Comparing coefficients of nested nonlinear probability models

Kohler, Ulrich; Karlson, Kristian Bernt; Holm, Anders

The Stata Journal

The *Stata Journal* publishes reviewed papers together with shorter notes or comments, regular columns, book reviews, and other material of interest to Stata users. Examples of the types of papers include 1) expository papers that link the use of Stata commands or programs to associated principles, such as those that will serve as tutorials for users first encountering a new field of statistics or a major new technique; 2) papers that go "beyond the Stata manual" in explaining key features or uses of Stata that are of interest to intermediate or advanced users of Stata; 3) papers that discuss new commands or Stata programs of interest either to a wide spectrum of users (e.g., in data management or graphics) or to some large segment of Stata users (e.g., in survey statistics, survival analysis, panel analysis, or limited dependent variable modeling); 4) papers analyzing the statistical properties of new or existing estimators and tests in Stata; 5) papers that could be of interest or usefulness to researchers, especially in fields that are of practical importance but are not often included in texts or other journals, such as the use of Stata in managing datasets, especially large datasets, with advice from hard-won experience; and 6) papers of interest to those who teach, including Stata with topics such as extended examples of techniques and interpretation of results, simulations of statistical concepts, and overviews of subject areas.

For more information on the *Stata Journal*, including information for authors, see the webpage

http://www.stata-journal.com

The *Stata Journal* is indexed and abstracted in the following:

- CompuMath Citation Index®
- Current Contents/Social and Behavioral Sciences®
- RePEc: Research Papers in Economics
- Science Citation Index Expanded (also known as SciSearch®)
- Scopus™
- Social Sciences Citation Index®

Comparing coefficients of nested nonlinear probability models

Ulrich Kohler
Wissenschaftszentrum Berlin
kohler@wzb.eu

Kristian Bernt Karlson
Department of Education
Aarhus University
kbk@dpu.dk

Anders Holm
Department of Education
Aarhus University
aholm@dpu.dk

Abstract. In a series of recent articles, Karlson, Holm, and Breen (Breen, Karlson, and Holm, 2011, http://papers.ssrn.com/sol3/papers.cfm?abstractid=1730065; Karlson and Holm, 2011, *Research in Stratification and Social Mobility* 29: 221–237; Karlson, Holm, and Breen, 2010, http://www.yale.edu/ciqle/Breen_Scaling %20effects.pdf) have developed a method for comparing the estimated coefficients of two nested nonlinear probability models. In this article, we describe this method and the user-written program `khb`, which implements the method. The KHB method is a general decomposition method that is unaffected by the rescaling or attenuation bias that arises in cross-model comparisons in nonlinear models. It recovers the degree to which a control variable, Z, mediates or explains the relationship between X and a latent outcome variable, Y^*, underlying the nonlinear probability model. It also decomposes effects of both discrete and continuous variables, applies to average partial effects, and provides analytically derived statistical tests. The method can be extended to other models in the generalized linear model family.

Keywords: st0236, khb, decomposition, path analysis, total effects, indirect effects, direct effects, logit, probit, primary effects, secondary effects, generalized linear model, KHB method

1   Introduction

Social scientists are often interested in research questions that need to be analyzed by comparing the estimated coefficients of nested linear regression models. One example, which we will use throughout this article, is the decomposition of total effects into direct and indirect effects. In studies of social mobility, sociologists analyze how the occupational position of parents affects the occupational position of their offspring. It is commonly hypothesized that the total effect of parents' occupational positions operates indirectly by influencing their children's educational attainment and more directly by inheriting economic capital or exploiting social capital (Blau and Duncan 1967; Breen 2004). In another example, political scientists seek to disentangle how much of the influence of long-term party identification on the voting decision is mediated by short-term

issues and candidate orientations (Miller and Shanks 1996). In studies of subjective well-being, economists have repeatedly posed the question of the extent to which the negative effect of unemployment can be explained by income losses resulting from unemployment (Frey and Stutzer 2002).

In the context of linear regression models, the comparison of estimated coefficients—and hence the decomposition of total effects into direct and indirect effects—is straightforward. Let there be the linear regression model

$$Y = \alpha_F + \beta_F X + \gamma_F Z + \delta_F C + \epsilon \tag{1}$$

where X is the variable whose effect is to be decomposed (it is the *key variable*), Z is a mediator in the sense that X is hypothesized to partly operate through it, and C is a concomitant variable used as a control variable for the decomposition. In this hypothesized situation, the regression coefficient β_F is commonly called the direct effect. The total effect of X is captured by the coefficient β_R of a reduced model that leaves out the mediator Z:

$$Y = \alpha_R + \beta_R X + \delta_R C + \varepsilon \tag{2}$$

The indirect effect is the difference between the total effect and the direct effects; that is,

$$\beta_I = \beta_R - \beta_F \tag{3}$$

Formulas for the standard errors of the coefficients of β_F, β_R, and β_I are readily available (Sobel 1982), and situations with more variables of type X, Z, and C do not pose any statistical problems.

The method just described is so common that it has also been used for other models of the generalized linear model family, in particular, in binary logit and probit models. However, comparing the effects of nested nonlinear probability models is not as straightforward as in linear models (Winship and Mare 1984). In nested nonlinear probability models, uncontrolled and controlled coefficients can differ not only because of confounding but also because of a rescaling of the model that arises whenever the mediator variable has an independent effect on the dependent variable. Stated differently, the inclusion of the mediator variable Z in a nonlinear probability model will alter the coefficient of X regardless of whether Z is correlated with X; it is a sufficient condition that Z is correlated with Y.

Several solutions have been proposed to deal with the problem of cross-model coefficient comparability. The solutions include Y standardization (Winship and Mare 1984; Long 1997); the use of average partial effects (Wooldridge 2002); and a decomposition method for binary response models developed by Erikson et al. (2005) and generalized by Buis (2010). However, Monte Carlo studies presented in Karlson, Holm, and Breen (2010) and Karlson and Holm (2011) show that the KHB method proposed by Karlson, Holm, and Breen is always as good as or better than these methods whenever it comes to recovering the degree of confounding net of rescaling; that is, the degree to which Z mediates the relationship between X and Y^* (that is, the underlying outcome of interest). In addition, the KHB method decomposes effects of both discrete

and continuous variables, can be extended to accommodate average partial effects, provides analytically derived statistical tests, and is computationally simple and intuitive (Karlson, Holm, and Breen 2010; Karlson and Holm 2011). In fact, the KHB method extends the decomposability properties of linear models to nonlinear probability models (Breen, Karlson, and Holm 2011).

In this article, we introduce the KHB method and the user-written Stata command `khb`, which implements the method. In section 2, we present the method and its formulas; in section 3, we present the `khb` command; and in section 4, we apply this command to data on inequality in educational attainment.

2   Method and formulas

In this section, we introduce the KHB method for a binary response model. A formal derivation, including proofs and a generalization, is given in Karlson, Holm, and Breen (2010), Breen, Karlson, and Holm (2011), and Karlson and Holm (2011).

2.1   Restating the problem

We begin with stating the problem of comparing coefficients of binary response models. Consider a latent variable model similar to (1),

$$Y^* = \alpha_F + \beta_F X + \gamma_F Z + \delta_F C + \epsilon \tag{4}$$

where Y^* is an unmeasured latent variable and ϵ is an error term. Likewise, we have a reduced model similar to (2),

$$Y^* = \alpha_R + \beta_R X + \delta_R C + \varepsilon \tag{5}$$

The sole difference between these two models is the mediator Z. Therefore the difference, $\beta_R - \beta_F$, can be interpreted as the indirect effect, just as in the linear regression model. However, because the dependent variable Y^* is unobserved, we cannot estimate the coefficients of these models.

If we measure a dichotomous variable Y with a value of 0 if Y^* is smaller than some threshold τ and a value of 1 if $Y^* \geq \tau$, then it is possible to develop an estimator for the regression coefficients. However, this estimator requires an assumption about the distribution and the variance of the error terms in (4) and (5) (Long 1997). If we assume a binary logit model, it can been shown that the resulting estimators b_F and b_R for the underlying coefficients are

$$b_F = \frac{\beta_F}{\sigma_F} \quad \text{and} \quad b_R = \frac{\beta_R}{\sigma_R} \tag{6}$$

where σ_F and σ_R are scale parameters, which are a function of the residual standard deviation of the underlying linear models. We only identify the underlying coefficients

of interest relative to a scale unknown to us. Thus we can never estimate β_F, β_R, σ_F, nor σ_R, only their respective ratios. It always holds that $\sigma_F \leq \sigma_R$, because adding a control variable Z to the model reduces the unexplained portion of the variance in Y^*.

It follows from (1) that the difference between b_F and b_R is affected by two differences, namely, differences in effects and differences in scale parameters:

$$b_R - b_F = \frac{\beta_R}{\sigma_R} - \frac{\beta_F}{\sigma_F} \tag{7}$$

Thus, had we followed standard practice in linear models and used (2) as an estimator for the indirect effect in a logit model, we would conflate mediation with rescaling. The difference is due to mediation—or confounding—to the degree at which X and Z are correlated and Z has an effect on Y^* independently of X.

Likewise, the difference is due to rescaling to the degree at which the restricted model has a larger residual standard deviation than the model that includes Z. Rescaling consequently occurs whenever Z has an effect on Y^* that is independent of X. Because the difference between b_F and b_R conflates mediation (or confounding) and rescaling, the question is how to achieve an estimate of the indirect effect not distorted by differences in scales.

2.2 The KHB method

A simple answer to that question is to use the KHB method. The idea is to extract from Z the information that is not contained in X. This is done by calculating the residuals of a linear regression of Z on X; that is,

$$R = Z - (a + bX) \tag{8}$$

where a and b are the estimated regression parameters of a linear regression. R is then used instead of Z for the reduced model; that is, instead of using (5), we use

$$Y^* = \widetilde{\alpha}_R + \widetilde{\beta}_R X + \widetilde{\gamma}_R R + \widetilde{\delta}_R C + \epsilon \tag{9}$$

Because R and Z differ only in the component in Z that is correlated with X, the full model of (4) is no more predictive than (9), and consequently, the residuals have the same standard deviation. It follows that $\widetilde{\sigma}_R = \sigma_F$, with $\widetilde{\sigma}_R$ being the standard deviation of the residuals of the estimated model from (9). Furthermore, we know that $\widetilde{\beta}_R = \beta_R$; hence, the difference between the estimated regression coefficients $\widetilde{b}_R$ and b_F of (9) and (4) can be rewritten as

$$\widetilde{b}_R - b_F = \frac{\widetilde{\beta}_R}{\widetilde{\sigma}_R} - \frac{\beta_F}{\sigma_F} = \frac{\beta_R - \beta_F}{\sigma_F}$$

The difference reflects the indirect effect as introduced below (3) divided by some common scale. Of course, rather than considering differences due to confounding, one may also consider the confounding ratio,

$$\frac{\widetilde{b}_R}{b_F} = \frac{\frac{\beta_R}{\sigma_F}}{\frac{\beta_F}{\sigma_F}} = \frac{\beta_R}{\beta_F} \tag{10}$$

where the scale parameter cancels out entirely. The same is true for the percentage difference of the estimated regression parameters

$$100 \times \frac{\widetilde{b}_R - b_F}{\widetilde{b}_R} = 100 \times \frac{\frac{\beta_R}{\sigma_F} - \frac{\beta_F}{\sigma_F}}{\frac{\beta_R}{\sigma_F}} = 100 \times \frac{\beta_R - \beta_F}{\beta_R} \tag{11}$$

which is a percentage measure of the degree to which Z mediates the $Z-Y^*$ relationship, called the confounding percentage.

2.3   Significance test for the difference in effects

One key variable and one confounder

When we wish to test whether the variable Z confounds X, we need to test the hypothesis $H_0 \colon \widetilde{b}_R - b_F = 0$ because it amounts to testing whether $\beta_R = \beta_F$. From Karlson, Holm, and Breen (2010), we have

$$\widetilde{b}_R - b_F = \frac{\gamma_F}{\sigma_F} b \tag{12}$$

where b is the regression coefficient of X on Z from (3). For (12) not to be zero, there needs to be a direct effect of the mediator (or confounder) on the outcome, $\frac{\gamma_F}{\sigma_F} \neq 0$, and a correlation between X and Z, that is, $b \neq 0$. Starting from these observations, a test statistic based on the delta method (Sobel 1982) can be derived for the indirect effect,

$$Z = \frac{\sqrt{N}\left(\widetilde{b}_R - b_F\right)}{\sqrt{\mathbf{a}'\boldsymbol{\Sigma}\mathbf{a}}} \sim N(0,1)$$

where $\mathbf{a}$ is the vector $\left(\gamma_F/\sigma_F, b\right)'$ and $\boldsymbol{\Sigma}$ is the variance–covariance matrix of γ_F and b; see Karlson, Holm, and Breen (2010) for a generalization.

Many key variables and many confounders

In the more general case of several key predictor variables and many confounders, it is still possible to evaluate the confounding effect of a vector of J confounders, $\mathbf{z}$, on a vector of key variables, $\mathbf{x}$. Usually, we want to know the confounding effect of $\mathbf{z}$ on one key variable conditional on the other key variables being in the model. Denote the logit or probit effect of the lth key variable, x_l, on y conditional on x, and denote the

residualized $\mathbf{z}$'s as $\widetilde{b}_{x_l.\widetilde{z}x(l)}$ ($\widetilde{b}_R$ in the previous section) where $x(l)$ denotes the set of $\mathbf{x}$'s without x_l. Denote the effect of x_l on y conditional on $\mathbf{z}$, and denote $x(l)$ as $b_{x_l.zx(l)}$ ($\widetilde{b}_F$ in the previous section). We then have that the confounding effect of $\mathbf{z}$ on x_l is $\widetilde{b}_{x_l.\widetilde{z}x(l)} - b_{x_l.zx(l)}$, measured on the same scale. The test statistic for the confounding effect of $\mathbf{z}$ on $x(l)$ is therefore

$$\frac{\widetilde{b}_{x_l.\widetilde{z}x(l)} - b_{x_l.zx(l)}}{\mathrm{sd}\left(\widetilde{b}_{x_l.\widetilde{z}x(l)} - b_{x_l.zx(l)}\right)}$$

where $\mathrm{sd}\left(\widetilde{b}_{x_l.\widetilde{z}x(l)} - b_{x_l.zx(l)}\right) = \sqrt{\mathbf{a}'\mathbf{\Sigma}\mathbf{a}}$ with $\mathbf{a} = \begin{pmatrix} \theta_{\mathbf{z}x_l.\mathbf{x}_l} \\ \mathbf{b}_{y\mathbf{z}.\mathbf{x}} \end{pmatrix}$ and $\mathbf{\Sigma} = \begin{pmatrix} \Sigma_{y\mathbf{z}.\mathbf{x}} & 0 \\ 0 & \Sigma_{\mathbf{z}x_l\mathbf{x}_{(l)}} \end{pmatrix}$. The vector $\theta_{\mathbf{z}x_l.\mathbf{x}_l}$ is a $J{\times}1$ vector of regression coefficients of x_l on $\mathbf{z}$ from a seemingly unrelated regression of $\mathbf{x}$ on $\mathbf{z}$, and the vector $\mathbf{b}_{y\mathbf{z}.\mathbf{x}}$ is a $J{\times}1$ vector of binary regression coefficients of $\mathbf{z}$ on y conditional on $\mathbf{x}$. The two block-diagonal elements in $\mathbf{\Sigma}$ are the $J{\times}J$ variances and covariances of $\mathbf{b}_{\mathbf{z}}$ from the regression of $\mathbf{x}$ and $\mathbf{z}$ on y and the $J{\times}J$ variance–covariance matrix from a seemingly unrelated regression of $\mathbf{x}$ on $\mathbf{z}$ using the elements relating x_l on $\mathbf{z}$, respectively.

3 The khb command

3.1 Syntax

khb *model-type depvar key-vars* || *z-vars* [*if*] [*in*] [*weight*]
   [, *options*]

model-type can be any of `regress`, `logit`, `ologit`, `probit`, `oprobit`, `cloglog`, `slogit`, `scobit`, `rologit`, `clogit`, `mlogit`, `xtlogit`, or `xtprobit`. Other models might also give output, but you should consider this output to be experimental for the time being.

depvar is the name of the dependent variable, *key-vars* is a varlist containing the names of the variables to be decomposed, and *z-vars* is a varlist holding the names of control variables of interest.

key-vars and *z-vars* may contain factor variables; see [U] **11.4.3 Factor variables**.

`aweight`s, `fweight`s, and `pweight`s are allowed if they are allowed for the specified *model-type*; see [U] **11.1.6 weight**.

options	Description
<u>conc</u>omitant(*varlist*)	concomitant variables
<u>dis</u>entangle	disentangle difference of effects by mediators
<u>sum</u>mary	summary of decomposition
or	exponentiate coefficients
vce(*vcetype*)	*vcetype* may be <u>robust</u> or <u>cluster</u> *clustvar*
ape	decompose *key-vars* using average partial (marginal) effects
continuous	treat dummy variables as continuous when using method ape
<u>nota</u>ble	suppress coefficient table
<u>ver</u>bose	show restricted and full model
<u>keep</u>	keep residuals of mediators
<u>xs</u>tandard	standardize *key-vars*
<u>zs</u>tandard	standardize *z-vars*
<u>out</u>come(*outcome*)	outcome used for decomposition when *model-type* is mlogit, ologit, or oprobit
<u>b</u>aseoutcome(*#*)	value of *depvar* that will be the base outcome when *model-type* is mlogit
group(*varname*)	necessary option for *model-type*s rologit and clogit
model_options	all options allowed for the specified *model-type*

3.2   Options

concomitant(*varlist*) specifies control variables that are not mediator variables. Factor variables are allowed; see [U] **11.4.3 Factor variables**.

disentangle requests a table that shows how much of the difference between the full and reduced models is contributed by each *z-var*.

summary requests a decomposition summary for all *key-vars*. By default, khb reports the effects of the full and reduced models, their difference, and their standard errors. With the summary option, khb also presents a table that shows the confounding ratios, the percentage reduction due to confounding, and the rescale factor. The confounding ratio measures the impact of confounding net of rescaling. The percentage reduction measures the percentage change in the coefficient of each *key-var* attributable to confounding net of rescaling. Finally, the rescale factor measures the impact of rescaling net of confounding.

or exponentiates the estimated coefficients, and hence shows odds ratios for logit models. The coefficient for the reduced model is then the product of the full model and the estimated difference.

vce(*vcetype*) specifies the type of standard error reported. The default is the Stata default for the specified *model-type*. Standard errors for indirect effects are estimated using a method discussed by Sobel (1982). The vce() option sets the standard errors for the effects of the reduced and full models and controls the type of standard error that enters into Sobel's method. The robust and cluster *clustvar* types are available; see [R] ***vce_option***.

ape is used to decompose the *key-vars* using average partial effects; for continuous X, this is the average of the derivatives of the predicted probability with respect to X, and for discrete X, this is the average of the discrete differences (see [R] **margins**). For *model-type*s ologit and oprobit, khb uses the average partial effect on the probability for the first outcome unless outcome() is specified; see [R] **ologit postestimation** for various ways to specify outcome(). With ape, the calculated difference is not constant across outcomes; this is a well-known property of ordered choice models (see Greene and Hensher [2010]).

continuous treats dummy variables equal to continuous variables. Average partial effects are by default based on unit effects for dummy variables. See [R] **margins** for details about this option.

notable suppresses the display of the coefficient table. This normally involves the summarize or disentangle option.

verbose is used to show the complete output of the full and restricted models that are used to estimate the decomposition. This is especially useful to detect problems that occur in the intermediate steps of the estimation.

keep is used to keep the residuals of the mediator variables, that is, the mediator variables net of confounding. These residuals are included as independent variables in the reduced model.

xstandard is used to standardize the *key-vars*.

zstandard is used to standardize the *z-vars*.

outcome(*outcome*) specifies the outcome for which the decomposition is to be calculated. This has an effect for multinomial response models (mlogit) and, if the ape option is specified, for ordered response models (ologit and oprobit). outcome() may be specified using

- #1, #2, ..., where #1 means the first category of the dependent variable, #2 means the second category, etc.;

- the values of the dependent variable; or

- the value labels of the dependent variable, if they exist.

baseoutcome(#) is for use with *model-type* mlogit. It specifies the value of *depvar* to be treated as the base outcome. The default is to choose the most frequent outcome.

`baseoutcome()` may be used together with `outcome()` to fully control the contrast for which the decomposition is done.

`group(`*varname*`)` is a necessary option for *model-type*s `rologit` and `clogit`; see [R] **rologit** and [R] **clogit**.

model_options: All options allowed for the specified *model-type* may be used.

4   Application

In this section, we give examples from educational sociology to show several applications of the `khb` command. Following Boudon (1974), researchers in this field are concerned with two ways in which social origin influences educational attainment, namely, "primary" and "secondary" effects (Breen and Goldthorpe 1997; Erikson et al. 2005; Erikson 2007; Jackson et al. 2007).

For our guiding example, the secondary effect is the direct effect, that is, the effect of social origin on educational attainment net of performance at school. The primary effect, on the other hand, is what we called the indirect effect above, that is, that part of the relationship between social origin and educational attainment that is due to uneven performance at school.

For the application, we use a subset of the Danish National Longitudinal Survey (DLSY).[1] We show the Stata commands for reproducing some of the analysis presented in Karlson and Holm (2011).

```
. use dlsy_khb
. describe
Contains data from dlsy_khb.dta
  obs:          1,896
 vars:              8                          17 Jan 2011 10:26
 size:         34,128
─────────────────────────────────────────────────────────────────────────────
              storage  display    value
variable name   type   format     label     variable label
─────────────────────────────────────────────────────────────────────────────
edu            byte    %20.0g     edu       Educational attainment
upsec          byte    %10.0g     yesno     Complete upper secondary
                                              education (Gymnasium)
univ           byte    %13.0g     yesno     Complete University education
fgroup         byte    %9.0g      fgroup    Father´s social group/class
fses           float   %9.0g                Father´s SES, standardized with
                                              mean 0 and sd 1
abil           double  %10.0g               Standardized ability measure,
                                              with mean 0 and sd 1
intact         byte    %9.0g      yesno     Intact family
boy            byte    %9.0g      yesno     Boy
─────────────────────────────────────────────────────────────────────────────
Sorted by:
```

---

1. Information on the DLSY is available online at http://www.sfi.dk/dlsy. We wish to thank the management at SFI—The Danish National Centre for Social Research for allowing us to distribute a subset of the DLSY with the online material of this article.

The subset contains 1,896 individuals born around 1954. These individuals were interviewed the first time in seventh grade and have been followed since then until around year 2000. The data contain information on university completion (`univ`), parental social status (`fses`), and academic ability (`abil`); `fses` and `abil` are standardized to have zero mean and unit variance. Using the `khb` command, we decompose the total effect of parental social status on university graduation into its direct part and an indirect part running through academic ability.

4.1 Basic use

The syntax diagram of `khb` requires the specification of four elements: the model type, the dependent variable, the variable to be decomposed (key variable), and the mediator variables. In our example, the dependent variable is university completion (`univ`). Because this variable is dichotomous, we choose `logit` as our *model-type*, although we could have also chosen `probit` or, if deemed relevant, any other Stata command for binary response models (like `cloglog`, `scobit`, or `slogit`, for example). We decompose the effect of parental social status (`fses`) on university completion. Finally, we use as mediator a measure of academic ability (`abil`). To separate the key variable—the variable whose effect is to be decomposed—from the mediators, the syntax requires two pipe symbols, `||`. In addition to these required elements, the command also has the `concomitant()` option, which allows for the addition of variables to be controlled for in both the full and the reduced models. In our example, we use this option to control for gender (`boy`) and intact family (`intact`).

```
. khb logit univ fses || abil, concomitant(intact boy)

Decomposition using the KHB-Method

Model-Type:  logit                              Number of obs   =    1896
Variables of Interest: fses                     Pseudo R2       =    0.19
Z-variable(s): abil
Concomitant: intact boy
```

univ	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
fses						
Reduced	.5459815	.0779806	7.00	0.000	.3931424	.6988206
Full	.3817324	.0778061	4.91	0.000	.2292353	.5342295
Diff	.1642491	.0293249	5.60	0.000	.1067734	.2217247

The output shows the estimated effect of the reduced model $(\widetilde{b}_R)$, the estimated effect of the full model (b_F), and the estimated difference between these two effects; see (12). For our guiding example, we call the estimated effect of the reduced model the total effect, the estimated effect of the full model the direct effect, and the estimated difference the indirect effect. We see that parental social status increases the log odds of completing university by 0.55. Controlling for academic ability, the effect of parental social status reduces to 0.38, leaving an indirect effect of 0.16.

The standard output of `khb` expresses the effects in terms of the estimated regression coefficients.[2] The KHB method ensures that the coefficients presented are measured on the same scale (and thus are not affected by the scale identification issue described earlier). However, the magnitude of logit coefficients is generally difficult to interpret, precisely because they are measured on "arbitrary" scales. This also holds for the interpretation in terms of total, direct, and indirect effects. Karlson, Holm, and Breen (2010) propose the confounding ratio and the confounding percentage, as defined by (10) and (11), to overcome these problems. Both measures can be easily calculated from the standard output of `khb`; however, the `summary` option provides the information directly.

In the following command, we use `summary` together with `notable` to save space:

```
. khb logit univ fses || abil, concomitant(intact boy) summary notable
Decomposition using the KHB-Method
Model-Type:  logit                              Number of obs   =     1896
Variables of Interest: fses                     Pseudo R2       =     0.19
Z-variable(s): abil
Concomitant: intact boy
Summary of confounding
        Variable │  Conf_ratio    Conf_Pct    Resc_Fact
       ──────────┼──────────────────────────────────────
            fses │   1.4302727       30.08    1.0602422
       ──────────┴──────────────────────────────────────
```

The total effect is 1.4 times larger than the direct effect, and 30% of the total effect is due to academic ability.

The `summary` option also shows the rescale factor, given by

$$\text{rescale factor} = \frac{\widetilde{b}_R}{b_R}$$

where b_R is the "naïve" estimator for the β_R of the model without Z, (5). The measure provides information about the size of the change in the scale parameter due to the inclusion of R or, in other words, due to the inclusion of Z net of confounding.

4.2   Comparing average partial effects

Average partial effects (Wooldridge 2002) are often used for reporting the effects of logit and probit models because of their natural interpretation on the probability scale. However, Karlson, Holm, and Breen (2010) show that simple comparisons of average partial effects across models with or without a confounder, Z, may be distorted in a range of scenarios encountered in real applications. Average partial effects may therefore not be well suited for decompositions of effects. Applying the KHB method to average partial effects resolves this problem (Karlson, Holm, and Breen 2010). This is an attractive property of the method because average partial effects are more interpretable than the

---

2. Odds-ratio decomposition can be obtained using the `or` option. In this situation, the total effect measured in odds ratios will be the product (not the sum) of the direct and indirect effects measured in odds ratios.

estimated coefficients of logit and probit models. `khb` therefore has the `ape` option, which requests the application of the KHB method on average partial effects:

```
. khb logit univ fses || abil, concomitant(intact boy) ape summary
Decomposition using the APE-Method
Model-Type:  logit                              Number of obs   =    1896
Variables of Interest: fses                     Pseudo R2       =    0.19
Z-variable(s): abil
Concomitant: intact boy
```

univ	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]
fses					
Reduced	.0384906	.0054429	7.07	0.000	.0278226 .0491585
Full	.0269113	.0054476	4.94	0.000	.0162343 .0375884
Diff	.0115792	.	.	.	. .

```
 Note: Standard errors of difference not known for APE method
Summary of confounding
```

Variable	Conf_ratio	Conf_Pct	Dist_Sens
fses	1.4302727	30.08	.95931864

On average, the probability of a youth's university completion increases by 3.9 percentage points for a standard-deviation change in the father's socioeconomic status.[3] After controlling for academic ability, this average increase is reduced to 2.7 percentage points. An increase of parental social status leads to higher academic ability, which is then translated into a higher probability of university graduation of 1.1 percentage points. Despite the arguably more interpretable values shown in the estimation table, the confounding ratios and confounding percentages in the summary table are always equal to those ascertained based on the regression coefficients.[4]

4.3   Disentangle contributions of mediators

If more than one mediator is used, the question arises as to which of the mediators contributes most to the confounding. The question can be answered using the `disentangle` option, which requests an additional table that shows the contribution of each mediator separately. In the following example, we use the concomitants (`intact` and `boy`) and combine `disentangle` with `summary` and `notable`:

---

3. The average partial effect is the average gradient of the function that links parental social status with the probability of university completion.
4. The summary table shows the distributional sensitivity instead of the rescale factor because the ratio in (9) is sensitive to the distributions of the included variables in the model.

```
. khb logit univ fses || abil intact boy, summary disentangle notable
Decomposition using the KHB-Method
Model-Type:  logit                          Number of obs    =    1896
Variables of Interest: fses                 Pseudo R2        =    0.19
Z-variable(s): abil intact boy
Summary of confounding
        Variable │  Conf_ratio    Conf_Pct   Resc_Fact
────────────────┼──────────────────────────────────────
            fses │   1.5207722       34.24   1.1317064


Components of Difference
      Z-Variable │      Coef     Std_Err     P_Diff  P_Reduced
────────────────┼──────────────────────────────────────────────
fses             │
            abil │  .1661177    .0301003      83.56      28.61
          intact │   .020142    .0144611      10.13       3.47
             boy │  .0125359     .011524       6.31       2.16
```

The first two columns of the disentangle table show the effect difference (indirect effect) due to each of the mediators along with their standard errors. The values in the first column sum up to 0.199, the overall confounding by all mediators together (that is, the sum of indirect effects). The third column expresses the contribution of each mediator to the indirect effect, and the last column shows how much of the total effect is due to confounding of the respective mediator; this last column sums up to 34.24, the overall confounding percentage. According to the results shown here, the degree of mediation is much larger for academic ability than for gender and an intact family.

4.4   More than one key variable

In the syntax diagram, the variable to be decomposed is termed the key variable. There can be more than one key variable in the same command. In this case, the command displays the decomposition for all key variables in one output. When specifying more than one key variable, we must decompose each *key-var* while controlling for all the others. In the example below, we perform the decomposition of the effects of boy and intact, using academic ability as mediator. For the decomposition of the gender effect, intact is controlled for in both the full and the reduced model. Likewise, for the decomposition of the intact-family effect, gender is controlled for in both equations.

```
. khb logit univ boy intact || abil, concomitant(fses) summary

Decomposition using the KHB-Method

Model-Type: logit                              Number of obs    =    1896
Variables of Interest: boy intact              Pseudo R2        =    0.19
Z-variable(s): abil
Concomitant: fses
```

univ	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]
boy					
Reduced	1.06178	.1848087	5.75	0.000	.6995613 1.423998
Full	.9821406	.1848351	5.31	0.000	.6198704 1.344411
Diff	.0796391	.133004	0.60	0.549	−.1810438 .3403221
intact					
Reduced	1.129767	.7386976	1.53	0.126	−.3180536 2.577588
Full	1.08391	.7386558	1.47	0.142	−.3638292 2.531648
Diff	.0458575	.1328438	0.35	0.730	−.2145116 .3062266

```
Summary of confounding
```

Variable	Conf_ratio	Conf_Pct	Resc_Fact
boy	1.0810873	7.50	1.0033213
intact	1.0423075	4.06	1.03542

We find that the effect of gender is mediated stronger by academic ability than having an intact family is.

4.5 Categorical variables

Categorical key variables and concomitants may be specified using factor-variable notation; see [U] **11.4.3 Factor variables**. The following example decomposes the effect of `fgroup`, a discrete social class measure, using a categorized version of academic ability as mediator:

```
. xtile catabil = abil, nquantiles(4)
. khb logit univ i.fgroup || i.catabil, concomitant(intact boy) summary
> disentangle
   (output omitted)
```

As shown in section 2.3, the standard errors of the regression of the mediators on the key variables are used in the formula for the standard errors of the effect difference. Huber/White/sandwich standard errors are used for that purpose, when the mediator only has values 0 and 1.

4.6 Ordered outcome

The decomposition of key variables on ordered outcomes can be done by specifying an ordered choice model like `ologit` or `oprobit` as *model-type*. Some care must be taken, however, when using the `ape` option for those models. It is a well-known feature of

ordered choice models that the average partial effect is not constant across outcomes (Greene and Hensher 2010). The default of `khb` with the `ape` option shows the decomposition using the average partial effect on the probability of the lowest outcome, but this can be changed with the `outcome()` option.

For an example, we use the variable `edu`, which is a three-level ordered discrete variable measuring educational attainment:

```
. tab edu
  Educational |
   attainment |      Freq.     Percent        Cum.
--------------+-----------------------------------
Compulsory schooling |      1,091       57.54       57.54
   Upper secondary |        633       33.39       90.93
        University |        172        9.07      100.00
--------------+-----------------------------------
        Total |      1,896      100.00
```

We now perform the decomposition with the `ape` and `summarize` options for each outcome, and we show the results in a table using Ben Jann's command `esttab` (Jann 2007):

```
. forv i = 1/3 {
  2.          quietly eststo: khb ologit edu fses || abil, outcome(`i´) ape
> summary
  3. }
. esttab, scalars("ratio_fses Conf.-Ratio" "pct_fses Conf.-Perc.")
--------------------------------------------------------------------
                         (1)            (2)            (3)
                         edu            edu            edu
--------------------------------------------------------------------
fses
Reduced              -0.103***      0.0643***      0.0385***
                     (-11.33)       (10.72)        (9.27)
Full                 -0.0755***     0.0472***      0.0283***
                     (-8.02)        (7.76)         (7.23)
Diff                 -0.0272        0.0170         0.0102
                         .              .              .
--------------------------------------------------------------------
N                        1896           1896           1896
Conf.-Ratio             1.360          1.360          1.360
Conf.-Perc.             26.48          26.48          26.48
--------------------------------------------------------------------
t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001
```

The estimations of the total, direct, and indirect effects differ between outcomes. However, all confounding ratios and confounding percentages are equal to those based on the decomposition of the regression coefficients. Hence, if researchers are interested in the relative measures, there is little to be gained by including the `ape` option. This property points to the generality of the KHB method.

4.7 Multinomial outcome

We now treat the ordinal variable `edu` as categorical to show how `khb` cooperates with multinomial logistic regression. The basic command is as simple as before: just change the *model-type* to `mlogit`.

```
. khb mlogit edu fses || abil, baseoutcome(2)

Decomposition using the KHB-Method

Model-Type:  mlogit                          Number of obs    =     1896
Variables of Interest: fses                  Pseudo R2        =     0.11
Z-variable(s): abil
Results for outcome Compulsory_schooling and base outcome Upper_secondary
```

edu	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
fses						
Reduced	-.4227091	.055406	-7.63	0.000	-.5313028	-.3141154
Full	-.3132967	.0549432	-5.70	0.000	-.4209834	-.20561
Diff	-.1094124	.0184393	-5.93	0.000	-.1455528	-.0732719

As noted above the table, the decomposition is only done for one outcome of the dependent variable using the base outcome of the multinomial regression. In our example, `khb` displays the decomposition of log odds for being in the first category of `edu` (compulsory schooling) instead of in the second category (upper secondary).

By default, `khb` inherits the default settings of `mlogit` so that the most frequent outcome becomes the base outcome. This can be changed with the `baseoutcome(#)` option. The default setting for the outcome is the alternative with the lowest level that is not the base outcome. The outcome can be changed with the `outcome()` option. In the following, we apply both options to show the decompositions for having an education level of 2 or 3 instead of 1. Again we exploit Ben Jann's `esttab` command (Jann 2007) to show the results in a single table.

```
. forv i = 2/3 {
  2.        quietly eststo: khb mlogit edu fses || abil, outcome(`i´)
> baseoutcome(1) summary
  3. }

. esttab, scalars("ratio_fses Conf.-Ratio" "pct_fses Conf.-Perc.")
```

	(1) edu	(2) edu
fses		
Reduced	0.423***	0.779***
	(7.63)	(9.30)
Full	0.313***	0.552***
	(5.70)	(6.68)
Diff	0.109***	0.227***
	(5.93)	(6.04)
N	1896	1896
Conf.-Ratio	1.349	1.411
Conf.-Perc.	25.88	29.15

```
t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001
```

4.8 A word of caution

The khb program is very general. It is written to cooperate with various standard Stata estimation commands. We have only performed tests of khb for regress, logit, ologit, probit, oprobit, cloglog, slogit, scobit, rologit, clogit, and mlogit, xtlogit, and xtprobit. Other models might also produce output, but this output should be considered experimental for the time being; the experimental status of such models is indicated by a note in the output.

```
. khb glm edu fses || abil, link(power 2) family(gaussian)
 Note: glm not supported. Output is experimental

Decomposition using the KHB-Method

Model-Type:  glm                          Number of obs   =    1896
Variables of Interest: fses               Pseudo R2       =      .
Z-variable(s): abil
```

	edu	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf. Interval]	
fses							
	Reduced	.4225422	.0413326	10.22	0.000	.3415317	.5035526
	Full	.3226696	.0423019	7.63	0.000	.2397593	.4055798
	Diff	.0998726	.0154422	6.47	0.000	.0696065	.1301388

As a side effect of its generality, the program does not provide sensible error messages for any situation that might arrive. Users should be aware that khb cannot provide any output if an intermediate step of estimating the full and reduced models returns an error. Moreover, khb inherits all problems that arise while performing these intermediate steps.

It is therefore sensible to investigate these intermediate steps. `khb` offers two ways to do this:

- The `verbose` option shows the output produced in the intermediate steps of estimating the full and reduced models. This is helpful if `khb` returns unclear error messages and to detect problems such as high discrimination or perfect multicollinearity.

- The `keep` option stores the residuals of (3). This is helpful for users who want to do specific diagnostics on the reduced model.

The KHB method solves the general problem of comparing effects between nested nonlinear regression models, and it will consequently be useful in many applications. The method allows for a complete analogy between the interpretation of effect differences in nonlinear models and the interpretation in linear models.

5   Acknowledgments

6   References

Blau, P. M., and O. D. Duncan. 1967. *The American Occupational Structure.* New York: Wiley and Sons.

Boudon, R. 1974. *Education, Opportunity, and Social Inequality: Changing Prospects in Western Society.* New York: Wiley.

Breen, R., ed. 2004. *Social Mobility in Europe.* Oxford: Oxford University Press.

Breen, R., and J. H. Goldthorpe. 1997. Explaining educational differentials: Towards a formal rational action theory. *Rationality and Society* 9: 275–305.

Breen, R., K. B. Karlson, and A. Holm. 2011. Total, direct, and indirect effects in logit models. http://papers.ssrn.com/sol3/papers.cfm?abstractid=1730065.

Buis, M. L. 2010. Direct and indirect effects in a logit model. *Stata Journal* 10: 11–29.

Erikson, R. 2007. Social selection in Stockholm schools: Primary and secondary effects on the transition to upper secondary education. In *From Origin to Destination: Trends and Mechanisms in Social Stratification Research*, ed. S. Scherer, R. Pollak, G. Otte, and M. Gangl, 58–77. Chicago: University of Chicago Press.

Erikson, R., J. H. Goldthorpe, M. Jackson, M. Yaish, and D. R. Cox. 2005. On class differentials in educational attainment. *Proceedings of the National Academy of Science* 102: 9730–9733.

Frey, B. S., and A. Stutzer. 2002. What can economists learn from happiness research? *Journal of Economic Literature* 40: 402–435.

Greene, W. H., and D. A. Hensher. 2010. *Modeling Ordered Choices: A Primer*. Cambridge: Cambridge University Press.

Jackson, M., R. Erikson, J. H. Goldthorpe, and M. Yaish. 2007. Primary and secondary effects in class differentials in educational attainment: The transition to A-level courses in England and Wales. *Acta Sociologica* 50: 211–229.

Jann, B. 2007. Making regression tables simplified. *Stata Journal* 7: 227–244.

Karlson, K. B., and A. Holm. 2011. Decomposing primary and secondary effects: A new decomposition method. *Research in Stratification and Social Mobility* 29: 221–237.

Karlson, K. B., A. Holm, and R. Breen. 2010. Comparing regression coefficients between models using logit and probit: A new method.
http://www.yale.edu/ciqle/Breen_Scaling%20effects.pdf.

Long, J. S. 1997. *Regression Models for Categorical and Limited Dependent Variables*. Thousand Oaks, CA: Sage.

Miller, W. E., and J. M. Shanks. 1996. *The New American Voter*. Cambridge, MA: Harvard University Press.

Sobel, M. E. 1982. Asymptotic confidence intervals for indirect effects in structural equation models. In *Sociological Methodology 1982*, ed. S. Leinhardt, 290–312. Washington D.C.: American Sociological Association.

Winship, C., and R. D. Mare. 1984. Regression models with ordinal variables. *American Sociological Review* 49: 512–525.

Wooldridge, J. M. 2002. *Econometric Analysis of Cross Section and Panel Data*. Cambridge, MA: MIT Press.

About the authors

Ulrich Kohler is a sociologist at the Wissenschaftszentrum Berlin (Social Science Research Center) who has used Stata for several years. His research interests include social inequality and political sociology. With Frauke Kreuter, he is author of the textbook *Data Analysis Using Stata.*

Kristian B. Karlson is a PhD Fellow in Sociology at SFI—Danish National Centre for Social Research and Aarhus University. His research interests lie within the fields of educational stratification and econometrics.

Anders Holm is professor in quantitative methods at the Centre for Strategic Educational Research at Aarhus University. His research interests include advanced econometrics, estimation of causal effects, and social stratification.