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Shape Index Descriptors Applied to Texture-Based Galaxy Analysis

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Abstract

A texture descriptor based on the shape index and the accompanying curvedness measure is proposed, and it is evaluated for the automated analysis of astronomical image data. A representative sample of images of low-redshift galaxies from the Sloan Digital Sky Survey (SDSS) serves as a testbed. The goal of applying texture descriptors to these data is to extract novel information about galaxies; information which is often lost in more traditional analysis. In this study, we build a regression model for predicting a spectroscopic quantity, the specific star-formation rate (sSFR). As texture features we consider multi-scale gradient orientation histograms as well as multi-scale shape index histograms, which lead to a new descriptor. Our results show that we can successfully predict spectroscopic quantities from the texture in optical multi-band images. We successfully recover the observed bi-modal distribution of galaxies into quiescent and star-forming. The state-of-the-art for predicting the sSFR is a color-based physical model. We significantly improve its accuracy by augmenting the model with texture information. This study is the first step towards enabling the quantification of physical galaxy properties from imaging data alone.

1. Introduction

This paper investigates a novel combination of texture descriptors and applies them for automated analysis of galaxy images. We follow the line of filter-based approaches to texture analysis. Specifically, we focus on derivative filters. We construct differential invariants from these filters and aggregate this information in histogram representations. Descriptors such as SIFT, HoG, and DAISY capture the local structure in images using first order differential structure in the form of gradient orientation histograms. We propose to extend these descriptors by a representation of the second order differential structure. To this end, we suggest using the shape index and the accompanying curvedness measure as the basis for our descriptor, since they provide a summary of the second order structure. The novelty of our approach lies in using localized shape index histograms combined with gradient orientation histograms both measured at multiple scales. For texture analysis, adding this higher order information will in some applications be necessary in order to improve the discriminative performance of texture representations—and quantifying physical properties of galaxies from imaging data is such an application.

Galactic structure (i.e. how the mass is generally distributed within galaxies) and morphology (i.e. how that mass is arranged on smaller scales) are important diagnostics of the formation and evolutionary mechanisms and timescales for galaxies. It is well known that this structure is correlated with other physical properties of the galaxies such as star-formation rate and dust content (e.g. [7]). However, the means to formalize these relationships are yet to be realized. Extremely large galaxy surveys from the ground, such as the SDSS, have compiled vast, homogeneous imaging of millions of galaxies. Furthermore, ever since the launch of the Hubble Space Telescope (HST) and the advent of adaptive-optics (AO) on large aperture ground-based telescopes enabling high physical-resolution images of galaxies, the study of galaxy structure and morphology has entered a data-rich era.

Galaxies are made of stars, gas and dust. Each of these components emits light over different wavelength ranges and with different intensities. To use the observed light, for example, to determine the mass of stars or the rate at which new stars are being formed, we need to be able to disentangle the various luminous contributions. To do so, astronomers build models of the emission for each source. Gas will primarily emit in emission lines, which appear at a set of discrete wavelengths associated with the emitting element. These emission lines can only be observed spectroscopically and give the most direct measurement of the rate at which new stars are being formed (SFR). Stars, on the other hand, emit continuum radiation over a large range of wavelengths. We can use models of populations of stars as a function of time to extract the mass and age of the stars.
in a galaxy. These models can be used for spectroscopy as well as (broad band) imaging in multiple filters (colors).

The mass and SFR of a galaxy can therefore be (coarsely) measured by comparing a set of models with the shape of the spectral energy distribution traced by multiple filters. The specific star formation rate (sSFR) is simply the current SFR divided by the mass of stars. Usually, even if the SFR is determined from emission lines spectroscopically, the mass is determined from the colors of the galaxy in multi-filter imaging. The dominant approach for estimating sSFR from imaging data alone is based on analysis of the color of the galaxy.

Our current knowledge of galaxies is built on imaging surveys and follow-up spectroscopy. Modern imaging surveys will acquire data in several band-pass filters and can be used to approximate galactic properties. However, better determinations of these quantities require deep spectroscopy covering a significant wavelength baseline. Furthermore, most surveys will only have a single band of high angular-resolution imaging (e.g. from space). In such resolved galaxy images, it is possible to use the structure as a proxy for internal dynamics that would require more time-consuming spectroscopic data to observe. Indeed, most of the future surveys will be imaging-only surveys that will not allow for spectroscopic follow-up observations of the vast majority of the observed galaxies. Therefore, being able to fully exploit the most well-resolved images as proxies for spectroscopic data is highly valuable.

Figure 1 illustrates examples of optical images of galaxy from the subset of the SDSS dataset that is used in this paper. The top row shows well-resolved galaxy images. Notice that the light profile of these galaxies contains intricate texture. This texture is caused by the distribution of stars and gas in the galaxy— an important cue for determining the sSFR. We propose to investigate the predictive power of texture when estimating sSFR from optical images. The bottom row of Fig. 1 illustrates problematic cases for our texture based analysis. These range from noise and nearby stars to faint distant galaxies which are poorly resolved in the images. At first glance, this may seem impossible. After all, making the leap from single-band or a few bands imaging data to spectroscopic quantities is a large jump. However, the properties of galaxies are correlated. We have known since the earliest galaxy surveys, that star-forming galaxies have more internal morphological structure due to dust obscuration and star-forming clumps than quiescent (elliptical) galaxies, which tend to be smoother.

There has been some prior work on automated analysis of optical images of galaxies [14, 9]. Much of this work, however, focuses on classification of galaxies based on morphology (e.g. [4]) and tends to ignore information found in the texture. Furthermore, these approaches have used somewhat standard image features as input. Here we present new image features which we believe can capture heretofore ignored information contained in resolved galaxy images.

The following section (§2) describes the galactic dataset we use in our experiments. The new texture features are introduced in §3. Section §4 explains how we perform regression in order to predict sSFR values from our features. The results are presented in §5 before we discuss their implications and future extensions of our work in §6.

2. Galaxy data

The primary data used for the current work are a sample of low-redshift galaxies drawn from the SDSS DR7, see Fig. 1. We use the $g$, $r$, and $i$-band images covering the wavelengths from 4000–5500, 5500–7000 and 7000–8500 Ångstroms, respectively. This sample is defined as all spectroscopic galaxies within the GAMA DR1 region [11] which also have entries in both the MPA-JHU and NYU-VAGC catalogs [11, 8]. The overlap with GAMA for these ∼12000 galaxies is of particular interest because that survey will acquire spectroscopy of fainter targets and higher quality imaging (including at different wavelengths) thus allowing us, eventually, to extend our analysis to more galaxies and to longer wavelengths.

The images for our galaxy sample were obtained using the skyview software provided by NASA/GSFC. For each galaxy position, as defined in the SDSS DR 7, we downloaded a 100 × 100 pixel region (covering 39.6′′ × 39.6′′) around that position. These images are not background subtracted and do not include an object segmentation map. We used SExtractor [6] on each image to generate and subtract an estimate of the background and to produce a segmentation map including both the target and neighboring galaxies. We have not applied any additional smoothing to the galaxy pixels at this stage because that is a core part of our following analysis. We however compress the intensity range by applying a logarithmic function of the intensities.
Discriminative information in textures may appear on several different scales—this is certainly the case for galaxy images—hence using a multi-scale representation appears to be a necessity when performing analysis of texture images. We use the linear scale-space representation \cite{20,29}, where the scale-space of an image $I : \Omega \rightarrow \mathbb{R}$, $\Omega \subset \mathbb{R}^2$ is defined as $L(x,y; \sigma) = (I \ast G)(x,y; \sigma)$, where $\ast$ denotes convolution with a Gaussian filter

$$G(x,y; \sigma) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right).$$

(4)

The parameter $\sigma > 0$ is the scale of the representation. In this representation we can compute image derivatives of order $n$ and $m$ by

$$L_{x^n y^m}(x,y; \sigma) = \left(I \ast \frac{\partial^{n+m}}{\partial x^n \partial y^m} G\right)(x,y; \sigma).$$

(5)
Image derivatives form the basic components of our descriptors, but we will introduce non-linearity in the features by applying functions of these derivatives.

Common descriptors such as SIFT, HoG and DAISY \cite{26,12,30} use first order differential structure in the form of gradient orientation histograms as the basis of the descriptor. In smooth scale space derivatives the gradient orientation may be defined as

$$\theta(x, y; \sigma) = \tan^{-1} \left( \frac{L_y(x, y; \sigma)}{L_x(x, y; \sigma)} \right),$$  \hspace{1cm} (6)

and the scale normalized gradient magnitude as

$$M(x, y; \sigma) = \sigma^2 \sqrt{L_x^2(x, y; \sigma) + L_y^2(x, y; \sigma)}. \hspace{1cm} (7)$$

We need to perform this scale normalisation in order to be able to compare \(M\) across different scales \(\sigma\) \cite{29}.

We also add a representation of the second order differential structure—namely the shape index and the accompanying curvedness measure \cite{21}. The shape index is based on the eigenvalues \(\kappa_1\) and \(\kappa_2\) of the Hessian matrix of the image function. It is defined as the angle between the vector of the eigenvalues \((\kappa_1, \kappa_2)\) and the first axis in this eigenvalue space. In terms of image derivatives we may express the shape index as

$$S(x, y; \sigma) = \frac{2}{\pi} \tan^{-1} \left( \frac{-L_{xx} - L_{yy}}{4L_x^2 + (L_{xx} - L_{yy})^2} \right). \hspace{1cm} (8)$$

The shape index represents the basic second order shapes with dark blobs \(S = -1\), over saddle points \(S = 0\), to bright blobs \(S = 1\), with valley- and ridge-like structure in between. For the detailed geometric interpretation see \cite{21}.

The curvedness is simply defined as the length of the eigenvalue vector \((\kappa_1, \kappa_2)\) and expresses how pronounced the second order structure is, similar to the role of the gradient magnitude for the first order structure. In terms of image derivatives the scale normalized curvedness may be defined as

$$C(x, y; \sigma) = \frac{1}{2} \sigma^2 \sqrt{L_{xx}^2 + 2L_x^2L_y + L_{yy}^2}. \hspace{1cm} (9)$$

The shape index is rotational invariant by design, contrary to gradient orientation which depends on the choice of coordinate system.

The exact spatial ordering of the texture is not necessarily important, hence it is common (e.g. \cite{26,12,30}) to introduce an agglomeration step such as statistical moments or histograms. Here we choose to use smooth histograms inspired by the concept of locally orderless images \cite{22}. This formulation makes the intrinsic parameters of the histogram representation explicit and provides a more robust estimate compared to the traditional histogram formulation.

We define a smooth histogram as a function of the feature \(f\) in question and its magnitude \(F\),

$$H(f_i) = \int F(x, y)A(x, y)B(f_i, x, y; f) \, dx \, dy , \hspace{1cm} (10)$$

where \(f_i\) denotes the histogram binning variable and will act as the bin center for a specific choice of binning aperture function \(B\). The function \(A\) localizes the descriptor to specific parts of the image. We propose to use the Gaussian function of \(\beta\) bin width as smooth bin aperture function for histograms of the shape index \(S(x, y; \sigma)\)

$$B_{\beta, \sigma}(S_i, x, y; S) = \exp \left( -\frac{(S(x, y; \sigma) - S_i)^2}{2\beta^2} \right). \hspace{1cm} (11)$$

The Gaussian bin aperture is not a good choice for gradient orientation histograms, since it does not incorporate the fact that \(\theta\) is periodic. A better choice is to use the von Mises density function as aperture function, since this is the extension of the Gaussian distribution to the unit circle. We therefore propose to use the following smooth bin aperture function for the gradient orientation \(\theta(x, y; \sigma)\)

$$B_{\beta, \sigma}(\theta_i, x, y; \theta) = \exp \left( \frac{1}{\beta} \cos (\theta(x, y; \sigma) - \theta_i - \theta_0) \right), \hspace{1cm} (12)$$

where \(\theta_0\) denotes a fiducial orientation.

As feature magnitude \(F\) for shape index we will use the curvedness measure \(C\) from \cite{9} and for the gradient orientation we will use the gradient magnitude \(M\) from \cite{7}. The rationale is that we would like local structure with a large magnitude to count more in the histogram. This also has the effect of reducing noise in the histograms caused by noise in the derivative measurements.

We propose to construct texture features by combining histograms of gradient orientation with histograms of shape index and to measure these histograms at different scales \(\sigma\). As a concrete discretization of this representation we choose an equidistant binning in the histograms and fix the number of bins to 8 for gradient orientation and to 9 for shape index histogram features. The bin width \(\beta\) is chosen such that with the specific choice of number of bins, we tile and cover the complete range of the feature. Equation \cite{10} weights each data point that is added to the histogram by its feature magnitude and each bin window, thus each point casts a vote in every bin of the histogram.

For our specific application to galaxy images we set \(\theta_0\) in the gradient orientation feature to be the fiducial orientation of the galaxy as defined in \cite{2} Furthermore, we choose the window function \(A\) in \cite{10} to be identical to the galaxy mask as outlined in \cite{2} This localizes the feature to include features from only galaxy pixels. In addition, a histogram at a specific scale \(\sigma\) is always normalized so that the bin counts \(H\) sum to one.
Notice that our gradient orientation histogram is similar to SIFT-like descriptors, except that we do not include a spatial pooling step (i.e. we only employ a single histogram for the region of interest).

Choosing measurement scales. Using the scale space representation we can compute features at a range of scales capturing pixel correlations across these scales. Selected scales should cover the range of characteristic scales for the particular galaxy image. The inner scale is given by the pixel scale, but since we want to compute derivatives up to second order we need to be careful with the numerics. By choosing the smallest inner scale to be \( \sigma_i = 0.88 \) pixels we will have less than 1% numerical error in the estimation of the second order derivatives \[29\]. This inner scale will measure geometry at near pixel level corresponding to 0.396".

We approximate the effective outer scale for a particular galaxy image with the Petrosian radius \( R_p \). For isotropic galaxies this will be a good estimate, however, for elongated ellipsoidal galaxies this will be a poor over-estimate. We have opted for the simple heuristics of picking the effective outer scale as a function of the Petrosian radius. Let \( w \) be the smallest of the image width and height measured in pixels. We then use the Petrosian radius as outer scale \( \sigma_o = R_p \) if 4\( R_p/w \leq 1 \), and otherwise choose \( \sigma_i \) such that 4\( \sigma_i/w = 1 \). In order to avoid artifacts in the computed scale space derivatives introduced by boundary effects, it is common to discard pixels that are close to the boundary. The heuristic ensures at least a one \( \sigma_o \) distance from the galaxy to the image boundary. This definition of the outer scale will measure geometry at galaxy scale. If \( \sigma_i > \sigma_o \), we discard the image from the analysis.

We sample the range of effective scales \([\sigma_i; \alpha\sigma_o]\) in exponentially growing steps. We found experimentally that \( \alpha = 0.2 \) is a good value for the fraction of the outer scale, which focuses the descriptor on the range of scales where relevant structure occurs in galaxy images. We note that this specific choice is application dependent. We choose to use 8 scale levels in the interest of minimizing the computational effort and at the same time achieving good results.

4. SSFR Prediction Experiments

We use regression to predict specific star formation rate (sSFR) from combinations of the texture descriptors outlined above.

Evaluation. We consider different models and feature combinations to predict the sSFR value for each galaxy image. We perform 10-fold cross validation (CV) on our subset of the SDSS dataset. As measure of the prediction error we report the root mean square error (RMSE) averaged over the 10 CV folds. We also report the standard deviation of the RMSE computed from the RMSE on each fold (when interpreting these values it has to be kept in mind that the CV folds are strictly speaking not fully independent).

Models. Because scatter plots indicated a near linear relation between our features and the sSFR, we consider a standard linear least squares regressor as predictor (Linear). To further improve the performance, we employ nonlinear regression techniques using the Shark machine learning library \[18\]. We initially considered Gaussian process regression with radial Gaussian kernels, where the bandwidth parameter of the kernel and the precision of the noise were adapted by grid-search as well as gradient-based optimization of the logarithmic marginal likelihood function (or evidence) \[27\]. However, because the Gaussian processes did not significantly improve over the linear regression, we apply multi-layer perceptron neural networks (MLP). Each MLP has a single hidden layer with 100 units with logistic activation functions and a linear output unit. We add shortcut connections linking the inputs directly to the output unit. The training data of each CV fold was further split into an MLP-training and an MLP-validation set using a 9:1 split ratio. The network was trained starting from small weights by minimizing the squared error on the MLP-training set using the iRProp\(^+\) first-order optimization algorithm \[19\]. The weight configuration with the smallest squared error on the MLP-validation set was considered to be the final hypothesis. This “early stopping” of a training process that increases the complexity starting from an (almost) linear model typically fosters good generalizing hypothesis (note that the actual number of hidden units is of lesser importance if chosen large enough, see \[5\]). In the following, we only report the linear regressor and MLP results.

As a baseline, we use the constant model predicting the sSFR value to be the average sSFR value of the training set (later referred to as Average).

We also include a color-based model of the sSFR which was provided together with the SDSS dataset (Color). The method is based on the approach described in \[16\] \[15\] \[28\] \[10\], which employs a physical model of the relations between sSFR and spectrum of a galaxy.

Finally, we augment the color-based physical model by our texture features. This is done by fitting the residuals of Color. We refer to the resulting additive models \[17\] as either Linear-AM or MLP-AM depending on whether linear regression or our neural network approach was used.

Features. As input features, we consider gradient orientation (GO) and shape index (SI) features as well as their combination (referred to as All). Each feature consists of histograms at 8 scale levels.

Furthermore, for reference we include the best results achieved using a feature set consisting of histograms of filter responses for second order directional derivatives and the Laplacian (2nd), i.e. the filters used in \[31\]. These features were implemented using the smooth histograms defined by \[10\] \[11\], and computed at multiple scales using the same choices as for our features.
Figure 2. Plot of RMSE (error bars indicate 1 standard deviation of the CV error) of Linear $gri$ (SI) across the 8 scale levels for the four masks. Notice for masks 1–2 the curve has a dip indicating that for single scale features an optimal scale exists.

Figure 3. Plot of the distributions of predicted sSFR values for different predictors and the ground truth for mask 1, using the $gri$ and shape index (SI) features. It is seen that all models but the linear recover the bimodal sSFR distribution.

5. Results and Discussion

Table 1 summarizes our results for different combinations of features extracted from either a single band ($g$, $r$, and $i$) or all bands ($gri$) and different regressors. The additive models (AM) yield more accurate predictions (2 standard deviations better) than the standard color-based predictor. Thus, the texture features provide information orthogonal to the color model.

Even in single bands the texture information is correlated with the sSFR value, see the Linear and MLP (All) results. Notice that we obtain slightly better accuracies in the $g$-band. However, the best texture-only results are obtained on the combined $gri$-bands.

Using gradient orientation features alone does not provide enough information in this particular application. Instead we need to include the shape index feature or use the shape index feature alone. We only include results for the Linear $gri$ predictor, but the tendency is the same for the single bands and the MLP predictor. This is consistent with similar observations made in [24], in which it is argued that increasing differential order of the features can be beneficial for discriminability. The results on the second order features $gri$ (2nd) are comparable to the (all) and (SI) results for mask 1 but with an increased variance, and for masks 2–3 these features are inferior to the shape index (SI) results.

Fig. 2 show the RMSE of the linear regressor based on shape index (SI) features using single scale levels applied to the combined $gri$ features. Remember that, due to our scale range selection procedure ($\S\,3$) for each image the exact scale used at each scale level will vary as a function of the galaxy size. Notice for masks 1–2 the curve has a dip, indicating that for single scale features an optimal scale exists. However the results of Table 1 show that by combining information at several scales simultaneously we are able to obtain better predictions than with a single scale.

The reason for the generally poor results on mask 4 is that these masks tend to only include the galaxy nuclei which usually appears as a bright saturated blob of light. Our texture features does therefore not provide much information at this part of the galaxy.

Our results also indicate that a linear model actually does a good job of fitting the data, but we do get a slight improvement by introducing the non-linear MLP.

To provide some additional insight Fig. 3 show histograms of the spectroscopic sSFR values together with the results of the predictors Linear $gri$ (SI), MLP $gri$ (SI), and MLP-AM $gri$ (SI). All predictors but the linear are able to recover the two known classes of star-forming and quiescent galaxies seen by the two modes in the histograms. Notice how the color-based predictor systematically underestimates the sSFR value (seen by the shift of the histogram to the left) and that the MLP has a tendency to push the modes towards the mean of the dataset. It is evident that the MLP does a better job at recovering the true sSFR distribution than the linear predictor. It can be nicely seen how fitting the residual (MLP-AM) corrects the Color model.

6. Conclusions

We propose to combine gradient orientation and shape index histograms measured at several scales to describe image texture. SIFT-like descriptors include a spatial pooling...
The dominating orientation may be estimated following the descriptors such as SIFT, HoG and DAISY [26, 12, 30].

extracted from a spatial pooling scheme such as used in the gradient orientation histogram. However, the descriptor can easily be reconfigured to be constrained to a local image region of interest (ROI). This allows SIFT descriptors to only include information from the galaxy pixels mask. Based on the mask we fix the outer scale used in the scale-space as well as the dominating orientation used in the gradient orientation histogram. However, the descriptor can easily be reconfigured to be constrained to a local image patch and even be extended to a collection of histograms extracted from a spatial pooling scheme such as used in descriptors such as SIFT, HoG and DAISY [26, 12, 30].

The power of the new descriptor is demonstrated in the application of predicting sSFR from imaging data. We obtain good results when using the texture features alone. By combining the color-based physical model with texture information, we outperform the state-of-the-art for sSFR prediction.

The success of the shape index feature can be explained by realizing that what distinguishes a quiescent galaxy from a star-forming one is the distribution of stars, gas, and dust. This leads to the presence or absence of blob-like structures, as well as the occurrence of ridge-like structures caused by spiral arms and stripe patterns formed by the distribution of gas and dust—the shape index is tuned to this type of second order structure.

A current limitation of the approach is that we extract features independently from each band image ignoring the natural correlation across bands. A future extension would be to extract color descriptors by extending the shape index descriptor to be based on the Hessian matrix of the 2D intensity manifold embedded in the spatio-color space. This strategy would also be readily applicable on other types of color image data.

One of the challenges for computer vision and machine learning in astrophysics is to take models and knowledge step collecting information from a grid of histograms tiling the region of interest (ROI). This allows SIFT descriptors to some extend code spatial structure in the ROI beyond first order differential structure. Our gradient orientation feature can be thought of as a single histogram SIFT descriptor. Contrary to general SIFT-like descriptors, we have the luxury of having a segmentation of the object of interest. Instead of applying a spatial pooling step we choose to increase the differential order.

The descriptor introduced in this paper is tuned towards the specific application, predicting the specific star-formation rate (sSFR) from galaxy images, by confining the descriptor to only include information from the galaxy pixels mask. Based on the mask we fix the outer scale used in the scale-space as well as the dominating orientation used in the gradient orientation histogram. However, the descriptor can easily be reconfigured to be constrained to a local image patch and even be extended to a collection of histograms extracted from a spatial pooling scheme such as used in descriptors such as SIFT, HoG and DAISY [26, 12, 30].

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One of the challenges for computer vision and machine learning in astrophysics is to take models and knowledge

Table 1. Summary of our results for different model-feature pairs applied to either single bands (g, r, and i) or all bands (gri) using four different masks (in decreasing size). The results are based on 6880 images passing the inclusion criteria. The numbers in the table indicate RMSE and cross validation standard deviation. Average refers to predicting the training data mean (i.e. an estimator of the data variance) and Color is the current state-of-the-art physical model (see §3). Linear and MLP denote linear and non-linear regression. Linear-AM and MLP-AM are the additive models combining Color with Linear and MLP, respectively. Gradient orientation (GO) and shape index (SI) features as well as their combination All and second order features (2nd) are considered. For more results see the supplementary material.

<table>
<thead>
<tr>
<th>Method</th>
<th>Band (features)</th>
<th>Mask 1</th>
<th>Mask 2</th>
<th>Mask 3</th>
<th>Mask 4</th>
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<td>0.88 ± 0.01</td>
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<tr>
<td></td>
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<td>0.63 ± 0.02</td>
<td>0.63 ± 0.01</td>
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<tr>
<td></td>
<td>i (all)</td>
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