked to insert personal descriptions in terms of modulus and argument in the text (for this, no feedback is available). The full Maple material can be consulted at the web address http://www.math.ku.dk/kurser/2013-14/blok2/matintrokem/selvstudium/, where the reader may explore the examples mentioned above, and more.

The functioning of the sheets produced so far is incomplete in at least two respects. The plainest is that feedback, which can be produced with Maple, is very limited and (as with most software) merciless on syntax and other formal errors, which are naturally common for beginners. Also, institutionalization (in the sense of Brousseau, 1997, p. 215) is independent of what students have actually achieved in a milieu.

To address these problems in part a discussion forum dedicated to the self-study part of the course was set up in the on-line platform of the course. Here students and teachers could post and answer any question or comment. The intention was that teachers monitored the discussion forum in a stipulated weekly time slot, and students were promised answers and rectifications to questions and problems, if possible immediately but else within a deadline.

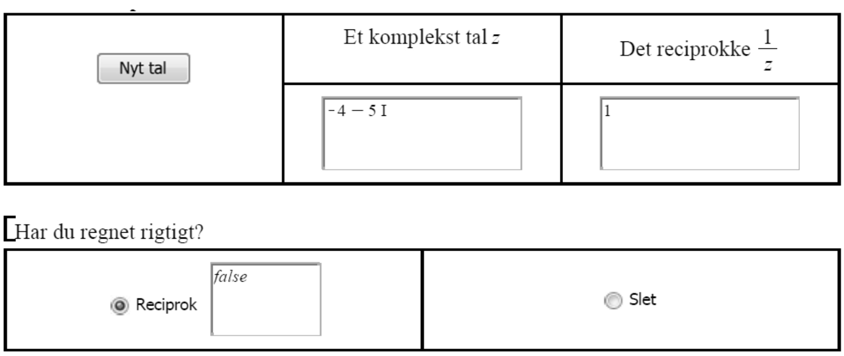

Figure 2: A simple drill applet to work with a technique for taking reciprocals

SOME OBSERVATIONS FROM A FIRST EXPERIMENTATION

Students were free to choose when to work with the material, and to work individually or with others. In their rather tight schedule only Friday afternoons were available to all students. Collaboration was encouraged in this slot. We reserved three afternoons for the monitoring of the discussion forum, corresponding to the three sub-modules into which the Maple sheets for the full self-study, corresponding to one week of teaching, were organized: (i) arithmetic and geometry of the complex plane, (ii) the quadratic equation, the exponential function, (iii) review of main points. Very few used the Friday slots and the discussion forum was practically unused (6 questions altogether). We have no solid evidence of the reason for lack of use of the discussion forum, but from focus group interviews, it appears that students preferred to confer with fellow students and instructor, as well as more informal channels such as *Facebook*.

A focus group test and interview

Following the first module a focus group interview was set up with four volunteer students commencing with four written tasks very similar to the drills of the first module sheets. Having collected the students' individual solutions to these tasks, we asked a number of questions about their experience with the sheets. The students reported to have worked with the sheets, in a combination of individual work and conferences with study groups. None reported difficulties; some even said the exercises were too easy. However, our written tasks suggested severe difficulties; for instance, not one of them had been able to compute $(1+2I)(3-I)$, and in at least two of the cases, this appeared to be related to an inappropriate mastery of the distributive law. From this experience at least two observations can be made.

The first is methodological: what students *say* can be strongly misleading as to what they are actually able to do. The reasons could be reluctance to admit difficulties to fellow students and lecturers as well as self-deception. The latter might in part be due to insufficient feedback from the sheets (in fact, one student noted that the applets may only let you know *that* you are wrong, not why).

The second point is specific to students' insufficient relationship with the distributive law (and other organisations of knowledge in the borderland between arithmetic and algebra). Whether or not it is rooted in CAS-use in secondary school, it becomes an obstacle to using the theoretical definition of operations in the setting of complex numbers. We had not anticipated such obstacles in our design.

Students' results

The most important measure of $R_I(x,O)$, from the viewpoint of students and institution (the university) alike, is that of summative assessments. In *MatIntro* naturally some of the items of the multiple choice tests are on complex numbers. Each test had one such task: in test 1, to calculate $\frac{(2+3I)(1-I)}{1+I}$; in test 2 to find a polynomial with roots $1+7I$ and $1-7I$. Texts and notes, but no computers, were allowed at the test. Students performed roughly 20% less well on these items than on the tests as a whole, but this is insignificant as the relative difficulty of items cannot be assumed to be uniform. However, the 250 students were organized in nine classes (for exercise sessions with instructors), and we observed significant variations among classes as concerns the ratio of *class average score on complex numbers items* and *average score on all items*. We suspected these could be related to class instructors' own initiatives to include complex number tasks in their teaching, even if they were not asked to. So, a survey on this was sent to them after the completion of the course. Indeed, it turned out that one instructor (whose class performed very well on complex numbers) had written and used a hand-out on complex numbers, based on the Maple sheets and providing an overview of most formulae and results on complex numbers to know about; these were also used by the instructors of a few other well performing classes. One could thus suspect that parts of the variations could be ascribed to these initiatives and the differences of focus. We do not have firm evidence to support our hypothesis that test oriented teaching caused the variations observed, but we can confirm that the "self-study of Maple sheets" were, in the end, far from the only source of media and milieus for students to learn about complex numbers.

Students' opinions

Most of our other evidence is gathered indirectly on students' impressions and opinions. These are in particular expressed in the anonymous on-line evaluation of the course, which is usually not very positive with students from the study programmes concerned. In fact, 71 respondents give mostly negative comments about the course as a whole, and also about the self-study (complex number) part. They criticize the lack of feedback, and some say directly that there should be accompanying lectures or videos of lectures. The idea of drills (built into the Maple sheets) seems, however, well accepted by those who mention them. Regular meetings between the lecturer and student representatives confirm these trends.

PERSPECTIVES AND FUTURE EXPERIMENT

The experiment with an "island" of self-study was imposed from the outside as an unexpected addition to the other constraints on this tightly packed introductory course for non-mathematics majors. The reaction of students and teachers alike seem to suggest that a satisfactory form of self-study is very hard to realize in these circumstances. In the absence of familiar teaching formats, they tend to replace a coherent organisation $O_{\mathbb{C}}$ of theory and practice related to complex numbers (as developed in the Maple sheets and foreseen in the course description) with a rather minimal set $o_{\mathbb{C}}$ of disconnected practices which appear in the most rigorously administered parts of the summative assessment (the multiple choice tests). This way, the institution as a whole seems to develop $R_I(x,o_{\mathbb{C}})$ rather than $R_I(x,O_{\mathbb{C}})$, as its way to measure $R_I(x,O_{\mathbb{C}})$ is in fact rather a minimal measure of $R_I(x,o_{\mathbb{C}})$.

One tempting way to proceed (under similar conditions) may thus be to accept that a generic student x in this institutional context I cannot be expected to develop more than $R_I(x,o_{\mathbb{C}})$. The design could be adapted to this situation by replacing most of the text with succinct expositions of task types and techniques, enriched with many more interactive drill items, examples and warnings on typical errors, and so on. This kind of approach seems to be endorsed, at least in deed, by many students and instructors.

This, however, would not be acceptable from the institutional point of view, for the reasons outlined in our exposition of "Conditions and constraints from the context". One could then try to pursue the (quite plausible) claim that the course is much too heavily packed with content, given the course time allotted. This makes the course very vulnerable to incidences as the one we described (one week disappearing), and one may also suspect that whether one accept it or not, outcomes of type $R_I(x,o_{\mathbb{C}})$ rather than $R_I(x,O_{\mathbb{C}})$ will be common unless the size of the total content organisation is reduced and adjusted more exactly to the students' programme. This, however, takes us away from the level of course design to the level of institutional politics.

Indeed, we are certain that the designed Maple sheets reflect the course goals adequately. They have been used with good results in two high school classes, in the spring of 2014 (with much more direct instruction on the use the sheets). This only strengthens our hypothesis that the framework (at the level of pedagogy) for their first implementation in *MatIntro* was inadequate, and that an island of unaccompanied "self-study" in such a course is very likely to result in side effects as those observed.

A third way out, which we shall try to pursue in the next run of the course, is therefore to make use of a new concession of the institution, allowing lectures on some of the Friday afternoons. They will be used to introduce the students to the sheets, and in particular how to make use of them (both as media and milieus). The class instructors will be asked to align with these lectures and use the course material rather than self-made cookbooks on how to pass the tests; in turn these latter will be amended to correspond more fully to both practical and theoretical levels of $O_{\mathbb{C}}$.

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