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Griffin, Tom; Grosvenor, Kevin John Torres; Horava, Petr; Yan, Ziqi

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Cascading Multicriticality in Nonrelativistic Spontaneous Symmetry Breaking

Tom Griffin, Kevin T. Grosvenor, Petr Hořava and Ziqi Yan

a Blackett Laboratory, Department of Physics, Imperial College London, SW7 2AZ, United Kingdom
b Berkeley Center for Theoretical Physics and Department of Physics University of California, Berkeley, California 94720-7300
c Physics Division, Lawrence Berkeley National Laboratory Berkeley, California 94720-8162

Without Lorentz invariance, spontaneous global symmetry breaking can lead to multicritical Nambu-Goldstone modes with a higher-order low-energy dispersion $\omega \sim k^n$ ($n = 2, 3, \ldots$), whose naturalness is protected by polynomial shift symmetries. Here we investigate the role of infrared divergences and the nonrelativistic generalization of the Coleman-Hohenberg-Mermin-Wagner (CHMW) theorem. We find novel cascading phenomena with large hierarchies between the scales at which the value of $n$ changes, leading to an evasion of the “no-go” consequences of the relativistic CHMW theorem.

I. INTRODUCTION

Some of the most pressing questions about the fundamental laws of the Universe (such as the cosmological constant problem, or the hierarchy between the Higgs mass and the Planck scale) can be viewed as puzzles of technical naturalness [1]. In this Letter, we study the interplay of technical naturalness with spontaneous symmetry breaking (SSB) in nonrelativistic systems.

SSB is ubiquitous in Nature. For relativistic systems and global continuous internal symmetries, the universal features of SSB are controlled by the Goldstone theorem. Much progress in SSB has also been achieved in the nonrelativistic cases, where the reduced spacetime symmetries allow a much richer behavior, still very much the subject of active research (see e.g. [2–8] and references therein). Important novelties emerge already in the simplest case of theories in the flat nonrelativistic spacetime $\mathbb{R}^{D+1}$ (covered with Cartesian coordinates $(t, \mathbf{x})$, $\mathbf{x} = (x^i, i = 1, \ldots, D)$) and with the Lifshitz symmetries of spatial rotations and spacetime translations. In such theories, the Nambu-Goldstone (NG) modes can be of two distinct types: Type A, effectively described by a single real scalar $\phi(t, \mathbf{x})$ with a kinetic term quadratic in the time derivatives; or Type B, described by two scalar fields $\phi_1, 2(t, \mathbf{x})$ which have a first-order kinetic term and thus form a canonical pair.

In [7–8], we showed that this Type A-B dichotomy is further refined into two discrete families, labeled by a positive integer $n$: Type $A_n$ NG modes are described by a single scalar with dispersion $\omega \sim k^n$ (and dynamical critical exponent $z = n$), while Type $B_{2n}$ modes are described by a canonical pair and exhibit the dispersion relation $\omega \sim k^{2n}$ (and dynamical exponent $z = 2n$). These two families are technically natural, and therefore stable under renormalization in the presence of interactions [7]. As usual, such naturalness is explained by a new symmetry. For $n = 1$, the NG modes are protected by the well-known constant shift symmetry $\delta \phi(t, \mathbf{x}) = b$. The $n > 1$ theories enjoy shift symmetries by a degree-$P$ polynomial in the spatial coordinates [7],

$$ \delta \phi(t, \mathbf{x}) = b + b_1 x^i + \cdots + b_i \cdots x^i \cdots x^i, $$

with suitable $P$. Away from the Type $A_n$ and $B_{2n}$ Gaussian fixed points, the polynomial shift symmetry is generally broken by most interactions. The lowest, least irrelevant interaction terms invariant under the polynomial shift were systematically discussed in [8] (see also [9]). Such terms are often highly irrelevant compared to all the other possible interactions that break the symmetry.

Having established the existence of the multicritical Type A and B families of NG fixed points, in this Letter we study the dynamics of flows between such fixed points in interacting theories. We uncover a host of novel phenomena, involving large, technically natural hierarchies of scales, protected again by the polynomial shift symmetries. As a given theory flows between the short-distance and the long-distance regime, it can experience a natural cascade of hierarchies, sampling various values of the dynamical critical exponent $z$ in the process. Such cascades represent an intriguing mechanism for evading some of the consequences of the relativistic Coleman-Hohenberg-Mermin-Wagner (CHMW) Theorem.

II. THE RELATIVISTIC CHMW THEOREM

Recall that in relativistic systems, all NG bosons are of Type $A_1$, assuming that they exist as well-defined quantum objects. Whether or not they exist, and whether or not the corresponding symmetry can be spontaneously broken, depends on the spacetime dimension. This phenomenon is controlled by a celebrated theorem, discovered independently in condensed matter by Mermin and Wagner [10] and by Hohenberg [11], and in high-energy physics by Coleman [12]. We therefore refer to this theorem, in the alphabetical order, as the CHMW theorem.

The relativistic CHMW theorem states that the spontaneous breaking of global continuous internal symmetries is not possible in $1 + 1$ spacetime dimensions. The
The proof is beautifully simple. $1 + 1$ is the “lower critical dimension,” where $\phi$ is formally dimensionless at the Gaussian fixed point. Quantum mechanically, this means that its propagator is logarithmically divergent, and we must regulate it by an infrared (IR) regulator $\mu_{\text{IR}}$:

$$
\langle \phi(x)\phi(0) \rangle = \int \frac{d^2 k}{(2\pi)^2} \frac{e^{ik\cdot x}}{k^2 + \mu_{\text{IR}}^2} = \frac{1}{2\pi} \log(\mu_{\text{IR}}|x|) + \text{const.} + \mathcal{O}(\mu_{\text{IR}}|x|).
$$

The asymptotic expansion in $[2]$, valid for $\mu_{\text{IR}}|x| \ll 1$, shows that as we take $\mu_{\text{IR}} \to 0$, the propagator stays sensitive at long length scales to the IR regulator. We can still construct various composite operators from the derivatives and exponentials of $\phi$, with consistent and finite renormalized correlation functions in the $\mu_{\text{IR}} \to 0$ limit, but the field $\phi$ itself does not exist as a quantum object. Since the candidate NG mode $\phi$ does not exist, the corresponding symmetry could never have been broken in the first place, which concludes the proof.

### III. NONRELATIVISTIC CHMW THEOREM

For nonrelativistic systems with Type $A_n$ NG modes, we find an intriguing nonrelativistic analog of the CHMW theorem. The scaling dimension of $\phi(t,x)$ at the $A_n$ Gaussian fixed point, measured in the units of spatial momentum, is

$$
[\phi(t,x)]_{A_n} = (D-n)/2.
$$

The Type $A_n$ field $\phi$ is at its lower critical dimension when $D = n$. Its propagator then requires an IR regulator. There are many ways how to introduce $\mu_{\text{IR}}$; for example, by modifying the dispersion relation of $\phi$, as in

$$
\langle \phi(t,x)\phi(0) \rangle = \int \frac{d\omega d^D k}{(2\pi)^{D+1}} \frac{e^{ik\cdot x - i\omega t}}{\omega^2 + |k|^{2D} + \mu_{\text{IR}}^{2D}},
$$

or

$$
\langle \phi(t,x)\phi(0) \rangle = \int \frac{d\omega d^D k}{(2\pi)^{D+1}} \frac{e^{ik\cdot x - i\omega t}}{\omega^2 + (|k|^2 + \mu_{\text{IR}}^2)^D}.
$$

Regardless of how $\mu_{\text{IR}}$ is implemented, as we take $\mu_{\text{IR}} \to 0$, the propagator again behaves logarithmically, both in space

$$
\langle \phi(t,x)\phi(0) \rangle \approx -\frac{1}{(4\pi)^{D/2} \Gamma(D/2)} \log(\mu_{\text{IR}}|x|) + \ldots
$$

for $|x|^D \gg t$, and in time,

$$
\langle \phi(t,x)\phi(0) \rangle \approx -\frac{1}{(4\pi)^{D/2} \Gamma(D/2)} \log(\mu_{\text{IR}}^D t) + \ldots
$$

for $|x|^D \ll t$. The propagator remains sensitive to the IR regulator $\mu_{\text{IR}}$. Consequently, we obtain the nonrelativistic multicritical CHMW theorem for Type $A$ modes:

The propagator of the Type $A_n$ would-be NG mode $\phi(t,x)$ at its lower critical dimension $D = n$ is logarithmically sensitive to $\mu_{\text{IR}}$, and therefore $\phi(t,x)$ does not exist as a quantum mechanical object. Consequently, no spontaneous symmetry breaking with Type $A_n$ NG modes is possible in $D = n$ spatial dimensions.

By extension, this invalidates all Type $A_n$ would-be NG modes with $n > D$, whose propagator would also be pathological at long distances. In contrast, the scaling dimension of the Type $B_{2n}$ fields is $[27]

$$
[\phi_{1,2}(t,x)]_{B_{2n}} = D/2,
$$

and the lower critical dimension is $D = 0$. Hence, in all spatial dimensions $D > 0$, the Type $B_{2n}$ NG modes are free of IR divergences and well-defined quantum mechanically for all $n = 1, 2, \ldots$. The nonrelativistic multicritical CHMW theorem for Type $B$ modes then simply states that the Type $B_{2n}$ symmetry breaking is possible in any $D > 0$ and for any $n = 1, 2, \ldots$.

### IV. CASCADING MULTICRITICALITY

Whereas in the relativistic case, all NG modes must always be of Type $A_1$, in nonrelativistic systems the existence of the Type $A_n$ and $B_{2n}$ families allows a much richer dynamical behavior.

For example, with the changing momentum or energy scales, a given NG mode can change from Type $A_n$ (or $B_{2n}$) to Type $A_{n'}$ (or $B_{2n'}$) with $n \neq n'$, or it could change from Type $A$ to Type $B$. The hierarchies of scales that open up in this process are naturally protected by the corresponding polynomial symmetries. One of the simplest cases is the Type $A_n$ scalar with $n > 1$, whose polynomial shift symmetry of degree $P$ is broken at some momenta scale $\mu$ to the polynomial shift symmetry of degree $P-2$, by some small amount $\varepsilon \ll 1$. This breaking modifies the dispersion relation to $\omega^2 \approx k^{2n} + \varepsilon \mu^2$, with $\varepsilon \approx \mu^2$. Here, as in [1], we identify $\mu$ as the scale of naturalness. At a hierarchically much smaller scale, $\mu_x \equiv \mu \sqrt{\varepsilon}$, the system exhibits a crossover, from Type $A_n$ above $\mu_x$ to Type $A_{n-1}$ below $\mu_x$. The technical naturalness of the large hierarchy $\mu_x \ll \mu$ is protected by the restoration of the polynomial shift symmetry of degree $P$ as $\varepsilon \to 0$.

In the special case of $n = D$, this crossover from Type $A_D$ to Type $A_{D-1}$ yields an intriguing mechanism for evading the naive conclusion of our CHMW theorem. For a large range of scales close to $\mu$, the would-be NG mode can exhibit a logarithmic propagator. The hierarchically smaller scale $\mu_x \ll \mu$ then serves as a natural IR regulator, allowing the NG mode to cross over to Type $A_{D-1}$ at very long distances. Therefore, the mode is well-defined as a quantum mechanical object, despite the large hierarchy across which it behaves effectively logarithmically.

An interesting refinement of this scenario comes from breaking the polynomial symmetries hierarchically, in
a sequence of partial breakings, from a higher polynomial symmetry of degree \( P \) to symmetries with degrees \( P' < P, P'' < P', \ldots \), all the way to constant shift. This gives rise to a cascading phenomenon, with a hierarchy of crossover scales \( \mu \gg \mu' \gg \mu'' \gg \ldots \), separating plateaux governed by the fixed points with the dynamical exponent taking the corresponding different integer values. Again, such cascading hierarchies are technically natural, and protected by the underlying breaking pattern of the polynomial symmetries.

Before we illustrate this behavior in a series of examples, it is worth pointing out one very simple yet important feature of large hierarchies in nonrelativistic theories. Consider a theory dominated over some range of scales by the dispersion relation \( \omega \approx k^n \), with \( n > 1 \). If we open up a large hierarchy of momentum scales \( \mu \gg \mu' \) (say by \( N \) orders of magnitude), this hierarchy of momentum scales gets magnified into an even larger hierarchy (by \( nN \) orders of magnitude) in energy scales.

V. EXAMPLE 1: A TYPE-A HIERARCHY

The first model that we use to illustrate the hierarchy is a relatively well-known system in \( 2 + 1 \) dimensions: the \( z = 2 \) Gaussian model of a single Lifshitz scalar field \( \phi(t, x) \), with a derivative 4-point self-interaction turned on [13, 14]:

\[
S_2 = \frac{1}{2} \int dt d^2x \left\{ \dot{\phi}^2 - (\partial^2 \phi)^2 - c^2 \partial_i \phi \partial_j \phi - g (\partial_i \phi \partial_j \phi)^2 \right\}.
\]

This action contains all the marginal and relevant terms of the \( z = 2 \) fixed point consistent with the constant shift symmetry and the reflection symmetry \( \phi \rightarrow -\phi \). At the \( z = 2 \) Gaussian fixed point, \( g \) is classically marginal, and breaks the polynomial shift symmetry of this fixed point to constant shift. Quantum corrections at one loop turn \( g \) marginally irrelevant [13].

This system allows a natural hierarchy of scales, stable under quantum corrections. At the naturalness scale \( \mu \), we can break the polynomial shift symmetry of the \( z = 2 \) fixed point to constant shifts by a small amount \( \varepsilon_0 < 1 \). This implies \( g \sim \varepsilon_0 \) and \( c^2 \sim \varepsilon_0 \mu^2 \), relations which can be shown to be respected by the loop corrections. In particular, \( c^2 \) can stay naturally small, much less than \( \mu^2 \). The dispersion relation changes from \( z = 2 \) close to the high scale \( \mu \), to \( z = 1 \) around the much lower scale \( \mu_1 \equiv \mu \sqrt{\varepsilon_0} \ll \mu \).

VI. EXAMPLE 2: A TYPE-A CASCADE

Our next example is a new model, which not only illustrates the cascading hierarchy with multiple crossovers, but also exhibits additional intriguing renormalization properties of independent interest.

We start with the Gaussian \( z = 3 \) fixed point of a single scalar \( \Phi(t, x) \) in \( 3 + 1 \) dimensions, and turn on derivative self-interactions and relevant terms as in our previous example. The free theory is

\[
S_3 = \frac{1}{2} \int dt d^2x \left\{ \dot{\phi}^2 - \zeta_3^2 (\partial_i \partial_j \phi \partial_i \partial_j \phi) (\partial_i \partial_j \partial_i \partial_j \phi) \right\}.
\]

At the classical level we can set \( \zeta_3^2 = 1 \) by the rescaling of space. The terms on the second line represent relevant Gaussian deformations away from the \( z = 3 \) fixed point. The spectrum of available self-interaction operators that are classically marginal or relevant at the \( z = 3 \) Gaussian fixed point is much richer than in our \( 2 + 1 \) dimensional example. We shall again restrict our attention only to the operators even under \( \Phi \rightarrow -\Phi \), and invariant at least under the constant shift. Up to total derivatives, which we ignore, there are three independent marginal 4-point operators \( O_{(a)}^{(4)}, a = 1, 2, 3 \), one marginal 6-point operator \( O_6 = (\partial_i \Phi \partial_i \Phi)^3 \), and one relevant 4-point operator

\[
W = (\partial_i \Phi \partial_i \Phi)^2.
\]

Among them, there is one unique operator \( O \) invariant under the linear shift symmetry up to a total derivative:

\[
O = 4 \partial_i \partial_j \partial_k \Phi \partial_i \partial_j \partial_k \Phi + 12 \partial_i \Phi \partial_i \partial_j \partial_j \partial_k \Phi \partial_k \Phi
\]

\[
= 4 \begin{array}{c}
\sum_{i=1,2,3} k_i^3
\end{array}
+ 12 \begin{array}{c}
\sum_{i=1,2}\sum_{j=3} k_i^3
\end{array}.
\]

The Feynman rules of this model involve one 4-vertex, which can be simplified using the momentum conservation \( k_4 = - \sum_{I=1,2,3} k_I \) to

\[
\sum_{a=k_1,k_2,k_3} \sum_{b=k_1,k_2,k_3} = -i\lambda \left[ k_1^2 k_2^2 k_3^2 + 2 (k_1 \cdot k_2) (k_2 \cdot k_3) (k_3 \cdot k_1) \right.

\]

\[
- k_1^2 (k_2 \cdot k_3)^2 - k_2^2 (k_3 \cdot k_1)^2 - k_3^2 (k_1 \cdot k_2)^2 \right].
\]

Note that in this vertex, each momentum appears quadratically, with no subleading terms. We can write
it even more compactly with the use of the fully anti-
symmetric $\varepsilon_{ijk}$ tensor; if for any three momenta $k, p, q$
we define $[kpq] \equiv \varepsilon_{ijk}k_ip_jq_k$, our vertex becomes simply
\[ -i\lambda[k_1k_2k_3]^2. \] (13)

This simple vertex structure is intimately related to the
underlying symmetries: When translated into momen-
tum space, the linear shift symmetry $\delta \phi(t, x) = \delta \phi_0(t, k)$ by
acts with this symmetry on any of
the legs of the vertex produces zero, as the vertex is purely quadratic in each of the outside momenta.

Quantum properties. This model has intriguing
renormalization group properties, which will be discussed
in detail elsewhere [15]. First, note that $\lambda$ satisfies a
non-renormalization theorem: Consider the 2N-point
function of $\Phi$, with $N \gg 1$, and with external momen-
ta $k_1, \ldots, k_{2N}$. Any 1PI diagram will be of the form
$k_1^4 \cdots k_N^4 \cdots k_{2N}^4 \cdot T_{i_1 \cdots i_{2N}}(k_1, \ldots, k_{2N})$. The
factor $T$ has no ultraviolet divergences, and with $c^2 \neq 0$ or $c_2^2 \neq 0$ it approaches a finite value at
$\varepsilon_1 = \ldots = k_{2N} = 0$. The special case of $N = 2$ implies that $\lambda$ does not get renormalized, at any loop order. Note also that none of the operators $W, O_0$ or $O^{(a)}_4$ that would break the linear shift symmetry are generated by the loop corrections.

The remarkable non-renormalization of $\lambda$ does not im-
ply that the effective self-interaction strength would be
independent of scale: There is a non-trivial renormaliza-
tion of the 2-point function. While the one-loop diagram
\[ \square \] is identically zero, the two-loop diagram \[ \square \square \] gives the generic behavior which persists at higher loops:
there is no wave-function renormalization, no renor-
malization of $c^2$, the loop corrections to $c_2^2$ are
quadratically divergent, and those to $c_3^2$ diverge logarithmically [15].

This log divergence effectively corrects the dynamical ex-
ponent of the ultraviolet fixed point away from the clas-
sical value $z = 3$. The modified scaling in turn implies
that the theory becomes effectively strongly coupled at
some finite scale $\mu_s$.

Having understood the quantum corrections, we can now
study cascading hierarchies of symmetry breaking in
this model, and confirm their technical naturalness. At
some high scale $\mu$, which will be our naturalness scale,
and which we keep below $\mu_s$, consider the following hier-
archical breaking of polynomial symmetries: First, break
the $P = 4$ symmetry of the $z = 3$ Gaussian fixed point
to the $P = 2$ symmetry of the $z = 2$ fixed point by some
small amount $\varepsilon_2 \ll 1$. Then break $P = 2$ to $P = 1$ by an
even smaller amount $\varepsilon_1 \ll \varepsilon_2$. This pattern corresponds to
\[ c_3^2 \approx 1, \quad c_2^2 \approx \mu^2 \varepsilon_2, \quad c^2 \approx \mu^4 \varepsilon_1, \quad \lambda \approx \varepsilon_1. \] (14)

The dispersion relation cascades from $z = 3$ at high en-
ergy scales, to $z = 2$ at intermediate scales, to $z = 1$ at
low scales. [28] Both the large hierarchies in [14] and
the cascading behavior of the dispersion relation are re-
spected by all loop corrections, and therefore are techni-
cally natural. This follows by inspection from the prop-
erties of the quantum corrections discussed above.

VII. EXAMPLE 3: A TYPE-A/TYPE-B HIERARCHY

So far, we focused on the cascading mechanism in the
Type A case. Type B systems can form their own hier-
archies, in the obvious generalization of the Type A cas-
cades exemplified above. There is no analog of the lower
critical dimension and the CHMW limit on $n$. Type A
NG modes can also exhibit a flow to Type B. This be-
behavior, albeit not new (see e.g. [13]), can be embedded as
one step into the more general technically natural hier-
archies of Type A or B as discussed above. In particular,
the crossover to Type B can provide a new IR regulator
of the Type A cascade at the lower critical dimension.

We shall illustrate this on the simplest Type A1 ex-
ample, although the full story is, of course, more general.
Consider two would-be Type A NG fields, $\phi_{1,2}(t, x)$, in
the vicinity of the $z = 1$ Gaussian fixed point
\[ S_1 = \frac{1}{2} \int dt dD x \{ \dot{\phi}_1^2 + \dot{\phi}_2^2 - c_1^2(\partial_i \phi_1)^2 - c_2^2(\partial_i \phi_2)^2 \}. \]

For simplicity, we will set $c_1 = c_2 = 1$, although this is
not necessary for our argument. Besides the rotations
and translations of the two scalars, note two independ-
ent $Z_2$ symmetries – the field reflection $R$: $(\phi_1, \phi_2) \rightarrow (\phi_1, -\phi_2)$, and the time reversal $T$: $t \rightarrow -t, \phi_{1,2}(t, x) \rightarrow \phi_{1,2}(-t, x)$. We can now turn on the Type B kinetic term,
\[ S' = S_1 + \Omega \int dt dD x \left( \phi_1 \dot{\phi}_2 - \phi_2 \dot{\phi}_1 \right). \] (15)

$\Omega$ now provides an IR regulator for the propagator. At
that scale, the field reversal $R$ and the time reversal $T$ are
broken to their diagonal subgroup. At energy scales be-
low $\Omega$, one of the would-be Type A NG modes sur-
vives and turns into the Type B NG mode, while the other
would-be Type A mode develops a gap set by $\Omega$. Note
that in 1 + 1 dimensions, the “no-go” consequences of the
relativistic CHMW theorem are again naturally evaded
by this hierarchy; a NG mode exists quantum mechani-
cally after all, and symmetry breaking is possible, despite
the fact that above the scale $\Omega$, the two would-be Type
A modes exhibit the logarithmic two-point function sug-
jecting that symmetry breaking may not be possible.

The hierarchy between the Type A and Type B behav-
ior is also protected by symmetries. In fact, the system
has multiple symmetries that can do this job. One can
rely on the breaking pattern of the discrete symmetries $R$
and $T$ mentioned above. If the Type A system is Lorentz
invariant, one can use Lorentz symmetry breaking to pro-
tect small $\Omega$. More interestingly, without relying on the
discrete or Lorentz symmetries, one can introduce a shift
symmetry linear in time, $\delta \phi_{1,2} = b_1 \phi_1 \phi$. While the Type A
kinetic term is invariant under this symmetry, the Type B
kinetic term is not. Breaking the linear shift symmetry to constant shifts allows the Type-A/Type-B crossover scale to be hierarchically smaller than the naturalness scale.

VIII. CONCLUSIONS AND OUTLOOK

We have seen that the multicritical Type $A_n$ and $B_{2n}$ NG modes can experience technically natural cascading hierarchies of scales, protected by a hierarchy of polynomial shift symmetries. Perhaps the most interesting case is Type A with $n = D$, which according to our CHMW theorem exhibits logarithmic sensitivity to the IR regulator. In the relativistic case, this would prevent the symmetry breaking. We have shown that the Type $A_D$ modes can experience a cascade to Type $A_n$ with $n < D$ (or to Type B), which provides a natural IR regulator, thus making the symmetry breaking possible after all.

Our original motivation for this study of technical naturalness and hierarchies in SSB came from quantum gravity and high-energy physics\cite{17, 18}, especially in the context of nonrelativistic gravity\cite{17, 18}. Besides extending our understanding of the general “landscape of naturalness,” we expect that our results could find their most immediate applications in two other areas: In condensed matter physics, and in effective field theory of inflationary cosmology\cite{19–21}. Both areas treat systems with nonrelativistic, Lifshitz-like symmetries similar to ours. In condensed matter, the multicriticality of NG modes will affect their thermodynamic and transport properties; for example, the Type $A_D$ modes at the lower critical dimension will exhibit specific heat linear in temperature $T$ over the range of $T$ dominated by the $z = D$ dispersion (up to log $T$ corrections due to self-interactions). In the context of inflation, our self-interacting scalar field theories represent a new nonrelativistic variation on the theme of the Galileon\cite{22}, an extension of the theme of the Galileon\cite{22}, and of the $z = 3$ cosmological scalar theory of Mukohyama\cite{23, 24}.

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[27] Here we have assumed that both components of the canonical Type B pair $\phi_1,2$ carry the same dimension. The more general case would only require the sum of their dimensions to equal $D$, still preventing a logarithmic IR divergence in $\langle \phi_1 \phi_2 \rangle$.
[28] We can also break the linear shift symmetry to constant shifts, by some amount $\varepsilon_0 \ll \varepsilon_1$. This would generate the remaining classically marginal operators $\mathcal{O}_4^{(a)}, \mathcal{O}_6$ and the relevant operator $\mathcal{W}$, with coefficients of order $\varepsilon_0$ in the units of $\mu$. 