Model Study of the Pressure Build-Up during Subcutaneous Injection
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Introduction

Diabetics are treated by several daily injections of insulin, most commonly delivered subcutaneously from where the insulin gets absorbed by the blood vessels. The injections are typically performed using an insulin pen or normal syringe. Due to the high frequency of injections for diabetics, increased patient convenience is of great importance and devices are therefore continuously improved to simplify treatment.

The force delivered by injection devices, either injection pumps or auto-injection devices, must overcome both the resistance in the injection system and the resistance in the body tissue to make room for the insulin bolus. The second part we refer to as the tissue counter pressure. We propose a model for the pressure evolution during a normal insulin injection. We then determine the flow permeability as well as the bulk modulus of the tissue, estimated to be of the order $10^{-11}$–$10^{-10}$ $m^2$ and $10^5$ Pa, respectively. The permeability is in good agreement with reported values for adipose porcine tissue. We suggest our model as a general way to estimate the pressure build-up in tissue during subcutaneous injection.

Materials and Methods

Ethical approval

The study involving human subjects was conducted in collaboration with Steno Diabetes Center A/S. While a nurse
administered insulin to the patient, the audible clicks, emitted from
the injection device, were recorded by microphone. Prior to the
investigation, The Committee on Health Research Ethics from the
Capital Region of Denmark reviewed the project. The conclusion
from the committee was that the project could be conducted
without further ethical approval because of its limited intervention.
The participating patients all had type 2 diabetes and received
regular control of their diabetes at Steno Diabetes Center. The
responsible nurse contacted the patients prior to their control visit
and questioned them about participation in the study. The head of
the Clinical Research Department, who was responsible for the
conduct of the study, approved the inclusion of patients without
written informed consent, after acceptance of this procedure by
The Committee on Health Research Ethics from the Capital Region of Denmark. No institutional review process was necessary
to take this decision. The only restriction to the selection of
subjects was that their insulin dose should be above 18 units to
have a sufficiently long sound file for the subsequent analysis. All
patients that were asked to participate gave their consent.

Experimental design
To measure the pressure build-up in human tissue, we use the
insulin pen FlexTouch from Novo Nordisk A/S, which is an auto
injecting device, where the drug is injected by the force from a
torsion spring. The spring is twisted when the patient dials the dose
and as the spring is untwisted during the injection the device emits
a click for each 10 μL of drug which has been delivered. By
recording the click signal with a microphone we predict the flow
rate. One example of a click signal and the calculated flow rate is
shown in Figure 1 for an injection of 240 μL insulin in air. It is
seen that the flow rate decreases throughout the injection as the
spring force is reduced for the last part of the click signal, seen as a
drop in the flow rate after about 1.2 s. Therefore the last part of
the click signal has been omitted in our analysis.

In order to confirm that the flow rate calculated from the click
signal match the actual flow rate a series of injections were
performed in air where the click signal was recorded by a
microphone and the injected mass was measured on a scale,
simultaneously. Figure 2 shows an example of the flow rate as
calculated from the click signal (red curve) and from the injected
mass (blue curve). In total the flow rate was measured for 70
injections, using 10 different pens, and the injected dose was varied
from 150–800 μL. On average the deviation between the flow
calculated from the click signal and the injected mass was about
2%. This uncertainty is related to the flow rate prediction and not
to the total dose delivered, which has been shown to be very
accurate for this device [17,18].

Data collection
This study includes 11 men with type 2 diabetes. All the
injections were given subcutaneously in the abdomen using a
6 mm 31G needle and performed by the same nurse. The patients
were treated with NovoRapid U100 from Novo Nordisk A/S and
the dose injected was set by the treatment scheme of the
individual. Therefore the dose varies from 180 μL up to 480 μL.
Both before and after the subcutaneous injection given to the
patient, we record the click signal from a series of similar injections
in air. The difference in the flow rate between the injections in air
and in tissue relates to the counter pressure. After the injections the
radius of each of the needles was calculated from Eq. (4) (see the
Results section), by measuring the flow rate through the needle for
a known pressure.

Results
Insulin injected subcutaneously distributes between the fat
lobulus and forms a depot, as seen from the histological cross
section of a 100 μL insulin injection shown in Figure 3 (left). The
insulin has been dyed to appear red and it is seen how the needle
has penetrated dermis (blue skin layer). For the model of the tissue
mechanics we consider the tissue as a porous elastic medium.
Figure 3 (right) shows the 3 dimensional structure of a similar
injection depot visualized by X-ray computed tomography, where
the insulin drug has been diluted with an iodine based contrast
agent in order to distinguish the fluid from the tissue, as described
in a previous study [19]. The extension of the depot is about 1 cm
and we have observed that the shape of the depot vary from
injection to injection. For simplicity we assume, that the tissue is
homogeneous and that the drug is distributed spherical symmet-
rically around the injection site.
Pressure Built Up during Injection

The injections were performed in adipose pig tissue and the injected volume was 100 μL. For histology the insulin has been dyed to appear red in the light microscopy image. The segmented tomographic reconstruction shows the 3 dimensional extension of the injection depot together with the injection channel and the backflow at the skin surface. The contrast between the tissue and the injected fluid is obtained by mixing the insulin drug with an iodine based contrast agent. The scale bar is 1 mm.

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In the description of the model we introduce a number of variables and constants, which we for clarity have listed in Table 1.

Mechanics of the injection device

The mechanical characteristics of the injection device is analysed through a series of test experiments where the drug is injected in air at atmospheric pressure. A simple schematic illustration of the injection device is shown in Figure 4.

The pressure drop in the syringe due to viscous forces (ΔP_s) is vanishingly small when compared with the pressure drop in the needle (ΔP_n). The pressure drop in the needle consists of a transient part, the entry length, over which the flow evolves to a steady state parabolic Poiseuille flow and a final part where the flow attains its parabolic shape (see Supporting Information S1). In the syringe the flow does not reach the full parabolic form during the short injections times considered here, however, an upper estimate of the pressure drop is achieved by assuming that the flow has reached a fully developed Poiseuille flow, which leads to ΔP_n ≈ 10^{-2} Pa. The pressure drop in the needle is estimated from the following expression derived in the Supporting Information S1, Eq. (S30),

\[ P_s - P_o = \frac{8\eta \ell}{\pi r_n^4} Q(t) + \frac{1.01 \rho_f}{\pi r_n^2} Q^2(t), \]  

(1)

where \( \eta \) is the dynamic viscosity of the insulin drug, \( Q(t) \) is the injection flow rate, \( \rho_f \) is the mass density of the drug and \( \ell \) and \( r_n \) denote the length and radius of the needle, respectively. We get for typical injection flow rates \( Q(t) \approx 100 \) μL/s a pressure drop in the needle \( \Delta P_n \approx 10^5 \) Pa. The pressure drop in the needle is therefore of the order of \( 10^7 \) times larger than that in the syringe. Therefore, we shall neglect the viscous forces inside the syringe in our calculations below.

From the Bernoulli Equation we obtain that the pressure at the inlet of the needle is approximately given by

\[ P_s \approx \frac{F(d)}{\pi R_0^2} - \frac{1}{2} \rho_f v_e^2, \]  

(2)

where \( F(d) \) is the force delivered by the spring, \( R_0 \) is the radius of the syringe and \( v_e \) denotes the average velocity over the cross section at the needle inlet, which can be determined as

\[ v_e = \frac{Q(t)}{\pi r_n^2}, \]  

(3)

Inserting Eq. (2) and (3) in Eq. (1), we obtain

\[ \frac{F(d)}{\pi R_0^2} - P_o = \frac{8\eta \ell}{\pi r_n^4} Q(t) \left( 1 + \frac{Q(t)}{5.29 \pi v_e \ell} \right). \]  

(4)

We now estimate the typical values of the second term inside the parentheses in Eq. (4). In Figure 5, we observe that the largest flow rates during the injection in air are approximately 150 μL/s.

Using the value of the kinematic viscosity of water \( v = 1.004 \times 10^{-6} m^2/s \) and that the length of the needle is \( \ell = 16.4 \) mm, we have that

\[ \frac{Q(t)}{5.29 \pi v \ell} \approx 0.55. \]  

(5)

As for the case of the fluid injection in the tissue, we also observe in Figure 5 that the largest flow rates are approximately 100 μL/s, which leads to

\[ \frac{Q(t)}{5.29 \pi v \ell} \approx 0.37. \]  

(6)

The second term in Eq. (4) therefore contributes significantly to the pressure drop between the syringe and the tissue/air and is included in our model.

Pressure in the tissue

In the derivation of an equation for the pressure evolution in tissue during injection we start from mass-conservation,

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = q(x,t), \]  

(7)

where \( \rho \) is the mass of the injected drug per unit volume \( V \), \( v \) is the pore flow velocity in a unit volume and the local source term \( q(x,t) \) is the mass of drug being injected per volume per time. A unit volume in the tissue is assumed to comprises of the tissue (\( V_i \)) and the injected drug (\( V_f \)), i.e.

\[ 1 = V_i/V + V_f/V. \]

The drug density in a unit volume of tissue can be written in terms of the mass density of the drug \( \rho_f \) as \( \rho = \rho_f V_f/V \), where \( V_f/V \) is the local volume fraction of the drug. We introduce a field for the local volume fraction of injected fluid \( \phi(x,t) = V_f/V \) in Eq. (7) and end up with the equation:

\[ \frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \mathbf{v}) = \frac{q(x,t)}{\rho_f}, \]  

(8)
where $\mathbf{v}$ is the pore averaged flow velocity of the injected fluid in a unit volume. Below we shall only consider the averaged velocity and therefore for convenience omit the tilde over $\mathbf{v}$.

The flow velocity is assumed to be given by the pressure gradient through Darcy's Law \[12\]
\[
\mathbf{v} = -\frac{K}{\eta} \nabla p,
\] (9)
in which $K$ is the permeability, $\eta$ the dynamic viscosity of the fluid and $p$ is the pressure.

Moreover we assume that displacement of the tissue by the injected fluid gives rise to a counter pressure, which will act to restore the tissue structure. This pressure is in the most simple approximation and for small deformations assumed to be linearly proportional to the difference in local tissue porosity before $(\phi_0)$ and after $(\phi)$ displacement by the injected fluid
\[
p(x, t) - p_0 = \beta (\phi(x, t) - \phi_0),
\] (10)
where $\beta$ is an effective bulk modulus describing the force needed to locally displace the tissue. Note that this form of the pressure-porosity relation is only valid for relatively small pressures, since for a very large pressure this relation might even lead to a porosity larger than unity, which would be highly non-physical. From Eq. (8)–(10), we can now derive the following equation for the pressure evolution in the tissue.

\[
\frac{\partial p}{\partial t} = \frac{K}{\eta} (\beta (\phi(x, t) - \phi_0) + p - p_0) \nabla^2 p + |\nabla p|^2 + \beta \frac{q}{p_f}.
\] (11)

A similar equation is frequently encountered in studies on flows in poro-elastic media [20,21] and in pattern formation studies of air injection in granular media [22], where we here have assumed that both the relative variation of volume and the advective contribution from the tissue motion are negligible.

We shall here take the tip of the needle as our center of coordinates. For simplicity, the tissue surrounding the needle tip is assumed to be homogeneous and isotropic and the injected fluid will therefore be distributed in the tissue in a spherical symmetric way. The equation for the pressure evolution is most conveniently solved in spherical coordinates where the angular components by symmetry can be disregarded. Since the source term is located at the tip of the needle, we remove this term from the equation by considering the pressure evolution outside a small sphere surrounding the tip. The radius of this sphere is taken to be equal to the radius of the needle $r_n$ and is therefore assumed to be infinitesimal relative to the scales on which the fluid are distributed. From Eq. (8) we then get from Gauss theorem, and from the fact that mass cannot accumulate in an infinitesimal volume, a boundary condition on the form,
where \( \nu_r \) is the radial component of the fluid velocity and the flow rate \( Q(t) \) is given by the equation:

\[
\dot{Q}(t) = 4\pi r^2 \phi \eta \nu_r (\phi r) = \frac{4}{3} \pi r^3 \dot{q}/\rho dV. \tag{12}
\]

If we now make use of Darcy’s law, we end with the following boundary condition for the pressure

\[
\left. \frac{\partial P}{\partial r} \right|_{r=r_g} = -\frac{\eta Q(t)}{4\pi r^2 \phi K}, \tag{13}
\]

where the porosity \( \phi \) is calculated from the pressure through Eq. (10).

**Experimental data on subcutaneous injection**

As examples of the flow data, Figure 5 shows the measurements from 3 of the 11 patients. From Eq. (4), we can directly estimate the function \( F(d) \) from the injections in air, since the atmospheric pressure, \( P_0 \), is known (note that variations in the atmospheric pressure have a negligible impact on our calculations and it is assumed that the friction force between the rubber piston and the syringe is independent on the flow rate). From our estimate of \( F(d) \), the drug injection rate and from Eq. (11) and (12), we can calculate the pressure evolution in the tissue once we know the model parameters \( K, \beta \) and \( \phi_0 \). The other parameters like the viscosity of water \( \eta \) and atmospheric pressure \( P_0 \) are known. The far field pressure in the tissue (away from the needle) is assumed to be equal to atmospheric pressure. In general, however, the pressure in subcutaneous tissue might be slightly below atmospheric pressure [13]. The pressure at the outlet of the needle tip is calculated from the function \( F(d) \) and the injection rate by using Eq. 4, where \( P_0 \) now is the pressure in the tissue and is a function of time. The average tissue counter pressure is shown in a box plot in Figure 6. Three patients has been omitted, as no change in the flow rate was observed indicating a low tissue resistance. We then compare the pressure \( P_0(t) \) in the tissue with that predicted by the model at the needle tip, \( p(r, t) \), in order to find the free model parameters through a best fit (least squares). Note that in our calculation of \( p(r, t) \), we use the flow rate as a boundary condition, given by Eq. (12). In Figure 7 we show \( P_0(t) \) together with the model estimate \( p(r, t) \) achieved by a best fit. The pressure evolution in radial distance from the needle tip and over time is presented in Figure 8. We see that the over-pressure is more or less localized in a sphere with a radius less than 5 mm, which is consistent with the distribution shown in Figure 3 (right).

In Figure 9A we show a box plot of the permeability computed from a best fit with the model to the patient data, again where three patients have been omitted. The typical permeability is from...
the model estimated to be of the order $10^{-11} - 10^{-10}$ m$^2$. From the interquartile range, we estimate the 95% confidence interval (the notch [23]) to be between $K = (8.9 \pm 4.7) \cdot 10^{-11}$ m$^2$.

Measurements of the permeability in porcine adipose tissue [24] gave values of the same order of magnitudes as reported in this article. The estimated values of the bulk modulus are shown in Figure 9B with a 95% confidence interval between the values $\beta = (0.89 \pm 0.13) \cdot 10^5$ Pa. Note that at the outlet of the needle, the...
pressure in the tissue does for a few patients reach levels which are not fully consistent with a constitutive equation on the form of Eq. (10), i.e. the tissue porosity reaches values close to unity or might slightly exceed it. One way to remedy this is to change the constitutive equation and include higher order elastic or plastic effects. That being said, the pressure in the tissue does immediately drop slightly away from the needle tip (see Figure 8) and Eq. (10) becomes a valid description. Finally the background porosity \( \phi_0 \) is in general very small and of the order \( \sim 0.01 \).

**Discussion**

Using a simple experimental setup we have been able to estimate the average tissue counter pressure during a subcutaneous injection with an insulin pen. The major advantages of this setup is that we do not modify the injection device and therefore are able to evaluate the pressure built up under normal injection conditions. We observe large variations in the counter pressure from a very low pressure, not detectable with this method, and up to pressures of about 800 mbar. All injections where performed with out complications, meaning that no skin reactions or pain during or after the injections were observed.

We have derived a model for the pressure evolution during injection in subcutaneous tissue, based on mass continuity as well as the basic laws of viscous flow in a porous-elastic medium. From application of the model to data on the insulin injections in diabetic patients, we have been able to determine flow permeability and bulk modulus of the tissue. Our model makes it possible to estimate how changes in the flow permeability and bulk modulus effect the pressure build-up in subcutaneous tissue during drug infusion, which is an important part of designing injection devices for insulin treatment. Control ensures that the device can deliver the full dose, that the back-flow through the injection channel is minimized and that injection pain or even tissue damage is reduced. Furthermore, our model, using the fitted parameters, is useful in general predictions of tissue pressure changes when mechanics of the injection device or the size of the needle is changed. Using the non-invasive method presented here the changes in counter pressure can easily be evaluated during clinical trials.

**Supporting Information**

**Supporting Information S1** Total pressure drop along the needle.  
(PDF)

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**Author Contributions**

Conceived and designed the experiments: MT MP DNS LT RF.  
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**References**