Elliptic flow of identified hadrons in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV

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Abstract: The elliptic flow coefficient ($v_2$) of identified particles in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV was measured with the ALICE detector at the Large Hadron Collider (LHC). The results were obtained with the Scalar Product method, a two-particle correlation technique, using a pseudo-rapidity gap of $|\Delta \eta| > 0.9$ between the identified hadron under study and the reference particles. The $v_2$ is reported for $\pi^\pm$, $K^\pm$, $K_S^0$, $p+\bar{p}$, $\phi$, $\Lambda+\bar{\Lambda}$, $\Xi^-+\Xi^+$ and $\Omega^-+\Omega^+$ in several collision centralities. In the low transverse momentum ($p_T$) region, $p_T < 3$ GeV/c, $v_2(p_T)$ exhibits a particle mass dependence consistent with elliptic flow accompanied by the transverse radial expansion of the system with a common velocity field. The experimental data for $\pi^\pm$ and the combined $K^\pm$ and $K_S^0$ results, are described fairly well by hydrodynamic calculations coupled to a hadronic cascade model (VISHNU) for central collisions. However, the same calculations fail to reproduce the $v_2(p_T)$ for $p+\bar{p}$, $\phi$, $\Lambda+\bar{\Lambda}$ and $\Xi^-+\Xi^+$. For transverse momentum values larger than about 3 GeV/c, particles tend to group according to their type, i.e. mesons and baryons. The present measurements exhibit deviations from the number of constituent quark (NCQ) scaling at the level of $\pm 20\%$ for $p_T > 3$ GeV/c.

Keywords: Collective flow, Heavy Ions, Particle correlations and fluctuations

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1 Introduction

Lattice quantum chromodynamics calculations predict a transition from ordinary nuclear matter to the Quark-Gluon Plasma (QGP) [1–4], in which the constituents, the quarks and the gluons, are deconfined. At low values of the baryochemical potential, a crossover transition is expected to take place at a temperature of about 150 MeV and at an energy density of about 0.5 GeV/fm$^3$ [5, 6]. These conditions are accessible in the laboratory by colliding heavy ions at ultra-relativistic energies. The study of the properties of this deconfined matter is the main goal of the heavy-ion collision program at the Large Hadron Collider (LHC). The existence of the QGP has been stipulated by observations at the Relativistic Heavy Ion Collider (RHIC) [7–10]. The first experimental results from the heavy-ion program at the LHC [11–25] have also provided evidence of the existence of the QGP in this new energy regime.
Anisotropic flow, which characterises the momentum anisotropy of the final state particles, can probe the properties, such as the ratio of shear viscosity to entropy density ($\eta/s$), of the system created in heavy-ion interactions. In nuclear collisions, the impact parameter vector and the beam axis define the reaction plane. It was recently realized that the overlap region of the colliding nuclei exhibits an irregular shape driven by the initial density profile of nucleons participating in the collision which is different from one event to the other. The symmetry plane of this irregular shape fluctuates around the reaction plane in every event. This spatial anisotropy of the overlap region is transformed into an anisotropy in momentum space through interactions between partons and at a later stage between the produced particles. The resulting anisotropy is usually characterised by the Fourier coefficients $v_n$ according to

$$v_n = \langle \cos \left[ n(\varphi - \Psi_n) \right] \rangle,$$  \hspace{1cm} (1.1)

where $\varphi$ is the azimuthal angle of particles, $n$ is the order of the flow harmonic and $\Psi_n$ is the angle of the spatial plane of symmetry of harmonic $n$ [28–32]. The second Fourier coefficient, $v_2$, measures the azimuthal momentum space anisotropy of particle emission relative to the second harmonic symmetry plane and is known as elliptic flow.

The study of anisotropic flow in nucleus-nucleus collisions at RHIC [7–10] contributed significantly in establishing that the produced system is described as a strongly coupled Quark-Gluon Plasma (sQGP) with a small value of the ratio of shear viscosity to entropy density ($\eta/s$), very close to the conjectured lower limit of $\hbar/4\pi k_B$, where $\hbar$ and $k_B$ are the reduced Planck and Boltzmann constants, respectively [33]. Recent anisotropic flow measurements for charged particles at the LHC [15–22] indicate that the system created in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV also behaves as a strongly interacting liquid. An additional constraint on the value of $\eta/s$ can be obtained by studying the flow coefficients of eq. (1.1) as a function of collision centrality and transverse momentum for different particle species [7–10]. An interplay of radial flow (i.e. azimuthally symmetric) and anisotropic flow leads to a characteristic mass dependence of $v_n(p_T)$ [34–37], first observed by the E877 Collaboration at the AGS for the case of directed flow ($v_1$) [38, 39] and by the NA49 Collaboration at SPS [40, 41]. This interplay was then studied in detail for $v_2$ at RHIC, where the characteristic mass ordering of the $v_2(p_T)$ (the $p_T$-differential $v_2$) for $p_T < 2$ GeV/c was reported [42–48].

The comparison of $v_2(p_T)$ measurements to hydrodynamic calculations in the low transverse momentum region has established that elliptic flow is built up mainly during the early, partonic stages of the system and is thus governed by the evolution of the QGP medium [7–10]. However, the hadronic rescattering that follows the QGP phase could also contribute to the development of $v_2$ [49]. The development of anisotropic flow at the partonic stage may be probed by studying particles with a small hadronic cross section, which are expected to be less affected by the hadronic stage and thus more sensitive to the early (partonic) stages of the collision. The $\phi$, $\Xi^{-}+\Xi^{+}$ and $\Omega^{-}+\Omega^{+}$ are argued to be such weakly coupled probes [50–54].

In addition, at RHIC energies, in the intermediate $p_T$ region ($2 < p_T < 6$ GeV/c) the $v_2(p_T)$ of baryons is larger than that of mesons. In [55], it was suggested that this
phenomenon can find an explanation in a picture where flow develops at the partonic level and quarks coalesce into hadrons during the hadronization. The proposed mechanism was argued to lead to the observed hierarchy in the values of $v_2(p_T)$, referred to as number of constituent quarks (NCQ) scaling, in the intermediate $p_T$ region where hydrodynamic flow might no longer be dominant and may compete with the corresponding contribution from jet fragmentation. The expectation was investigated by several studies of the quark coalescence picture both experimentally [42–48] and theoretically [56–59].

In [60], we presented the first measurements of $v_2(p_T)$ for identified $\pi^\pm$, p and $\bar{p}$ at the LHC in the range $3 < p_T < 20 \text{ GeV}/c$. In the present article, the $v_2(p_T)$ of identified particles is reported for $0.2 < p_T < 6.0 \text{ GeV}/c$ measured in Pb-Pb collisions at the centre of mass energy per nucleon pair $\sqrt{s_{NN}} = 2.76 \text{ TeV}$ with the ALICE detector [61–63] at the LHC. Results on $v_2(p_T)$ for identified mesons ($\pi^\pm$, $K^\pm$, $K^0_S$, $\phi$) and baryons ($p$, $\Lambda$, $\Xi^-$, $\Omega^-$, and their antiparticles), measured in $|y| < 0.5$ (where $y$ is the rapidity of each particle) are presented. The $v_2(p_T)$ values of particles and antiparticles were measured separately and were found to be compatible within the statistical uncertainties. Thus, in this article the $v_2(p_T)$ for the sum of particles and antiparticles is reported. For the reconstruction of the decaying particles presented in section 3, the following channels were used: $K^0_S \rightarrow \pi^+ + \pi^-$, $\phi \rightarrow K^+ + K^-$, $\Lambda \rightarrow p + \pi^-$ ($\bar{\Lambda} \rightarrow \bar{p} + \pi^+$), $\Xi^- \rightarrow \Lambda + \pi^-$ ($\bar{\Xi}^+ \rightarrow \bar{\Lambda} + \pi^+$), and $\Omega^- \rightarrow \Lambda + K^-$ ($\bar{\Omega}^+ \rightarrow \bar{\Lambda} + K^+$). The results are obtained with the Scalar Product method described briefly in section 4, and in detail in [64, 65], using a pseudo-rapidity gap of $|\Delta \eta| > 0.9$ between the identified hadrons under study and the charged reference particles (see section 4 for details). This method suppresses the contribution to $v_2(p_T)$ from correlations not related to the symmetry plane, i.e. non-flow effects, such as correlations arising from jets and resonance decays. The $v_2(p_T)$ is reported for different centralities of Pb-Pb collisions, which span the range 0–60% of the inelastic cross section [66], where the contribution from non-flow effects is small as compared to the collective flow signal.

2 Experimental setup

ALICE [63] is one of the four major experiments at the LHC. It is particularly designed to cope with the large charged-particle densities present in central Pb-Pb collisions [11]. ALICE uses a right-handed Cartesian system with its origin at the second LHC Interaction Point (IP2). The beam direction defines the $z$-axis, the $x$-axis is horizontal and points towards the centre of the LHC, and the $y$-axis is vertical and points upwards. The apparatus consists of a set of detectors located in the central barrel positioned inside a solenoidal magnet which generates a 0.5 T field parallel to the beam direction, and a set of forward detectors. The central detector systems allow for full azimuthal coverage for track reconstruction within a pseudo-rapidity window of $|\eta| < 0.9$. The experimental setup provides momentum resolution of about 1 to 1.5 % for the momentum range covered in this article, and particle identification (PID) over a broad momentum range.

For this analysis, the charged particles were reconstructed using the Time Projection Chamber (TPC) [67] or the combination of the TPC and the Inner Tracking System (ITS) [63]. The TPC is the main tracking detector of the central barrel. The detector
provides full azimuthal coverage in the pseudo-rapidity range $|\eta| < 0.9$. The ITS consists of six layers of silicon detectors employing three different technologies. The two innermost layers, positioned at $r = 3.9$ cm and 7.6 cm, are Silicon Pixel Detectors (SPD), followed by two layers of Silicon Drift Detectors (SDD) ($r = 15$ cm and 23.9 cm). Finally the two outermost layers are double-sided Silicon Strip Detectors (SSD) at $r = 38$ cm and 43 cm.

Charged particles were identified using the information from the TPC and the Time of Flight (TOF) detector [63]. The TPC provides a simultaneous measurement of the momentum of a particle and its specific ionisation energy loss ($dE/dx$) in the gas. The detector provides a sufficient separation (i.e. better than 2 standard deviations) for the hadron species at $p_T < 0.7$ GeV/c and the possibility to identify particles on a statistical basis in the relativistic rise region of $dE/dx$ (i.e. $2 < p_T < 20$ GeV/c) [68]. The $dE/dx$ resolution for the 5% most central Pb-Pb collisions is 6.5% and improves for peripheral collisions. The TOF detector surrounds the TPC and provides a $3\sigma$ separation between $\pi$-K and K–p up to $p_T = 2.5$ GeV/c and $p_T = 4$ GeV/c, respectively [68]. This is done by measuring the arrival time of particles with a resolution of about 80 ps. The start time for the TOF measurement is provided by the T0 detectors, two arrays of Cherenkov counters positioned at opposite sides of the interaction points covering $4.6 < \eta < 4.9$ (T0-A) and $-3.3 < \eta < -3.0$ (T0-C). The start time is also determined using a combinatorial algorithm that compares the timestamps of particle hits measured by the TOF to the expected times of the tracks, assuming a common event time $t_{ev}$ [68, 69]. Both methods of estimating the start time are fully efficient for the 60% most central Pb-Pb collisions.

A set of forward detectors, the VZERO scintillator arrays [70], were used in the trigger logic and for the centrality and reference flow particle determination for the Scalar Product method described in section 4. The VZERO consists of two systems, the VZERO-A and the VZERO-C, positioned on each side of the interaction point, and cover the pseudo-rapidity ranges of $2.8 < \eta < 5.1$ and $-3.7 < \eta < -1.7$ for VZERO-A and VZERO-C, respectively. For more details on the ALICE experimental setup, see [63].

3 Event sample, track selection and identification

3.1 Trigger selection and data sample

In this analysis approximately $15 \times 10^6$ Pb-Pb events were used. The sample was recorded during the first LHC heavy-ion data taking period in 2010 at $\sqrt{s_{NN}} = 2.76$ TeV. Minimum bias Pb-Pb events were triggered by the coincidence of signals from the two VZERO detectors. An offline event selection exploiting the signal arrival time in VZERO-A and VZERO-C, with a 1 ns resolution, was used to discriminate background (e.g. beam-gas) from collision events. This reduced the background events in the analysed sample to a negligible fraction ($< 0.1\%$). All events retained in the analysis have a reconstructed primary vertex position along the beam axis ($V_z$) within 10 cm from the centre of the detector. The vertex was estimated using either tracks reconstructed by the TPC or by the global tracking, i.e. combining information from all tracking detectors (the TPC and the ITS).

The data were grouped according to fractions of the inelastic cross section and correspond to the 60% most central Pb-Pb collisions. The 0–5% interval corresponds to the
most central (i.e. small impact parameter) and the 50–60% interval to the most peripheral (i.e. large impact parameter) collisions in the analysed sample. The centrality of the collision was estimated using the distribution of signal amplitudes from the VZERO scintillator detectors (default analysis). The systematic uncertainty due to estimating the centrality of the collision is determined using the charged particle multiplicity distribution of TPC tracks, and the number of ITS clusters. Details on the centrality determination can be found in [66].

3.2 Selection of $\pi^\pm$, $K^\pm$ and $p+\bar{p}$

Primary charged pions, kaons and (anti-)protons were required to have at least 70 reconstructed space points out of the maximum of 159 in the TPC. The average $\chi^2$ of the track fit per TPC space point per degree of freedom (see [68] for details) was required to be below 2. These selections reduce the contribution from short tracks, which are unlikely to originate from the primary vertex, to the analysed sample. To further reduce the contamination from secondary tracks (i.e. particles originating either from weak decays or from the interaction of other particles with the material), only particles within a maximum distance of closest approach (DCA) between the tracks and the primary vertex in both the $xy$-plane ($d_{xy} < 2.4$ cm) and the $z$ coordinate ($d_z < 3.0$ cm) were analysed. The selection leads to an efficiency of about 80% for primary tracks at $p_T > 0.6$ GeV/$c$ and a contamination from secondaries of about 5% at $p_T = 1$ GeV/$c$ [71]. These values depend strongly on particle species and transverse momentum [71]. The $v_2(p_T)$ results are reported for $|y| < 0.5$ and $0.2 < p_T < 6.0$ GeV/$c$ for $\pi^\pm$, $0.3 < p_T < 4.0$ GeV/$c$ for $K^\pm$ and $0.3 < p_T < 6.0$ GeV/$c$ for $p+\bar{p}$.

For the identification of $\pi^\pm$, $K^\pm$ and $p+\bar{p}$ over the wide $p_T$ range, the combination of information from the TPC and the TOF detectors was used. In particular, the identification was based on a two-dimensional correlation between the response of the TPC and the TOF. The particles were selected by requiring their signal to lie within three standard deviations (3σ) of both the $dE/dx$ ($\sigma_{TPC}$) and TOF ($\sigma_{TOF}$) resolutions. For some particles (particularly kaons) with $p_T > 3$ GeV/$c$ where the relevant bands for different particle species start to overlap, the requirement was changed to $2\sigma$. This identification strategy results in a purer sample as compared to previous analyses reported by ALICE (see e.g. [71]). It is adopted since it reduces the need for potential corrections due to particle misidentification that could introduce additional uncertainties to the measurement of $v_2$. An example of a correlation plot between the number of standard deviation from the expected signal of the TPC and the TOF detectors for three different transverse momentum intervals in the 5% most central Pb-Pb collisions is presented in figure 1. The resulting purity, estimated using Monte-Carlos (MC) simulations but also data-driven methods (e.g. selecting pions and (anti)protons from $K^0$ and $\Lambda(\bar{\Lambda})$ decays) was larger than 90% for $\pi^\pm$, $K^\pm$ and $p+\bar{p}$ throughout the analysed transverse momentum range.

Finally, since the contamination from secondary protons created through the interaction of particles with the detector material can reach values larger than 10% for $p_T < 1$ GeV/$c$, only $\bar{p}$ were considered for $p_T < 3$ GeV/$c$, while for higher values of $p_T$ a combined measurement of $p$ and $\bar{p}$ was used.
3.3 Reconstruction of $K^0_S$ and $\Lambda+\bar{\Lambda}$

The measurement of $K^0_S$, $\Lambda$ and $\bar{\Lambda}$ was performed using their weak decays in the following channels: $K^0_S \rightarrow \pi^+ + \pi^-$ (branching ratio 69.2%) and $\Lambda \rightarrow p + \pi^-$, $\bar{\Lambda} \rightarrow \bar{p} + \pi^+$ (branching ratio 63.9%) [72]. The identification of these particles is based on the reconstruction of the secondary vertex exhibiting a characteristic V-shape, called V0, defined by the trajectories of the decay products.

For all three particles, the decay products of the V0 candidates were required to have a minimum $p_T$ of 0.1 GeV/$c$, while the criteria on the number of TPC space points and on the $\chi^2$ per TPC space point per degree of freedom were identical to those applied for primary particles. In addition, a selection of secondary particles based on a minimum DCA to the primary vertex of 0.1 cm was applied. Furthermore, a maximum DCA of 0.5 cm between the decay products at the point of the V0 decay was required to ensure that they are products of the same decay. The decay tracks were reconstructed within $|\eta| < 0.8$. Finally, for the $\Lambda+\bar{\Lambda}$ candidates with low values of transverse momentum, a particle identification cut to select their $p+\bar{p}$ decay products was applied that relied on a 3-σ band around the expected energy loss in the TPC, defined by a parameterization of the Bethe-Bloch curve. The selection parameters are summarised in table 1.

To reduce the contamination from secondary and background particles, mainly from other strange baryons affecting $\Lambda$ and $\bar{\Lambda}$, a minimum value of the cosine of the pointing angle ($\cos \theta_p \geq 0.998$) was required. The pointing angle is defined as the angle between the momentum vector of the V0 candidate and the vector from the primary to the reconstructed V0 vertex [73]. To further suppress the background, only V0 candidates whose decay length was within three times the $c\tau$ value of 2.68 cm for $K^0_S$ and 7.89 cm for $\Lambda$ ($\bar{\Lambda}$) [73] were analysed. In addition, the radial position of the secondary vertex reconstruc-
Table 1. Selection criteria for the decay products of the V0 candidates.

<table>
<thead>
<tr>
<th></th>
<th>(K^0_S) and (\Lambda(\bar{\Lambda})) decay products</th>
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</thead>
<tbody>
<tr>
<td>TPC space points</td>
<td>(\geq 70)</td>
</tr>
<tr>
<td>(\chi^2) per TPC space point per d.o.f.</td>
<td>(\leq 2)</td>
</tr>
<tr>
<td>DCA to primary vertex</td>
<td>(\geq 0.1) cm</td>
</tr>
<tr>
<td>DCA between decay products</td>
<td>(\leq 0.5) cm</td>
</tr>
<tr>
<td>(p_T)</td>
<td>(\geq 0.1) GeV/c</td>
</tr>
<tr>
<td>(</td>
<td>\eta</td>
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<tr>
<td>TPC PID compatibility selection for (p+\bar{p}) decay products of (\Lambda+\bar{\Lambda})</td>
<td>(\leq 3) (\sigma)</td>
</tr>
</tbody>
</table>

Table 2. Topological selections for the \(K^0_S\) and \(\Lambda(\bar{\Lambda})\).

<table>
<thead>
<tr>
<th></th>
<th>(K^0_S) and (\Lambda(\bar{\Lambda})) candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decay length</td>
<td>(\leq 3\epsilon\tau)</td>
</tr>
<tr>
<td>(\cos\theta_p)</td>
<td>(\geq 0.998)</td>
</tr>
<tr>
<td>Decay radius</td>
<td>(\geq 5) cm</td>
</tr>
<tr>
<td>(</td>
<td>y</td>
</tr>
<tr>
<td>(q_T) ((K^0_S) only)</td>
<td>(\geq 0.2</td>
</tr>
</tbody>
</table>

Charged pions and pion-(anti-)proton pairs were then combined to obtain the invariant mass \((m_{\text{inv}})\) for \(K^0_S\) and \(\Lambda(\bar{\Lambda})\), respectively. Examples of such distributions for two of the lowest transverse momentum intervals used in this analysis for the 10–20% centrality of Pb-Pb collisions at \(\sqrt{s_{NN}} = 2.76\) TeV are shown in figure 2 (a) and (b) for \(K^0_S\) and \(\Lambda\), respectively. These distributions are fitted with a sum of a Gaussian function and a third-order polynomial to estimate the signal and the background in the mass peak. The signal to background ratio in the mass peak depends on the transverse momentum and on the event centrality and is better than 5 for both particles. The \(v_2(p_T)\) results are reported for \(|y| < 0.5\) and \(0.4 < p_T < 6.0\) GeV/c for \(K^0_S\) and \(0.6 < p_T < 6.0\) GeV/c for \(\Lambda\) and \(\bar{\Lambda}\).

The Armenteros-Podolanski variables are the projection of the decay charged-track momentum on the plane perpendicular to the V0 momentum \((q_T)\) and the decay asymmetry parameter defined as \(\alpha = (p_{L+} - p_{L-})/(p_{L+} + p_{L-})\), where \(p_L\) is the projection of the decay charged-track momentum on the momentum of the V0.

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1The Armenteros-Podolanski variables are the projection of the decay charged-track momentum on the plane perpendicular to the V0 momentum \((q_T)\) and the decay asymmetry parameter defined as \(\alpha = (p_{L+} - p_{L-})/(p_{L+} + p_{L-})\), where \(p_L\) is the projection of the decay charged-track momentum on the momentum of the V0.
Figure 2. Invariant mass distributions in the 10–20% centrality interval of Pb-Pb collisions for reconstructed decaying particles: (a) K$_0^S$, (b) $\Lambda$+$\overline{\Lambda}$, (c) $\phi$, (d) $\Xi^-$+$\Xi^+$, and (e) $\Omega^-$+$\Omega^+$.  

### 3.4 Reconstruction of $\phi$ 

The $\phi$-meson was reconstructed via its hadronic decay channel: $\phi \rightarrow K^+ + K^-$ (branching ratio 48.9%) [72]. The selections applied for the decay products were identical to those of primary $K^\pm$, described in section 3.2. The $\phi$-meson yield was extracted from the invariant mass ($m_{inv}$) reconstructed from the unlike-sign kaon pairs. 

The combinatorial background was evaluated using the like-sign kaon pairs in each $p_T$ and centrality interval. The like-sign background $m_{inv}$ distribution is normalised to the corresponding distribution of unlike-sign pairs in the region above the $\phi$-meson mass ($1.04 < m_{inv} < 1.09$ GeV/$c^2$). An example of an invariant mass distribution before the like-sign subtraction for $0.6 < p_T < 1.2$ GeV/$c$ is given in figure 2 (c) for the 10–20% centrality interval of Pb-Pb collisions. The remaining background was estimated using a third-order polynomial. 

These invariant mass distributions were then fitted with a relativistic Breit-Wigner distribution, describing the signal in the mass peak. The $v_2(p_T)$ results for the $\phi$-meson are reported for $|y| < 0.5$ and $0.6 < p_T < 6.0$ GeV/$c$ for the centrality intervals covering the 10–60% of the inelastic cross section. For the 10% most central Pb-Pb collisions, the extraction of the signal over the large combinatorial background resulted into large uncertainties using the currently analysed data sample.
The measurement of $\Xi^-$+$\Xi^+$ and $\Omega^-$+$\Omega^+$ was performed using the following decay channels: $\Xi^-$ $\rightarrow$ $\Lambda$+$\pi^-$, $\Xi^+$ $\rightarrow$ $\Lambda$+$\pi^+$ (branching ratio 99.9%) and $\Omega^-$ $\rightarrow$ $\Lambda$+$K^-$, $\Omega^+$ $\rightarrow$ $\Lambda$+$K^+$ (branching ratio 67.8%) [72]. The reconstruction of $\Xi^-$+$\Xi^+$ and $\Omega^-$+$\Omega^+$ is performed based on the cascade topology of the decay, consisting of the V-shape structure of the $\Lambda$ decay and a charged particle not associated to the V0, referred to as bachelor track i.e. $\pi^-$ and $K^-$ for the case of $\Xi^-$+$\Xi^+$ and $\Omega^-$+$\Omega^+$, respectively.

To reconstruct $\Xi^-$+$\Xi^+$ and $\Omega^-$+$\Omega^+$ candidates, topological and kinematic criteria were applied to first select the V0 decay products and then to match them with the secondary, bachelor track. The track selection criteria, summarised in tables 3–5, for the reconstruction of $\Xi^-$+$\Xi^+$ and $\Omega^-$+$\Omega^+$ follow the procedure described in [75]. The cuts that contributed significantly to the reduction of the combinatorial background were the predefined window around the $\Lambda$+$\Xi$ mass, the DCA cut between the V0 and the bachelor track, and the V0 and cascade pointing angle with respect to the primary vertex position. Finally, for all three decay tracks, a particle identification cut for the pion, kaon or proton hypotheses was applied using their energy loss in the TPC. This was done by selecting particles within three standard deviations from the Bethe-Bloch curve for each mass hypothesis.

Examples of invariant mass distributions for two of the lowest transverse momentum intervals used in this analysis before the background subtraction for $\Xi^-$+$\Xi^+$ and $\Omega^-$+$\Omega^+$ for the 10–20% centrality class of Pb-Pb collisions can be seen in figure 2 (d) and (e). These distributions are fitted with a sum of a Gaussian function and a third-order polynomial.
Selection of bachelor tracks

<table>
<thead>
<tr>
<th>Selection criteria for secondary, bachelor tracks.</th>
</tr>
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<tbody>
<tr>
<td>DCA of bachelor track to primary vertex $\geq 0.03$ cm</td>
</tr>
<tr>
<td>$p_T$ $\geq 0.15$ GeV/c</td>
</tr>
<tr>
<td>Pseudo-rapidity $</td>
</tr>
<tr>
<td>Number of TPC space points $\geq 70$</td>
</tr>
<tr>
<td>$\chi^2$ per TPC space point per d.o.f. $&lt; 2$</td>
</tr>
<tr>
<td>$n\sigma dE/dx$ (TPC PID) $3$</td>
</tr>
</tbody>
</table>

Table 5.

Extraction of $v_2(p_T)$

The $v_2(p_T)$ was calculated with the Scalar Product (SP) [64, 65], a two-particle correlation method, using a pseudo-rapidity gap of $|\Delta \eta| > 0.9$ between the identified hadron under study and the reference flow particles. The applied gap reduces correlations not related to the symmetry plane $\Psi_n$, such as correlations due to resonance decays and jets, known as non-flow effects.

The SP method is based on the calculation of the $Q$-vector [65], computed from a set of reference flow particles (RFP) and defined as:

$$\vec{Q}_n = \sum_{i \in \text{RFP}} w_i e^{i n \varphi_i},$$

where $\varphi_i$ is the azimuthal angle of the $i$-th reference flow particle, $n$ is the order of the harmonic and $w_i$ is a weight applied for every RFP.

The default results were obtained by dividing each event into three sub-events A, B and C using three different detectors. The reference flow particles were taken from sub-events A and C, using the VZERO-A and VZERO-C detectors, respectively. Each of the VZERO arrays consists of 32 channels and is segmented in four rings in the radial direction, and each ring is divided in eight sectors in the azimuthal direction. They cover the pseudo-rapidity ranges of 2.8 < $\eta$ < 5.1 and -3.7 < $\eta$ < -1.7 for VZERO-A and VZERO-C, respectively. Since these detectors do not provide tracking information, the amplitude of the signal from each cell, which is proportional to the number of particles that cause a hit, was used as a weight $w_i$. A calibration procedure [68, 70] was performed prior to the usage of these signals, to account for fluctuations induced by the performance of the hardware, and for different conditions of the LHC for each data taking period. The particles under
study (i.e. $\pi^\pm$, $K^\pm$, $K^0_s$, $p$, $\bar{p}$, $\phi$, $\Lambda$, $\Xi^-$, $\Xi^+$, $\Omega^-$ and $\Omega^+$) were taken from sub-event B within $|y| < 0.5$ as described in section 3, using the region covered by the mid-rapidity detectors.

The $v_2$ was then calculated using the unit flow vector $\vec{u}_B^2 = e^{2i\phi_B}$ measured in sub-event B according to

$$v_2 = \sqrt{\frac{\langle \langle \vec{u}_B^2 \cdot \vec{Q}_A^* \rangle \rangle \langle \langle \vec{u}_B^2 \cdot \vec{Q}_C^* \rangle \rangle}{\langle \vec{Q}_A^2 \cdot \vec{Q}_C^* \rangle}}$$

where the two brackets in the numerator indicate an average over all particles of interest and over all events, $M_A$ and $M_C$ are the estimates of multiplicity from the VZERO-A and VZERO-C detectors, and $\vec{Q}_2^A$, $\vec{Q}_2^C$ are the complex conjugates of the flow vector calculated in sub-event A and C, respectively. The non-uniformity of the detector azimuthal efficiency is taken into account in the SP method by applying the inverse of the event-averaged signal as a weight for each of the VZERO segments [68, 70], together with a recentring procedure (i.e. subtraction of the average centroid position of each sector) [68].

To investigate the dependence of the results on the applied pseudo-rapidity gap and the possible residual contribution from non-flow effects, the analysis was repeated taking the particles under study from $y > 0$ (or $y < 0$) and the reference particles from $-3.7 < \eta < -1.7$ i.e. VZERO-C (or $2.8 < \eta < 5.1$ i.e. VZERO-A). The results were consistent with the default ones within the uncertainties.

4.1 Reconstruction of $v_2(p_T)$ with the invariant mass method

For the $v_2(p_T)$ measurement of $K^0_s$, $\phi$, $\Lambda$ ($\bar{\Lambda}$), $\Xi^-$ ($\bar{\Xi}$), and $\Omega^-$ ($\bar{\Omega}$), the $v_2$ versus invariant mass ($m_{\text{inv}}$) method [76, 77] was used. The $v_2(p_T)$ of the particles of interest ($v_{2}\text{Sgn}(p_T)$) is extracted from the total $v_{2}\text{Tot}(p_T)$ of all pairs or triplets contributing to the invariant mass window and from background ($v_{2}\text{Bg}(p_T)$) contributions, measured with the SP method, weighted by their relative yields according to

$$v_{2\text{Tot}}(m_{\text{inv}}, p_T) = v_{2\text{Sgn}}(p_T) \frac{N_{\text{Sgn}}(m_{\text{inv}}, p_T)}{N_{\text{Tot}}(m_{\text{inv}}, p_T)} + v_{2\text{Bg}}(m_{\text{inv}}, p_T) \frac{N_{\text{Bg}}(m_{\text{inv}}, p_T)}{N_{\text{Tot}}(m_{\text{inv}}, p_T)},$$

where $N_{\text{Tot}}$ is the total number of candidates, and $N_{\text{Bg}}$ and $N_{\text{Sgn}}$ are the yields of the background and signal respectively. The relative yields are determined from the fits to the invariant mass distributions shown in figure 2 for each transverse momentum interval.

For a given $p_T$, the observed $v_{2}\text{Sgn}$ is determined by fitting simultaneously the invariant mass distribution and the $v_{2\text{Tot}}(m_{\text{inv}})$ according to eq. (4.3). The value of $v_{2}\text{Bg}$ in the peak region is obtained by interpolating the values from the two sideband regions. Figure 3 shows these fits for each decaying particles in a given characteristic $p_T$ range in the 10–20% centrality interval of Pb-Pb collisions.

5 Systematic uncertainties

The systematic uncertainties in all results were determined by varying the event and particle selections and by studying the detector response with Monte-Carlo (MC) simulations. The
contributions from different sources, described below, were estimated for every particle species and centrality separately, as the maximum difference of $v_2(p_T)$ extracted from the variations of the cut values, relative to the main result extracted using the default selection criteria described in section 3. The ranges of each individual contribution over all centralities, expressed in percentages of the measured values, are summarized in table 6 for $\pi^\pm$, $K^\pm$ and $p+\bar{p}$ and table 7 for the decaying particles. The total systematic uncertainty was calculated as the quadratic sum of these individual contributions.

The event sample was varied by (i) changing the cut on the position of the primary vertex along the beam axis ($V_z$) from ±10 cm to ±7 cm, (ii) changing the centrality selection criteria from the signal amplitudes of the VZERO scintillator detectors to the multiplicity of TPC tracks, and the number of ITS clusters. For all species and centralities, the resulting $v_2(p_T)$ was consistent with results obtained with the default cuts. Results from runs with different magnetic field polarities did not exhibit any systematic change in $v_2(p_T)$ for any particle species for any centrality.

In addition, the track selection criteria, such as the number of TPC space points and the $\chi^2$ per TPC space point per degree of freedom were varied, for both primary hadrons (i.e. $\pi^\pm$, $K^\pm$ and $p+\bar{p}$) and the daughters of decaying particles. No systematic deviations in the values of $v_2(p_T)$ relative to the results obtained with the default selection were found. To estimate the uncertainties for the decaying particles, the ranges of the

\[ \text{ALICE} \]
\[ \text{Pb-Pb} \quad \sqrt{s_{NN}} = 2.76 \text{ TeV} \]
\[ \text{Centrality 10-20} \% \]
Table 6. Summary of systematic errors for the $v_2(p_T)$ measurement for $\pi^\pm$, $K^\pm$ and $p+\bar{p}$. Percentages given are fractions of the measured values.

<table>
<thead>
<tr>
<th>Error source</th>
<th>$\pi^\pm$</th>
<th>$K^\pm$</th>
<th>$p+\bar{p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex position</td>
<td>$\leq 0.1%$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Centrality estimator</td>
<td>$\leq 0.1%$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Magnetic field polarity</td>
<td>$\leq 0.1%$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of TPC space points</td>
<td>$\leq 0.1%$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi^2$ per TPC space point</td>
<td>$\leq 0.1%$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Particle identification</td>
<td>$5-15%$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contamination</td>
<td>$\leq 5%$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Feed-down</td>
<td>$\leq 0.1%$</td>
<td>$5%$</td>
<td></td>
</tr>
<tr>
<td>Selection of reference particles</td>
<td>$\leq 0.1%$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Local track density</td>
<td>$\leq 15%$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Error source</th>
<th>$\phi$</th>
<th>$K_S^0$</th>
<th>$\Lambda+\bar{\Lambda}$</th>
<th>$\Xi^-+\Xi^+$</th>
<th>$\Omega^-+\Omega^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex position</td>
<td>$\leq 0.1%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Centrality estimator</td>
<td>$\leq 0.1%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Magnetic field polarity</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of TPC space points</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi^2$ per TPC space point</td>
<td>$\leq 0.1%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decay length</td>
<td>n/a</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decay vertex (radial position)</td>
<td>n/a</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Armenteros-Podolanski variables</td>
<td>n/a</td>
<td>$\leq 0.1%$</td>
<td>n/a</td>
<td>n/a</td>
<td></td>
</tr>
<tr>
<td>DCA decay products to primary vertex</td>
<td>n/a</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DCA between decay products</td>
<td>n/a</td>
<td>$\leq 10%$</td>
<td>n/a</td>
<td>n/a</td>
<td></td>
</tr>
<tr>
<td>Pointing angle $\cos\theta_{p}$</td>
<td>n/a</td>
<td>$\leq 10%$</td>
<td>n/a</td>
<td>n/a</td>
<td></td>
</tr>
<tr>
<td>Particle identification</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$5-15%$</td>
</tr>
<tr>
<td>Contamination</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\leq 5%$</td>
</tr>
<tr>
<td>Signal and background estimation</td>
<td>$5-10%$</td>
<td>$\leq 0.1%$</td>
<td>$\leq 0.1%$</td>
<td>$5-10%$</td>
<td></td>
</tr>
<tr>
<td>Feed-down</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\leq 0.1%$</td>
</tr>
<tr>
<td>Selection of reference particles</td>
<td>$5%$</td>
<td>$1-5%$</td>
<td>$\leq 0.1%$</td>
<td>$1-5%$</td>
<td></td>
</tr>
<tr>
<td>Local track density</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\leq 0.1%$</td>
</tr>
</tbody>
</table>

Table 7. Summary of systematic errors for the $v_2(p_T)$ measurement for the decaying particles. Percentages given are fractions of the measured values (the notation n/a stands for non-applicable).
cuts for the decay length, the radial position of the decay vertex, the correlation between
the Armenteros-Podolanski variables, and the DCA of the decay products to the primary
vertex were varied by as much as three times the default values. These variations did not
affect the measured result. Differences were observed for the cases of $K^0_S$ and $\Lambda(\bar{\Lambda})$, when
changing the requirement on the minimal distance between the two daughter tracks (DCA)
and the pointing angle $\cos \theta_p$. These differences resulted in systematic uncertainties on the
measured $v_2(p_T)$ of $\leq 10\%$ for both $K^0_S$ and $\Lambda+\bar{\Lambda}$.

Systematic uncertainties associated with the particle identification procedure were
studied by varying the number of standard deviations (e.g. between 1-$4\sigma$) around the
expected energy loss in the TPC (similarly for the TOF) for a given particle species. Fur-
thermore, the contamination of the kaon and proton samples was studied in collision data
by selecting pions and (anti)protons from $K^0_S$ and $\Lambda(\bar{\Lambda})$ decays, respectively, and then deter-
mining the number that passed the kaon selections. The resulting uncertainties related to
the particle (mis-)identification on the extracted $v_2(p_T)$ values depend weakly on centrality,
increase with transverse momentum and are in the range 5–15\% for all particle species.

The feed-down from weakly decaying particles was found to be a significant factor only
for $p+p$. Its contribution was determined by selecting $p(\bar{p})$ from $\Lambda(\bar{\Lambda})$ decays and measuring
their anisotropy with the SP method. It was found that the systematic uncertainty in the
extracted $v_2(p_T)$ resulting from this source was at maximum 5\% for all centralities.

The systematic uncertainty originating from the signal extraction and the background
description, used in the method described in section 4.1, was studied by extracting the yields
with a simple bin-counting method. The uncertainty was further investigated by using
different functions to describe the signal (e.g. Breit-Wigner, Gaussian and double Gaussian)
and background (e.g. polynomial of different orders) in the invariant mass distribution. In
addition, for the case of the $\phi$-meson, a subtraction of the background estimated with the
mixed events method was used. The mixed events were formed by combining tracks from separate events belonging to the same centrality interval, with a reconstructed primary
vertex position along the beam axis within $\pm 2$ cm) from the value of the original event. The
corresponding systematic uncertainties in the extracted $v_2(p_T)$ from the previous sources
were below 0.1\% for $K^0_S$ and $\Lambda(\bar{\Lambda})$. For the $\phi$-meson, $\Xi^-/(\Xi^+)$ and $\Omega^-/(\Omega^+)$ they were found
to be in the range 5–10\%.

The systematic uncertainties originating from the selection of reference flow particles
were extracted by measuring $v_2(p_T)$ with reference particles estimated either with the three
sub-event method described in section 4, or using two sub-events with either the VZERO-A
or the VZERO-C detectors separately. This resulted in a systematic uncertainty in the
extracted $v_2(p_T)$ up to 5\% for the $\phi$-meson, $\Xi^-/(\Xi^+)$ and $\Omega^-/(\Omega^+)$. The

Finally, due to the anisotropy of particle production there are more particles in the
direction of the symmetry plane than in the direction perpendicular to the plane. Conse-
quently, the detector occupancy varies as a function of the angle relative to the symmetry
plane. The track finding and track reconstruction are known to depend slightly on the
detector occupancy. A local track density dependent efficiency would reduce the recon-
structed $v_2$ for all charged tracks proportional to the modulation of the efficiency. In order
to investigate how a variation in occupancy affects the efficiency for track finding and track
reconstruction, dedicated Monte-Carlo events using a generator without any physics input (i.e. a so-called toy-model) with the particle yields and ratios, momentum spectra, and flow coefficients (e.g. \(v_2(p_T)\), \(v_3(p_T)\)) measured in data for every centrality interval were generated. The ALICE detector response for these events was determined using a GEANT3 \[78\] simulation. The occupancy dependence of the tracking and matching between the TPC and the TOF contributed to the systematic uncertainty of \(v_2(p_T)\) for \(\pi^\pm\), \(K^\pm\) and \(p^+\bar{p}\) with less than 10\%, independent of momentum. An additional contribution of less than 6\% of the measured \(v_2(p_T)\) for \(p_T > 2.5\text{ GeV}/c\) for the same particles resulted from the sensitivity of the TPC \(dE/dx\) measurement to the local track density. The analysis of the MC events did not indicate any additional systematic effect related to the detector occupancy for the other particle species and was in agreement with a numerical calculation of the particle reconstruction efficiency as a function of the total event multiplicity.

6 Results and discussion

Figure 4 presents the \(p_T\)-differential \(v_2\) for all identified particles measured in Pb-Pb collisions at \(\sqrt{s_{NN}} = 2.76\text{ TeV}\). To illustrate the development of \(v_2\) as a function of centrality for \(\pi^\pm\), \(K^\pm\), \(K^0_s\), \(p+\bar{p}\), \(\phi\), \(\Lambda+\bar{\Lambda}\), \(\Xi^-+\Xi^+\) and \(\Omega^-+\Omega^+\), the results are grouped by particle species in different panels. The error bars correspond to statistical uncertainties, while the hollow boxes around each point indicate the systematic uncertainties. The same convention for these uncertainties is used for the rest of the figures in this article. The systematic uncertainties in many cases are smaller than the marker size.

The value of \(v_2(p_T)\) progressively increases from central to peripheral collisions up to the 40–50\% centrality interval for all particle species. This is consistent with the picture of the final state anisotropy driven by the geometry of the collision, as represented by the initial state eccentricity which increases for peripheral collisions. For more peripheral events (i.e. 50–60\%), the magnitude of \(v_2\) does not change significantly within the systematic uncertainties compared to the previous centrality interval. According to \[85\], this might originate from a convolution of different effects such as the smaller lifetime of the fireball in peripheral compared to more central collisions that does not allow \(v_2\) to further develop. The authors also attributed this effect to the less significant (compared to more central events) contribution of eccentricity fluctuations and to final state hadronic effects. The transverse momentum dependence of \(v_2\) exhibits an almost linear increase up to about 3\text{ GeV}/c. This initial rise is followed by a saturation and then a decrease observed for all particles and centralities. The position of the maxima depends on the particle species and on the centrality interval.

Figure 5 presents the same data points shown in figure 4, arranged into panels of different event centrality selection, illustrating how \(v_2(p_T)\) develops for different particle species within the same centrality interval. The panels are arranged by decreasing centrality from left to right and top to bottom. The top left plot presents results for the 5\% most central Pb-Pb collisions, while the most peripheral interval presented in this article, the 50–60\% centrality, is shown in the bottom right plot.
Figure 4. The $p_T$-differential $v_2$ for different centralities of Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV grouped by particle species.

A clear mass ordering is seen for all centralities in the low $p_T$ region (i.e. $p_T \leq 3$ GeV/$c$), attributed to the interplay between elliptic and radial flow [34–37]. Radial flow tends to create a depletion in the particle $p_T$ spectrum at low values, which increases with increasing particle mass and transverse velocity. When introduced in a system that exhibits azimuthal anisotropy, this depletion becomes larger in-plane than out-of-plane, thereby reducing $v_2$. The net result is that at a fixed value of $p_T$, heavier particles have smaller $v_2$ value compared to lighter ones. In addition, a crossing between the $v_2$ values of baryons (i.e. p, Λ, Ξ and Ω and their antiparticles) and the corresponding values of pions and kaons is observed, that takes place between 2 and 3.5 GeV/$c$, depending on the particle species and centrality. It is seen that the crossing between e.g. $\pi^\pm$ and $p+\bar{p}$ happens at lower $p_T$ for peripheral than for central collisions. For more central collisions, the crossing point moves to higher $p_T$ values,
since the common velocity field, which exhibits a significant centrality dependence \cite{71}, affects heavy particles more. For higher values of \( p_T \) (\( p_T > 3 \text{ GeV}/c \)), particles tend to group according to their type, i.e. mesons and baryons. This feature will be discussed in detail in section 6.3.

Figure 5 also shows how \( v_2 \) develops for \( K^\pm \) and \( K^0 \) as a function of transverse momentum for different centralities. A centrality and \( p_T \) dependent difference is observed in these two measurements. In particular, the \( v_2(p_T) \) for neutral kaons is systematically lower than that of their charged counterparts. The difference between the two measurements reaches up to two standard deviations in central, and is on the level of one standard deviation in peripheral Pb-Pb collisions. A number of cross checks performed using data (e.g. calculating the \( v_2(p_T) \) of kaons identified via the kink topology of their leptonic decay, studies of feed-down corrections) as well as analysis of the dedicated MC simulations described in

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{The \( p_T \)-differential \( v_2 \) for different particle species grouped by centrality class of Pb-Pb collisions at \( \sqrt{s_{NN}} = 2.76 \text{ TeV} \).}
\end{figure}
section 5 did not reveal an origin for the difference. Additionally, no physics mechanism (e.g. feed-down from φ, larger mass for K_S^0 than K^± by about 4 MeV/c^2) responsible for the difference could be found. Therefore, for the remaining figures of this article, the v_2(p_T) results for K^± and K_S^0 were considered as two independent measurements of kaon flow. Following the description given by the PDG (section 5 of [72]) we averaged the two sets of data points in the overlapping p_T region (i.e. for p_T < 4 GeV/c) using the statistical and the total (uncorrelated) systematic uncertainties in every transverse momentum interval as a weight. The uncertainty on the average was obtained from the individual uncertainties added in quadrature and the differences between the two measurements assigned asymmetrically. With this procedure, the averaged values for kaons are closer to the K_S^0, which have a smaller error (and hence larger weight), but the assigned asymmetric error covers both the original charged and neutral data points. For p_T > 4 GeV/c, only the K_S^0 data points were used and their uncertainty has not been modified. Since our studies did not identify a common underlying effect (e.g. from charged particle tracking) as the source of the difference, the additional uncertainties were not propagated to other particles.

Among all particle species, the φ-meson is of particular interest since its mass is close to that of p and Λ. It provides an excellent testing ground of both the mass ordering and the baryon-meson grouping at low and intermediate p_T, respectively. The v_2 values of the φ-meson in figure 5 indicate that for p_T < 3 GeV/c it follows the mass-ordered hierarchy. However, for higher p_T values the φ data points appear to follow the band of baryons for central events within uncertainties. For peripheral collisions though, the v_2 values of the φ-meson shift progressively to the band of mesons. This is congruous with the observation that the (p+p)/φ ratio, calculated from the transverse momentum spectra, is almost constant as a function of transverse momentum in central Pb-Pb events, while for peripheral collisions the ratio decreases with increasing p_T, as reported in [79].

Finally, the multi-strange baryons, i.e. Ξ^−+Ξ^+, and Ω^−+Ω^+, provide another interesting test of both the mass ordering and the baryon-meson grouping. Similar to all other particle species, a mass ordering is reported at low p_T values. At intermediate p_T values, both particles seem to follow the band formed by the other baryons, within the statistical and systematic uncertainties.

6.1 Comparison with hydrodynamic calculations

It has been established that hydrodynamic [80–82] as well as hybrid models (hydrodynamic system evolution followed by a hadron cascade model) [83–85] describe the soft particle production at both RHIC and the LHC fairly well.

In figure 6, the v_2 measurements for two centrality intervals, the 10–20% in the left column and the 40–50% interval in the right column, are compared to hydrodynamic calculations coupled to a hadronic cascade model (VISHNU) [83–85]. The usage of such a hybrid approach provides the possibility of investigating the influence of the hadronic stage on the development of elliptic flow for the different particle species. It also provides an excellent testing ground for the particles that are estimated to have small hadronic cross section (φ, Ξ) and are thus expected not to be affected by this stage. VISHNU uses the MC-KLN model [86] to describe the initial conditions, an initial time after which the hydrodynamic
Figure 6. The $p_T$-differential $v_2$ for different particle species in (a), (b), (e), and (f), measured with the scalar product method with a pseudo-rapidity gap $|\Delta\eta| > 0.9$ in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, compared to theoretical, hydrodynamic calculations coupled to a hadronic cascade model [80–82]. The panels (c), (d), (g), and (h) show the dependence of the ratio of the experimental points to a fit over the theoretical calculations as a function of $p_T$. The left and right plots present the comparison for the 10–20% and 40–50% centrality intervals, respectively. The low transverse momentum points for $p+\bar{p}$ are out of scale in panels (c) and (d).
evolution begins at $\tau_0 = 0.9 \text{ fm}/c$ and a value of $\eta/s = 0.16$, almost two times the lower bound of $1/4\pi$ (for $\hbar = k_B = 1$). The transition from the hydrodynamic description to the microscopic evolution of the hadronic matter is done at a temperature of $T = 165$ MeV. More information about the hadronic cascade model can be found in [87, 88]. These theoretical calculations are represented in figure 6 by the different curves with the line colour matching that of the experimental measurement for each species.

Figures 6-(a), (b), (e), (f), present the $p_T$-differential $v_2$ for different particle species, while figure 6-(c), (d), (g) and (h) show the ratio of the measurement to a fit to the theoretical calculations as a function of $p_T$. It is seen that VISHNU gives a qualitatively similar picture with a similar mass ordering to that seen experimentally for most particle species.

For more central collisions the measured $v_2(p_T)$ for the $\pi^\pm$ is systematically above the theoretical calculations for $p_T < 2\text{ GeV}/c$, whereas the kaon measurement is described fairly well for the same range. In addition, the model calculations appear to underestimate significantly the elliptic flow for protons, but overestimate $v_2$ of $\Xi^- + \Xi^+$. This multi-strange baryon is estimated to have small hadronic cross sections and thus could be unaffected from the hadronic rescattering in the later stages of the collision [50–53]. Furthermore, for $\Lambda + \bar{\Lambda}$, the model does not preserve the mass ordering observed in the experiment and overestimates the $v_2$. This could indicate that the implementation of the hadronic cascade phase and the hadronic cross-sections within the model need further improvements.

Finally, the $\phi$-meson was argued to reflect the properties of the early partonic stages in the evolution of the system, being less affected by the hadronic interactions. The latter is suggested by phenomenological calculations to stem from the small hadronic interaction cross section of the $\phi$-meson [54]. It is seen that VISHNU systematically overestimates $v_2(p_T)$ and expects that the measurement does not follow the mass ordering for $p_T < 2\text{ GeV}/c$. This might be an indication that the $\phi$-meson’s hadronic cross section is underestimated in these calculations.

For peripheral collisions, the model calculations agree better with the results for $\pi^\pm$, K and $\Lambda + \bar{\Lambda}$. However, VISHNU under-predicts the $v_2(p_T)$ values of $p + \bar{p}$ and over-predicts the values for K, $\phi$ and $\Xi^- + \Xi^+$.

### 6.2 Comparison with RHIC results at $\sqrt{s_{NN}} = 0.2$ TeV

The mass ordering in the $p_T$-differential $v_2$ and the qualitative agreement with hydrodynamic calculations were first reported in Au-Au collisions at RHIC energies by both STAR [42–44] and PHENIX experiments [45–48]. In addition, one of the first experimental observations at the LHC [15] was that the $p_T$-differential $v_2$ for inclusive charged particles remains almost unchanged between RHIC and LHC for several centrality intervals. On the other hand, the integrated $v_2$ values at the LHC were about 30% higher compared with RHIC. The comparison of the $v_2(p_T)$ values for different particle species in these two different energy regimes could provide additional insight into the dynamics of anisotropic flow and the effect of radial expansion of the system.

Figure 7 presents the comparison between the measurements for $\pi^\pm$, K and $p + \bar{p}$ performed at the LHC and the results from Au-Au collisions at $\sqrt{s_{NN}} = 200\text{ GeV}$ from STAR [44] and from PHENIX [48]. The comparison is based on the 10–20% centrality
interval, one of the most central classes, where the values of the transverse expansion velocity extracted from a blast-wave fit to the identified particle spectra are $0.57 \pm 0.01\text{(stat.)}$ at RHIC [10] and $0.639 \pm 0.004\text{(stat.)} \pm 0.022\text{(syst.)}$ at the LHC [71]. The $v_2(p_T)$ from STAR is calculated using the two particle cumulant analysis (i.e. $v_2\{2\}(p_T)$) [44], while PHENIX reconstructed $v_2(p_T)$ using the event plane method with a pseudo-rapidity gap of $|\Delta \eta| > 1.0$ [48]. These two measurements have different sensitivity to non-flow effects, which makes a quantitative comparison difficult.

At low values of transverse momentum ($p_T < 1.5\text{ GeV}/c$) the $v_2(p_T)$ reported from STAR and ALICE exhibits qualitatively similar behavior. On the other hand, for $p_T > 1.5\text{ GeV}/c$ for $\pi^\pm$ and $K^\pm$ and for $p_T > 2.5\text{ GeV}/c$ for $p+p$, the $v_2$ measurements at the LHC are significantly higher than those at the lower energies. Although this direct quantitative comparison might be subject to e.g. different non-flow contributions, spectra, radial flow, the qualitative picture that emerges from the $p_T$-differential $v_2$ appears similar at the LHC and RHIC.

### 6.3 Test of scaling properties

One of the experimental observations reported at RHIC was that at intermediate values of transverse momentum, particles tend to group based on their hadron type [42, 43, 45–47] i.e. baryons and mesons. It was also reported that if both $v_2$ and $p_T$ are scaled by the number of constituent quarks ($n_q$), the various identified hadron species approximately follow a common behaviour [42, 43, 45–47]. The PHENIX Collaboration suggested extending the scaling to the lower $p_T$ region by plotting elliptic flow as a function of the transverse kinetic energy defined as $KE_T = m_T - m_0$, where $m_T = \sqrt{p_T^2 + m_0^2}$ is the transverse mass [45–47]. Initially, this representation was observed to work well at RHIC energies. However, recent publications report deviations from this scaling for Au-Au collisions [48].

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**Figure 7.** The comparison of the $p_T$-differential $v_2$ for $\pi^\pm$, $K$ and $p+p$ for the 10–20% centrality class of Pb-Pb and Au-Au collisions at the LHC and RHIC, respectively. The RHIC points are extracted from [44] (STAR) and [48] (PHENIX).
Such baryon versus meson grouping triggered significant theoretical debate over its origin. The effect was successfully reproduced by models invoking quark coalescence as the dominant hadronization mechanism in this momentum range [56–59]. Thus, the number of constituent quark (NCQ) scaling of $v_2$ has been interpreted as evidence that quark degrees of freedom dominate in the early stages of heavy-ion collisions when collective flow develops [56–59].

To test the scaling properties of $v_2$, $v_2/n_q$ is plotted as a function of $p_T/n_q$ in figure 8 for $\pi^{\pm}$, K, p+\(\bar{p}\), $\phi$, $\Lambda+\bar{\Lambda}$, and $\Xi^{-}+\bar{\Xi}^{+}$. In the intermediate transverse momentum region (i.e. $3 < p_T < 6$ GeV/$c$ or for $p_T/n_q > 1$ GeV/$c$), where the coalescence mechanism is argued to be dominant [42–48, 55–59], the measurements at the LHC indicate that the scaling is only approximate. The magnitude of the observed deviations seems to be similar for all centrality intervals.
To quantify the deviation, the $p_T/n_q$ dependence of $v_2/n_q$ for $p$ and $\bar{p}$ is fitted with a seventh order polynomial function and the ratio of $(v_2/n_q)/(v_2/n_q)_{\text{Fit}p}$ for each particle species is calculated. The corresponding $p_T/n_q$ dependence of this double ratio is presented in figure 9 for the various centrality intervals. Figure 9 illustrates that for $p_T/n_q > 1 \text{ GeV}/c$ the data points exhibit deviations from an exact scaling at the level of $\pm 20\%$ with respect to the reference ratio for all centrality intervals.

Figure 10 presents the $(m_T - m_0)/n_q$ dependence of $v_2/n_q$. In this representation, introduced to extend the scaling to low values of transverse momentum, the data points illustrate significant deviations for $(m_T - m_0)/n_q < 0.6 - 0.8 \text{ GeV}/c^2$. For the intermediate region the scaling, if any, is approximate for all centrality intervals. To quantify these deviations, in figure 11 the $(m_T - m_0)/n_q$ dependence of $v_2/n_q$ for $p$ and $\bar{p}$ are fitted with a seventh order polynomial function and the double ratio of $(v_2/n_q)/(v_2/n_q)_{\text{Fit}p}$ for each particle species is then formed. It is seen that there is no scaling for $(m_T - m_0)/n_q < \ldots$
Figure 10. The \((m_T - m_0)/n_q\) dependence of \(v_2/n_q\) for \(\pi^\pm\), K, p+\(\bar{p}\), \(\phi\), \(\Lambda+\bar{\Lambda}\), and \(\Xi^-+\bar{\Xi}^+\) for Pb-Pb collisions in various centrality intervals at \(\sqrt{s_{NN}} = 2.76\) TeV.

0.6 – 0.8 GeV/c\(^2\), while for higher values there are deviations at the level of \(\pm 20\%\) with respect to the reference ratio for all centrality intervals.

Figure 12 presents the comparison of the \(p_T/n_q\) dependence of the double ratio of \(v_2/n_q\) for \(\pi^\pm\), K relative to a fit to \(v_2/n_q\) of p and \(\bar{p}\) for both the LHC and RHIC energies. The RHIC data points are extracted from [48]. It is seen that the deviations at intermediate values of transverse momentum are qualitatively similar at the two energy regimes. However, there are differences in the \(p_T/n_q\) evolution of this double ratio for \(\pi^\pm\) and K between ALICE and PHENIX.

Figure 13 presents the comparison of the \((m_T - m_0)/n_q\) dependence of the double ratio of \(v_2/n_q\) for \(\pi^\pm\), K relative to a fit to \(v_2/n_q\) of p and \(\bar{p}\) between ALICE and PHENIX [48]. As in figure 12, the deviations are qualitatively similar at the two energy regimes but the \((m_T - m_0)/n_q\) evolution of the double ratio is different for \(\pi^\pm\) and K at the LHC and RHIC.
Figure 11. The \((m_T - m_0)/n_q\) dependence of the double ratio of \(v_2/n_q\) for every particle species relative to a fit to \(v_2/n_q\) of \(p\) and \(\bar{p}\) (see text for details) for Pb-Pb collisions at \(\sqrt{s_{NN}} = 2.76\) TeV.

7 Conclusions

In summary, the first measurements of \(v_2\) as a function of transverse momentum for \(\pi^\pm\), \(K^\pm\), \(K^0_S\), \(p+\bar{p}\), \(\phi\), \(\Lambda+\bar{\Lambda}\), \(\Xi^-+\bar{\Xi}^+\) and \(\Omega^-+\bar{\Omega}^+\) for various centralities of Pb-Pb collisions at \(\sqrt{s_{NN}} = 2.76\) TeV were reported. The second Fourier coefficient was calculated with the Scalar Product method, using a pseudo-rapidity gap of \(|\Delta\eta| > 0.9\) between the identified hadron under study and each of the reference flow particles.

A distinct mass ordering was found for all centralities in the low transverse momentum region i.e. for \(p_T < 3\) GeV/c, which is attributed to the interplay between elliptic and radial flow that modifies the \(v_2(p_T)\) according to particle mass. The \(v_2(p_T)\) for heavy particles appears to be shifted to higher \(p_T\) with respect to the \(v_2(p_T)\) values of light particles. In this transverse momentum range, the experimental points for \(\pi^\pm\) and \(K\) are described fairly well for peripheral collisions by hydrodynamic calculations coupled to a hadronic
Figure 12. The $p_T/n_q$ dependence of the double ratio of $v_2/n_q$ for $\pi^\pm$, K relative to a fit to $v_2/n_q$ of p and $\bar{p}$ (see text for details) in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. The LHC points are compared with the results from Au-Au collisions at $\sqrt{s_{NN}} = 0.2$ TeV from [48].

Figure 13. The $(m_T - m_0)/n_q$ dependence of the double ratio of $v_2/n_q$ for $\pi^\pm$, K relative to a fit to $v_2/n_q$ of p and $\bar{p}$ (see text for details) in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. The LHC points are compared with the results from Au-Au collisions at $\sqrt{s_{NN}} = 0.2$ TeV from [48].

cascade model (VISHNU) indicating that a small value of $\eta/s$ (close to the lower bound) is favoured. However, for central collisions and for heavy particles, the same theoretical calculations tend to overestimate (i.e. $\Lambda$, $\Xi$) or underestimate (i.e. p) the measured $v_2$. VISHNU fails to describe the measured $v_2$ of $\phi$, which could be an indication that this particle has a larger hadronic cross section than its current theoretical estimate.

In the intermediate transverse momentum region (i.e. $3 < p_T < 6$ GeV/c), where at RHIC there was evidence that coalescence is the dominant hadronization mechanism, our data exhibit deviations from the number of constituent quark (NCQ) scaling at the level of $\pm 20\%$. 
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References


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