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Abstract: The total cost of ownership is calculated for a combined heating and humidification coil of an air-handling unit taking into account investment and operation costs simultaneously. This total cost represents the optimization function for which the minimum is sought. The parameters for the cost dependencies are the physical dimensions of the coil: length, width and height. The term “coil” is used generically since in this setup it generates heating as well as humidification in a single unit. The first part of the paper deals with the constructive optimization and finds the relationship between the dimensions for a minimum cost. The second part of the paper takes the results of the constructive optimization further and, based on the data derived in our previous papers, analyzes the minimum total cost for the humidification coil while balancing the amount of water used to humidify the air and modify its temperature.

Keywords: thermoeconomic optimization, exergoeconomic analysis, combined humidification and heating of air

1 Introduction

Thermoeconomics is the mixture of thermodynamic and economic considerations within engineering design. It was stated by some of the founders of the thermoeconomics school, El-Sayed [1, 2] and later on by Valero [3, 4], von Spakovský and Frangopoulos [5], that the criteria involved in the thermal systems design optimization may be

- economic – such as total capital investment, total annual leveled costs, averaged annual profits;
- technological – such as thermodynamic efficiency, power, production time or rate, fuel consumption rate; and
- environmental – such as rates of pollutants emitted.

Most early writings had the full intuitive understanding that there is a trade-off between thermodynamic efficiency and economic investment, i.e. that the operating expenses should be included when considering investment cost, but the exact balanced quantification of these two components necessary for a proper optimization was not yet fully developed (see, e.g. [6]). A somewhat alternative view is that thermodynamic efficiency may be used as an approximation to a full economic analysis if complete data is not available or is uncertain [7].

There are a number of ways in which thermodynamics can influence economic optimization, e.g.:

1. Thermodynamics puts constraints on the interchangeabilities (trade-offs) assumed in economic theory [8–10]. That may seriously shift the economic optimal point. However, it is basically a standard economic theory with thermodynamics simply changing the boundary conditions.

2. For any process one may achieve a given output (amount produced, power produced, etc.) either with a small piece of equipment which is pushed to its limit and thus runs rather inefficiently (large friction and gradient losses) or by a large unit running so slowly that it is close to equilibrium and thus very

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efficient. In the former instance, investment costs are small while operating costs are high, while in the latter the opposite is the case. The economical optimum point will be located somewhere in between, i.e. the sizing of the equipment which, all counted, will be the cheapest [11–14]. This will most likely not be the same setpoint as the one listed in the manufacturer’s material.

3. In the exergoeconomic view, destruction of input exergy is minimized using methods developed by, e.g. Tsatsaronis, Valero, and Frangopoulos [3, 15–19]. It is still being used (see, e.g. [20, 35, 36]). This is a world where not money but exergy is assumed to be the ultimate carrier of value. This type of analysis can be done with or without including the costs of producing and decommissioning the equipment and/or product itself, i.e. a total lifetime analysis spearheaded by Hirs and Cornelissen in Enschede [21, 22] and by Long and Berry in Chicago a little earlier [23–25].

A number of other studies optimize the physical design of the equipment based on performance and thermodynamic and physical constraints and only afterwards append an economic evaluation of the physically optimal designs (see, e.g. [26]) as they analyze usually already existing equipment in view of improving its performance and operating or investment costs. By contrast we have performed a combined thermoeconomic optimization, where the thermodynamic and economic performances are simultaneously balanced against one another.

In this paper, we choose the point view of item 2 earlier because of its unambiguous thermodynamic definition and realism. The aim of the paper is not to present explicit construction details of a concrete unit but rather to present two important concepts: (i) Optimizing investment and operating costs on an equal footing based on thermodynamic performance which of course is a consequence of the investment and (ii) combining heating and humidification into a single unit with direct contact between the heating water and the air to be heated and humidified. In Section 2 we set up the optimization problem. In Section 3 we provide some general observations about the optimal solution. In Section 4 we present the results of the specific example of combined heating and humidification, including dependencies on system size and hot water temperature. These findings are discussed in Section 5 and conclusions are drawn in Section 6.

2 Setting up the optimization problem

The way the design variables are defined is the foundation of a successful optimization and of obtaining effective solutions. The goal of an optimization problem can be either a maximum or a minimum of the objective function, depending on the criterion selected for the analysis (cost, thermodynamic efficiency, etc.).

The mathematical model for an optimization problem consists of:

- an objective function to be minimized/maximized;
- a set of equality constraints; and/or
- a set of inequality constraints.

Our thermoeconomic optimization method uses as objective function the total annual cost of a system, consisting of the amortized capital investment, the cost of resources and the operating and maintenance costs. The constraints (boundary conditions) are at least as essential as the objective function. Just a small innocently looking change of constraints can totally change the optimization result [27]. Here we specify the environmental conditions of temperatures, humidity, etc. as well as a fixed output (amount of conditioned air) as our constraints.

The system under study is the combined heating and humidification HVAC (Heating, Ventilation, and Air Conditioning) system presented in [28, 29], where outside air is passed over the surface of a counter-current flow of hot water (see Figure 1). This achieves simultaneous heating and humidification of the air for inside office use. The geometry of the coil and the flow rates are adjusted to obtain the desired inside air quality. The goal of the optimization is to find the minimum total annual cost of the system consisting of the amortized cost of the initial investment and the operating cost (including maintenance).
We are aware that currently no equipment is on the market, which achieves both heating and humidification at the same time within the same device. The standard is to have two separate units, one for each process. One of our aims is to advance the concept of a combined unit and analyze its most economic dimensions. Thus, we do not get into details about possible fins, materials, etc. in our model. That is left for future developments. Similarly, we disregard a number of fine details in the model, which are typically much smaller than the main components and thus will not affect our conclusions (e.g. end areas of the coil, cost of pumping the water, connecting piping, etc.).

In our present model, the variables with respect to which the optimization is conducted are the dimensions of the coil: width, height and length. We consider a rectangular shape of the coil for simplicity of demonstration. For practical applications, the manufacturers can of course increase the thermal exchange area by various constructive compact shapes. Further, we vary the air speed and the water speed as well as the heating water temperature. Thus, the objective function may be written as

$$I_{tot} = I_{invest} + I_{operation}, \quad (1)$$

where $I_{invest}$ is the annual amortized investment cost of the system and $I_{operation}$ is the annual operating cost. The investment is assumed to be proportional to the surface area of the coil, i.e. circumference times length, and it is amortized over the expected lifetime $n$ years of the equipment with the interest rate $r$ using the basic formula:

$$Z_n = \frac{r}{1 - (1 + r)^{-n}}, \quad (2)$$

thus leading to the expression

$$I_{invest} = 2C_{coil}L(y + z)Z_n. \quad (3)$$

Here $C_{coil}$ is the surface unit cost of the coil, $L$ is its length, $y$ its width and $z$ its height. Since the length of such a coil is always much longer than its height and width, we have disregarded the end surface areas.

The operating cost is considered distributed evenly over the year for simplicity. Limited periods of operation could be introduced by a simple proportionality constant, which would not change our conclusions. The operating cost is composed of the cost for heating the water and running the air through the coil. The cost of pumping the hot water is considered small in comparison to heating the water and to circulating the air and is therefore included in the cost for heating the humidification water. Thus

$$I_{operation} = I_{water} + I_{air}. \quad (4)$$

The operating cost for water is taken to be

$$I_{water} = m_w C_{hw}, \quad (5)$$

with $m_w$ being the water mass velocity (kg/m$^2$ s) and $C_{hw}$ the unit cost of heating (and pumping) the water from an assumed supply temperature of 10 °C to the specified entry temperature $T_{hw}$. In principle, the heating requirement is proportional to the product of water depth and (linear) flow velocity, but as the results show, this product is independent of coil dimensions $y, z, L$. Thus, we have absorbed this geometric product in $C_{hw}$ for simplicity.

The operating cost for circulating the air is a bit more complicated. In its simple form it is

$$I_{air} = \Delta p \, V C_e, \quad (6)$$
where $\Delta p$ is the pressure drop along the coil, $V$ is the volumetric flow of air (m$^3$/s) and $C_e$ the unit cost of electricity used to operate the fan. The complicated factor here is $\Delta p$ since the air flow is turbulent. We choose to use the Darcy–Weisbach phenomenological equation [30] to express the pressure loss in the air flow,

$$\Delta p = f \frac{L \rho v^2}{d} , \quad (7)$$

where $f$ is the dimensionless friction factor of the fluid, $\rho$ is its density, $v$ is the average linear fluid velocity and $d$ is the pipe diameter. The friction factor $f$ is not a constant but depends on the dimensions of the pipe and the velocity of the fluid flow, known to high accuracy within certain flow regimes through empirical expressions. For this turbulent regime, we use the Blasius approximation [31]

$$f = 0.316 \text{Re}^{-1/4} , \quad (8)$$

where

$$\text{Re} = \frac{\rho v d}{\mu} \quad (9)$$

is the Reynolds number and $\mu$ is the dynamic viscosity of the air. The equivalent pipe diameter $d$ is given by Chesy for conduits without specified form as [32]

$$d = 4 \frac{X}{P} , \quad (10)$$

in which $X$ is the cross-sectional area of the pipe and $P$ is its perimeter. For our rectangular coil that becomes

$$d = \frac{2yz}{y+z} . \quad (11)$$

Using the same $z$ in this equation as in eq. (3) we have implicitly disregarded the height of the water stream since it is minimal compared to that of the air flow. All combined we arrive at the objective function for the problem,

$$I_{\text{tot}} = 2C_{\text{coil}} L (y+z) \frac{r}{1 - (1+r)^{-\frac{3}{2}}} + m_w C_{\text{hw}} + C_e 0.316 \frac{L \rho v^{1/4} (y+z)^{3/4}}{(yz)^3} . \quad (12)$$

The optimal control equations for this objective function consist of the evolution equations along the coil for $T_{\text{air}}(x)$, $T_{\text{water}}(x)$, air humidity $m_v(x)$, and water mass velocity $m_w(x)$.

### 3 Optimal solution

For our optimal solution we fix four boundary conditions: The outside and inside air temperatures, the outside relative humidity and the temperature of the heating water, while we vary the coil geometry (length, width and height) in order to achieve the cheapest possible annual cost of operation. Along the process we also investigate the influence of the air and water flows. This means that the water mass velocity $m_w$ is a result of the optimization.

Before embarking on a numerical example, let us look at some general features of the solution. The optimum will occur where

$$\frac{dI_{\text{tot}}}{dy} = 0, \frac{dI_{\text{tot}}}{dz} = 0, \frac{dI_{\text{tot}}}{dL} = 0. \quad (13)$$

The two former requirements result in the relations
respectively. The annual cost of the hot water does not exist as an explicit expression but is determined by the necessary water flow \( m_w \) through the solution of the optimal control equations for the system \([28, 29]\). In these solutions the \( z \) (height) dependence is so weak that we have neglected it in eq. (15). That cannot be done with the \( y \) (width) dependence in eq. (14), although the dependence is only moderate. If one proceeded to assume that also \( dI_{\text{water}}/dy = 0 \) in eq. (14), one would arrive at

\[
z = y \tag{16}
\]

when combining eqs. (14) and (15), i.e. a square cross section of the coil. Thus, a good first-order approximation would be

\[
z = y(1 + \epsilon(y, L)), \tag{17}
\]

where \( \epsilon \) is small compared to 1.

With respect to the length optimum we observe from eq. (12) that the first and last terms (investment and air flow) are proportional to \( L \). That leaves just the middle term (hot water supply) to locate the minimum. Again there is no explicit expression for \( I_{\text{water}} \), so its \( L \) dependence must also come from the solution of the optimal control equations. However, this is much stronger than the \( y \) dependence and cannot be linearized. Consequently, the full solution can only be obtained numerically.

4 Numerical example

We illustrate this thermoeconomic optimization with the example presented in \([28, 29]\), a combined heating and humidification coil where direct counter-current contact between the air and water delivers both effects (see Figure 1). In the present paper, we extend that thermodynamic optimization with the economic components derived above. We have used reference values for all the material constants (see Table 1) and fix the environmental conditions at \( T_{\text{outside}} = 0 \, ^\circ\text{C}, T_{\text{inside}} = 20 \, ^\circ\text{C} \), and relative outside humidity \( \phi = 70\% \). Assigning specific values to the width and height of the coil, the temperature of the heating water, the air flow and the water speed, the time evolution equations for the two flows are calculated for a wide range of coil lengths following \([28, 29]\), and the minimum total annual cost is determined. Varying each of these parameters one at a time we find their respective influences on the objective, the total annual cost.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>Interest rate</td>
<td>2</td>
<td>% p.a.</td>
</tr>
<tr>
<td>( n )</td>
<td>Duration of annuity</td>
<td>15</td>
<td>year</td>
</tr>
<tr>
<td>( C_{\text{coil}} )</td>
<td>Unit cost of fabrication of coil</td>
<td>50</td>
<td>euro/m²</td>
</tr>
<tr>
<td>( C_{\text{hw}} )</td>
<td>Unit cost of hot water</td>
<td>0.03</td>
<td>euro/kWh</td>
</tr>
<tr>
<td>( C_e )</td>
<td>Unit cost of electricity</td>
<td>0.20</td>
<td>euro/kWh</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Density of water</td>
<td>1,000</td>
<td>kg/m³</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Kinematic viscosity of air</td>
<td>( 1.983 \times 10^{-5} )</td>
<td>Pa s</td>
</tr>
</tbody>
</table>
For each set of parameters, curves like those in Figure 2 are obtained. We observe that the investment cost increases linearly with coil length while the hot water cost goes down, simply because the long coil lengths result in a lower effluent water temperature. The cost of driving the fan, on the other hand, is generally much lower than the two above. These competing trends result in a distinct minimum for the total annual cost.

Figure 3 shows the dependence of these optima on the temperature of the hot water supply which drives the whole process. It is clear that an increased temperature is advantageous, even though the heating then becomes more expensive, because it is possible to use a shorter coil and thus save on the investment. The idea is to use wastewater from some other process in the plant. The water temperature in the calculations is therefore kept low, and in particular no steam is applied.

In Figure 4, we consider the influence of pipe cross section. The solid $I_{tot}$ and $L$ curves are obtained for equal width and height while fixing the height at 1 m and only allowing the width to vary gives almost indistinguishable results (dashed). In other words, what primarily matters is the width of the water surface. Clearly the lower height of the duct lumen will increase the fan cost, but that is so small compared to the investment and water heating costs that it is hardly visible.

The final Figures 5 and 6 investigate the effects of water speed and air flow. Water speed is clearly immaterial (within bounds). A larger speed simply results in a lower optimal water depth such that the
Figure 4: Optimal total annual operating cost (red) and optimal coil length (black) versus coil width and height. The solid curves are for equal width and height, the dashed curves for a fixed height of $z = 1$ m varying only the width $y$. $T_{\text{outside}} = 0 \degree \text{C}, T_{\text{inside}} = 20 \degree \text{C}, T_{\text{water}} = 70 \degree \text{C}$.

Figure 5: Optimal total annual operating cost (red) and optimal coil length (black) versus linear water speed. $T_{\text{outside}} = 0 \degree \text{C}, T_{\text{inside}} = 20 \degree \text{C}, T_{\text{water}} = 70 \degree \text{C}, y = z = 2$ m.

Figure 6: Optimal total annual operating cost (red) and optimal coil length (black) versus air flow. $T_{\text{outside}} = 0 \degree \text{C}, T_{\text{inside}} = 20 \degree \text{C}, T_{\text{water}} = 70 \degree \text{C}, y = z = 2$ m.
volumetric flow remains the same. The surface area, temperatures, etc. and thus performance are unaffected by this. By contrast, the airflow, the deliverable of the system, has a large influence. Sending more air through a given cross section of duct drives the exchange harder and thus requires a longer coil to deliver the same interior air temperature. The total annual cost goes up even faster than the length because also more hot water is needed to drive the system.

5 Discussion

Our thermoeconomic analysis returned to the combined processes of heating and humidification, as initially defined in [28, 29]. We investigated the humidification coil of an air-handling unit as a generic reference since it would be the equipment available on the market most likely to benefit from such an analysis. However, the optimization results can be used for any other device whose purpose is to humidify and heat at the same time.

In general, this combination is not used sufficiently in practice for the purpose of heating and humidifying buildings. The role played by humidification for human health as well as for the conservation of different materials and products (in libraries, museums, communication centers, hospitals, cereal processing, electrical and electronics products, furniture, etc.) is well known, and humidification is widely applied and controlled. Investigation of humidification in combination with heating has been done in the medical field, for instance, for the mechanical ventilation systems of intensive care units and anesthesia purposes [33, 34]. Here, conditioning of the air for humans resulted from a combination of heating and humidification and this method should be feasible for other systems as well, like entire buildings. But when designing heating systems for buildings, engineers often do not consider the humidification in the grand picture. In most applications, humidification requirements would be handled apart from the heating/cooling system as an add-on. There is a wealth of choices on the market for humidification devices. We are making the case here that standard practice is always humidification separate from heating. It is sufficient to have a look at the vast literature published by the manufacturers of humidification equipment. They are always listed separate from the heating systems and mainly operated with steam, which does not change the temperature of the air other than negligibly. The hot water, on the other hand, does change the temperature of the air sensibly. Therefore, both heating and humidification can be achieved this way.

Steam is rather expensive to obtain, especially for small- and medium-sized humidifiers, as it will normally be produced by means of electricity or direct firing as opposed to using hot water which often is a by-product from another process. Manufacturers often claim that steam is the most cost-efficient method but without showing clear detailed cost comparisons between the various solutions and of course without considering a comparison between separate humidification as opposed to combined heating and humidification.

One could argue that the maintenance required by hot water systems is demanding. The most important needs are a continuous effort to prevent scaling and, most importantly, to fight bacteria growth in the water (either by thermal or chemical means). However, depending on the application, the degree to which the relative humidity has to be controlled, the desired indoor temperature and the availability of the hot water as a by-product from an industrial process, the combined heating and humidification may still be an attractive choice.

The second main point of the paper is that simultaneous optimization of investment and operating costs is essential for a proper optimum. In this connection, Figures 2 and 4 are most illustrative. They show that any increase in equipment size (coil length, width and height) of course increases the thermal efficiency as well as the annual investment cost, but in the region of small equipment this saving on investment is more than offset by increased operating costs. This is a quantitative statement of the cost of pushing the equipment. Only a simultaneous optimization of all cost components can achieve this best trade-off.
6 Summary and conclusion

For a generic heating and humidification coil we first examined its geometry, keeping in mind that the water surface in the coil is not necessarily a continuous straight segment of water film as in an evaporative pan, but could be a water film on fills in a more compact geometry amounting to the same area. Of course, in this case the airflow pressure loss will be modified accordingly.

The result of this optimization formulation was a consequence of the pressure loss in the airflow through the coil and it showed that a nearly square cross section yields the cheapest geometry, considering the costs for both investment and operation.

Then in numerical examples we carried out a sensitivity study to see how the total cost for investment and operation of the humidification and heating coil attains a minimum while varying the hot water temperature and the length of the coil. The results showed that the hotter the water the shorter the coil length for which the minimum total cost occurs, which is obviously related to the cost of investment.

In this instance, we selected four boundary conditions: The outdoor and indoor air temperatures, the outdoor relative humidity and the temperature of the heating water, while we varied the coil geometry. Clearly, the hot water temperature could be modified such as to adjust the resulting indoor relative humidity instead.

We believe that the combined process of heating and humidification is not sufficiently exploited in practice and envisage new further optimizations that could shed more light on the advantages of this combined process – either by looking into an existing system to apply the theoretical principles we have presented here or by imagining new ones. The applications can vary substantially, with a registry from residential to industrial, depending on the indoor temperatures and relative humidity to be achieved.

Nomenclature

*Latin symbols*

- \( C_{\text{coil}} \): unit cost of fabrication of coil
- \( C_{\text{hw}} \): unit cost of hot water
- \( C_e \): unit cost of electricity
- \( d \): pipe diameter
- \( n \): duration of annuity
- \( r \): interest rate
- \( I_{\text{tot}} \): total cost of investment and operation – objective function
- \( I_{\text{Invest}} \): the annual amortized investment cost of the system
- \( I_{\text{operation}} \): the annual operating cost
- \( I_{\text{air}} \): the annual cost of pumping the air
- \( I_{\text{water}} \): the annual cost of heating (and pumping) the water
- \( f \): dimensionless friction factor of air
- \( X \): cross-sectional area of the coil
- \( P \): perimeter of the coil
- \( L \): length of the coil
- \( m_v \): mass of water vapor per volume of air
- \( m_w \): water mass velocity
- \( \Delta p \): pressure differential driving the air flow
- \( \text{Re} \): Reynolds number
- \( T \): temperature
- \( v \): average linear fluid velocity
- \( V \): volumetric flow of air
- \( y \): width of coil
- \( z \): height of air inside coil
- \( Z \): amortization factor of investment over lifetime of device
Subscripts

- outside: related to outdoor environmental conditions
- inside: related to indoor environmental conditions
- air: related to air
- \( v \): related to vapor
- \( w \): related to water

Greek symbols

- \( \varphi \): relative humidity of air
- \( \mu \): kinematic viscosity of air
- \( \rho \): density of fluid

References