Growth or stagnation in pre-industrial Britain? A revealed income growth approach

Groth, Christian; Persson, Karl Gunnar

Publication date:
2016

Jan 2016 No.264

Growth or stagnation in pre-industrial Britain?
A revealed income growth approach

Christian Groth and Karl Gunnar Persson

WORKING PAPER SERIES

Centre for Competitive Advantage in the Global Economy

Department of Economics

Growth or stagnation in pre-industrial Britain?
A revealed income growth approach

Christian Groth and Karl Gunnar Persson[1]
Department of Economics, University of Copenhagen

Version 6.3. January 28, 2016

Abstract

The extent of growth in pre-industrial Europe in general and in Britain in particular has attracted intense scholarly focus. Growth or Malthusian stagnation? No consensus has evolved. Reconstructions of national income from 1300 and up to the Industrial Revolution come to opposing conclusions and so do econometric studies. Applying Engels' law, we suggest a new approach in which income growth is revealed by changes in occupational structure. Data needed for this approach are less contested than the wage and output series used in the existing literature. We find that pre-industrial Britain exhibited secular rise in the standard of living.

Key words: Malthusian stagnation; Engel's law; Revealed income growth; Pre-industrial productivity growth; Structural change.

JEL classification: E24, N13, O11, O41, O47.

[1] chr.groth@econ.ku.dk (corresponding author); karlgunnar.person@econ.ku.dk. We are grateful to Leigh Shaw-Taylor for advice and helpful discussion on the evolution of the occupational structure in Britain, and we wish to thank John Dodgson, Paul Sharp, and participants at the MEHR seminar, University of Copenhagen, the CAGE Seminar on British Economic Growth, British Academy, the EHES Conference, Pisa University, and the Karl Farmer Festakt, Graz University, October 2015, for useful comments.

Introduction

Was the British economy before the Industrial Revolution in a Malthusian state, exhibiting fluctuations in income per capita but with no upward trend (Clark et al., 2012), or was there slow but positive income growth? That question relates to an earlier discussion of the nature of medieval and Early Modern Britain and Europe in which Postan (1966) and Le Roy Ladurie (1977) set the tone arguing that the Medieval and Early Modern economy was trapped in Malthusian stagnation, “histoire immobile”. Clark’s work relates to that earlier discussion and he maintains that “all the major implications of the Malthusian model hold true for the world in the years before 1800” (2007, p. 39). Recently the debate regarding Britain has been revitalized by a number of studies, helas, without any sign of an emerging consensus view. Two ambitious historical national account reconstructions of English/British[2] economic development over the very long run have been published (Broadberry et al., 2015, Clark 2010) but they arrive at conflicting results. Broadberry et al. estimate national income and per capita income growth using output data while Clark is using income data, that is, wage, capital income, and land rent data. In principle results from these reconstructions should be similar if not identical, but they differ profoundly. Clark suggests that per capita income was stationary from the 14th century until the 19th century but Broadberry et al. find a modest but positive long run growth, which accelerates in the mid-17th century, that is, well before the Industrial Revolution.

Historical reconstructions of national income are based on the same methodology as modern national accounting but have to rely on fragile and fragmentary data. There is a number of time series for which robustness and representativeness can be questioned, which probably explains part of the differing results. For example, the results from Clark depend crucially on the particular real wage series he is using and the controversial assumption that the number of working days was constant over time. Wage series are not used at all in the Broadberry et al. national income estimates but reconstructions from output data are also troubled by the uncertainty of the data. To reach an estimate of agricultural output, for example, you need to establish the acreage, its division between tillage and pasture, the size of herds, the relative importance of different crops and their yields. Each step in this estimation procedure is done with a margin of uncertainty. As a consequence, various estimates using similar methodology but different assumptions regarding yields and acreage arrive at widely different estimates of agricultural output around 1700 and later in the 18th century. Estimates of the calorie consumption per capita differ by a factor of 1.7 (Kelly and Ó Gráda, 2013).

A number of recent econometric studies of England/Britain focus on the fundamental Malthusian view that real wages were stationary in the long run and that technological progress only led to transitory income increases and a permanently larger population. Focusing on Early

[2] We follow the convention using data for England up to 1700 and Britain, that is England, Scotland, and Wales, after that date.

Modern Britain, Møller and Sharp (2014) applied co-integration analysis to test whether real wages and population behaved according to the Malthusian thesis. That is, wage increases above "subsistence" should only be transitory, triggering off an increase in population which ultimately will bring down wages. Unlike other recent studies, such as Nicolini (2007) and Crafts and Mills (2009), Møller and Sharp find that wages were *not* stationary. While higher wages stimulated population growth, wages were not affected negatively by population growth. They invoke Smithian and Boserupian mechanisms in which population growth fosters technological progress and gains from division of labour. Crafts and Mills (2009), however suggest that wages in Preindustrial Britain were stationary despite the fact that they use the same wage data as Møller and Sharp. Results are evidently sensitive to econometric methodology.

In the absence of a consensus view, the motivation for this paper is to offer an alternative approach which does not use controversial or contested data. The approach needs little data, in fact "just" occupational and wage premium data, which are comparatively robust, together with six parameters. What we propose is a *revealed income growth* approach. *Revealed* indicates that we derive income changes from observed changes in occupational structure which in turn are linked to changes in consumption patterns. The logic builds on Engel's law. This is the empirically supported claim that as income increases, a falling share of income is used for consumption of food. A general income increase will thus imply a shift in the production structure leading to an increasing share of industry and services in production. There are a number of earlier attempts to measure income growth by looking at changes in occupational structure and consumption patterns. Wrigley (1967) analyzed pre-industrial England in a pioneering article. Persson (1991) generalized Wrigley's intuition analyzing Medieval "Low Countries" and Tuscany. And Allen (2000) used a similar method and applied it to pre-industrial Europe. However, none of these attempts build on a coherent and fully articulated economic model. One purpose of this paper is to take steps filling that void.

The paper is organized as follows. In Section 1 the basics of the model is exposed. Section 2 derives the analytical properties of the model and develops a convenient growth-indicating formula. In Section 3 we present the data used in the calibration of the model. Section 4 presents the numerical results, and sensitivity checks are carried out. In Section 5 we conclude. Some microeconomic aspects and proofs have been relegated to the appendices.

1. The model

There are two production sectors, Sector 1, interpreted as agriculture, and Sector 2, interpreted as industry and services, for convenience sometimes called the "urban" sector. The reservation indicated by the quotation marks is motivated by the fact that some of the industrial activities certainly took place outside towns.

1.1 Production

Sector 1 produces basic food, agrarian capital goods (think of cattle), and intermediates (think of wool), the latter demanded by Sector 2. The production technology is Cobb-Douglas, using labour, capital (produced within the sector itself), land, and intermediate goods delivered from Sector 2 as inputs. Sector 2 produces more refined consumption goods, including luxury goods, as well as urban capital goods (think of buildings and weaving equipment) and intermediates (say bricks and metal goods), the latter demanded by Sector 1. The production technology is Cobb-Douglas, using labour, capital (produced within the sector itself) and intermediate goods delivered from Sector 1 as inputs. The role of land as input in Sector 2 is diminutive, hence ignored. The technology in both sectors has constant returns to scale. What we call "output" of a sector is really *net output* of the sector in the sense of the output remaining after subtracting input of raw materials produced by the sector itself. The terms "raw materials" and "intermediate goods" are synonyms and refer to inputs used up in the single production process. In contrast, "capital goods" are durable means of production. Foreign trade is ignored.

With Q_{1t} and Q_{2t} denoting (net) output of sector-1 and sector-2 goods, respectively, in period *t*, the aggregate production functions are thus given by

$$Q_{1t} = A_{1t} X_{2t}{}^{\alpha} L_{1t}{}^{\beta} K_{1t}{}^{\gamma} Z^{1-\alpha-\beta-\gamma}, \qquad 0<\alpha<\alpha+\beta<\alpha+\beta+\gamma<1, \tag{1.1}$$

and

$$Q_{2t} = A_{2t} X_{1t}{}^{\theta} L_{2t}{}^{\varepsilon} K_{2t}{}^{1-\theta-\varepsilon}, \qquad 0<\theta<\theta+\varepsilon<1, \tag{1.2}$$

where X_{it} is input of intermediates *from* sector i, L_{it} is input of labor (standardized man-years) and K_{it} input of capital *in* sector i, Z is land (a constant), and A_{it} is total factor productivity in sector i, i = 1, 2. Time is discrete, t = 0, 1, 2,…, T, and the period length is one year, which is also our time unit.

An important feature of the described technology is the input-output interdependencies between the two sectors implied by the intermediates. We apply Cobb-Douglas specifications also regarding these inputs, thereby assuming that between all pairs of inputs within a sector, the elasticity of substitution equals one. In particular regarding agrarian intermediates in urban production this may entail too high substitutability. If we think of cloth production, the elasticity of substitution between wool and weaving equipment or wool and labour is likely to be small. Fixed technological proportionality between output of cloth and required input of wool might be a better approximation. We have chosen the Cobb-Douglas specification throughout for tractability reasons. But we shall keep this caveat in mind in the discussion later.

Competitive forces are operating within sectors and there is free mobility of produced goods across sectors. Let p_t denote the relative price of sector-2 goods in terms of sector-1 goods, and let the agrarian good be the numeraire throughout. Then value added (gross) in the two sectors are

$$\begin{aligned} Q_{1t} - p_t X_{2t} &\equiv Y_{1t} = w_{1t} L_{1t} + r_{1t} K_{1t} + r_{zt} Z, \\ p_t Q_{2t} - X_{1t} &\equiv Y_{2t} = w_{2t} L_{2t} + r_{2t} K_{2t}, \end{aligned} \tag{1.3}$$

where r_{zt} is land rent, w_{1t} and w_{2t} are the agrarian and urban wage rates, respectively, and r_{1t} and r_{2t} are the gross returns, or "rental rates", per unit of agrarian and urban capital, respectively. There is no economy-wide labour market. Hence w_{1t} and w_{2t} will generally differ. We think of production in the agrarian sector as typically carried out by farmer-entrepreneurs who rent land from large land owners and produce agricultural goods, using agrarian capital, family labor and in general a few hired agrarian workers; the capital is financed by inheritance, own saving, and to a limited degree leasing and loans within the sector. Production in the urban sector is governed by merchant-entrepreneurs who own some urban production capital and hire urban workers to produce manufactures and services; the capital may be financed by inheritance, own saving, and to a limited degree leasing and loans within the sector. To what extent land is traded rather than inherited from father to son is immaterial for our analysis.

Available data indicate a persistent "urban premium" or "skill premium" (w_{2t} / w_{1t} > 1).[3] Even if this premium reflects a higher skill level in urban production, this skill need not be due only to costly craft's apprenticeship. Part of the background may be that costless learning by doing and learning by watching is an appendage to being an urban citizen. Should we then not expect the urban premium to be eliminated through migration? No. In the short and medium run, mobility of labour was limited (curbed supply of apprenticeships for instance). Hence, we consider the sectoral labor supplies (until further notice equal to the employment levels L_{1t} and L_{2t}), along with the respective capital stocks, as state variables and thereby at any time determined by previous history.[4] In the longer run, however, the pulling force from a higher wage level in urban areas tends to partly erode the barriers to migration. At the same time total factor productivities may be rising in both sectors, perhaps faster in the urban sector, and on net leave the urban premium more or less unchanged.

[3] Clark (2010), Table 1.

[4] This means that from an aggregate production point of view, at any given point in time, subject to the proviso that there is "full employment", the only "choice variables" are X_{1t} and X_{2t}. In Appendix D we briefly consider implications of taking open or disguised unemployment into account.

Economically motivated migration in the longer run from agrarian areas to towns and formation of new towns are thus consistent with the model.[5] But for our purpose there is no reason to rule out alternative or complementary explanations of the empirically observed growth in L_{2t} / L_{1t} in pre-industrial Great Britain (along with a rising total population). For instance, at least theoretically it is conceivable that the higher urban standard of living enabled by the urban premium leads to higher fertility and/or lower mortality in urban areas, thus resulting in growth in L_{2t} / L_{1t} even in the absence of migration. Empirically, however, the disease burden seems to have been larger in cities, so that this particular alternative explanation does not seem plausible. Nevertheless we underline the susceptibility to alternative explanations, because the point of the simple bookkeeping to be carried out below is that the actual evolution of L_{2t} / L_{1t}, whatever its origin, is tantamount to a rising general wage level and, under reasonable parameter values, a rising per capita income.

From now on we omit the explicit dating of the variables unless needed for clarity. We assume that market forces tend to come through sooner or later so as to allow us to describe prices, regional wages, and allocation of goods as being competitively determined. Entrepreneurs choose inputs with the aim of maximizing profits, taking total factor productivity and wages and prices as given. Profit maximization in the agrarian sector thus implies

$$
\begin{aligned}
&\partial Q_1 / \partial X_2 = \alpha A_1 X_2{}^{\alpha-1} L_1{}^{\beta} K_1{}^{\gamma} Z^{1-\alpha-\beta-\gamma} = \alpha Q_1 / \mathrm{X}_2 = p, \\
&\partial Q_1 / \partial L_1 = \beta A_1 X_2{}^{\alpha} L_1{}^{\beta-1} K_1{}^{\gamma} Z^{1-\alpha-\beta-\gamma} = \beta Q_1 / L_1 = w_1, \\
&\partial Q_1 / \partial K_1 = \gamma A_1 X_2{}^{\alpha} L_1{}^{\beta} K_1{}^{\gamma-1} Z^{1-\alpha-\beta-\gamma} = \gamma Q_1 / K_1 = r_1, \\
&\partial Q_1 / \partial Z = (1-\alpha-\beta-\gamma) A_1 X_2{}^{\alpha} L_1{}^{\beta} K_1{}^{\gamma} Z^{-\alpha-\beta-\gamma} = (1-\alpha-\beta-\gamma) Q_1 / Z = r_z.
\end{aligned}
\tag{1.4}
$$

And profit maximization in the urban sector implies

$$
\begin{aligned}
&p\partial Q_2 / \partial X_1 = p\theta A_2 X_1{}^{\theta-1} L_2{}^{\varepsilon} K_2{}^{1-\theta-\varepsilon} = p\theta Q_2 / \mathrm{X}_1 = 1, \\
&p\partial Q_2 / \partial L_2 = p\varepsilon A_2 X_1{}^{\theta} L_2{}^{\varepsilon-1} K_2{}^{1-\theta-\varepsilon} = p\varepsilon Q_2 / L_2 = w_2, \\
&p\partial Q_2 / \partial K_2 = p(1-\theta-\varepsilon) A_2 X_1{}^{\theta} L_2{}^{\varepsilon} K_2{}^{-\theta-\varepsilon} = p(1-\theta-\varepsilon) Q_2 / K_2 = r_2.
\end{aligned}
\tag{1.5}
$$

In a partial equilibrium perspective these equations describe input demands at firm level for given output and factor prices. A remark on interpretation of the "rental rates" r_1 and r_2 seems pertinent, though. With well-developed capital markets, even if only within each sector, r_1 and r_2 would appear as market-determined factor-prices satisfying the no-arbitrage conditions $r_1 = \hat{r}_1 + \delta_1 \cdot 1$ and $r_2 = \hat{r}_2 p + \delta_2 p - \Delta p$, respectively, where $\hat{r}_1$ and $\hat{r}_2$ are equilibrium interest rates in the respective sectors and δ_1 and δ_2 are the *physical* depreciation rates for the two kinds of

[5] Cf. Appendix E.

capital, while Δp measures the capital gain on urban capital. But in view of the very fragmented capital markets in the pre-industrial era, it seems unworldly to assume that entrepreneurs faced such clearly defined market-determined and impersonal capital costs. Fortunately, however, the equation system (1.4)-(1.5) is valid even without well-developed capital markets. Owing to constant returns to scale, ignoring the third lines in (1.4) and (1.5), the remaining equations fully characterize profit maximization in the two sectors, given the price vectors $(1, p, w_1, r_z)$ and $(p, 1, w_2)$, respectively. These equations actually *imply* the third lines in (1.4) and (1.5), respectively, when we interpret the "rental rates" r_1 and r_2 as just representing the *obtained* returns per unit of capital of each kind, given the *other* prices and the inputs of intermediates, labour, and capital in the respective sectors.

In a full-employment general equilibrium perspective the interpretation of the system (1.4)-(1.5) is that it describes (a) the equilibrium output and factor prices and returns appearing on the right-hand sides and (b) the traded quantities of intermediates, X_1 and X_2, that in the given period are consistent with the demand side of the economy and the given supply factors (i.e., the total factor productivities, the agrarian and urban capital and labour supplies, and the amount of land).[6]

1.2 Some national income accounting

In view of (1.3), (1.4) and (1.5), value added, and thereby income, in the agrarian and urban sectors can be written

$$\begin{aligned} Y_1 &= (1-\alpha)Q_1, \\ Y_2 &= (1-\theta)pQ_2. \end{aligned} \tag{1.6}$$

Gross national product, GNP, is

$$Y \equiv Y_1 + Y_2 = w_1 L_1 + w_2 L_2 + r_1 K_1 + r_2 K_2 + r_z Z, \tag{1.7}$$

where the last equality follows from (1.3) and indicates GNP as the sum of labour income, gross operating profit, and land rent.

Output in each sector is used partly as raw material in the other sector, partly as investment, I_1 and I_2, within *own* sector and partly for consumption, C_1 and C_2. Thus

$$\begin{aligned} Q_1 &= X_1 + I_1 + C_1, \\ Q_2 &= X_2 + I_2 + C_2, \end{aligned} \tag{1.8}$$

[6] At least in the short run all these factors are given.

where investment gives rise to changed capital stocks next period according to $\Delta K_1 = I_1 - \delta_1 K_1$ and $\Delta K_2 = I_2 - \delta_2 K_2$. Moreover, from (1.3) and (1.8) follows

$$Y \equiv Y_1 + Y_2 = Q_1 - X_1 + p(Q_2 - X_2) = C_1 + I_1 + p(C_2 + I_2) = C_1 + pC_2 + I_1 + pI_2, \tag{1.9}$$

reflecting that in our closed economy, GNP equals the value of aggregate consumption and aggregate gross investment. We ignore the public sector and taxation.

1.3 Households

Let N denote the size of the total working-age population. Suppose all households are of the same size. To ease notation, we normalize this size to be 1 working-age person so that the number of households equals the size of the working-age population. In year t each household inelastically supplies h units of labour[7] to non-homework activities; a possible time-dependence of h is implicit. Under full employment we thus have

$$L_1 + L_2 \equiv L = hN. \tag{1.10}$$

There are three categories of households: land owners, entrepreneurs, and workers. Each of the two latter categories is sub-divided into an agrarian and urban sub-class. In spite of household incomes varying extensively across these categories, we assume households have the same consumption *function*. Consider an arbitrary household with annual (gross) income $y > 0$. Let b denote the necessary "basic food" per year, a constant. Although b may exceed the biological minimum, we shall for convenience refer to b as subsistence minimum. If "residual income" $y - b$ is positive, a constant fraction σ of this is saved and the remainder is split into a constant fraction, m, used for agrarian consumption goods while the rest is used for urban consumption goods. So annual consumption of agrarian goods by the household is

$$c_1 = \begin{cases} y & \text{if } y < b \qquad (b > 0), \\ b + m(1-\sigma)(y-b) & \text{if } y \geq b \qquad (0 \leq m < 1, 0 < \sigma < 1). \end{cases} \tag{1.11}$$

Consumption of urban goods and saving, respectively, are given by

$$\begin{aligned} c_2 &= \begin{cases} 0 & \text{if } y < b, \\ (1-m)(1-\sigma)(y-b)/p & \text{if } y \geq b, \end{cases} \\ s &= \begin{cases} 0 & \text{if } y < b, \\ \sigma(y-b) & \text{if } y \geq b. \end{cases} \end{aligned} \tag{1.12}$$

[7] By a unit of labour is meant a standardized man-year, i.e. a certain number of hours per worker per year.

This behavior is a key element in the model and reflects Engel's law claiming that the expenditure on basic food falls as a share of income when income rises (above *b*).[8]

We assume society can at least "reproduce" itself. As a substantial part of the population has essentially only labour income, we identify reproducibility with the condition

$$w_1 h \geq b \quad \text{and} \quad w_2 h \geq b. \tag{1.13}$$

This condition ensures that no social class is below subsistence minimum, and so aggregate consumption demands are

$$\begin{aligned} C_1^d &= bN + m(1-\sigma)(Y - bN), \\ pC_2^d &= (1-m)(1-\sigma)(Y - bN), \end{aligned} \tag{1.14}$$

while aggregate (gross) saving is

$$S = \sigma(Y - bN)\,. \tag{1.15}$$

We see that the income elasticity of consumption of agrarian goods is below one while the income elasticity of consumption of urban goods is above one. Moreover, a rising per capita income leads to a rising aggregate saving-income ratio. These features are in accordance with the informed view on the pre-industrial era.

In spite of not explicitly incorporating wealth and expected future earnings in the consumption/saving decision, we believe the above description gives a reasonable approximation of aggregate consumption and saving in a society with little developed capital markets and likely high correlation between incomes and wealth.

We imagine that households are either fully "agrarian", working full-time in agrarian production, or fully "urban", working full-time in urban production. This allows us to speak of "agrarian" households and "urban" households. We denote the number of each type N^1 and N^2, respectively. Thereby $N^1 + N^2 \equiv N$ and, in line with the assumption (1.10),

$$L_1 / N^1 = L_2 / N^2 = L / N = h. \tag{1.16}$$

Thus h signifies the general labor participation rate in the economy.

Given absence of economy-wide labour and capital markets, we also assume that the households of each type get all their income from the similarly indexed production sector and invest all their saving within that sector. Investment in the respective sectors thereby satisfies

[8] Appendix A briefly accounts for the link from preferences to the consumption behaviour (1.11)-(1.12).

$$I_1 = S^1 = \sigma(Y_1 - bN^1),$$
$$pI_2 = S^2 = \sigma(Y_2 - bN^2), \tag{1.17}$$

where S^j, $j = 1, 2,$ is the aggregate saving by households of type j. Thus $S^j = Y_j - C^{jd}$, where C^{jd} is the value of aggregate consumption demand by households of type j.[9] With C_i^{jd} denoting the quantity of Sector-*i* consumption goods demanded by households of type j, we have $C^{jd} \equiv C_1^{jd} + pC_2^{jd}$, $j = 1, 2$. Finally, $S^1 + S^2 = S$.

2. Analytics

For fixed t the following variables are predetermined: population, total factor productivities, and capital and labour supplies in each sector. Under full employment L_1 and L_2 equal the respective predetermined labour supplies and are in that sense given, while the wage rates w_1 and w_2 and adjust endogenously. By appropriate substitutions we shall reduce the model to two independent equations involving the wage rate w_1 and the wage ratio w_2 / w_1. The measurement unit for land is chosen such that $Z = 1$. We let k_i denote the capital-labour ratio in sector i, that is, $k_i \equiv K_i / L_i$, $i = 1, 2$. Let us first consider the relations between the variables as seen from the supply side.

2.1 The supply side

By the first line in (1.4), $X_2 = \alpha Q_1 / p$. Substituting this into (1.1) and isolating Q_1 yields

$$Q_1 = (\alpha^\alpha A_1 L_1^{\beta+\gamma} k_1^\gamma p^{-\alpha})^{1/(1-\alpha)}. \tag{2.1}$$

The second line in (1.4) gives $w_1 = \beta Q_1 / L_1$. By substituting (2.1) into this, we have

$$w_1 = \beta(\alpha^\alpha A_1 L_1^{-(1-\alpha-\beta-\gamma)} k_1^\gamma p^{-\alpha})^{1/(1-\alpha)}. \tag{2.2}$$

The second line in (1.5) gives $w_2 = \varepsilon p Q_2 / L_2 = \varepsilon p A_2 (X_1 / L_2)^\theta k_2^{1-\theta-\varepsilon}$, where the last equality comes from (1.2). Substituting into this that $X_1 / L_2 = \theta w_2 / \varepsilon$ (from combining the two first lines of (1.5)) and isolating w_2 yields

[9] In order not to confuse the sectors to which production and employment belong with the two "types" of households both of which split their consumption budget on both sectors' produce, we use subscripts for the sectors and superscripts for the household types. Then, for instance, C^{1d} refers to the value of agrarian households' aggregate consumption while C_1^d is aggregate demand for agrarian output.

$$w_2 = \varepsilon(\theta^{\theta} A_2 k_2^{1-\theta-\varepsilon} p)^{1/(1-\theta)}. \tag{2.3}$$

The asymmetry exhibited by the appearance of L_1 in (2.2) but neither L_1 nor L_2 in (2.3) is due to land being a production factor only in the agrarian sector. This factor is in fixed supply, thus resulting in diminishing returns to the ensemble of variable factors: intermediates, labor, and capital. In contrast, the urban sector has constant returns to scale with respect to these factors taken together.

In view of (2.2) and (2.3), the relationship between the urban-rural wage ratio (the urban premium) and *p* is given by

$$\frac{w_2}{w_1} = \frac{\varepsilon(\theta^{\theta} A_2 k_2^{1-\theta-\varepsilon} p^{(1-\alpha\theta)/(1-\alpha)})^{1/(1-\theta)}}{\beta(\alpha^{\alpha} A_1 L_1^{-(1-\alpha-\beta-\gamma)} k_1^{\gamma})^{1/(1-\alpha)}}. \tag{2.4}$$

Isolating *p* in (2.2) and substituting into (2.4) gives

$$\frac{w_2}{w_1} = \frac{\varepsilon(\alpha\beta^{(1-\alpha)/\alpha} A_1^{1/\alpha} k_1^{\gamma/\alpha} \theta^{\theta} A_2 k_2^{1-\theta-\varepsilon})^{1/(1-\theta)}}{L_1^{(1-\alpha-\beta-\gamma)/[\alpha(1-\theta)]}} w_1^{-\frac{1-\alpha\theta}{\alpha(1-\theta)}}. \tag{2.5}$$

For unchanged $A_1,\ A_2,\ k_1,\ k_2,$ and L_1, we thus have a *negative* equilibrium relationship between the urban premium w_2 / w_1 and the agrarian wage rate w_1. To get the intuition behind this, we go back to (2.2) and (2.3). Equation (2.2) exhibits a negative association between *p* and w_1 for given L_1. The explanation is that a higher price *p* of urban goods reduces use of urban goods as input in the agrarian sector, and due to technological complementarity this reduces the marginal productivity of labor, hence the wage, in that sector. On the other hand, Equation (2.3) exhibits a positive association between *p* and w_2. The explanation of this is that a higher *p* means higher marginal value products of the inputs in the urban sector. Combining these two features, we see that at the same time as a higher price of urban goods goes hand in hand with a lower w_1, it goes hand in hand with a higher w_2. This explains the negative relationship in (2.5) between the urban premium w_2 / w_1 and the agrarian wage rate w_1. We shall call (2.5) the *supply-side wage relation.* For later use, we note that this supply-side relation implies an absolute value of the partial elasticity of w_2 / w_1 with respect to w_1 equal to $(1-\alpha\theta)/(\alpha(1-\theta)$ > 1.

Observe also that for given w_1 and L_1, the urban premium is an increasing function of the total factor productivity, as well as the capital-labour ratio, in *both* sectors. The intuitive reason is as follows. Naturally, a higher total factor productivity as well as a higher capital-labour ratio in the *urban* sector generate a higher marginal productivity of labour within the sector, thus higher w_2 for given w_1. But why is it that *if* we observe a higher total factor productivity or a higher capital-labour ratio in the *agrarian* sector, at the same time as we observe that w_1 and L_1 are unchanged, *then* a higher w_2 can be inferred? The reason is that the two observations are only compatible if there is less input of intermediates delivered by the urban sector, cf. the second line in (1.4). In turn, this must be due to a higher price of these. This higher price of urban goods amounts to a higher marginal value product of labor in the urban sector, hence higher w_2 for the given w_1, as was to be explained.

2.2 Taking final demand into account

The amount of agrarian goods available for consumption is linked to w_1 and the urban premium w_2 / w_1 in the following way:

$$\begin{aligned} C_1 &= Q_1 - X_1 - I_1 = Q_1 - \frac{\theta}{\varepsilon} w_2 L_2 - \sigma(Y_1 - bN^1) = Q_1 - \frac{\theta}{\varepsilon} w_2 L_2 - \sigma((1-\alpha)Q_1 - bN^1) \\ &= \frac{1-\sigma(1-\alpha)}{\beta} w_1 L_1 - \frac{\theta}{\varepsilon} w_2 L_2 + \sigma b(N - N^2) = \left[\frac{1-\sigma(1-\alpha)}{\beta}(1-\ell) - \frac{\theta w_2 / w_1}{\varepsilon} \ell \right] w_1 L + \sigma b(1-\ell)N, \end{aligned} \tag{2.6}$$

where we have used (1.8), (1.5), (1.17), (1.6), (1.4), the identities $N \equiv N^1 + N^2$ and $\ell \equiv L_2 / L$ (the urban employment share), and finally (1.16).

Starting from (1.14) and using similar substitutions, we find the consumption demand for agrarian goods to be

$$\begin{aligned} C_1^d &= (1-m(1-\sigma))bN + m(1-\sigma)Y = (1-m(1-\sigma))bN + m(1-\sigma)\left[(1-\alpha)Q_1 + (1-\theta)pQ_2\right] \\ &= (1-m(1-\sigma))bN + m(1-\sigma)\left[\frac{1-\alpha}{\beta}(1-\ell) + \frac{(1-\theta)w_2 / w_1}{\varepsilon} \ell \right] w_1 L. \end{aligned} \tag{2.7}$$

We see that if w_1 and w_2 are raised by the same factor, then the consumption demand for agrarian goods is raised by *less* than that factor. This disproportionality reflects Engels' law, implying that the income elasticity of consumption of agrarian goods is below one while that of consumption of urban goods is above one.

By market clearing, the right-hand sides of (2.6) and (2.7) are equal. Hence, after rearranging,

$$\begin{aligned} w_1 \frac{L}{N} &= \frac{b\beta\left((1-m)(1-\sigma) + \sigma\ell\right)}{1-(1-\alpha)M - \left[1-(1-\alpha)M + \beta\varepsilon^{-1} J w_2 / w_1\right]\ell}, \quad \text{where} \\ M &\equiv \sigma + m(1-\sigma) \in (0,1) \quad \text{and} \quad J \equiv \theta + (1-\theta)m(1-\sigma) \in (0,1). \end{aligned} \tag{2.8}$$

This is our second equation linking the two endogenous variables, the urban premium w_2 / w_1 and the agrarian wage rate w_1, in equilibrium. Since $M < 1$, the term in square brackets is positive. Hence, for a given participation rate L / N and a given urban employment share ℓ, the equation shows that taking final demand into account, there is a *positive* relationship between w_2 / w_1 and w_1 in equilibrium. We shall call (2.8) the *demand-side wage relation*.

The explanation of this positive relationship lies in the asymmetry between (2.7) and (2.6). Imagine circumstances, for instance total factor productivities or capital-labour ratios, change so as to raise w_1 and w_2 by the *same* factor. Then the demand for agrarian consumption goods is raised *less* than the supply. This is implied by the unaffected term in (2.7) being larger than that in (2.6). Indeed, the difference between the two is $(1-m(1-\sigma))bN - \sigma b(N - N^2)$ $= (1-m)(1-\sigma)bN + \sigma bN^2 > 0$. To eliminate this excess supply of agrarian consumption goods, we imagine that circumstances instead change such that also p increases and, while doing so, makes w_2 rise by a factor *greater* than that by which w_1 rises, cf. (2.3). This will have two effects. *First*, it will reduce the *supply* of agrarian consumption goods. This is because the higher p increases the marginal value products of inputs in urban production, thereby stimulating the demand for intermediates from the agrarian sector, leaving less agrarian output available for consumption. Indeed, when a higher p leads to a higher w_2 / w_1, (2.6) shows that the supply of agrarian consumption goods is reduced compared with the situation where w_2 / w_1 is kept unchanged. *Second*, the *demand* for agrarian consumption goods will be stimulated in a situation with a higher w_2 / w_1 accompanying a given rise in w_1, everything else equal. This is because income in this situation automatically rises *more* than if w_2 / w_1 is kept unchanged along with the given rise in w_1. The higher income implies a boost to consumption in general, including consumption of agrarian goods, cf. (2.7). We conclude that an increase in w_2 / w_1 along with the rise in w_1 is needed to maintain clearing in the market for agrarian consumption goods. This is a first message summed up in equation (2.8).

A second message is that for unchanged participation rate L / N and unchanged urban premium w_2 / w_1, a higher urban employment share ℓ unambiguously reveals a higher agrarian wage rate w_1. In view of the unchanged ratio w_2 / w_1, this also implies a higher urban wage rate w_2. The intuition is that a higher urban employment share ℓ reflects a disproportionate expansion of urban goods relative to agrarian goods. According to Engels' law the demand for urban goods will only follow suit if there is a sufficient rise in income per capita.

2.3 Temporary equilibrium

Figure 1 illustrates determination of temporary equilibrium in an arbitrary year in the pre-industrial era. For fixed A_1, A_2, k_1, k_2 and L_1, the solid downward-sloping curve in Figure 1 represents the supply side wage relation (2.5); the tails of this curve have the two coordinate axes as asymptotes. For fixed L / N and ℓ, the solid upward-sloping curve represents the demand side wage relation (2.8); the upper horizontal stippled line indicates the asymptote for this curve. Any point $(w_1, w_2 / w_1)$ consistent with temporary equilibrium must be situated on both these solid curves. These curves will always intersect, and do so at one point only. This point, with coordinates $(w_1*, w_2*/w_1*)$, is denoted E in the figure and represents the temporary equilibrium in the year considered. It is an equilibrium in the sense that agents optimize given the circumstances, and goods and labour markets clear. It is temporary in the sense that in the next year, circumstances will generally be different, among other things as a consequence of the currently chosen actions. Capital stocks, for instance, change as a result of saving and investment.

Figure 1 about here

A little comparative statics may be useful. The dotted downward-sloping curve below the solid one in Figure 1 shows the position of the supply relation (2.5) for a larger agrarian employment, L_1', but unchanged A_1, A_2, k_1, and k_2. If simultaneously the urban employment is proportionally increased so as to leave the urban employment *share* unchanged at the level ℓ, the new equilibrium would be at the point E' in the figure. This represents what we may call a "population increase/scarcity of land effect". If instead the urban employment share is simultaneously *increased* to ℓ', the upward-sloping curve representing (2.8) shifts rightward to the dotted position marked ℓ'. The corresponding new equilibrium, denoted E'' in the figure, unambiguously has lower urban premium than the original equilibrium point E.[10] But whether it has w_1 lower or higher than w_1* depends on how much the urban employment share has risen meanwhile (the figure exhibits the former case). Considering, in addition, an increase in A_1, A_2, k_1, or k_2, the new equilibrium point will necessarily have higher values of both w_1 and w_2 / w_1 than the point E''. If the urban premium w_2 / w_1 is *unchanged* relative to the original equilibrium point E, the fact that the ℓ' curve *must* be to the right of the original ℓ curve ensures that the final equilibrium point E''' is to the right of E so that $w_1*''' > w_1*$.

Over the pre-industrial era from the sixteenth to the late eighteenth century the agrarian labour force has been rising in absolute terms, but declining in relative terms, i.e. the urban employment share ℓ has been rising. Meanwhile the urban premium w_2 / w_1 has been roughly constant at a value above 1 (Broadberry et al., 2015, Table 9.10, Clark 2010, Table 1). With unchanged participation rate L / N, such an evolution corresponds to a horizontal rightward shift from E to the point E''' in the figure. This means that both w_1 and w_2 have been increased and

[10] The shift from E' to E'' implies a percentage fall in $w_2 = (w_2 / w_1) w_1$ by $(1-\alpha\theta)/(\alpha(1-\theta)) - 1 > 0$ for every per cent w_1 is increased, cf. (2.5).

have been so in roughly the same proportion. This outcome reveals that a *sufficient* rise in one or several of the productivity-increasing variables A_1, A_2, k_1, or k_2 must have taken place with a per capita income effect large enough to more than overcome the "population increase/scarcity of land effect" mentioned above. Otherwise, the observed disproportionate rise in the urban labour force, not producing basic food, could not have occurred without decreasing w_2 / w_1. Moreover, in view of diminishing returns to capital, the ultimate source of continuing rises in income must be rising total factor productivities, A_1 and/or A_2.

With a moderate simultaneous increase in L / N, cf. Section 3, we get a slightly smaller rightward shift of the equilibrium position because the increase in L / N curbs the rightward shift of the upward-sloping (2.8) curve. The increase in L / N amounts to more work per capita per year and this accounts for a part of the rise in labour income per capita per year.

2.4 Existence of temporary equilibrium

It remains to check the parameter conditions needed for this "story" to be internally consistent. Since the story relies on the aggregate consumption function (1.14), it requires the reproducibility condition (1.13) satisfied. What are the necessary and sufficient conditions for existence of a temporary equilibrium satisfying the reproducibility condition (1.13)?

To answer this, consider Figure 2. This figure is similar to Figure 1 except that its focus is on the question of internal consistency. Observe the three points D, F, and G in the positive quadrant of Figure 2. The point D is fixed and has coordinates $(bN / L, 1)$. Thus D represents the case where both w_1 and w_2 are at the lower bound $b / h = bN / L$, cf. (1.13) and (1.16). The point F is some point on the stippled vertical line $w_1 = bN / L$. More precisely, F is the point of intersection between this line and an arbitrary member of the family of downward-sloping supply side (2.5)-curves. In the example shown in Figure 2 this member is marked by "L_1, A_1, A_2, k_1, k_2" and places F above D. Finally, the point G is a point on the stippled hyperbola defined by $w_2 / w_1 = (bN / L) w_1^{-1}$, hence passing through the point D. More precisely, G is the point of intersection between this hyperbola and the arbitrary (2.5)-curve through which F was defined. When a particular (2.5)-curve is chosen, both F and G are thus fixed. Another downward-sloping (2.5)-curve might be situated *below* the dotted one that goes through the point D shown in the figure. In that case F would be placed below D as well, and G would be placed to the left of D. The dotted (2.5)-curve through the point D in Figure 2 illustrates a case where G and D coincide. Note that the solid (2.5)-curve intersects the stippled hyperbola *from below* when moving from right to left. This property holds for any (2.5)-curve, i.e. independently of the position of its associated F (Lemma B.1 of Appendix B).

Define the right-hand side of equation (2.5) as a function $f(w_1, L_1, A_1, A_2, k_1, k_2)$, when convenient abbreviated to $f(w_1, \cdot)$. The point F has abscissa $w_1 = bN/L$, and so the ordinate of F can be written

$$\frac{w_2}{w_1} = \frac{\varepsilon(\alpha\beta^{(1-\alpha)/\alpha} A_1^{1/\alpha} k_1^{\gamma/\alpha} \theta^{\theta} A_2 k_2^{1-\theta-\varepsilon})^{\frac{1}{1-\theta}}}{L_1^{(1-\alpha-\beta-\gamma)/[\alpha(1-\theta)]}} \left(bN/L\right)^{-\frac{1-\alpha\theta}{\alpha(1-\theta)}}$$
$$\equiv f(bN/L, L_1, A_1, A_2, k_1, k_2) \equiv f(bN/L, \cdot) \equiv f(1,\cdot)\left(bN/L\right)^{-\frac{1-\alpha\theta}{\alpha(1-\theta)}}. \tag{2.9}$$

From Lemma B.1 of Appendix B follows that the point G, as defined above, always exists, is unique, and has abscissa

$$\overline{x} = f(bN/L, \cdot)bN/L. \tag{2.10}$$

LEMMA 1. Let the predetermined variables L_1, A_1, A_2, k_1, k_2, and L/N be given. Assume $f(bN/L, L_1, A_1, A_2, k_1, k_2) \geq 1$. Then the point F in Figure 2 is not below the point D. In addition, $\overline{x} \geq bN/L$, and there exists numbers $\underline{\ell}$, $\hat{\ell}$, and $\overline{\ell}$ with the following properties: (i)

$$0 < \underline{\ell} = \frac{1-(1-\alpha)M - \beta(1-m)(1-\sigma)}{1-(1-\alpha)M + \beta\sigma + \beta\varepsilon^{-1}Jf(bN/L,\cdot)} \leq \hat{\ell} = \frac{1-(1-\alpha)M - \beta(1-m)(1-\sigma)}{1-(1-\alpha)M + \beta\sigma + \beta\varepsilon^{-1}J}$$
$$\leq \overline{\ell} = \frac{1-(1-\alpha)M - \beta(1-m)(1-\sigma)bN/(\overline{x}L)}{1-(1-\alpha)M + \beta\sigma bN/(\overline{x}L) + \beta\varepsilon^{-1}Jf(\overline{x},\cdot)} < 1. \tag{2.11}$$

(ii) Considering the dotted upward-sloping curves in Figure 2 representing (2.8), the one passing through the point F has urban employment share $\ell = \underline{\ell}$, the one passing through the point D has urban employment share $\ell = \hat{\ell}$, and the one passing through the point G has urban employment share $\ell = \overline{\ell}$.

(iii) If $f(bN/L, L_1, A_1, A_2, k_1, k_2) = 1$, both F and G coincide with D, and $\underline{\ell} = \hat{\ell} = \overline{\ell}$.

(iv) If $f(bN/L, L_1, A_1, A_2, k_1, k_2) > 1$, F is above D, G is to the right of D, and $\underline{\ell} < \hat{\ell} < \overline{\ell}$.

Proof. See Appendix B.

Observe that both $f(bN/L,\cdot)$ and $f(1,\cdot)$ are time dependent, as they are dependent on L_1, A_1, A_2, k_1, k_2, and L/N. Hence, also $\underline{\ell}$ and $\overline{\ell}$ are time dependent.

Figure 2 about here

PROPOSITION 1. Let the predetermined variables L_1, A_1, A_2, k_1, k_2, and L/N be given, and let the numbers $\underline{\ell}$, $\hat{\ell}$, and $\overline{\ell}$ be defined as in (2.11). Then there are three mutually exclusive cases, (i), (ii), and (iii), characterised as follows:

Case (i): $f(bN/L, L_1, A_1, A_2, k_1, k_2) < 1$. No temporary equilibrium satisfying the reproducibility condition (1.13) exists.

Case (ii): $f(bN/L, L_1, A_1, A_2, k_1, k_2) = 1$. A temporary equilibrium satisfying the reproducibility condition (1.13) exists and is unique. It has urban employment share ℓ equal to $\hat{\ell}$ given in (2.11) and its wage rates satisfy $w_2 = w_1 = bN/L$ (both wage rates are at subsistence minimum).

Case (iii): $f(bN/L, L_1, A_1, A_2, k_1, k_2) > 1$. For every $\ell \in [\underline{\ell}, \overline{\ell}]$ a unique temporary equilibrium satisfying the reproducibility condition (1.13) and having urban employment share equal to this ℓ exists. Graphically the equilibrium is represented in Figure 2 by a point on the segment FG of the downward-sloping supply-side curve (2.5) going through the point F (which is situated above the point D in view of (iv) of Lemma 1). If $\ell = \underline{\ell}$, the equilibrium has $w_2 > w_1 = bN/L$. If $\ell = \overline{\ell}$, the equilibrium has $w_1 > w_2 = bN/L$. Finally, if $\ell \in (\underline{\ell}, \overline{\ell})$, the equilibrium has both wage rates above subsistence minimum (as exemplified by for instance the point E in the figure).

Proof. See Appendix B.

In the generic case (iii) of Proposition 1, a temporary equilibrium satisfying the reproducibility condition (1.13) *requires* an urban employment share $\ell \in [\underline{\ell}, \overline{\ell}]$. What this rules out is both a "too large" and a "too small" ℓ. On the one hand, a "too large" ℓ would imply low marginal productivity of labour in the urban sector, hence low w_2. This is a direct effect of a high L_2 and an indirect effect of a low L_1 limiting the availability of intermediates, X_1, delivered by the agrarian sector. Consequently, the subsistence requirement $w_2 L/N \geq b$ would be violated and the aggregate consumption function (1.14) could not be upheld. On the other hand, a "too small" ℓ would have a symmetric effect on marginal productivity of labour in the agrarian sector, hence violating the subsistence requirement $w_1 L/N \geq b$. In brief, $\ell \notin [\underline{\ell}, \overline{\ell}]$ means that at least one of the sectors in the economy is not productive enough to feed its population. The constraint $\ell \in [\underline{\ell}, \overline{\ell}]$ is thus a reflection of the inter-sectoral mutual dependence which is typical for a dual economy.

In our calibrations below we concentrate on a more narrow set of temporary equilibria, namely those that are consistent with an urban premium w_2/w_1 in the empirically relevant range $[1,\ 1.5]$, cf. Section 3.2. For any w_2/w_1 in this interval, we shall make sure that the parameter values we suggest on the basis of the data in Section 3 are within the bounds required for existence of a temporary equilibrium that has an urban employment ratio ℓ equal to the empirically observed one and at the same time satisfies the reproducibility condition (1.13). Sufficient for this is that for all $w_2 / w_1 \in [1,\ 1.5]$,

$$\frac{1-(1-\alpha)M-\beta(1-m)(1-\sigma)}{1-(1-\alpha)M+\beta\sigma+\beta\varepsilon^{-1}Jw_2 / w_1} \leq \ell < \frac{1-(1-\alpha)M}{1-(1-\alpha)M+\beta\varepsilon^{-1}Jw_2 / w_1}, \tag{2.12}$$

where M and J are functions of given parameters as defined in (2.8). We shall call this double inequality (2.12) the *consistency condition*. Other than the urban premium w_2 / w_1 and the urban employment share ℓ, the condition involves only parameters.

The lower weak inequality in (2.12) is equivalent to the right-hand side in (2.8) being weakly above subsistence minimum b which is in turn equivalent to ℓ being weakly above $\underline{\ell}$ in (2.11) with $f(bN/L,\cdot)$ replaced by w_2 / w_1. The upper strict inequality in (2.12) is equivalent to the requirement that the urban employment has not become so high as to make it impossible to maintain the imposed value of w_2 / w_1. Indeed, as appears from the horizontal stippled line in Figure 2 which is asymptotic to the upward-sloping (2.8)-curve for $\ell = \overline{\ell}$, $\overline{\ell}$ could become so large (and still less than 1) that the corresponding (2.8)-curve is below the horizontal stippled line w_2 / w_1 = 1 for *all* values of $w_1 < \infty$. In fact, the upper bound for ℓ in (2.12) equals the limiting value of $\overline{\ell}$ in (2.11) with $f(\overline{x},\cdot)$ replaced by w_2 / w_1 and the agrarian wage rate $\overline{x}$ approaching infinity. The intuition is that, everything else equal, a still higher ℓ implies a still lower marginal productivity of urban labour and a still higher marginal productivity of agrarian labour, thus making it still harder for rising total factor productivities or rising capital-labour ratios to maintain a non-reduced wage ratio w_2 / w_1. At some upper point regarding ℓ, this even becomes impossible.

2.5 Analytical results

We are now ready to consider properties of *sequences* of temporary equilibria. We assume a *constant* urban premium $w_2 / \mathrm{w}_1 \geq 1$ and values of ℓ within the constraint given by the consistency condition (2.12). In view of (1.16), the agrarian labor participation rate equals the economy-wide labor participation rate, L/N. We therefore have:

Result 1. By equation (2.8) follows that a rising fraction of the labor force being employed in the urban sector reveals a rising labour income per capita in the agrarian sector, $w_1 L_1 / N^1\ (= w_1 L / N)$. Moreover, if the labor participation rate is constant or only "modestly" increasing, a rising agrarian wage rate w_1 is revealed.

The economy-wide labour income per capita is

$$\frac{w_1 L_1 + w_2 L_2}{N} = \frac{L_1 + (w_2 / \mathrm{w}_1) L_2}{L}\frac{w_1 L}{N} = \left(1 + (\frac{w_2}{w_1} - 1)\ell\right)\frac{w_1 L}{N}. \tag{2.13}$$

Result 2. By equation (2.13) and Result 1 follows that a rising fraction of the labor force being employed in the urban sector reveals a rising economy-wide labour income per capita. If $w_2 / \mathrm{w}_1 > 1$, a rising ℓ reveals that the economy-wide labour income per capita is rising *faster* than the agrarian labour income per capita, $w_1 L_1 / N^1 (= w_1 L / N)$.

Conclusions regarding growth in *total* income (i.e. including operating profit and land rent) per capita and *total* income per unit of labor ("labour productivity") depend on the values assigned to the parameters and are therefore postponed until we have considered the choice of these values on the basis of empirical information.

To make clear for what parameters we need numbers, we here present the basic *growth indicator* to be used. Let time 0 represent for instance our initial year 1522 CE and let *t* represent a year a century later, say. Considering per capita "basic" wage income[11] $w_1 L / N$ in year *t* relative to that in year *0*, we get from (2.8)

$$\frac{w_{1t} L_t / N_t}{w_{10} L_0 / N_0} = \frac{1 - M + \sigma \ell_t}{1 - M + \sigma \ell_0} \cdot \frac{1 - (1-\alpha)M - \left[1 - (1-\alpha)M + \beta \varepsilon^{-1} J w_2 / w_1\right]\ell_0}{1 - (1-\alpha)M - \left[1 - (1-\alpha)M + \beta \varepsilon^{-1} J w_2 / w_1\right]\ell_t}, \tag{2.14}$$
$$\text{with } M \equiv \sigma + m(1-\sigma) \text{ and } J \equiv \theta + (1-\theta)m(1-\sigma).$$

As long as the consistency condition (2.12) is satisfied, when $\ell_t > \ell_0$, the right-hand side of (2.14) exceeds 1, thus indicating growth.

The ratio (2.14) is seen to be independent of the subsistence minimum *b* which thus does not have to be estimated.[12] Likewise, the ratio is independent of the agrarian and urban output elasticities with respect to capital, γ and $1-\theta-\varepsilon$, respectively. While these elasticities naturally enter the supply-side relationship (2.5), they do not enter the key demand-side relationship (2.8). In view of the assumed logarithmic preferences, saving is independent of the sectoral returns to capital which naturally *do* depend on these elasticities, cf. (1.4) and (1.5). Similarly, the ratio in (2.14) is independent of the share of land rent in agrarian income, $1-\alpha-\beta-\gamma$. Moreover, to estimate the growth factor, w_{1t} / w_{10}, of the agrarian wage rate, we need not appraise the labour participation rates as such, only their ratio $(L_t / N_t)/(L_0 / N_0)$.

[11] Basic in the sense that on top of this, urban workers possibly receive a premium.

[12] Yet, as we shall see in Section 3.1, some judgement about the ratio of Y / N to b, i.e. the ratio of per capita income to subsistence minimum, around 1688 will be needed to fix another parameter, the marginal saving rate σ.

Besides, as argued in Appendix D, even under conditions of open or "disguised" unemployment, the growth formula (2.14) as well as the empirical inferences to be drawn from it in Section 4 can be upheld.

Finally, the principle behind the growth indicator (2.14) does not require that the parameter values at time 0 and time t are the same. Nevertheless, in the present application we have no compelling clues as to their change within the pre-industrial era of Britain and so we assume them constant.

3. Data

We focus on Britain's pre-industrial era spanning the 16th, 17th, and first half of the 18th century.

3.1 Output elasticities and household behavior

Regarding the technological output elasticities with respect to inputs, the parameters to which we have to assign values are:

- The (partial) elasticities of agrarian output with respect to intermediate goods from industry and services, α, and with respect to labour, β.
- The (partial) elasticities of output in industry and services with respect to intermediate goods from the agricultural sector, θ, and with respect to labour, ε.

Our estimates of the elasticities α and θ are guided by the input-output table for 1688, constructed by Dodgson (2013) on the basis of Gregory King's economic data from the 1690s. In Table 1 we have aggregated the original 17-sector table to a three-sector table.

To our knowledge no other input-output tables for pre-industrial England/Britain exist. Aggregating services and industry to an "urban sector" and transforming the table into an input-*net* output table, we arrive at the values 0.08 and 0.37 for the elasticities with respect to intermediates, α and θ, respectively. Motivated by the table's suspicious total absence of industrial inputs to the agricultural sector, we make a slight upward adjustment of the value of α to 0.09. Anyway, the results turn out to be not sensitive to α.

The results are however sensitive to the intermediates parameter in urban production, θ. For our baseline case we choose a downward adjustment of the value of θ from the mentioned 0.37 to 0.20 and leave the possibility of other values for sensitivity analysis. This downward adjustment is motivated by the caveat stated in Section 1.1 concerning our simplifying assumption that also intermediates enter in a Cobb-Douglas way. Indeed, the probably too high elasticity of substitution between intermediates and other inputs, that is implied by this assumption, generates an upward bias in the per capita income growth revealed by a given rise in the urban employment share. Since θ measures the elasticity of urban output with respect to intermediates from agriculture, one way to counteract this bias is to make a downward adjustment of θ. A lower θ means that the given disproportionate rise in urban employment, combined with no simultaneous fall in the urban-rural wage ratio, reveals lower growth in real wages (as indicated by considering the effect in (2.14) of a lower $J \equiv \theta + (1-\theta)m(1-\sigma)$ for given ℓ_0 and ℓ_t, recalling that $m(1-\sigma)$ < 1).

Table 1. A three-sector input-output table for 1688. £millions. England.

	Agriculture	Industry	Services	Consumption	Investment	Exports	Total
Agriculture	4.45	14.14	1.0	6.48	0.25	0.19	26.51
Industry	0	17.56	0.84	30.94	3.04	2.72	55.09
Services	1.51	4.56	0	9.61	0.10	0	15.80
Imports	0.16	2.14	0				2.30
Indirect taxes	0.05	1.48	0				1.53
Value added	20.34	15.21*	13.96				49.51
Total	26.51	55.09	15.80	47.03	3.39	2.91	

Source: Dodgson (2013), based on Gregory King (c. 1696). * A misprint concerning Value added in branch 3 (Food, Drink, and Tobacco) in Dodgson's Table 1 leads to a total Value added for Industry of 15. 59, but 15.21 is the correct number. Private communication with Dodgson.

Concerning the output elasticities with respect to labour in the two sectors, the guidance from the input-output table is more indirect. For the urban sector we choose ε so as to generate a labour income share in the sector in accordance with the standard competitive split of factor income of a modern economy into 2/3 to labour and 1/3 to capital. That is, we set ε equal to (1-0.20)2/3 = 0.53. Then consistency with, first, the distribution of value added across sectors shown in Table 1, second, the proportionality between average and marginal productivity of labour implied by the Cobb-Douglas technologies, and third, an urban-rural wage ratio of 1.25 (as argued in Section 3.2) requires labour's agrarian output elasticity, β, to be 0.51. The implied labour income share in the agrarian sector equals 0.51/(1- 0.09) = 0.56. This can be seen as a "compromise" between a half-and-half divide, as often stipulated by share-cropping contracts, and the economy-wide labour income share equal to 0.60 suggested by Clark (2010). At any rate, compared to this number, both our lower value for the agrarian sector and higher value for the urban sector draw revealed growth in a conservative direction.

Regarding households' behavior we need to assign values to σ, i.e. the marginal propensity to save out of income over and above the subsistence minimum, and to *m*, i.e. the marginal propensity to consume basic food out of the resulting "free" consumption budget. As we shall see, the results are sensitive to the magnitude of *m*. An overestimated value, in combination with fixed output elasticity parameters, will inflate the revealed income growth estimates. As our baseline value we have chosen *m* = 0.05, which is perhaps in the lower end of the plausible interval. With *m* = 0.05 (and $\sigma = 0.10$, a value we will motivate in a moment), the marginal propensity to consume basic food out of income over and above subsistence minimum equals $m(1-\sigma)$ = 0.045. This may seem a low figure compared to estimates for underdeveloped economies. However these contemporary estimates include food in a much broader sense than what corresponds to our consumption basket *b*, i.e. processed goods from the alimentary industry. Anyway, with a greater *m*, *higher* income growth would be revealed. So choosing *m* = 0.05 draws the estimate of growth in a conservative direction.[13]

According to Table 1, the aggregate gross investment-income ratio around 1688 was 0.067, which is within the range of the received opinion in the literature. Ignoring foreign trade, this number also represents the aggregate gross *saving*-income ratio, S/Y, around 1688. Because there is only saving out of income above the subsistence threshold, to generate this *average* saving rate, the marginal propensity to save, σ, must exceed 0.067. Indeed by (1.15),

$$\frac{S}{Y} = \sigma\left(1 - \frac{1}{(Y/N)/b}\right) = 0.067 \text{ in year } 1688\,. \tag{3.1}$$

Available estimates of Britain's ratio of Y/N to "subsistence'' income refer to 1870. Broadberry et al. (2015, pp. 373, 399-400) suggest a ratio in the range 6.4 to 8.0, and Pritchett (1997) suggests a ratio in the range 8.7 to 11.0. We have settled for 8.7. Given this value in 1870 and given an annual per capita growth rate for the period 1759 to 1870 of 0.64 per cent (as suggested by Broadberry et al., 2015, Table 11.01), a backward calculation leads to a value of $(Y/N)/b$ in 1759 equal to 4.28. The calculation further backward to 1688 depends on the so far unknown revealed annual growth rate of Y/N over the period 1688 to 1759, and this growth rate *depends* on what value for σ we choose. Given baseline values of the other parameters, this mutual dependency leads to the value $\sigma \simeq 0.10$ (Appendix C). The implied baseline value of $(Y/N)/b$ in 1688 is 3.10; the baseline value of $(Y/N)/b$ in 1522 becomes 1.79.[14]

[13] Although pertaining to a later period, 1770-1850, evidence of a quite low *m* is provided in Clark et al. (1995).

[14] These two numbers are implied by the growth rates reported in the second to last row of Table 4 below.

3.2 Quasi-parameters for the numerical analysis

While the urban-to-rural wage ratio w_2 / w_1 is endogenous in the theoretical model, in the data it seems to be relatively stable over time. At least the ratio between skilled and unskilled wages has been fairly constant over long stretches of time, at around 1.5 (Clark 2010, Table 1). However, urban workers were not exclusively skilled. So we have in our baseline case opted for a somewhat lower but still constant ratio w_2 / w_1 = 1.25, which we treat as a quasi-parameter in the numerical analysis.

Table 2. Values chosen for parameters and quasi-parameters. Baseline case.

Elasticity of agrarian output with respect to intermediates from industry and services	α = 0.09
Elasticity of agrarian output with respect to labour	β = 0.51
Elasticity of output in industry and services with respect to agrarian intermediates	θ = 0.20
Elasticity of output in industry and services with respect to labour	ε = 0.53
The marginal propensity to consume agricultural goods out of the consumption budget	m = 0.05
The marginal propensity to save	σ = 0.10
The urban-agrarian wage ratio	w_2 / w_1 = 1.25
Growth factor for the participation rate, $\dfrac{L_t / N_t}{L_0 / N_0}$	1522-1688: 1.3381 1688-1759: 1.0427

An additional quasi-parameter that enters the growth indicator (2.14) is the growth factor for the participation rate *L/N* (the number of standardized man-years relative to working-age population). Labour force participation rates are not a well-researched area, but there is an emerging consensus that the number of working days increased in the Early Modern Period up to the Industrial Revolution. The magnitude of that increase is a matter of debate, though. An increase in the number of constant-hours working days make the growth rate of income per unit of labour smaller than the growth rate of income per capita. Considering various estimates (see Broadberry et al., 2015, Table 6.02) we have settled for an increase in annual constant-hours working days from 210 in 1522 to 281 in 1688 and 293 in 1759. This gives rise to the growth factors for L/N indicated in the bottom row of Table 2. About hours worked per workday there is very little precise data. However, daylight was the limit and hours varied seasonally. We assume the annual average number of hours per workday is constant over the whole period.

Table 2 gives an overview of our baseline values for parameters and quasi-parameters.

3.3 Occupational distribution

Change in the occupational distribution of the labour force is the factor revealing growth. A variety of sources are available for the assessment of these changes. Robust census data, however, arrive comparatively late, not until the 19th century. Before that, probate inventories, poll tax returns, muster rolls, and parish registers have been used as well as the so-called social tables edited by Gregory King (for 1688), Joseph Massie (for 1759), and Patrick Colquhoun (for 1801/03).

Female labour force participation and occupational distribution are more difficult to determine than male occupational distribution. There is a consensus that the female labour force was proportionately less active in agriculture and more active in services than the male labour force. The first robust female labour force distribution data are from the 1851 census. Shaw-Taylor and Wrigley (2014) argue that it's comparatively low agricultural share of the female labour force, 16.8 per cent, can plausibly be used also for the early 19th century. However, further back in history Shaw-Taylor and Wrigley estimate the agricultural share of the female labour force to be higher. They suggest a doubling by the early 18th century.

Our baseline estimates in Table 3 of the occupational share of industry and services in the total labour force, men and women, are based mainly on Broadberry et al. (2015), but differ for the years 1759 and 1688. Broadberry et al. use the early 19th century agricultural share of the female labour force also for these years while we are inclined to follow the mentioned Shaw-Taylor and Wrigley (2014) view. However, for 1522 we find Broadberry et al.'s much higher estimates of the agricultural share of the female labour force plausible. It is based on occupational data in the poll tax returns from 1381.

Table 3 indicates a remarkable change of the occupational structure over three centuries. Owing to the mentioned difference regarding the distribution of the female labour force in 1759 and 1688, the rise in the estimated industry and services employment share over the period 1522-1688 is somewhat smaller in the baseline row than in the Broadberry et al. row.

In the calibration below we use the data from 1522 and up to 1759. The reason we do not include the subsequent period is that for Britain the assumption of zero net imports of agricultural goods ceases to be an acceptable approximation. Wool was a major export product from Britain although it was increasingly manufactured and exported as cloth. Eventually Britain turned into a wool importer. However grain was exported in considerable volumes in the first half of the 18th century. By around 1780 Britain becomes a net importer of grain (Sharp, 2010) and there is a significant increase in the imports of colonial goods such as sugar, coffee, and tea. Under these circumstances the drift towards a more industrial occupational structure can no longer be solely ascribed to changes in the composition of domestic demand. Not controlling for positive net imports of food would lead to an overestimation of the revealed income growth.

Table 3. Employment in industry and services as a share of total employment, 1522-1801. Per cent.

Year	1522	1570	1688	1710	1759	1801
Baseline	44.4		57.6		62.4	68.3
Broadberry et al.	44.4		61.1		63.2	68.3
1570: Clark et al. 1710: S-T & W		47.0		53.1		

Note and sources: The baseline row is primarily based on male occupational shares from Broadberry et al. (2015, Tables 9.03-9.06). Concerning female labour, in the spirit of Shaw-Taylor and Wrigley (2014), we assume that the share of industry and service occupation in the female labour force is 75 and 66 % in 1759 and 1688, respectively. For 1522 we follow Broadberry et al. and assume that the share is 65.5 %. The B et al. row is from Table 9.01 in Broadberry et al. (2015). The female labour force is throughout the whole period assumed to make up 30 % of the total labour force. The last row is based on male occupational data from a 20-year period centered around 1570 in Clark et al. (2012) and assuming the industry and service share of the female labour force to be 66 %, the same as in our baseline estimate for 1688. The 1710 estimate is from Shaw-Taylor and Wrigley (2014, Table 2.6). See text for more details.

Although we have argued that occupational data are less controversial than other pre-industrial data, there is a margin of uncertainty here as well. Hence the last row of Table 3 includes alternative estimates. Clark et al. (2012) uses occupational data found in probate inventories. For the mid-16th and 17th centuries their results amount to a very high, perhaps implausibly high, urban share for the male labour force and as a consequence a small increase over the 100 years covered in their analysis, two percentage points only. However their sample is regionally biased and include only males. When we adjust for the latter, using the same distribution of the female labour force as in our baseline estimate for 1688 (see note to Table 3), we end up with a service and industry share of total employment equal to 47 per cent in 1570, as indicated in Table 3. Clark et al. have no numbers for the 18th century but Shaw-Taylor and Wrigley (2015, Table 2.6) provide two estimates, an upper bound and a lower bound estimate, for the year 1710 and we use the average of the two, i.e. 53 %, cf. Table 3. Table 5 below reports the implied revealed growth over the period 1570-1710.

4. Results

Until further notice we report results based on our baseline calibration, including $m = 0.05$ and $w_2 / w_1 =$ 1.25 (constant). In all cases, in combination with the observed occupational distribution, the applied values of parameters and quasi-parameters are consistent with the double inequality (2.12).

4.1 Stagnation in per capita income rejected

Based on the growth indicator (2.14), Table 4 shows an average compound growth rate in agrarian *per capita* labour income over the period 1522-1688 of 0.34 per cent per year and over the period 1688-1759 of 0.45 per cent per year. By correcting for the increases in the participation rate L / N, cf. Table 2, we find the corresponding numbers for the growth rate of the agrarian real wage *per unit of work* to be 0.17 per cent per year in the first period and 0.39 per cent per year in the second.

For the economy-wide *per capita* labour income, Table 4 displays a growth rate of 0.36 per cent per year over the period 1522-1688 and 0.47 per cent per year over the period 1688-1759. The corresponding numbers for the growth rate of labour income *per unit of work* are 0.19 and 0.41 per cent per year, respectively. These numbers indicate that over the whole period 1522-1759, economy-wide labour income has grown by a factor equal to 2.54 when measured on a per capita basis and has almost doubled when measured per unit of work (growth factor = 1.83).

The two last rows in Table 4 are about *total* income per capita (average "standard of living"). In view of equation (1.7), total income (including gross operating profit and land rent) per capita is

$$\frac{Y}{N} = \frac{w_1 L_1 + r_1 K_1 + \hat{r} Z + w_2 L_2 + r_2 K_2}{L} \frac{L}{N} = \left[\frac{1-\alpha}{\beta} + \left(\frac{1-\theta}{\varepsilon} \frac{w_2}{w_1} - \frac{1-\alpha}{\beta} \right) \ell \right] \frac{w_1 L}{N}, \tag{4.1}$$

where the second equality comes from equation (1.4) and (1.5). With our baseline parameter values, we get a growth rate of Y / N equal to 0.35 per cent per year over 1522-1688 and 0.45 per cent per year over 1688-1759. The corresponding growth rates of total income *per unit of work,* Y / L, are 0.17 and 0.40 per cent per year, respectively. Of course, these numbers express *trend* growth and do not rule out sizeable fluctuations around the trend path due to harvest failure, civil war, bouts of disease etc. For the whole period 1522-1759 these results suggest that economy-wide *total* income has grown by a factor 2.46 when measured per capita and has almost doubled when measured per unit of work. To be precise, the implied growth factor for economy-wide *labor productivity* over the whole period is 1.79.

Table 4. Results for baseline case. Compound annual growth. Per cent.

Period	1522-1688	1688-1759
Agrarian real per capita labour income, $w_1 L / N$	0.34	0.45
Agrarian real wage per unit of work, w_1	0.17	0.39
Economy-wide real per capita labour income, $(w_1 L_1 + w_2 L_2) / N$	0.36	0.47
Economy-wide real wage per unit of work, $(w_1 L_1 + w_2 L_2) / L$	0.19	0.41
Real income per capita, *Y/N*	0.35	0.45
Real income per unit of work, *Y/L*	0.17	0.40

Sources: see text.

Fundamentally, real income per capita growth has two sources, increased productivity and more hours per worker per year (implying an increase in *L/N*). The increase in hours worked is revealed by the increasing number of working days mentioned earlier and is associated with what is known as the *industrious revolution* (de Vries, 1994). A measure of the latter is given by the difference between growth in income per capita and growth in income per unit of work. Comparing the two last rows of Table 4, we see that the *industrious revolution effect* amounts to about half of the growth in income per capita in the first period and to only one ninth in the second period.

We may allow for a rise in "necessities", *b*, along with the industrious revolution. Then the left-hand side of the growth formula (2.14) is replaced by

$$\frac{w_{1t} L_t / (b_t N_t)}{w_{10} L_0 / (b_0 N_0)},$$

and so the growth rates reported in the first row of Table 4 applies to this expression. When $b_t / b_0 > 1$, the revealed growth of agrarian per capita labour income, $w_1 L / N$, is thus *higher* than the reported numbers.

4.2 Sensitivity and comparisons

One might suspect that these notable growth rates are driven by an unreasonably high marginal propensity to consume basic food, *m.* Indeed, with a higher *m*, it takes a larger income growth to generate a given observed growth in urban employment. But if we change our value for *m* from 0.05 to 0.00, the estimated growth rates of per capita income are only reduced modestly, cf. the second row of Table 5. The growth rate of Y / N becomes 0.30 per cent per year over the period 1522-1688 and 0.37 per cent per year over the period 1688-1759.[15]

Alternatively one might suspect that our notable growth rates are driven by a high urban premium. Indeed, a higher urban premium means that, by mere structural change, a given observed rise in the urban employment share *automatically* results in a higher calculated *economy-wide* productivity growth. Within the model it nevertheless turns out that reducing the value for w_2 / w_1 from 1.25 to 1.00, the revealed growth rates are only modestly diminished. The growth rate of for instance Y / N becomes 0.28 per cent per year over the period 1522-1688 and 0.35 per cent per year over 1688-1759, cf. row 3 of Table 5.

How sensitive are the revealed-growth numbers with regard to the values for the marginal propensity to save, σ, and the output elasticities α, β, θ, and ε? In the fourth to eighth row of Table 5 we vary each of these five parameters separately in the direction that reduces growth. We change the parameter values by 20 per cent except for β which is only reduced by 10 per cent. This is to avoid violation of the lower bound in the consistency condition (2.12). The ninth row shows the effect of varying the five parameter values *simultaneously* in the described way but now only by four per cent each. Throughout the effects are modest.

Overall Table 5 indicates that revealed growth is somewhat sensitive to changes in the parameter values in a growth-reducing direction. This downward sensitivity of growth is not dramatic, however, and so our rejection of stagnation seems robust. Changing the parameter values in the *opposite* direction will imply *higher* revealed growth, the latter becoming more and more sensitive to further parameter changes in the same direction.[16] One will soon be left in a region with unbelievable high growth. As long as we impose non-reduced w_2 / w_1, this property is

---

[15] In her study of the British Industrial Revolution, 1780-1850, Stokey (2001) similarly considers Engel preferences with *m* equal to nil.

[16] The parameter α is an exception here, in that the effect on revealed growth of reducing α is almost imperceptible.

Table 5. Sensitivity analysis and comparisons. Compound annual growth in per capita income, per cent.

	Period	1522-1688	1688-1759
1	*Y/N*, baseline case	0.35	0.45
2	*Y/N*, baseline case except $m = 0$	0.30	0.37
3	*Y/N*, baseline case except $w_2 / w_1 = 1.00$	0.28	0.35
4	*Y/N*, baseline case except lower $\sigma = 0.079$	0.34	0.44
5	*Y/N*, baseline case except higher $\alpha = 0.108$	0.35	0.45
6	*Y/N*, baseline case except lower $\beta = 0.459$	0.31	0.40
7	*Y/N*, baseline case except lower $\theta = 0.160$	0.31	0.38
8	*Y/N*, baseline case except higher $\varepsilon = 0.636$	0.29	0.36
9	*Y/N*, baseline case except simultaneous change to $\sigma = 0.095,\ \alpha = 0.094,\ \beta = 0.490,\ \theta = 0.192,$ and $\varepsilon = 0.551$	0.31	0.40
10	GDP per capita as presented in Broadberry et al. (2015)	0.12	0.37
11	*Y/N*, baseline case except using the "B et al." row of Table 3	0.48	0.23
12	*Y/N*, baseline case except using the "Clark et al." for 1570 and "S-T & W" for 1710 in Table 3	1570-1710 0.18	

Note and sources: See text. Row 10 is based on Broadberry et al. (2015), Appendix 5.3; the numbers 0.12 and 0.37 are compound annual growth rates calculated by us on the basis of nine-year GDP-per-head averages centered around 1522, 1688 and 1759, respectively.

inherent in the growth formula (2.14) where the denominator, for given $\ell_t > \ell_0$ and non-reduced w_2 / w_1, diminishes faster than the numerator when we for instance decrease ε and/or increase β. Indeed, for a given rise in the urban employment share, it takes a *higher* productivity increase to overcome the downward pressure on w_2 / w_1 due to diminishing returns to urban labour the *smaller* is ε, that is, the *faster* are these diminishing returns.[17] Moreover, the higher is ℓ already,

[17] The absolute elasticity of marginal productivity of L_2 with respect to L_2 is $|\varepsilon - 1|$, cf. (1.5).

the higher is the growth revealed by a further percentage point increase in ℓ *not* accompanied by a reduced w_2 / w_1. The importance of the non-reduced w_2 / w_1 derives from its implication that the revealed growth must take place proportionately in *both* wage rates. The rising upward sensitivity of revealed growth is counteracted if one allows w_2 / w_1 to be reduced in the process.

Row 10 and 11 of Table 5 offer a both reassuring and puzzling comparison with the growth estimates by Broadberry et al. (2015). The Broadberry group concludes with lower annual growth, in particular for the 1522-1688 period. Our results suggest healthy growth also in this first period. In the Broadberry analysis the break in the GDP per capita growth comes in the mid-17th century (Broadberry et al. 2015,Table 5.07). What is puzzling from our perspective is that the drift towards a rising share of industry and services employment in the 1522-1688 period is nevertheless stronger, in percentage points, in the Broadberry et al. data in row 2 of Table 3 above than in the baseline row of the table. Furthermore, the Broadberry data for the 1500-1650 period (their Table 5.07 and Appendix 5.3) indicate a small fall in total GDP per capita along with a sharp fall in agrarian output relative to total population, about 20 per cent, and a 10 per cent *increase* in industrial per capita output.

To address the incongruence between their and our results for this sub-period, in row 11 of Table 5 we report growth rates calculated on the basis of the revealed growth formula (2.14) and our baseline parameter values listed in Table 2, but using Broadberry et al.'s estimate of the evolution of the employment share of industry and services (as shown in the second row of Table 3 above). This gives high income growth in the 1522-1688 period and fairly low income growth in the 1688-1759 period, contrary to received opinion in the literature, including Broadberry et al. An explanation of this paradox might be that Broadberry et al. have exaggerated the increase in industry and services employment in the first period because they possibly underestimate the share of female labour in agriculture at the end of that period, 1688, as we discussed in Section 3.3.

The uncertainty over the precise magnitude of sectoral employment rates calls for additional robustness valuation. This is the motivation for calculating the growth estimate in row 12 of Table 5 on the basis of the 1570 and 1710 industry and services employment shares reported in the last row of Table 3. We regard these shares as making up an "upper bound" estimate for 1570 and a "lower bound" estimate for 1710. On this basis revealed annual compound growth in per capita income over the period 1570-1710 turns out to be a low 0.18 %. We consider this as the lower endpoint of the plausible range for this period. The corresponding growth factor is 1.29.

4.3 Discussion

As to the big picture of pre-industrial Britain, our results are in line with the "moderate but positive growth view" articulated by Broadberry et al. (2015),[18] while at variance with the "stagnation view" of Clark (2007, 2010) and Clark et al. (2012), cited in the introduction. Our results also seem at variance with interpretations of the data for the pre-industrial period expressed by leading theorists. For instance Hansen and Prescott (2002, p. 1205) claim: "Prior to 1800, living standards in world economies were roughly constant over the very long run: per capita wage income, output, and consumption did not grow". And according to Galor (2005, p. 180): "… the average growth rate in each of these regions [including "Western Europe" - authors' insertion] was nearly zero. This state of stagnation persisted until the end of the 18th century across all regions".

Diverging views elicit reflection. Could our results be biased as a consequence of the assumption that all goods entering the "subsistence minimum" *b* are produced in the agrarian sector? Could it be that the observed rise in urban employment just reflects the expansion of an urban alimentary industry that produces slightly cheaper substitutes for some of the agrarian "necessities" and so gradually take over a larger and larger share of the market for "necessities"?[19] In that case, the urban employment expansion need not reflect *sustained* rise in per capita income.

Considering what is known about the composition of the produce of the urban sector, we are inclined to rule out this alternative interpretation. The industry sector introduced new and refined goods, for instance books, newspapers, spectacles, pocket-watches, and glass windows. Also increased differentiation of old products, such as kitchen- and tableware, furniture and apparel, took place. A large number of new urban occupations emerged. In London alone there were about 150 separate occupations in the late Medieval period but by c.1700 there were more than 700 (Persson and Sharp 2015, Table 2.1). Elite consumer preferences were increasingly imitated by the middle classes and skilled workers in the 16th to 18th centuries. A substantial but not very rich farmer's probate inventory revealed, for example, several pieces of high-quality clothes, which evidently were tailor-made, and other items typically found among the gentry, such as a chafing dish (Dyer 2012, 18-19). Cabinetmakers responded to increased demand for furniture, were literate, and would send their sons, and sometimes their daughters, to private tuition or to grammar schools. This gave rise to new occupations: tutors and schoolteachers. Between 1500 and 1750 Britain developed from an almost illiterate society to one in which half the male population had at least elementary literacy skills and some numeracy skills. The increased

[18] A recent national income reconstruction of pre-industrial Holland (Van Zanden and van Leuwen, 2012) indicates slow but persistent growth in agreement with Broadberry et al.

[19] We thank Nicolai Kaarsen for raising this question at the MEHR seminar.

commercialization also expanded the transport sector benefitting the shipbuilding industry, carriage makers and wheelwrights, occupations also known for their high literacy and numeracy.

5. Conclusion

In this paper we have developed a two-sector model intended as a tool for assessing income growth in a pre-industrial society with limited foreign trade. A key ingredient in the model is Engel's law, that is, the empirical regularity that as income increases, a falling share of income is used for consumption of food. At the analytical level we find that a rising fraction of the labor force being employed in industry and services unequivocally reveals a rising per capita labour income.

We have applied the framework to England/Britain in the period 1522-1759. A calibration of the model is carried out, requiring a limited number of empirical observations, primarily changes in the occupational structure, which are uncontroversial or at least not seriously contended. The baseline calibration, building on a marginal propensity to consume basic food slightly less than 5 per cent, suggests a compound growth rate of 0.35 per cent per year in *income per capita* over the period 1522- 1688 and of 0.45 per cent per year over the period 1688 - 1759. Taking the estimated slight annual rise in the participation rate into account, we end up with growth in *income per unit of work* (productivity) of 0.17 per cent per year in the first period and 0.40 per cent per year in the second. Robustness checks indicate sensitivity to several of the uncertain parameter values. However, even with downward adjustments of the urban-rural wage ratio to one and of the marginal propensity to consume basic food to zero, is the hypothesis of stationary per capita income rejected.

These results lend support to the conclusion reached by Broadberry et al. (2015) of modest positive growth in GDP per head in pre-industrial Britain well before the Industrial Revolution. We argue, more controversially, that even the period from the early 16th century to the mid-17th century experienced perceptible positive growth. This is contrary to the slightly negative growth in this period estimated by the Broadberry group.

Our results have wider repercussions because they challenge a longstanding Malthusian tradition in the interpretation of European and British economic history, known as "l'histoire immobile", the stationary history. Today this view is associated with Clark (2007) but it goes back to economic historians like Postan (1966) and Le Roy Ladurie (1974). More specifically our results challenge the prevailing view of stationary real wages well into the Industrial Revolution (Phelps Brown and Hopkins 1955, 1956; Clark 2010). Can we explain why our results differ from received opinion?

There are four major reasons why our results differ. *First* we refer to real wages and real income of the entire labour force while the much cited and used British real wage series are based on nominal day wages of a small fraction of the labour force. Furthermore the size as well as representativeness of that fraction can vary over time. We do not rely on these series at all in our estimates. *Second*, traditional historical national accounts arrive at GDP per capita estimates by controlling for population growth. However, population levels before the mid-16th century are far from fully researched. As a rule population estimates at early dates are reached by backward interpolation from some robust benchmark estimate. An assumed too low population level at some initial year will generate a too high population growth which will affect income per capita growth numbers negatively. In our per capita growth estimates we do not use or need population level numbers. The *third* reason is that the often used real wage deflator with constant commodity composition or fixed weights may exaggerate inflation because it does not adjust for changes in the consumption pattern over time as a response to changes in income and relative prices. *Finally*, the real wage deflator tends to exaggerate inflation because as a rule it does not control for quality improvements in goods and services. Fixed expenditure weights and neglect of quality improvements are major problems in modern real national income accounting since these factors generate spurious inflation, and there is no reason to believe it was not a problem in the past as well (Nordhaus 1999).

The revealed income growth approach does not use real wage or GDP deflators. The real wage – and productivity – change is instead detected by the behavioral response of the economic agents: when incomes increase, there is a change in the consumption pattern and occupational distribution. We derive the change in real wages and productivity from that occupational shift. It is difficult to imagine what other forces than a substantial rise in per capita income could in the pre-industrial era cause such considerable changes in the occupational structure as the data indicate.

6. Appendix. For online publication

A. Behavior of a single household

Let $y_i > 0$ be (gross) income of household no. *i* in the year considered. Labour supply of the household is inelastic. The household cares about current consumption of the family and, if residual income, $y_i - b$, is positive, its preference is to save a part "for the future". More precisely, the household in any year solves the problem:

$$\max u(c_{1i}, c_{2i}, s_i) = \begin{cases} c_{1i} - b & \text{if } c_{1i} \leq b, \\ ((c_{1i} - b)^m \; c_{2i}{}^{1-m})^{1-\sigma} s_i{}^{\sigma} & \text{if } c_{1i} > b, \end{cases}$$
$$\text{s.t.} \quad c_{1i} + pc_{2i} + s_i \leq y_i,$$
$$c_{1i} \geq 0, c_{2i} \geq 0, s_i \geq 0,$$

where s_i is (gross) saving and the preference parameters m and σ satisfy $0 < m < 1$ and $0 < \sigma < 1$, respectively. Also households that have inherited wealth are assumed to have this consumption-saving behavior. Thereby they at least keep the inherited wealth intact and, at "retirement" or death, transfer a typically enlarged wealth to the next generation.

In view of the reproducibility assumption (1.13), the relevant case to consider is $y_i \geq b$. We get the solution

$$c_{1i} = b + m(1-\sigma)(y_i - b),$$
$$pc_{2i} = (1-m)(1-\sigma)(y_i - b),$$
$$s_i = \sigma(y_i - b).$$

For any distribution of household income in the economy satisfying the reproducibility assumption, aggregation over i gives the aggregate consumption and saving functions in (1.14) and (1.15).

B. Proofs for Section 2

LEMMA B.1. Consider Figure 2 of the main text. Let the hyperbola in $\mathbb{R}^2_{++}$, defined by $w_2 / w_1 = \varphi(w_1) \equiv (bN / L)w_1^{-1}$, be denoted *H*. It has the following properties:

(i) H goes through the point D in Figure 2.

(ii) Every (2.5)-curve has one point in common with *H.* This common point, denoted G in Figure 2, has abscissa $\bar{x} \lesseqgtr bN / L$ for $f(bN / L, \cdot) \lesseqgtr 1,$ respectively.

(iii) Every (2.5)-curve in $\mathbb{R}^2_{++}$ crosses *H* from below when moving from the right to the left in the figure.

Proof. (i) D is the point $(bN / L, 1)$. Since $\varphi(bN / L) = 1,$ D is a point on *H*. (ii) Consider an arbitrary (2.5)-curve in $\mathbb{R}^2_{++}$. By definition of the point G, G's abscissas is a w_1 such that $\varphi(w_1) = f(w_1, \cdot)$. This equation is equivalent to $(bN / L)w_1^{-1} = f(1, \cdot)w_1^{-\frac{1-\alpha\theta}{\alpha-\alpha\theta}}$. Solving for w_1 gives

$$w_1 = \left(\frac{f(1,\cdot)}{bN/L} \right)^{\frac{\alpha(1-\theta)}{1-\alpha}} = \left(\frac{f(bN/L,\cdot)(bN/L)^{\frac{1-\alpha\theta}{\alpha(1-\theta)}}}{bN/L} \right)^{\frac{\alpha(1-\theta)}{1-\alpha}} = f(bN/L,\cdot)^{\frac{\alpha(1-\theta)}{1-\alpha}} bN/L,$$

where the second equality follows from (2.9). This unique solution is the abscissa, $\bar{x}$, referred to in the claim. Since $\alpha \in (0,1)$ and $\beta \in (0,1)$, it follows that $\bar{x} \lesseqgtr bN/L$ for $f(bN/L,\cdot) \lesseqgtr 1$, respectively. (iii) Select an arbitrary (2.5)-curve in $\mathbb{R}^2_{++}$. Its point of intersection with the vertical line $w_1 = bN/L$ is by definition the point F associated with the selected (2.5)-curve. As $f(bN/L,\cdot)$ is the ordinate of F, it follows from (ii) that $\bar{x} \lesseqgtr bN/L$ for F below, coinciding with, or above D in Figure 2, respectively. Q.E.D.

The last part of (iii) of Lemma B.1 can also be seen as an implication of the fact that the elasticity of w_2/w_1 with respect to w_1 along the hyperbola is -1 while it is $-(1-\alpha\theta)/(\alpha-\alpha\theta)$ <-1 along the (2.5)-curve.

Proof of Lemma 1. (i) and (ii). Assume $f(bN/L,\cdot) \geq 1$. In this case the point F in Figure 2, i.e. the point $(bN/L, f(bN/L,\cdot))$, is not below the point D in the figure. Consider the curve representing the demand-side relation (2.8) and passing through the point F. By (2.8), the ℓ associated with this curve must satisfy the equation

$$\frac{bN}{L}\frac{L}{N} = \frac{b\beta\big((1-m)(1-\sigma)+\sigma\ell\big)}{1-(1-\alpha)M-\big[1-(1-\alpha)M+\beta\varepsilon^{-1}Jf(bN/L,\cdot)\big]\ell}, \tag{6.1}$$

the solution of which is $\underline{\ell}$ as defined in (2.11). If instead the requirement is that the (2.8)-curve passes through D, i.e. the point $(bN/L,1)$ in Figure 2, the needed ℓ must satisfy (6.1) with $f(bN/L,\cdot)$ replaced by 1. This gives the solution $\hat{\ell}$ as defined in (2.11). Finally, if instead the requirement is that the curve should pass through G, i.e. the point $(\bar{x}, f(\bar{x},\cdot))$ in Figure 2, the needed ℓ must satisfy (6.1) with bN/L on both sides replaced by $\bar{x}$. This gives the solution $\bar{\ell}$ as defined in (2.11). (iii) Assume $f(bN/L,\cdot)=1$. Then $\underline{\ell}=\hat{\ell}$ in view of (2.11), and F has coordinates $(bN/L,1)$. Thereby F coincides with D. Moreover, as $f(bN/L,\cdot)=1$, we have $\underline{\ell}=\hat{\ell}$ in view of (2.11). In addition, $f(bN/L,\cdot)=1$ implies $\bar{x}=bN/L$ in view of (2.10). Hence, $bN/(\bar{x}L)=1$ whereby (2.11) shows that $\bar{\ell}=\hat{\ell}$ and that the point G coincides with the point D. (iv) Assume $f(bN/L,\cdot)>1$. Then F is above D, and $\underline{\ell}<\hat{\ell}$ in view of (2.11). By (2.10), G has abscissa $\bar{x}>bN/L$ so that G is to the right of D. Moreover, $\bar{x}>bN/L$ implies $\bar{\ell}>\hat{\ell}$ by (2.11). Q.E.D.

Proof of Proposition 1. (i) In this case the point F is below the point D in Figure 2. Hence, for $w_1 = bN/L$, (2.9) implies $w_2 = f(bN/L, L_1,\cdot)w_1 < w_1 = bN/L$. Moreover, for all $w_1 > bN/L$, (2.9) implies $w_2 = f(w_1,\cdot)w_1 < (bN/L)w_1^{-1}w_1 = bN/L,$ where the inequality follows from (ii) of Lemma B.1. Hence, there is no way of satisfying the reproducibility condition (1.13).

(ii) In this case, by (iii) of Lemma 1, both F and G coincide with D in Figure 2, and it holds that $\underline{\ell} = \hat{\ell} = \overline{\ell}.$

(iii) In this case, by (iv) of Lemma 1, the conclusion follows by inspection of Figure 2. Q.E.D.

C. Calibration of the marginal saving rate

The potential difficulty in the calibration of the marginal saving rate σ encountered in Section 3.1 is solved the following way. As basis for the calibration we have, first, equation (3.1) for the year 1688, where both σ and *(Y/N)/b* of that year are unknown, second, the suggested value, 4.28, of *(Y/N)/b* in the year 1759, obtained by backward calculation from 1870, using the Broadberry et al. (2015) data. To "discount" this suggested 1759-value of *(Y/N)/b* further back to 1688, we need the revealed growth factor of (*Y/N*)/*b* from 1688 to 1759. In view of (2.14) and (4.1), this so far unknown growth factor can be written

$$\frac{Y_t/N_t}{Y_0/N_0} = \frac{(1-\alpha)/\beta + \big((1-\theta)(w_2/w_1)/\varepsilon - (1-\alpha)/\beta\big)\ell_t}{(1-\alpha)/\beta + \big((1-\theta)(w_2/w_1)/\varepsilon - (1-\alpha)/\beta\big)\ell_0} \cdot g(0,t,\sigma) \equiv G(0,t,\sigma),$$

where $g(0,t,\sigma)$ is the growth factor of w_1L/N as given by (2.14). The growth factor $G(0,t,\sigma)$ of (*Y/N*)/*b* thus depends on the unknown parameter σ. Consequently, discounting by $G(0,t,\sigma)$, the 1688-value of *(Y/N)/b* depends on σ the following way:

$$\frac{Y/N}{b}\Big|_{1688} = 4.28 \cdot G(0,t,\sigma)^{-1}, \tag{6.2}$$

where 0 stands for 1688 and *t* for 1759. The corresponding curve is shown in Figure 3, given baseline values for the other parameters. The abscissa, 0.0989, of the point of intersection with the curve representing (3.1) is our calibrated value of σ. The corresponding solution for *(Y/N)/b* in 1688 is 3.10.

Figure 3 about here

D. Open and disguised unemployment

The analysis in the main text has proceeded as if there were always full employment. However, from an empirical point of view neither in the agrarian nor in the urban sector can unemployment and "disguised unemployment", also called "surplus labour", be ruled out. Moreover, seasonal unemployment is probably more prevalent in the agrarian sector. In this appendix we briefly address the question whether a reinterpretation of our formal analysis is possible so as to incorporate such aspects.

First, the analysis allows for disguised unemployment being present in both sectors if we simply assume there is no trend over time in the fractions of the employment levels L_1 and L_2 that make up just disguised unemployment. The only effect of disguised unemployment is then to prompt constant percentage reductions in the total factor productivities A_1 and A_2.

Second, to take the possibility of open unemployment into account, we skip the unconditional equality between employment and labour supply and in its place impose weak inequality: $L_1 \leq \bar{L}_1$ and $L_2 \leq \bar{L}_2$. A pragmatic and not unrealistic approach is then to assume that the wage rate in sector i is at any time t determined as the maximum of the competitive wage level w_{it}^c and a "minimum wage level" which we may denote $\bar{w}_{it}$. So

$$w_{it} = \max(w_{it}^c, \bar{w}_{it}), \qquad i = 1, 2. \tag{6.3}$$

In each sector the wage is thus flexible upwards but becomes inflexible downwards when at the level $\bar{w}_{it}$. This lower bound on the sectoral wage rate can be explained by an nutritional efficiency-wage argument or social norms supported by workers' refusal to undercut one another even in the face of open unemployment. A time index on the lower bound is included to allow for possible cultural and historical ingredients and for changes over time in the "normal" number of workdays (and thereby man-hours) per worker per year.

Compared with the full employment case analysed in the text, with unemployment in sector i, the roles of L_{it} and w_{it} are temporarily reversed. The given subsistence wage $\bar{w}_{it}$ in (6.3) then becomes binding, and L_{it} adjusts endogenously in (1.4) or (1.5), interpreted as equilibrium conditions. Extending our notion of temporary equilibrium to include a state of the economy satisfying these conditions, the analytical conclusions of Section 2 go through and the growth indicator (2.14) together with the empirical inferences remain unaffected. Finally, the likely circumstance that seasonal unemployment is more prevalent in agriculture than in the urban sector just means that the rising urban employment share is strengthened in efficiency terms. The revealed per capita income growth is not affected.

E. Migration

As suggested in the text, a plausible element in the dynamic "story" behind the observed change in occupational distribution is migration from agrarian to urban sectors, induced by better job opportunities including the urban wage premium. Let F_t denote the net inflow per year to urban from agrarian sectors. Further, let the biological population growth rate in the agrarian area from *t* to *t+1* be n_{1t}. Then the evolution of the agrarian population N_t^1 is described by the first-order difference equation

$$N_{t+1}^1 = (1+n_{1t})N_t^1 - F_t.$$

The migration may take the simple form

$$F_t = \begin{cases} \lambda_1 (w_{2t} / w_{1t} \text{ -}1) N_t^1 & \text{if } w_{2t} / w_{1t} \geq 1 \quad (\lambda_1 > 0), \\ \lambda_2 (w_{2t} / w_{1t} \text{ -}1) N_t^2 & \text{if } w_{2t} / w_{1t} < 1 \quad (\lambda_2 > 0), \end{cases}$$

where N_t^2 is the urban population in period *t* and λ_1 and λ_2 are adjustment speeds,[20] and only the case $w_{2t} / w_{1t} \geq 1$ is relevant in the present context. At the same time, total factor productivities may be rising so as to maintain a more or less constant urban premium $w_2 / w_1 > 1$, along with a slow but persistent rise in w_{1t}, cf. (2.5), as well as a rising urban employment share ℓ_t.

References

Allen, R. A. (2000), Economic structure and agricultural productivity in Europe, 1300-1800, *European Review of Economic History*, 4 (1), 1-25.

Broadberry, S., B. Campbell, A. Klein, M. Overton, and B. van Leeuwen (2015), *British Economic Growth 1270-1870*, Cambridge University Press.

Clark, G. (2007), *A Farewell to Alms. A brief economic history of the world*, Princeton University Press.

Clark, G. (2010), The macroeconomic aggregates for England 1209-2008*, Research in Economic History,* 27, 51-140*.*

[20] These may be time-dependent.

Clark, G., J. Cummins, and B. Smith (2012), Malthus, wages, and preindustrial growth, *Journal of Economic History*, 72 (2), 364-392.

Crafts, N., and T.C. Mills (2009), From Malthus to Solow: How did the Malthusian economy really evolve? *Journal of Macroeconomics*, 31, 68-93.

De Vries, J. (1994), The Industrial Revolution and the Industrious Revolution, *Journal of Economic History*, 54 (2), 249-270.

Dodgson, J. (2013), Gregory King and the economic structure of early modern England: An input-output table for 1688, *The Economic History Review*, 66 (4), 993-1016.

Dyer, C. (2012), *A Country Merchant 1495-1520, Trading and Farming by the End of the Middle Ages,* Oxford: Oxford University Press.

Galor, O. (2005), From stagnation to growth: Unified growth theory. In: *Handbook of Economic Growth*, vol. 1A, ed. by P. Aghion and S. N. Durlauf, Elsevier B. V.

Hansen, G.D., and E. Prescott (2002), Malthus to Solow, *American Economic Review*, 92 (4), 1205-1217.

Hersch, J., and H.-J. Voth (nd), Sweet diversity: Colonial goods and welfare gains from trade after 1492, draft.

Kelly, M., and C. Ó Gráda (2013), Numerare Est Errare: Agricultural output and food supply in England before and during the Industrial Revolution, *Journal of Economic History,* 73 (4), 1132-1163.

Le Roy Ladurie, E. (1974), *Les paysans de Langedoc*, S.E.V.P.E.N. , Paris.

Nicolini, E. A. (2007), Was Malthus Right? A VAR analysis of economic and demographic interactions in pre-industrial England, *European Review of Economic History*, 11, 99-121.

Nordhaus, W. D. (1999), Quality change in price indexes, *Journal of Economic Perspectives*, 12 (1), 59-68.

Persson, K.G. (1991), Labour productivity in Medieval Tuscany and the 'Low Countries'. In: *Land, labour and livestock. Historical studies in European agricultural productivity*, edited by B. M. S Campbell and M. Overton, Manchester University Press, 124-143.

Persson, K.G., and P. Sharp (2015), *An Economic History of Europe, Knowledge, Institutions and Growth, 600 to the Present,* Cambridge: Cambridge University Press.

Møller, N. F., and P. Sharp (2014), Malthus in cointegration space: Evidence of a post-Malthusian pre-industrial England, *Journal of Economic Growth*, 19 (1),105-140.

Phelps Brown, K.H., and S. V. Hopkins (1955), Seven centuries of building wages, *Economica*, 22, 195-216.

Phelps Brown, K. H., and S.V. Hopkins (1956), Seven centuries of the prices of consumables, compared with builder's wage rates, *Economica*, 23, 296-314.

Postan, M. M. (1966), Medieval agrarian society in its prime: England. In: *The Cambridge Economic History of Europe, vol. 1. The Agrarian Life of the Middle Ages*, 2nd edition, edited by M. M. Postan, Cambridge University Press.

Pritchett, L. (1997), Divergence. Big time, *Journal of Economic Perspectives,* 11 (2), 3-17.

Sharp, P. (2010), '1846 and All That': the rise and fall of British wheat protection in the nineteenth century, *Agricultural History Review*, 58 (1), pp 76-94.

Shaw-Taylor, L., and E. A. Wrigley (2014), Occupational structure and population change. In: *The Cambridge Economic History of Modern Britain,* vol. 1, 1700-1870, edited by R. Floud, J. Humphries, and P. Johnson, Cambridge University Press, 53-88.

Stokey, N. L. (2001), A quantitative model of the British industrial revolution, 1780-1850, *Carnegie-Rochester Conference Series on Public Policy*, 55, 55-109.

Wrigley, E. A. (1967), A simple model of London's importance in changing English society and economy, 1650-1750, *Past and Present*, 37, 44-70.

Van Zanden, J. L., and B. van Leuwen (2012), Persistent but not Consistent: The Growth of National Income in Holland, 1347-1807, *Explorations in Economic History*, 49, 119-130.

Figures below:

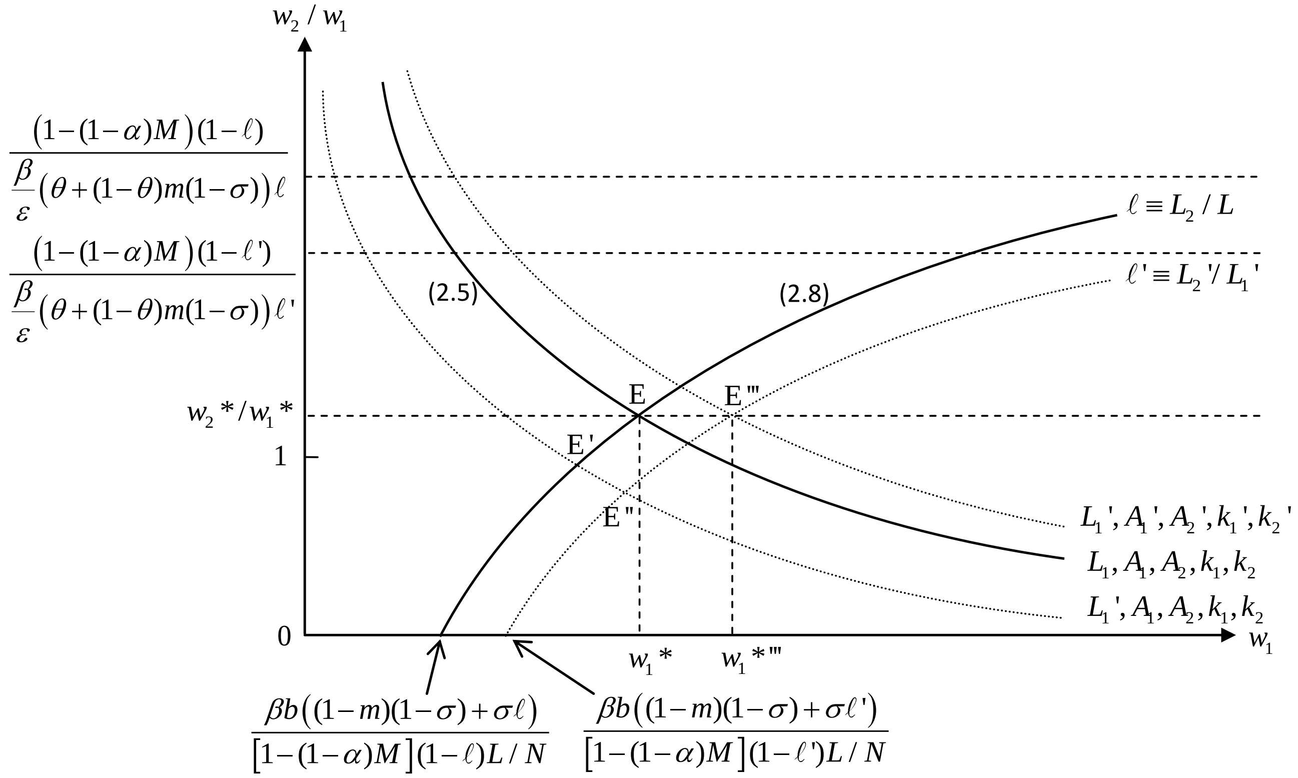

Figure 1. The supply-side wage relation (2.5), the demand-side wage relation (2.9), and the resulting wage constellation in temporary equilibrium. *Note:* $M \equiv \sigma + m(1-\sigma)$; L/N fixed; $L_1' > L_1$; $\ell' > \ell$; $A_1' \geq A_1$, $A_2' \geq A_2$, $k_1' \geq k_1$, and $k_2' \geq k_2$ with at least one strict inequality. The point E''' refers to a period subsequent to the period to which the point E refers.

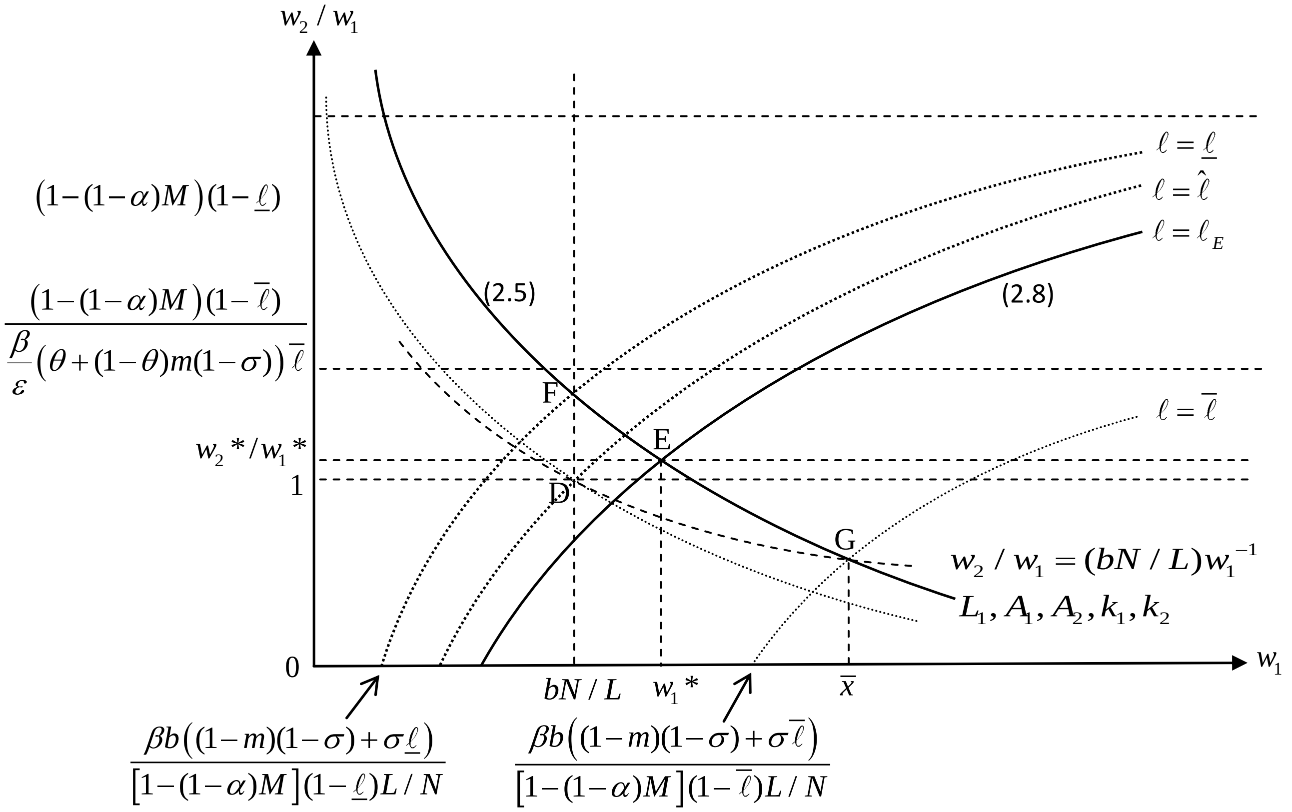

Figure 2. Existence of temporary equilibrium. Given L_1, A_1, A_2, k_1, k_2, and *L/N* such that $f(bN/L,\cdot)>1$, each point on the curve segment FG satisfies the reproducibility condition (1.13) and represents a temporary equilibrium with a specific urban employment ratio in the interval $\left[\underline{\ell},\overline{\ell}\right]$. If for instance $\ell=\ell_E\in(\underline{\ell},\overline{\ell})$, the temporary equilibrium is represented by the point E.
Note: $M\equiv\sigma+m(1-\sigma)$.

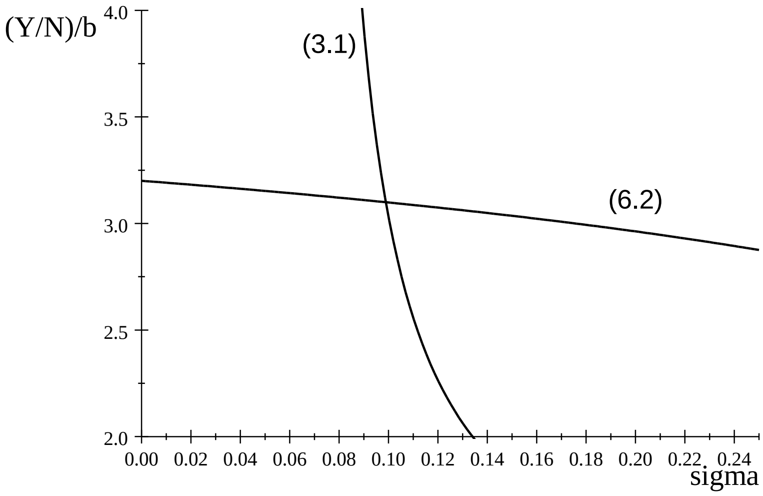

Figure 3. The calibrated marginal saving rate σ consistent with both (3.1) and (6.2).