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Schultz’s Hypothesis Revisited – Small Scale Joint Production in the Eastern Amazon

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Abstract

This research empirically investigates the well known ‘poor-but-efficient’ hypothesis formulated by Schultz (1964) assuming that small scale farmers in developing countries are reasonably efficient in allocating their scarce resources by responding positively to price incentives. Deviating from Schultz it is assumed here that scale effects explain a considerable proportion of small scale farmers’ relative efficiency. The theoretical underpinnings of the scale efficiency concept are briefly reviewed before a normalized generalized Leontief profit function is modeled by using its output supply and input demand system to capture the joint production of cassava flour and maize by a sample of small scale farmers in the Bragantina region of the Eastern Amazon, Brazil. The discussion on theoretical consistency and functional flexibility is considered by imposing convexity on the GL profit framework. The empirical results confirm our revised hypothesis that small farmers in traditional development settings are ‘poor-but-allocatively efficient’ by clearly suggesting considerable inefficiency with respect to the scale of operations.

Keywords: Efficiency, Joint Production, Small Scale Farming, Schultz Hypothesis.

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Contents

Preface .............................................................................................................................................. 3

1. Introduction .................................................................................................................................. 4

2. Scale and Economic Efficiency ................................................................................................. 7

3. Small Scale Agriculture in the Eastern Amazon ................................................................. 9

4. Modelling ................................................................................................................................... 11

5. Data, Variables and Estimation ............................................................................................... 20

6. Results and Discussion ............................................................................................................. 24

7. Conclusions ............................................................................................................................... 31

References ...................................................................................................................................... 33

Appendix ......................................................................................................................................... 39
Preface

This research work has been done by assistant professor Johannes Sauer and senior researcher Arisbe Mendoza-Escalante, Center for Development Research, University of Bonn, Germany.

An anonymous reviewer has reviewed the paper and secretary Inger Sommer edited the final version.

Institute for Food and Resource Economics
Production and Technology Department, October 2006

Mogens Lund
1. Introduction

Schultz’s (1964) ‘poor-but-efficient’ hypothesis – i.e. small farmers in traditional agricultural settings are reasonably efficient in allocating their resources by responding positively to price incentives – can be fairly considered as one of the enduring themes in rural development economics over the past three decades. Although challenged from some fronts (Myrdal, 1968; Bhagwati/Chakravorty, 1969; Shapiro; 1983; Chakravorty, 1984; Adams, 1986 and more recently e.g. by Ball/Pounder, 1996; Duflo, 2006 and Ray, 2006) it has been widely accepted by both economists and policy makers (see e.g. Hayami/Ruttan, 1985; Stiglitz, 1989; Nerlove, 1999; Ruttan, 2003; Abler/Sukhatme, 2006). With respect to the long-term effectiveness of the individual development strategy applied on small-scale farming the level of efficiency of those farming activities has important implications: If farmers are reasonably efficient, then an additional increase in efficiency requires the usage of more productive inputs and/or the application of a more productive technology to shift the production frontier upwards. If on the other hand current inputs and/or technology could be used more productive, an improvement in the institutional setting - e.g. input markets, infrastructure endowment, available extension systems, management and training services - should be targeted to increase the efficiency on farm level. Hence, the two broad approaches - technology development and transfer versus more efficient use of available technology and resources on the individual farm level - can be considered as a continuum in the process of development (Ali and Bayerlee, 1991; Schultz, 1975). Assuming efficiency of small-scale farming could be based on the notion that farmers in a more traditional agricultural setting depend largely on their own resources and consequently managed to adjust their coordination and management efforts in the long-run to the most efficient use of these resources. Assuming on the other side inefficiency in a more dynamic and developed agricultural setting could be based on the reasoning that the individual producer find it more difficult to adjust the allocative decisions to a continuously changing production environment: “Farmers in this situation are likely to be in a continual state of disequilibrium, and there will be high returns to improving their information and skills to help them to adjust more rapidly and reduce technical and allocative errors.” (Ali and Byerlee, 1991, p. 2). Most recently, development economists have questioned the efficient but poor hypothesis again by pointing to the detrimental influence of household decisions and land tenancy arrangements on efficient economic behaviour (Ball/Poulder, 1996; for an overview see Abler/Sukhatme, 2006). However, many empirical contributions to this discussion treat efficiency as a black-box concept and lack the explicit consideration of the scale of agricultural production and based on this the notion of other policy options than simply correcting
input prices and/or modernizing production technology (see e.g. Taylor/Shonkwiler, 1986; Cotlear, 1987; Flinn/Ali, 1986; Bravo-Ureta/Evansen, 1994; Admassie/Heidhues, 1996, Otsuka, 2006). According to production theory ‘overall’ allocative or technical efficiency can be decomposed into ‘pure’ allocative or technical efficiency as well as scale efficiency (see Chambers, 1988 or Coelli et al., 1998). Hence a very poor performance of a small farmer relative to others operating on the production frontier can be simply due to the small scale of his/her agricultural operations and vice versa a good performance relative to others can be simply due to the large scale of his/her operations compared to the peer group average. Considering also the scale effects on efficiency could deliver a more precise picture of the relative economic efficiency of small scale farms in developing areas. If this could be empirically verified then a viable policy option in both a more traditional as well as a more dynamic setting would be to enhance overall economic performance on the firm level by delivering incentives for an increase in the scale of operations and forming bigger production units by fostering farm cooperations and/or mergers.

To measure quantitatively such inefficiencies due to scale in a stochastic setting requires other approaches than the commonly applied error components model. The shadow price approach based on a flexible profit function allows for investigating beside input and output oriented allocative inefficiency also scale related inefficiency by accounting for possible price distortions in the relevant input and output markets. We formulate a flexible generalized Leontief shadow profit function framework to impose functional consistency (convexity) and remain a flexible estimation.

The empirical analysis uses data on small scale farmers in the Bragantina region (Pará State) of the Eastern Amazon in Brazil. Here 80% of the total agricultural production originates from smallholders mainly depending on available natural resources and living in poor conditions (Serrão/Homma 1993). Vosti et al. (2002, p.6) note that, “(...) agricultural productivity and poverty among smallholders is of particular importance in Amazonia because smallholdings have an important presence in demographic and economic terms.” (see also Costa, 2000; Fearnside, 1994; Hurtienne, 2002; and Kitamura, 1982). While the government has undertaken some reform measures aiming to affect the incentive structure and productivity of the smallholder farming system in the last decades (e.g. the National Programme for the Strengthening of Family Agriculture, PRONAF and the credit program FNO-Especial by the Ministry of Integration) the analysis of the characteristics and the choice of inputs and technologies in the region has received little empirical attention in the literature so far (Costa, 2000). In the Bragantina region farmers generally grow several crops on the same field mak-
ing a disaggregation of the data with respect to crop-specific input information impossible. Thus, a joint production approach seems appropriate to adequately reflect the case of agricultural production in the region.

This paper is structured as follows: Section II contains a brief reconsideration of the concept of scale and scale efficiency in production economics followed by section III describing the case of small-scale farming in the Bragantina region of the Eastern Amazon in Brazil. Section IV introduces the shadow price approach to efficiency measurement as well as outlines the different model(s) applied. The data and the variables used in the empirical analysis as well as the estimation procedure applied are described in section V. Section VI discusses the results and finally section VII concludes the analysis.
2. Scale and Economic Efficiency

As is well known the concept of returns to scale (rts) reflects the degree to which a proportional increase in all inputs increases output. We refer to constant, increasing, or decreasing rts as a proportional increase in all inputs results in the same, in a more than proportional, or less than proportional increase in output. This basic economic concept refers to a long-run factor-factor relationship where output may be increased by simply changing all factors by the same proportion i.e. by altering the scale of the operation (Chambers, 1988; Coleman/Young, 1995). Hence, the observation that a farm has increased its productivity from one year to the next does not imply that the improvement has been resulted from pure technical and/or pure allocative efficiency improvements alone, but may have been (also) due to technical change or the exploitation of scale economies or from some combination of these three factors. Consequently, beside technical inefficiency failure to maximize profit – i.e. maximize output and minimize cost - in a given period has a systematic allocative inefficiency component, which can involve an inappropriate input mix, an inappropriate output mix (i.e. the scope of production in the case of multiple outputs) and an inappropriate scale. For a farm to be profit efficient it requires technical efficiency and both input and output allocative efficiency to be achieved at the proper scale. Based on an output-oriented measure of technical efficiency the overall measure of profit efficiency PE can be decomposed as (see Kumbhakar/Lovell, 2000)

\[
PE(y, x, p, w) = \left\{ \begin{array}{l}
TE_o(x, y) \times AE_o(x, y, p) \times \left[ \frac{r(x, p)}{p^T y(p, w)} \right] \times p^T y(p, w) \\
- \left[ AE_i(y, x, w) \right]^{-1} \times \left[ \frac{c(y/TE_o(x, y), w)}{w^T x(p, w)} \right] \times w^T x(p, w)
\end{array} \right\} / \pi(p, w)
\]

[1]

where $TE_o(x, y) \leq 1$ and $AE_o(x, y, p) \leq 1$ are output-oriented technical and allocative efficiency respectively having an impact on profit-maximizing revenue $p^T y(p, w)$, input-oriented allocative efficiency $AE_i(y, x, w) \geq 1$ increases profit-maximizing expenditure $w^T x(p, w)$, and finally $\left[ \frac{r(x, p)}{p^T y(p, w)} \right] \times p^T y(p, w)$ and $\left[ \frac{c(y/TE_o(x, y), w)}{w^T x(p, w)} \right] \times w^T x(p, w)$ constitute the measure of scale efficiency. It is evident that $PE = 1$ if, and only if, all five efficiency related terms are unity. Maximum profit is attained by the farm as technical efficiency is reached, the right input mix with respect to the input prices $w$ is
used, the right output mix with respect to \( p \) is produced, and the farm is operated at the right scale in light of \((p,w)\).

**Proposition:** The overall economic efficiency of a small scale agricultural enterprise can only be adequately assessed by also investigating its relative scale efficiency.

To conclude, the more theoretical explanations revealed that the economic efficiency of small scale agricultural operations are inherently related to the scale of the farm at that particular point in time. Hence, to capture these different efficiency components we have to focus on the measurement of farms’ profit efficiency and consider possible effects of price distortions on their allocative decisions.
3. Small Scale Agriculture in the Eastern Amazon

Unlike most other parts of Amazonia Bragantina has a long settlement history beginning in the mid 19th century. Land use in the region dates back at least 100 years and has gone through several phases. It was the first area of extensive and planned settlement in the Amazonian lowland (Baar et al., 2004). Settlement and agricultural activities in the Bragantina region resulted also in vast deforestation and today the region is an agricultural landscape comprised of a variety of secondary vegetation and annual cropping, plantation crops and pastures (Burger, 1991; Denich, 1991). Smallholder agricultural production in general depends heavily on environmental resource conditions that are largely exogenously determined. Even though environmental conditions (i.e. physical soil characteristics) in the Bragantina are classified as being quite homogeneous, variations in climatic conditions - primarily in terms of rainfall - reflect the intra regional heterogeneity of the Bragantina (see Appendix, map 1). The physical and climatic conditions as well as the kind of technology used for land preparation can significantly influence the farmers’ income (see Sherlund et al., 2002). In terms of demographic characteristics the population in the Bragantina has increased by 32% in fifteen years (1980 to 1995). This implies a growing demand for food which is reflected by more land being cultivated and a decrease in the fallow areas.

A further constraint faced by smallholders in the region is structural poverty. Bragantina is the fifth poorest micro-region in Pará state in terms of annual per capita income. The average annual per capita income in the study area was about 1558 Reais (US$ 577) in 2002 (see Mendoza-Escalante, 2005). The average income of the poorest 25% of all households was approximately US$ 90 which is about 22 times less than the income of the most wealthiest 25% in the sample indicating a very unequal income distribution: The calculated gini coefficient of 0.64 indicates a slightly higher inequality than was found by Verner in 2004 for the Pará state (0.60). Farming income is the most important source of total household income (about 70%). However, most poor farmers do not depend on agriculture alone but also on off-farm earnings amounting to about 30% of their total income compared to only 10% for the wealthier ones. Public pensions seem to be an important source of income for poor households and even more for mid income households: Data from the survey used for this study revealed that about 50% of the households in the mid income level receive pensions averaging to an amount of more than 20% of their total income. The poorest 25% receive about 13% of their total income from the state.
Despite governmental programs aiming to address smallholdings’ production constraints (e.g. PRONAF and FNO-Especial) the sample indicates that access to services such as agricultural extension and credit is strikingly low in the region. Subsidized credit is on average being used by only 23% of all farmers. Technical assistance is only significant for the wealthier group of farmers. These numbers suggest that lacking access to capital, technical assistance and credit is a severe constraint for small scale farming in the region which holds especially for the poorest farms. On the other side the use of machinery (especially mechanized plowing for land preparation) as well as fertilizer is relatively high. With respect to the survey on average about 40% and 70% of the households reported the use of fertilizer and mechanization, respectively. The land endowment varies quite a lot over the region even if one considers that large-scale farms play no significant role. Annual crops are the most important source of income for all income groups. Both annual and perennial crops are cultivated as cash crops. Yet, the poorest 25% depend largely on annual crops accounting for 65% of their total value of production. The relatively low level of investment into perennial crops by the poorer households could be due to the capital constraints. Livestock husbandry plays only a minor role in the households surveyed.
4. Modelling

The previous sector descriptions suggest the following research hypothesis as a reference point for the subsequent modelling details:

_Hypothesis: The constraints to small scale agricultural production in the study region are scale dependent. The scale of production can be therefore expected to account for a relatively large proportion of the economic inefficiency of such farms._

Different approaches exist to model efficiency frontiers, whereas the majority of stochastic applications uses the error components model. In contrast to the error components model the shadow price approach enables us to consider non-observable shadow price ratios as the relevant ones for producer decisions in distorted agricultural markets. Such can be assumed with respect to agricultural production in the Brazilian Bragantina region (see e.g. Almeida/Uhl, 1995).

**The Shadow Price Approach**

Hopper (1965) already reported a high efficiency of resource allocation and crop mix for Indian farmers and like Tax (1953) before him, Hopper found the small-scale farms in the sample to be “poor but efficient”. Beside being a kind of predecessor to Schultz (1964) his statistical tests of different allocative efficiency hypotheses can be also regarded as a first attempt to explicitly model shadow parameters. However, beginning with the study of Lau and Yotopoulos (1971) a vast shadow price literature has been emerged in the last decades. In the single-output case a shadow profit function following the output-oriented approach is given by

$$\pi(p^*, w^*; \beta) = \max_x \left\{ p \phi f(x; \beta) - \sum_n \theta_n w_n x_n \right\}$$

where \( y = \phi f(x; \beta) \), with \( 0 < \phi \leq 1 \) capturing the effect of output-oriented technical inefficiency, \( p \) and \( w \) as the output and input prices, \( y \) and \( x \) as the output and input quantities respectively as well as \( p^* \) and \( w^* \) as the shadow output and input prices. To maximize shadow profit requires \( \partial f(x; \beta) / \partial x_n = (\theta_n w_n / \phi p) \), with \( n = 1, \ldots, N \) capturing the effects of systematic input allocative inefficiency. Hence, \( p^* = \phi p \) and \( w^* = \theta_n w_n \), \( n = 1, \ldots, N \). In the shadow profit function model all \( N \) input allocative inefficiency parameters \( \theta_n, n = 1, \ldots, N \) can be identified and no price normalization

Schultz’s Hypothesis Revisited – Small Scale Joint, FOI 11
is required at this stage. However, the linear homogeneity property of \( \pi(p^*, w^*; \beta) \) in \( (p^*, w^*) \) must be imposed through parametric restrictions. The majority of empirical studies consequently follow the seminal work by Lau and Yotopoulos (1971) who derived a normalized shadow profit function from the shadow profit function given in [2] as

\[
\frac{\pi(p^*, w^*; \beta)}{p} = \phi \max_x \left\{ f(x; \beta) - \sum_n \left( \frac{\theta_n w_n x_n}{\phi p} \right) x_n \right\} = \phi \left[ (w/p)^*; \beta \right] \tag{3}
\]

which is homogeneous of degree 0 in \( (p^*, w^*) \). The shadow price ratios used for the normalization of the profit function contain both technical and systematic allocative inefficiencies. Applying Hotelling’s Lemma on [3] generates the system of observed output supply and input demand equations

\[
y = \phi \pi \left[ (w/p)^*; \beta \right] - \phi \sum_n \left( \frac{\partial \pi \left[ (w/p)^*; \beta \right]}{\partial (w/p)^*_n} \right) \left( \frac{\partial \pi \left[ (w/p)^*; \beta \right]}{\partial (w/p)^*_n} \right)
\]

\[
= \phi \pi \left[ (w/p)^*; \beta \right] - \phi \sum_n \left( \frac{\partial \pi \left[ (w/p)^*; \beta \right]}{\partial (w/p)^*_n} \right) \left( \frac{\partial \pi \left[ (w/p)^*; \beta \right]}{\partial (w/p)^*_n} \right)
\]

\[
-x_n = \frac{\partial \pi \left[ (w/p)^*; \beta \right]}{\partial (w/p)^*_n} = \frac{\phi \pi \left[ (w/p)^*; \beta \right]}{\theta_n} \frac{\partial \pi \left[ (w/p)^*; \beta \right]}{\partial (w/p)^*_n}, \quad n=1, \ldots, N
\] \tag{4}

[4] and [5] generate observed normalized profit

\[
\frac{\pi}{p} = y - \sum_n \left( \frac{\partial \pi \left[ (w/p)^*; \beta \right]}{\partial (w/p)^*_n} \right) x_n = \phi \pi \left[ (w/p)^*; \beta \right] + \phi \sum_n \frac{1-\theta_n}{\theta_n} \left( \frac{\partial \pi \left[ (w/p)^*; \beta \right]}{\partial (w/p)^*_n} \right)
\] \tag{6}

Estimation can be performed by either using the system of (N+1) equations given by [4] and [5] or by using the normalized profit function in [6] as well as N observed profit share equations following [4] and [5]. Based on duality theory Lovell and Sick-
les (1983) developed a multi-product model by building on a normalized profit function. We base our efforts to model joint production by small scale farmers on this multi-product structure and use a flexible functional form.

**Functional Flexibility and Theoretical Consistency**

According to Diewert (1973) a flexible functional form provides a second order approximation to the real production structure by an arbitrarily chosen set of parameters. Hence, a functional form can be denoted as flexible if its shape is only restricted by theoretical consistency. This implies the absence of unwanted a priori restrictions and is paraphrased by the metaphor of “providing an exhaustive characterization of all (economically) relevant aspects of technology.” (Fuss/McFadden, 1978). Nevertheless, Diewert and Wales (1987) noticed the fundamental trade-off between functional flexibility and theoretical consistency, i.e. that in a production context the theoretical curvature conditions – convexity with respect to a profit function – are frequently not satisfied by the estimated function. Based on these seminal works different contributions point to the crucial importance of considering the consistency of the estimated efficiency frontier with basic microeconomic requirements as monotonicity with respect to the inputs as well as convexity of the profit function (see e.g. Ryan/Wales, 1998 and Sauer, 2006). Monotonicity of the estimated profit function – i.e. positive first derivatives with respect to all input and output prices - holds as all inputs and outputs are positive for all observations in the sample. The necessary and sufficient condition for a specific curvature consists in the definiteness of the bordered Hessian matrix as the Jacobian of the derivatives \( \frac{\partial \Pi}{\partial w_i(p_i)} \) with respect to \( w_i \) and \( p_i \): if \( \nabla^2 (w, p) \) is positive definite, \( \Pi \) is convex, where \( \nabla^2 \) denotes the matrix of second order partial derivatives with respect to the shadow translog profit model defined by [3]. The Hessian matrix is positive definite at every unconstrained local maximum. The condition of convexity is related to the fact that this property implies a concave cost function based on a quasi-concave production function and consequently a convex input requirement set (see in detail e.g. Chambers, 1988).

**The Model – A Consistent Generalized Leontief Profit Frontier**

We now consider a small scale farmer employing inputs \( x = (x_1, \ldots, x_n) \geq 0 \) to produce outputs \( y = (y_1, \ldots, y_m) \geq 0 \). The set of technologically feasible input-output vectors is given by the production possibilities set \( T \) assumed to satisfy the regularity conditions i.1 to i.4:

\[
    \begin{align*}
    \text{i.1} & \quad T \text{ is nonempty, if } (y,-x) \in T \text{ then } y \geq 0 \text{ and } x \geq 0 \\
    \text{i.2} & \quad T \text{ is closed and bounded from above}
    \end{align*}
\]
T is convex, and

if \((y, -x) \in T\) then \((y', -x') \in T\) for all \(0 \leq y' \leq y\) and \(x' \geq x\)

Assuming well functioning output and input markets the farmer takes output prices \(p = (p_1, \ldots, p_m) > 0\) and input prices \(w = (w_1, \ldots, w_n) > 0\) as exogenously given and adjusts inputs and outputs to consequently maximise

\[
\max_{y, x} \{py - wx : (y, -x) \in T\}.
\]

By assuming that \((y', -x')\) solves this maximisation problem the farm’s profit function can be formulated as \(\pi(p, w) = py' - wx'\) by satisfying i.5 to i.8:

\[
\begin{align*}
\text{i.5} & \quad \pi(p, w) \text{ is real valued and defined for } (p, w) > 0 \\
\text{i.6} & \quad \pi(p, w) \text{ is nondecreasing in } p \text{ and nonincreasing in } w \\
\text{i.7} & \quad \pi(\lambda p, \lambda w) = \lambda \pi(p, w) \text{ for all } \lambda > 0, \text{ and} \\
\text{i.8} & \quad \pi(p, w) \text{ is a convex function in } (p, w)
\end{align*}
\]

where the duality between a function adhering to [i.1] to [i.4] and such adhering to [i.5] and [i.8.] becomes obvious. Following again Hotelling’s Lemma the farm’s profit maximising output supply as well as input demand equations are directly obtained from the profit function for all differentiable \((p, w) > 0\) by

\[
\Delta_p \pi(p, w) = y(p, w) \quad \text{and} \quad \Delta_w \pi(p, w) = -x(p, w).
\]

The pioneering generalized Leontief function (GL) leads off the extensive literature on second order flexible functional forms motivated by the endavour to make the progresses of duality theory empirically applicable. The dual cost function can be formulated as

\[
c(y, w; \beta) = y \sum_{n} \sum_{k} \beta_{nk} w_n \frac{1}{n} w_k \frac{1}{k} = y \left[ \sum_{n} \beta_{nn} w_n + 2 \sum_{n} \sum_{k>n} \beta_{nk} w_n \frac{1}{n} w_k \right]
\]

Since it does not treat input and output related variables symmetrically, several multi-output generalizations are possible. Based on the flexible generalized Leontief profit function framework, we go beyond the Lovell/Sickles model to consistently model allocative and scale efficiency by imposing curvature correctness on the estimated frontier. The GL is linearly homogenous in input and output prices by construction, however, by globally imposing curvature and monotonicity the property of second order flexibility is lost.

a) Basic model: Due to the previously described setting of small scale farming in the Bragantina region we now leave the model of perfect markets and consequently as-
sume that a small scale farmer optimizes his/her production with respect to shadow price ratios. Supposing further that the underlying profit function takes the GL form, with $M = N = 2$ for produced outputs (cassava flour, maize) and applied inputs (labour, fertilizer) as well as controlling for the fixed input (land) $c$ and other exogenous factors $z_i$ (biomass, soil pH, phosphorus content, fallow age, precipitation, market distance, household size, education of household head, type of ownership, share of hired labor, farm location) we obtain

$$\pi(p, w, \beta, \theta) = \frac{\beta_{11} p_1 + \beta_{12} \theta_{12} p_1^2 \beta_{22} p_2^2 + \beta_{13} \theta_{13} p_1 \beta_{23} p_2^2 \beta_{24} p_2 \beta_{24} p_2^2 \beta_{32} p_2^2 \beta_{33} \beta_{22} p_2 + \beta_{23} \theta_{23} p_2^2 \beta_{24} \theta_{24} p_2^2 \beta_{31} \beta_{32} \theta_{31} \beta_{32} \theta_{32} \beta_{33} \beta_{34} \theta_{34} p_1^2 \beta_{41} \theta_{41} p_1 \beta_{42} \theta_{42} p_2 \beta_{43} \theta_{43} p_2 \beta_{44} p_2 + \chi c + \sum_{i=2}^{31} \chi_i z_i}{\beta_{ij} = \beta_{ji} \forall j \neq i \text{ and } \theta_{ij} = \theta_{ji} \forall j \neq i \text{. As outlined above observed price ratios are replaced with shadow price ratios } \theta_{ij} \left(\frac{p_{ij}}{w_{ij}}\right) \text{, } i \neq j \text{ and } \theta_{ij} \left(\frac{p_{ij}}{w_{ij}}\right) \text{, } i \neq j \text{.}$$

The GL profit function is homogeneous of degree +1 in $(p, w)$ by construction. Its functional shape is convex in $(p, w)$ if $\beta_{ij} \leq 0 \forall j \neq i \text{. By applying Hotelling’s Lemma and assuming that the individual farmer optimizes with respect to shadow price ratios, the system of profit-maximizing output supply and input demand equations is generated}$

$$y_1 = \beta_{11} + \beta_{12} \left[\theta_{12} \left(\frac{p_1}{p_2}\right)\right] + \beta_{13} \left[\theta_{13} \left(\frac{p_1}{w_1}\right)\right] + \beta_{14} \left[\theta_{14} \left(\frac{p_1}{w_2}\right)\right] + \chi c + \sum_{i=2}^{31} \chi_i z_i$$

$$y_2 = \beta_{22} + \beta_{12} \left[\theta_{12} \left(\frac{p_1}{p_2}\right)\right] + \beta_{23} \left[\theta_{23} \left(\frac{p_2}{w_1}\right)\right] + \beta_{24} \left[\theta_{24} \left(\frac{p_2}{w_2}\right)\right] + \chi c + \sum_{i=2}^{31} \chi_i z_i$$

With respect to the production of cassava flour one could alternatively specify a more household oriented production function. However, the joint usage of labor as well as the unavailability of product specific input data makes it reasonable to follow the chosen agricultural production function based modelling approach here.
where $\theta_{ij}$ denotes the shadow parameter with respect to the systematic price ratio $i,j$.

The system is estimated by using nonlinear iterative seemingly unrelated regression procedures (ITSUR) and imposing the cross-equation parameter restrictions. Technical inefficiency could be introduced in [9] to [12] by simply replacing the intercepts with $(\beta_{jj} - \phi_j)$, $j = 1, \ldots, 4$. However, here technical inefficiency would be nonneutral and could only be determined for groups of producers, consequently we only model allocative inefficiency with respect to inputs and outputs as well as scale.

\[ b) \text{ Consistent model 1 - global convexity imposed:} \text{ Although our GL specification of } \pi(p, w) \text{ satisfies i.5 and i.7 by construction, monotonicity in outputs and inputs (i.6) as well as convexity in output and input prices (i.7) have to be checked and imposed respectively. Monotonicity holds for every observation in the sample as all show positive output and input quantities. Correct curvature is given as the } \beta_{ij} \leq 0 \forall j \neq i \text{. This can be imposed on the system of profit-maximizing output supply and input demand equations by applying the following restrictions on [9] to [12]} \]

\[ \beta_{ij} = -\left(d_{ij}^2\right) \forall j \neq i \]  

and consequently:

\[ y_i = \beta_{i1} + \left[-\left(d_{i1}^2\right)\right]\left[\theta_{i2} \left(\frac{p_1}{p_2}\right)\right]^\frac{1}{2} + \left[-\left(d_{i3}^2\right)\right]\left[\theta_{i3} \left(\frac{p_1}{w_i}\right)\right]^\frac{1}{2} + \left[-\left(d_{i4}^2\right)\right]\left[\theta_{i4} \left(\frac{p_i}{w_2}\right)\right]^\frac{1}{2} + \chi_i c + \sum_{i=2}^{31} \chi_i z_i \]
\[ y_2 = \beta_{22} + \left[ -\left( d_{12}^2 \right) \right] \left[ \theta_{12} \left( \frac{p_1}{p_2} \right) \right]^\frac{1}{2} + \left[ -\left( d_{23}^2 \right) \right] \left[ \theta_{23} \left( \frac{p_2}{w_1} \right) \right]^\frac{1}{2} + \left[ -\left( d_{24}^2 \right) \right] \left[ \theta_{24} \left( \frac{p_2}{w_2} \right) \right]^\frac{1}{2} + \]
\[ \chi_i c + \sum_{i=2}^{31} \chi_i z_i \]  
\[[15]\]

\[ -x_1 = \beta_{33} + \left[ -\left( d_{13}^2 \right) \right] \left[ \theta_{13} \left( \frac{p_1}{w_1} \right) \right]^\frac{1}{2} + \left[ -\left( d_{23}^2 \right) \right] \left[ \theta_{23} \left( \frac{p_2}{w_1} \right) \right]^\frac{1}{2} + \left[ -\left( d_{34}^2 \right) \right] \left[ \theta_{34} \left( \frac{w_1}{w_2} \right) \right]^\frac{1}{2} + \]
\[ \chi_i c + \sum_{i=2}^{31} \chi_i z_i \]  
\[[16]\]

\[ -x_2 = \beta_{44} + \left[ -\left( d_{14}^2 \right) \right] \left[ \theta_{14} \left( \frac{p_1}{w_2} \right) \right]^\frac{1}{2} + \left[ -\left( d_{24}^2 \right) \right] \left[ \theta_{24} \left( \frac{p_2}{w_2} \right) \right]^\frac{1}{2} + \left[ -\left( d_{34}^2 \right) \right] \left[ \theta_{34} \left( \frac{w_1}{w_2} \right) \right]^\frac{1}{2} + \]
\[ \chi_i c + \sum_{i=2}^{31} \chi_i z_i \]  
\[[17]\]

where \( \theta_{ij} \) denotes again the shadow parameter with respect to the systematic price ratio \( i,j \). Here \( \beta_{ii}, \theta_{ij}, d_{ij} \), and \( \chi_i \) are estimated by using nonlinear iterative seemingly unrelated regression procedures (ITSUR) and imposing again the cross-equation parameter restrictions.

c) Consistent model 2 - consistent systematic allocative efficiency imposed: The preceding analysis is based on three independent market price ratios as well as six independent shadow price ratios. As we have consequently used six independent parameters \( \theta_{ij} \) to model systematic allocative inefficiency in the preceding analysis it remains highly unlikely that producers are consistent in their deviating perceptions of the output and input market price ratios. Hence, the preceding models permit inconsistent allocative inefficiency. Consistent systematic allocative inefficiency can be nevertheless modeled as a constrained version of the model 1 or 2 by imposing the following parametric restrictions

\[ \theta_{ik} = \theta_{ij} \ast \theta_{jk}, i < j < k \]  
\[[18]\]

resulting in:
\[ \theta_{13} = \theta_{12} \ast \theta_{23} \]  
\[ \theta_{14} = \theta_{12} \ast \theta_{23} \ast \theta_{34} = \theta_{13} \ast \theta_{34} \]  
\[ \theta_{24} = \theta_{23} \ast \theta_{34} \]

and hence reducing the number of independent allocative inefficiency parameters to three. By adhering to theoretical consistency of the underlying functional form this finally generates model 3

\[ y_1 = \beta_{11} + \left[ -\left( d_{12}^2 \right) \right] \left[ \theta_{12} \left( \frac{p_1}{p_2} \right) \right]^{\frac{1}{2}} + \left[ -\left( d_{13}^2 \right) \right] \left[ \left( \theta_{12} \ast \theta_{23} \right) \left( \frac{p_1}{w_1} \right) \right]^{\frac{1}{2}} + \left[ -\left( d_{14}^2 \right) \right] \left( \theta_{12} \ast \theta_{23} \ast \theta_{34} \right) \left( \frac{p_1}{w_2} \right) + \chi_1 c + \sum_{i=2}^{31} \chi_i z_i \]  
\[ y_2 = \beta_{22} + \left[ -\left( d_{12}^2 \right) \right] \left[ \theta_{12} \left( \frac{p_1}{p_2} \right) \right]^{\frac{1}{3}} + \left[ -\left( d_{23}^2 \right) \right] \left[ \theta_{23} \left( \frac{p_2}{w_1} \right) \right]^{\frac{1}{2}} + \left[ -\left( d_{24}^2 \right) \right] \left( \theta_{23} \ast \theta_{34} \right) \left( \frac{p_2}{w_2} \right) + \chi_1 c + \sum_{i=2}^{31} \chi_i z_i \]  
\[ -x_1 = \beta_{33} + \left[ -\left( d_{13}^2 \right) \right] \left( \theta_{12} \ast \theta_{23} \right) \left( \frac{p_1}{w_1} \right) + \left[ -\left( d_{23}^2 \right) \right] \left( \theta_{23} \left( \frac{p_2}{w_1} \right) \right) + \left[ -\left( d_{34}^2 \right) \right] \left( \frac{w_1}{w_2} \right) + \chi_1 c + \sum_{i=2}^{31} \chi_i z_i \]  
\[ -x_2 = \beta_{44} + \left[ -\left( d_{14}^2 \right) \right] \left( \theta_{12} \ast \theta_{23} \ast \theta_{34} \right) \left( \frac{p_1}{w_2} \right) + \left[ -\left( d_{24}^2 \right) \right] \left( \theta_{23} \ast \theta_{34} \right) \left( \frac{p_2}{w_2} \right) + \left[ -\left( d_{34}^2 \right) \right] \left( \theta_{34} \left( \frac{w_1}{w_2} \right) \right) + \chi_1 c + \sum_{i=2}^{31} \chi_i z_i \]
where $\theta_{ij}$ denotes again the shadow parameter with respect to the systematic price ratio $i,j$ now restricted according to [19] to [21]. The system is again estimated by using nonlinear iterative seemingly unrelated regression procedures (ITSUR) and imposing beside the cross-equation parameter restrictions also the specified equality constraints. The resulting shadow profit frontier is globally convex and consistent with respect to systematic allocative efficiency.

**Partial profit effects of systematic allocative inefficiency**

If, and only if, all $\theta_{ij} = 1$, the effect of systematic allocative inefficiency on profit equals zero. If at least one $\theta_{ij} \neq 1$, the effect of systematic allocative inefficiency (i.e. output allocative inefficiency, input allocative inefficiency, and scale inefficiency) can be considered as producer specific, depending on the prices ratios perceived by the individual producer.

(i) Accordingly, the partial effect of systematic output allocative inefficiency on profit can be calculated by

$$
\pi(p, w; \beta, \theta) - (\pi|\theta_{12} \neq 1) = \beta_{12} p_1^{\frac{1}{2}} p_2^{\frac{1}{2}} \left[ 2 - \left( \theta_{12}^{\frac{1}{2}} + \theta_{12}^{\frac{1}{2}} \right) \right]$$

Equation [26] is positive unless $\theta_{12} = 1$ and hence the observed output mix chosen by the individual producer does not maximize profit.

(ii) The partial effect of systematic input allocative inefficiency on profit can be calculated by

$$
\pi(p, w; \beta, \theta) - (\pi|\theta_{34} \neq 1) = \beta_{34} w_1^{\frac{1}{2}} w_2^{\frac{1}{2}} \left[ 2 - \left( \theta_{34}^{\frac{1}{2}} + \theta_{34}^{\frac{1}{2}} \right) \right]$$

which is positive unless $\theta_{34} = 1$. If $\theta_{34} \neq 1$ the observed input mix does not maximize profit.

(iii) The partial effect of systematic scale inefficiency on profit is given by

$$
\pi(p, w; \beta, \theta) - (\pi|\theta_{13} \neq 1, \theta_{14} \neq 1, \theta_{24} \neq 1) = \sum_i \sum_j \beta_{ij} q_i^{\frac{1}{2}} q_j^{\frac{1}{2}} \left[ 2 - \left( \theta_{ij}^{\frac{1}{2}} + \theta_{ij}^{\frac{1}{2}} \right) \right]
$$

where $i = 1, 2$ and $j = 3, 4$. If $(\theta_{13}, \theta_{14}, \theta_{23}, \theta_{24}) \neq (1,1,1,1)$ the observed output-input ratios by the individual producer are not conducive for maximizing profit.
5. Data, Variables and Estimation

The data used in this study has been collected by two surveys conducted in the Braganțina region as part of the project SHIFT ENV 44 (‘Studies on Human Impact on Forests and Floodplains in the Tropics’). This region comprises 14 municipalities with a total area of about 11,609 km\(^2\) and accounts for about 5-6% of the total population of the Pará state but only for 1% of the total area. With respect to agricultural production it is one of the most important zones in the state. A total of 271 households from 22 villages were included in the study which contains 91 households from seven villages of the municipality of Igarapé-Açu, 90 households from three villages belonging to the municipality of Castanhal and 91 households from twelve villages of the municipality of Bragança. This survey was carried out between October 2002 and December 2002 covering the 2001/2002 cropping season. The sampling was done in two stages involving a sample stratification in the first (i.e. a proportionate stratification by using the category village to build the sampling fractions) and a random selection in the second stage (see Mendoza-Escalante, 2005). The questionnaire consisted mainly of questions related to the farming system and farm-household characteristics. In addition, from each of the 271 farmers, plot or parcel specific information was collected from two randomly selected plots used for annual cropping (between December 2002 and February 2003). The second survey was carried out in the municipalities of Barcarena and Igarapé-Açu. Here a total of 57 households from 10 villages (41 households from 8 villages belonging to the municipality of Igarapé-Açu, and 16 households from two villages of the municipality of Barcarena) were included. This survey was carried out between May 2003 and June 2003 also covering the 2001/2002 cropping season. The farmers surveyed were selected with respect to the use of fire-free technologies. The questionnaire mainly included questions related to the farming system and farm-household characteristics. In addition, from each of the 57 farmers, plot or parcel specific information was collected from two plots (i.e. one parcel with fire-free technology and another parcel with slash-and-burn agriculture) for the purpose of comparison. Here a total of 101 parcels were investigated.

Based on these surveys a final sample of 194 small scale farmers were selected jointly producing cassava flour and maize in the study period. Table 1 summarizes the descriptive statistics for the variables used. Some of these variables are self-explanatory,

\(^2\) SHIFT ENV 44 project: Smallholders in the Amazon: Interactions between Ecosystem and Social System in the Use and Protection of Tropical Rain Forests, financed by the German Ministry of Education, Science, Research and Technology (BMBF).
but some require additional explanations. The aggregate fertilizer quantity represents the sum of the NPK fertilizer in kilograms used on the plot. This is justified by the fact that information provided in the survey about the quantities of specific chemicals, turned out to be for the majority on different types of NPK amounts. Thus, given this shortcoming, all chemicals were included in the same homogeneous group. This was done by extracting the percentage of NPK from castor oil and poultry dung, followed by the summation of all the NPK quantities measured as total amount applied in kilograms. The representative price was simply the 2002 average price of the three different NPKs’ traded in local markets. Total labour is defined as the number of man-days (family and hired labour) used in agricultural activities for the specific plot. The wage rate per man-day was calculated from the wage bill of hired labour. Land is proxied by plot size. Control variables for the dry weight of above ground biomass in the plot, for the soil pH, and for the available phosphorus in the soil were included as the results of the different biotests conducted for the soil samples. The age of the respective fallow was included to account for its quality. The average amount of rainfall in the dry months was included as well as the distance from the community to the next market center. Control variables for the size of the household, the education of the household head, for the case if the land is owned or rented by the respective farmer as well as if the specific farm hires seasonal labor or not. Dummy variables are used to account for the location of the individual farm with respect to the village and the relevant municipality.

### Table 1. Descriptive Statistics

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>MEAN</th>
<th>STDEV</th>
<th>MIN</th>
<th>MAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cassava Flour (kg)</td>
<td>3,294.021</td>
<td>3,616.609</td>
<td>90</td>
<td>27,000</td>
</tr>
<tr>
<td>Maize (kg)</td>
<td>474.892</td>
<td>563.59</td>
<td>15</td>
<td>4,000</td>
</tr>
<tr>
<td>CassFlour Price (Reais/kg)</td>
<td>0.774</td>
<td>0.097</td>
<td>0.683</td>
<td>0.893</td>
</tr>
<tr>
<td>Maize Price (Reais/kg)</td>
<td>0.235</td>
<td>0.032</td>
<td>0.187</td>
<td>0.273</td>
</tr>
<tr>
<td>Total Labour (mandays)</td>
<td>69.241</td>
<td>57.517</td>
<td>11.75</td>
<td>340</td>
</tr>
<tr>
<td>Fertilizer NPK (kg)</td>
<td>13.56</td>
<td>54.996</td>
<td>0</td>
<td>500</td>
</tr>
<tr>
<td>Wage (Reais/manday)</td>
<td>8.068</td>
<td>1.344</td>
<td>6</td>
<td>16.25</td>
</tr>
<tr>
<td>NPK Price (Reais/kg)</td>
<td>0.967</td>
<td>0.148</td>
<td>0.683</td>
<td>1.063</td>
</tr>
<tr>
<td>Land – Plot size (ha)</td>
<td>1.026</td>
<td>0.935</td>
<td>0.301</td>
<td>9.01</td>
</tr>
<tr>
<td>Biomass (g/plot)</td>
<td>0.555</td>
<td>0.216</td>
<td>0.33</td>
<td>1.27</td>
</tr>
<tr>
<td>pH</td>
<td>4.552</td>
<td>1.237</td>
<td>4.03</td>
<td>6.53</td>
</tr>
<tr>
<td>Phosphorus (mg/100g soil)</td>
<td>0.532</td>
<td>0.458</td>
<td>0.147</td>
<td>3.285</td>
</tr>
<tr>
<td>Age of fallow (years)</td>
<td>13.629</td>
<td>10.338</td>
<td>1</td>
<td>60</td>
</tr>
<tr>
<td>Precipitation (mm/month)</td>
<td>47.227</td>
<td>32.201</td>
<td>12</td>
<td>86</td>
</tr>
<tr>
<td>Market distance (km)</td>
<td>24.557</td>
<td>13.226</td>
<td>4</td>
<td>62</td>
</tr>
<tr>
<td>Household size (n)</td>
<td>6.299</td>
<td>2.827</td>
<td>1</td>
<td>17</td>
</tr>
<tr>
<td>Education of head (years)</td>
<td>3.758</td>
<td>2.558</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>Ownership (1-ownership, 0-other)</td>
<td>0.618</td>
<td>0.487</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Hired Labor (1-yes, 0-no)</td>
<td>0.851</td>
<td>0.357</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Village (1-3 located in Igarapé-Açu, 4-9 located in Castanhal, 10-20 located in Bragança)</td>
<td>10.041</td>
<td>6.843</td>
<td>1</td>
<td>20</td>
</tr>
</tbody>
</table>

1: The single characteristics for this variable are included as dummy variables in the estimation models.
As mentioned above the different models were estimated by applying an iterative seemingly unrelated regression procedure (SURE). These models consist of several equations which appear to be unrelated, i.e. a system of standard linear regression models. However, they are related due to the facts that some explanatory variables are the same and that the disturbances are correlated across equations. Each of our \(m\) standard linear regression models is based on \(n\) cross-sectional observations where the \(m\) matrices of regressors \(X_j\) may contain common variables and the \(m\) regression coefficients \(\beta_j\) are generally unrestricted. By putting these \(m\) separate and seemingly unrelated linear regressions together, possible correlations between the \(m\) disturbance terms are considered

\[
\text{Cov}(\varepsilon_j, \varepsilon_k) = \sigma_{jk} I_n \quad \text{for } j \neq k
\]  

[29]

and the variance-covariance matrix of the system disturbance term \(\varepsilon\) is

\[
\text{Var}(\varepsilon) = \sum_{mn \times nm} \begin{pmatrix}
\sigma_1^2 I_n & \sigma_{12} I_n & \ldots & \sigma_{1m} I_n \\
\sigma_{21} I_n & \sigma_2^2 I_n & \ldots & \sigma_{2m} I_n \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_{m1} I_n & \sigma_{m2} I_n & \ldots & \sigma_m^2 I_n
\end{pmatrix}
\]  

[30]

where \(\sigma_j^2\) is the disturbance variance in the \(j\)-th equation and \(\sigma_{jk}\) is the covariance between the disturbance terms of the \(j\)-th and \(k\)-th equations. Using the Kronecker product notation this can be further simplified (see e.g. Greene, 2000). As the system is a generalized linear regression model, the Generalized Linear Square (GLS) estimator resp. Feasible Generalized Linear Square (FGLS) estimator can be used to estimate the regression coefficient \(\beta\). The feasible GLS consists of two steps (see in detail e.g. Greene, 2000): (1) the Ordinary Least Square (OLS) estimation is used to compute the OLS residuals, and (2) the FGLS estimators are computed as follows

\[
\beta^- = \left[ X' \left( S_o^{-1} \otimes I_n \right) X \right]^{-1} X' \left( S_o^{-1} \otimes I_n \right) y
\]  

[31]

where \(S_o = \begin{bmatrix} s_{jk} \end{bmatrix}\). The variance-covariance matrix of the GLS estimator can be derived by
\[ Var(\beta^-) = \left[ X'\left(S_o^{-1} \otimes I_{n}\right)X \right]^{-1} \]

As Greene (2000) notes, the Oberhofer/Kmenta (1974) conditions are met for the SURE model, so maximum likelihood estimates can be obtained by iterating the FGLS procedure: once the FGLS estimate of the second step is computed, the corresponding residuals are computed and the first step to get another set of estimates of \( s_{jk} \) is repeated, which are then used to estimate the second step again and so on. Iteration of the two steps of the FGLS procedure usually helps to improve the efficiency of the estimation and hence, it is well known that maximum likelihood enjoys no advantage over FGLS in its asymptotic properties.
6. Results and Discussion

Table 2 summarizes the parameter estimates and model statistics for the different models estimated.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model 1 (Basic)</th>
<th>Model 2 (Convex)</th>
<th>Model 3 (Convex, Efficiency Consistent)</th>
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</thead>
<tbody>
<tr>
<td>$\beta_{\text{eff}}$</td>
<td>0.0087</td>
<td>0.1608</td>
<td>0.3822**</td>
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<td>$\beta_{\text{fm}}$</td>
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<td>$\beta_{\text{fl}}$</td>
<td>1.6398***</td>
<td>-3.9E-08</td>
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<td>$\beta_{\text{ffert}}$</td>
<td>-0.0598</td>
<td>-0.0006</td>
<td>-0.0012</td>
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<tr>
<td>$\beta_{\text{num}}$</td>
<td>0.826***</td>
<td>1.1199***</td>
<td>1.1686***</td>
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<tr>
<td>$\beta_{\text{ml}}$</td>
<td>-0.3819***</td>
<td>-0.0778*</td>
<td>-0.0373</td>
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<tr>
<td>$\beta_{\text{mfert}}$</td>
<td>-0.1317</td>
<td>-0.1417*</td>
<td>-0.0301</td>
</tr>
<tr>
<td>$\beta_{\text{ll}}$</td>
<td>-4.7399</td>
<td>-0.3941**</td>
<td>-0.4356***</td>
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<td>$\beta_{\text{fert}}$</td>
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<td>-3.1E-08*</td>
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<td>1.9308</td>
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<td>$\chi_c$</td>
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<td>0.8593***</td>
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<td>$\chi_{\text{bio}}$</td>
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<tr>
<td>$\chi_{\text{pH}}$</td>
<td>0.0731***</td>
<td>0.0710**</td>
<td>0.0296***</td>
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<td>$\chi_{\text{phos}}$</td>
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<td>0.0051</td>
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<td>$\chi_{\text{rain}}$</td>
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<td>0.0507***</td>
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<td>$\chi_{\text{dist}}$</td>
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<td>3.5779***</td>
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<td>1.2311**</td>
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<tr>
<td>$\chi_{\text{vill05}}$</td>
<td>3.5566***</td>
<td>3.9188***</td>
<td>3.5239</td>
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<tr>
<td>$\chi_{\text{vill06}}$</td>
<td>2.4624***</td>
<td>2.7457***</td>
<td>2.3351*</td>
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<tr>
<td>$\chi_{\text{vill07}}$</td>
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<td>2.1581*</td>
<td>2.0457</td>
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<tr>
<td>$\chi_{\text{vill08}}$</td>
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<td>2.4286***</td>
<td>2.1341</td>
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* ** ***: significance at 10, 5, and 1%-level; despite dummy variables are included for all villages, the problem of perfect collinearity is avoided as by model specification no intercepts are included.
The model statistics show significant fits for all estimated models. Due to the cross-sectional data set used the adjusted $R^2$ values are relatively modest showing the highest values for the unconstrained basic model (model 1). The t-statistics reveal the most significant parameter estimates again for the unconstrained model 1 followed by model 2 restricted for correct curvature. These findings confirm earlier empirical studies with respect to a trade-off between statistical significance and theoretical consistency of the frontier estimates (see e.g. Sauer, 2006). The quasi fixed input land is significant in all models showing more or less the same magnitude and a positive effect on the level of profit. All other control variables show consistent signs over the three models whereas the variables for soil pH, precipitation and the average market distance show the highest significance. However the direction of influence on profit is not always consistent with theory (see biomass, market distance, share of hired labor). The estimates of the village dummies are consistent over all three models showing significant positive values for the farms belonging to villages located in the municipalities of Igarapé-Açu and Castanhal (villages 1 to 9) but significant negative values for those located in the municipality of Bragança (villages 10 to 20). These findings could be predominantly due to the more favourite climatic conditions (i.e. precipitation, soil moisture) as well as infrastructural endowments of these villages (see also Appendix, map 1). The shadow price parameters $\theta_{fm}$, $\theta_{fl}$, $\theta_{fert}$, $\theta_{ml}$, $\theta_{mfert}$ and $\theta_{lfert}$ contain the information on the systematic allocative efficiency with respect to the output and input price ratios experienced by the farmer. The parameters’ estimates translated into systematic relative efficiency scores are given in table 3.

Table 3. Systematic Allocative Efficiency per Price Pair

<table>
<thead>
<tr>
<th>Price Pair</th>
<th>Model 1 (Basic)</th>
<th>Model 2 (Convex)</th>
<th>Model 3 (Convex, Efficiency Consistent)</th>
</tr>
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<tbody>
<tr>
<td>Flour/Maize</td>
<td>0.7818</td>
<td>0.0280</td>
<td>0.0094</td>
</tr>
<tr>
<td>Labour/Flour</td>
<td>0.1529</td>
<td>0.9829</td>
<td>2.3056E-05</td>
</tr>
<tr>
<td>Flour/Fertilizer</td>
<td>0.0029</td>
<td>0.0029</td>
<td>1.4229E-05</td>
</tr>
<tr>
<td>Labour/Maize</td>
<td>0.1537</td>
<td>0.0094</td>
<td>0.0024</td>
</tr>
<tr>
<td>Maize/Fertilizer</td>
<td>0.0142</td>
<td>0.0426</td>
<td>0.0015</td>
</tr>
<tr>
<td>Fertilizer/Labour</td>
<td>0.1336</td>
<td>0.1336</td>
<td>0.6171</td>
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</table>
Relatively high differences in the systematic efficiency values were found for the three models estimated. The closer the value is to unity the lower the difference between observed and latent shadow prices. It becomes clear from the compilation in table 3 that the shadow price ratios are neither all efficient nor all inefficient. Hence, the empirical results suggest that only analysing overall allocative efficiency is misleading and does not show the real sources of inefficient profit maximisation behaviour of small scale farmers. Hence, we subsequently take a farm specific perspective by differentiating between pure allocative and scale inefficiency for each farm. Table 4 summarizes the results for the whole sample of small scale farmers over the different models estimated.

| Table 4. Farm Specific Pure Allocative and Scale Efficiency |

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><em>(Basic)</em></td>
<td><em>(Convex)</em></td>
<td><em>(Convex, Efficiency Consistent)</em></td>
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<tr>
<td>Overall Allocative Efficiency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>0.9435</td>
<td>0.8681</td>
<td>0.8499</td>
</tr>
<tr>
<td>min</td>
<td>0.0331</td>
<td>0.0124</td>
<td>0.0288</td>
</tr>
<tr>
<td>max</td>
<td>0.9983</td>
<td>0.9958</td>
<td>0.9840</td>
</tr>
<tr>
<td>p-value</td>
<td>3.6550E-18</td>
<td>5.6495E-08</td>
<td>9.4535E-15</td>
</tr>
<tr>
<td>Pure Allocative Inefficiency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>7.1323E-05</td>
<td>0.0015</td>
<td>0.0019</td>
</tr>
<tr>
<td>min</td>
<td>0</td>
<td>4.0885E-05</td>
<td>4.9842E-05</td>
</tr>
<tr>
<td>max</td>
<td>0.0049</td>
<td>0.0251</td>
<td>0.0307</td>
</tr>
<tr>
<td>p-value</td>
<td>3.6551E-18</td>
<td>5.6500E-08</td>
<td>9.4540E-15</td>
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<tr>
<td>Scale Inefficiency</td>
<td></td>
<td></td>
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<tr>
<td>mean</td>
<td>0.0564</td>
<td>0.1311</td>
<td>0.1491</td>
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<tr>
<td>min</td>
<td>0.0016</td>
<td>0.0039</td>
<td>0.0157</td>
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<tr>
<td>max</td>
<td>0.9661</td>
<td>0.9736</td>
<td>0.9588</td>
</tr>
<tr>
<td>p-value</td>
<td>3.3186E-18</td>
<td>0.0048</td>
<td>4.1101E-15</td>
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The mean overall allocative efficiency on farm level is relatively high for the three models (0.849 – 0.943) with a wide range of farms’ performance. The scores for the pure allocative inefficiency per farm show a relatively low mean value (7.13E-05 - 1.5E-03) whereas those for the scale inefficiency per farm show a considerably higher mean value (0.056 – 0.149) with again a wide range of farms’ performance. This simply means that the mean allocative inefficiency due to an inappropriate scale of farm operations accounts for the largest part of overall allocative inefficiency on farm level. The mean farm in the sample of small scale Brazilian farmers could increase its efficiency by up to 15% for the efficiency and curvature consistent model 3 by simply
adjusting the input/output ratios. Figure 1 shows the distribution of the scale inefficiencies over the sample of small scale farms for the efficiency and curvature consistent model 3: The majority of farms show a scale inefficiency in the range of up to 20% and increasing returns to scale for all input/output relations - flour/labour, flour/fertilizer, maize/labour, as well as maize/fertilizer.

**Figure 1. Scale Inefficiency / Model 3**

![Graph showing scale inefficiency distribution](image)

(scin = scale inefficiency in %)

The corresponding absolute profit loss due to output allocative inefficiency, input allocative inefficiency as well as scale inefficiency is summarized by table 5.
Table 5. Farm Specific Profit Effects

<table>
<thead>
<tr>
<th></th>
<th>Model 1 (Basic)</th>
<th>Model 2 (Convex)</th>
<th>Model 3 (Convex, Efficiency Consistent)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Partial profit effect of output allocative inefficiency</strong> (in Brazilian Reais per plot)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>1.19E-04</td>
<td>0.824</td>
<td>1.005</td>
</tr>
<tr>
<td>min</td>
<td>9.98E-05</td>
<td>0.692</td>
<td>0.844</td>
</tr>
<tr>
<td>max</td>
<td>1.38E-04</td>
<td>0.956</td>
<td>1.165</td>
</tr>
<tr>
<td>p-value</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>[0.05; 0.95]</td>
<td>[1.17E-04; 1.21E-04]</td>
<td>[0.810; 0.839]</td>
<td>[0.987; 1.02]</td>
</tr>
<tr>
<td><strong>Partial profit effect of input allocative inefficiency</strong> (in Brazilian Reais per plot)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>8.794</td>
<td>9.39E-09</td>
<td>5.22E-12</td>
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<tr>
<td>min</td>
<td>6.437</td>
<td>6.88E-09</td>
<td>3.82E-12</td>
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<tr>
<td>max</td>
<td>13.017</td>
<td>1.39E-08</td>
<td>7.73E-12</td>
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<td>p-value</td>
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</tr>
<tr>
<td>[0.05; 0.95]</td>
<td>[8.661; 8.927]</td>
<td>[9.25E-09; 9.54E-09]</td>
<td>[5.15E-12; 5.31E-12]</td>
</tr>
<tr>
<td><strong>Partial profit effect of scale inefficiency</strong> (in Brazilian Reais per plot)</td>
<td></td>
<td></td>
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<tr>
<td>mean</td>
<td>36.812</td>
<td>78.007</td>
<td>116.116</td>
</tr>
<tr>
<td>min</td>
<td>3.827</td>
<td>9.838</td>
<td>6.444</td>
</tr>
<tr>
<td>max</td>
<td>96.668</td>
<td>258.738</td>
<td>297.091</td>
</tr>
<tr>
<td>p-value</td>
<td>0.088</td>
<td>0.079</td>
<td>0.078</td>
</tr>
<tr>
<td>[0.05; 0.95]</td>
<td>[10.886; 79.850]</td>
<td>[24.831; 142.193]</td>
<td>[16.191; 246.626]</td>
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From this compilation the relatively large amount of foregone profit due to an inappropriate scale of farms’ operations is again evident. Accordingly the average farm in the sample could increase its profit in absolute terms by approximately 37-116 Reais per plot and year (i.e. 36-112 Reais per ha and year) cultivated whereas the average total profit is about 595-778 Reais per plot and year (i.e. 577-755 Reais per ha and year). Hence, the empirical findings for a sample of small scale farmers in the Bragantina region of the Eastern Brazilian Amazon confirmed the preceeding theoretical considerations on the relative importance of scale economies with respect to an overall judgement whether agricultural operations are efficient or not. Our analytical hypothesis based on the formulated theoretical proposition is therefore confirmed for the sample of small scale farmers.

The existing empirical literature on peasants’ efficiency reports quite mixed results with respect to the efficiency of the scale of agricultural operations. The vast majority of studies incorporates scale as a technical or allocative inefficiency explaining factor and does not explicitly consider the measurement of scale efficiency (see Ali/Byerlee, 1991 and Barrett, 1997). Wang et al. (1996) e.g. found a positive influence of farm size on the technical as well as allocative efficiency of farms in China, whereas the
opposite was reported by Flinn/Ali (1986) for small scale farms in Pakistan. No significant scale effect was found e.g. by Huang/Bagi (1984) for peasants in India. However, the systematic scale errors – i.e. the failure to use profit maximising levels of inputs – found for the sample of small scale farmers in the Eastern Amazon could be due to different factors: an existing capital constraint, limited access to inputs constraining the farmer’s ability to adjust output volumes, risk averse investment behaviour by the peasant, inadequate information with respect to market developments, formal and/or informal institutional barriers (e.g. tenancy, traditional consumption patterns), missing output markets, or multi-value based decision making (see also Myrdal, 1968). Barrett (1997) nevertheless questions the use of empirical findings of farm-level inefficiencies caused by variables beyond the farmer’s control as well as doubts the relevance of an industry level related concept of scale optimality for small scale agriculture in developing countries. The current discussion of the ‘Efficient but Poor’ hypothesis offers different starting points for an explanation of prevailing scale inefficiency among small scale agriculture in a developing country setting as the Bragantina region: Prevailing structural poverty can be interpreted as a major hurdle for lacking optimization behaviour among farmers by applying Ray’s concept of an aspiration window. The latter suggests that farmer’s investment behaviour is affected by the gap between the aspired standard of living and the one the farmer and his/her family already has. Accordingly individual farmer’s effort to invest in enhancing the production is minimal when this aspiration gap is large because it is viewed as too great to overcome, and similarly when the gap is small because there is little to aspire by increasing investment (Ray, 2006, Duflo, 2006). This reasoning builds on Ruttan (2003) and contradicts Schultz’s emphasis on the responsiveness of farmers implying that they immediately seek to identify and correct the optimization errors made. Banerjee and Newman (1994) have stressed that scarcity constraints with respect to investment resources - as is the case for the farmers in the Brazilian sample despite governmental programs - can explain the persistence of inefficient choices made by poor households. Linked to this and following Ball and Pounder (1996) as well as Stiglitz (1989) the revealed scale inefficiency over the sample could be finally due to prevailing market failure with respect to input and output markets. However, the limitations of the used cross-sectional data set should be kept in mind.3

3 We are grateful to an anonymous referee for valuable suggestions with respect to the results’ discussion.
7. Conclusions

The ‘small-but-efficient’ hypothesis with respect to the economic performance of small scale farmers in traditional development settings is still largely recognized by agricultural and development economists. However, the discussion on the efficiency of small farmers in developing countries lacks the explicit consideration of farm size as well as different forms of efficiency and based on this the notion of other policy options than simply correcting input prices and/or modernising production technology. Hence, by generating empirical evidence on small scale farmers in the Bragança region of the Brazilian Eastern Amazon the aim of this research was to show that from a production economics point of view a more differentiated picture emerges as one considers the different parts of allocative efficiency. By modelling a multi-product shadow profit function based on a flexible generalized Leontief functional form we capture joint production and possible price distortions in the output markets for cassava flour and maize as well as in the input markets for labour and fertilizer. Land is considered as a quasi-fixed factor in the short run production environment and different soil and household related control variables are included in the model. We account for the discussion on theoretical consistency and curvature correctness and estimate different models with respect to convexity as well as consistent efficiency imposed. The basic research hypothesis assumes a significant effect of the farm scale on the overall allocative efficiency of the farm.

The empirical findings revealed that small scale farmers in the Bragança region are relatively efficient with respect to their purely allocative decisions on joint production. In so far existing evidence on smallholders producing different crops in other regions was confirmed. However, the analysis of scale efficiency delivered evidence for high increasing returns to scale and consequently a relatively low scale efficiency for the farms in the sample. These results confirm our hypothesis that the scale of the agricultural operations plays a crucial role in determining the relative economic efficiency of the respective farm. Systematic scale inefficiency was found for the sample of Brazilian small scale farmers in the range of up to 20%, i.e. that these farmers are constrained by their scale of production with respect to an improvement in their relative economic performance. Allocative decisions with regard to the choice of the adequate input/output ratios are inefficient and have to be improved to increase the overall economic efficiency of the small scale farms. Following current interpretations of persisting inefficiency in small scale farming it remains unclear what can be suggested as an effective policy measure beyond the usual advice of addressing input and output market failures to enable growing farms to efficiently use production factor as
well as foster farm level cooperations. Hence, despite being based on a relatively limited set of cross sectional data the empirical evidence suggests the revision of the ‘poor-but-efficient’ hypothesis in the sense that small-scale farmers in a more traditional setting are allocatively efficient but at the same time scale inefficient.
References


Appendix

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<td>Recent Productivity Developments and Technical Change in Danish Organic Farming – Stagnation?</td>
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<td>Prices and Species Diversity – Stochastic Modelling of Environmental Efficiency</td>
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